Ringdown Beyond Kerr

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Based on arXiv:2206.10653 with Asad Hussain

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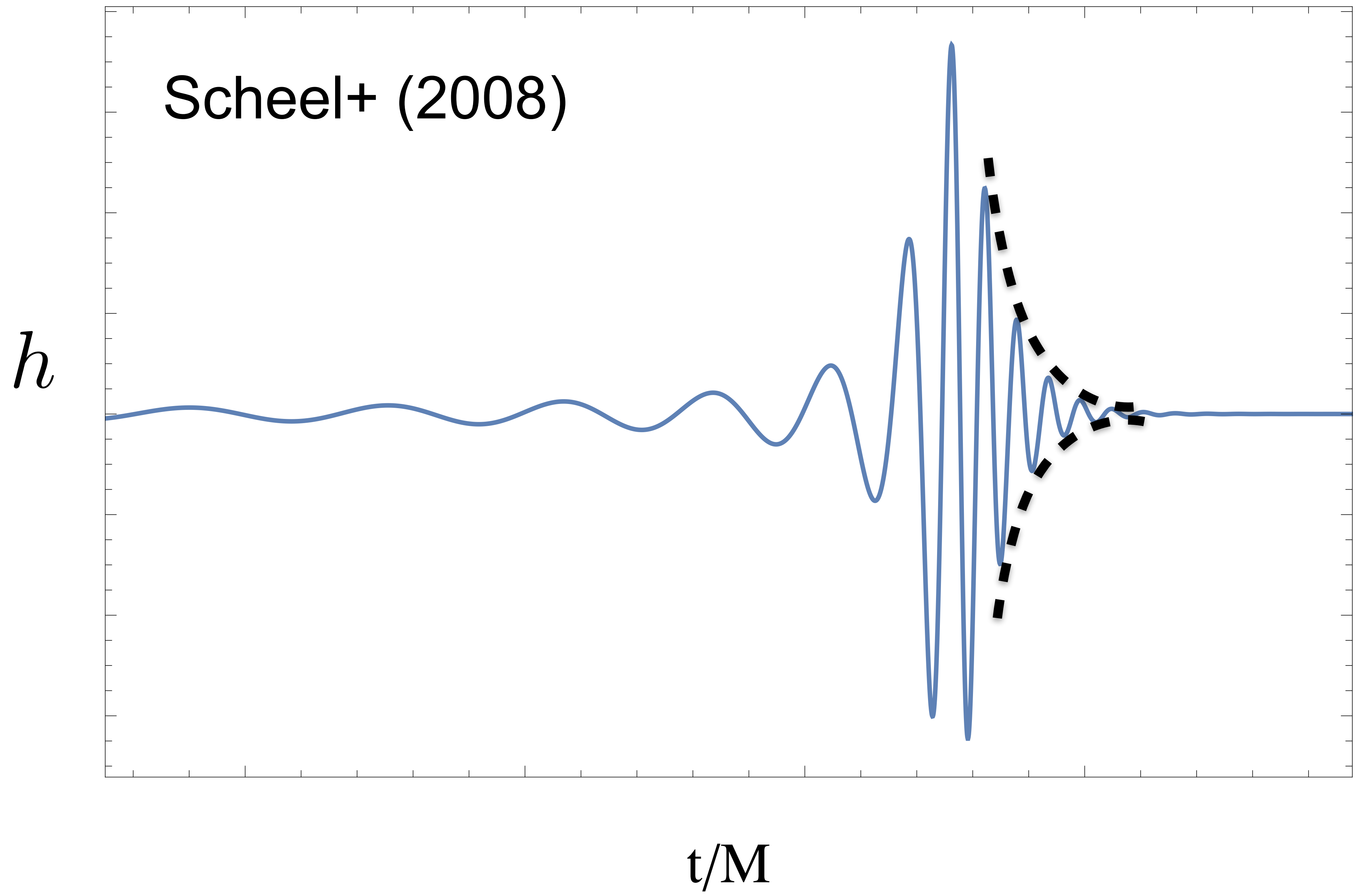
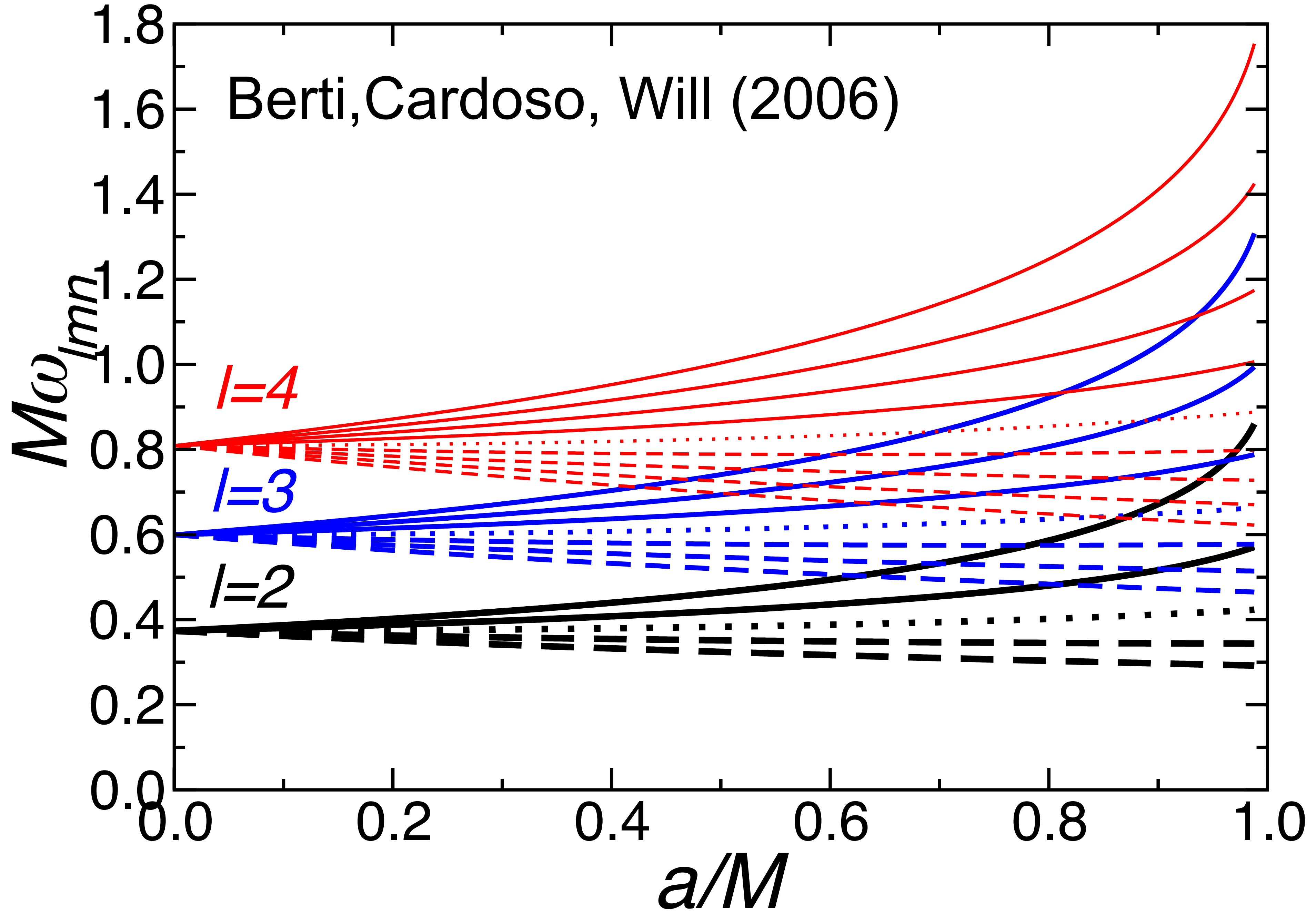
Motivation

- Ringdown signal following merger a sum of damped sinusoids
  - Complex frequencies are the QNM spectrum of BHs
  - Completely determined by mass and spin of remnant BH
- BH spectroscopy
  - Measuring two QNMs allow for tests of GR





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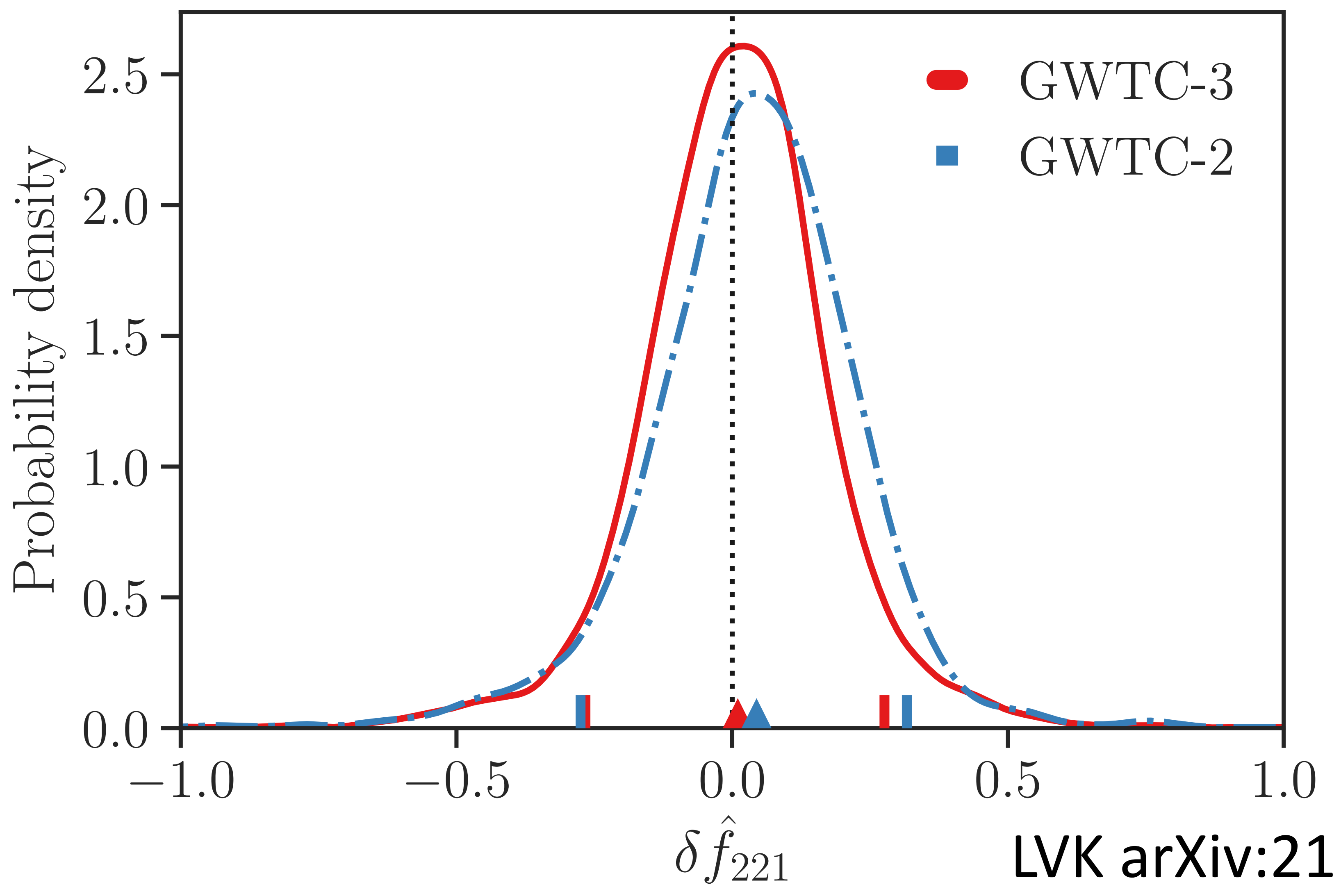
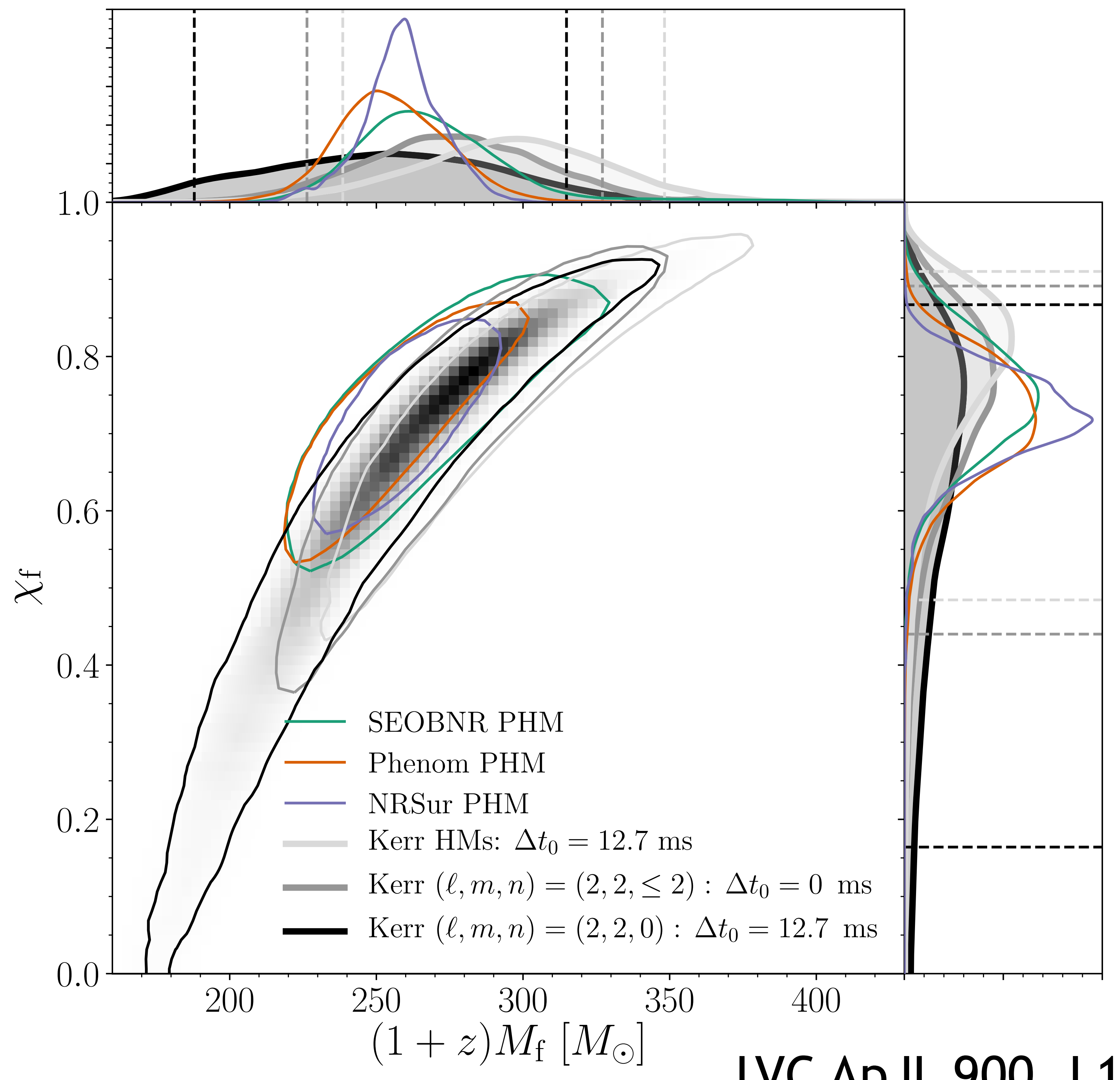
Motivation

- Strongest null tests from combining events
- Computing QNMs in theories beyond GR is tractable
  - Allows for stronger tests, better use of population
- Much work using expansions in small spin e.g. McManus+ (2019), Cano, Fransen & Hertog (2020)
- But merged black holes have $\chi \sim 0.7$



LVK arXiv:2112.06861



LVC ApJL 900, L13 (2020)

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Gravitational perts for Kerr

- Scalar wave equation separates, metric perts don't separate or decouple

$$G_{ab}(g) = \kappa_0 \eta T_{ab} \qquad\qquad g_{ab} = g_{ab}^{(0)} + \eta h_{ab}$$

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- Teukolsky (1973): Use Newman-Penrose eqns to decouple scalar quantites

$$s = 0 : \quad \Phi \qquad \Phi$$

$$s = \pm 1 : \quad F_{\mu\nu} \longrightarrow \phi_0,\ \phi_2 \longrightarrow \mathcal{O}_s[\psi_s] = 4\pi T_s$$

$$s = \pm 2 : \quad C_{\mu\nu\rho\sigma} \qquad \Psi_0,\ \Psi_4$$

Gravitational perts for Kerr

- Master eqn separates into ODEs

$$\psi_{slm\omega} = e^{-i\omega t} e^{im\phi} R_{slm\omega}(r) S_{slm\omega}(\theta)$$

- Apply appropriate boundary conditions: discrete eigensolutions ω_{lmn}

- Operator picture

$$\mathcal{S}_s^{ab} \mathcal{E}_{ab}[h] = \mathcal{O}_s[\psi_s]$$

- Metric can be reconstructed

$$h_{ab}[\Psi_H, \bar{\Psi}_H] \longrightarrow h_{ab}[\psi_s, \bar{\psi}_s]$$

Black holes beyond GR

- Focus on theories which perturb off GR in decoupling limit

$$S = S_{EH} + \int d^4x\sqrt{-g}[\mathcal{L}_\vartheta + \epsilon\mathcal{L}_{\rm int} + \mathcal{L}_{\rm matter}]$$

Hussain, AZ arXiv: 2206.10653

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- Solve order by order for equilibrium solution

$$\vartheta_A = 0 \longrightarrow G_{ab}(g^{(0)}_{cd}) = 0 \longrightarrow g_{ab} = g^{(0)}_{ab}$$

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Black holes beyond Kerr

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$$g_{ab} = g_{ab}^{(0)}$$

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$$\vartheta_A = 0 + \epsilon \vartheta_A^{(1)}$$

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Black holes beyond Kerr

- Solve order by order for equilibrium solution

$$T_{ab}^{\vartheta(2,0)}[\vartheta^{(1)}, \vartheta^{(1)}] + V_{ab}^{\mathrm{int}(1,0)}[\vartheta^{(1)}]$$

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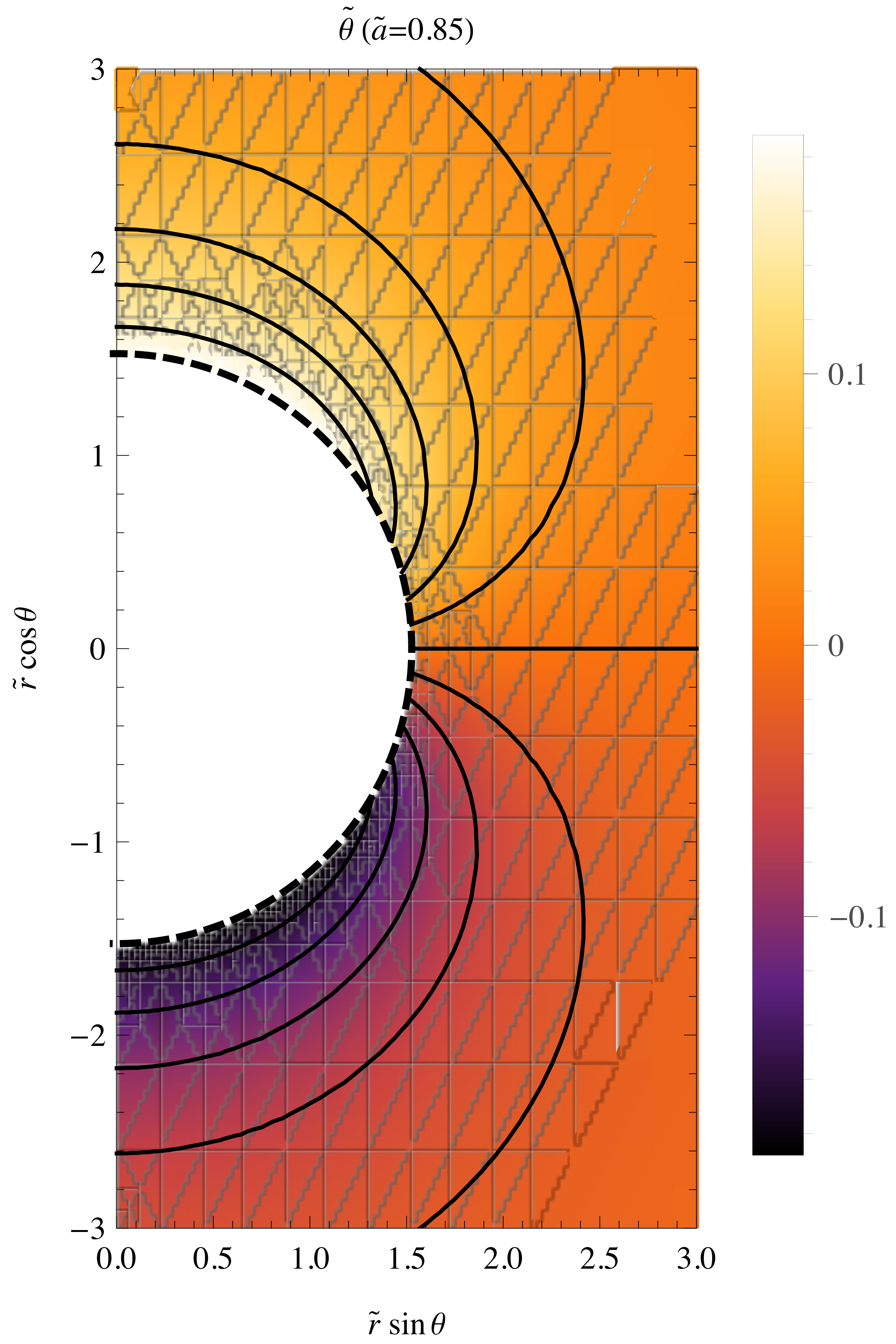
Quadratic gravity example: dCS

- Dynamical Chern-Simons: couple total derivative to scalar field, new length scale $\epsilon \sim \ell^2/M^2$

$$\mathcal{L}_{\text{int}} = \vartheta \mathcal{R}_{\text{dCS}} \qquad \mathcal{R}_{\text{dCS}} = -\frac{1}{8}{}^*RR := -\frac{1}{8}{}^*R^{abcd}R_{abcd}$$



Stein arXiv:1407.2350

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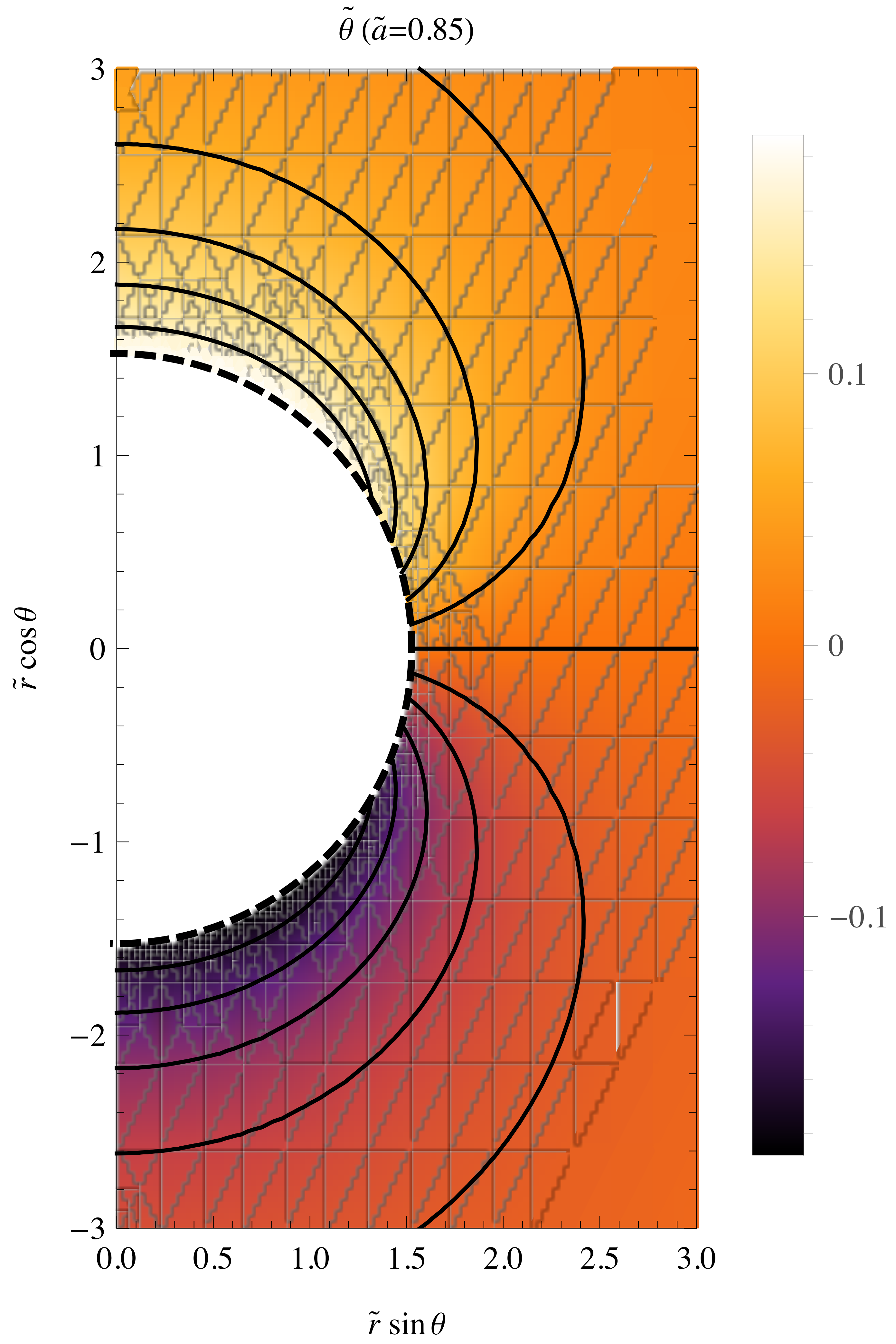
$$\Box^{(0)}\vartheta^{(1)} = \frac{1}{8}({}^*RR)^{(0)}$$

$$V_{ab}^{\rm int} = 2g_{c(a}g_{b)d}\epsilon^{edfg}\nabla_h({}^*R^{ch}{}_{fg}\nabla_e\vartheta^{(1)})$$



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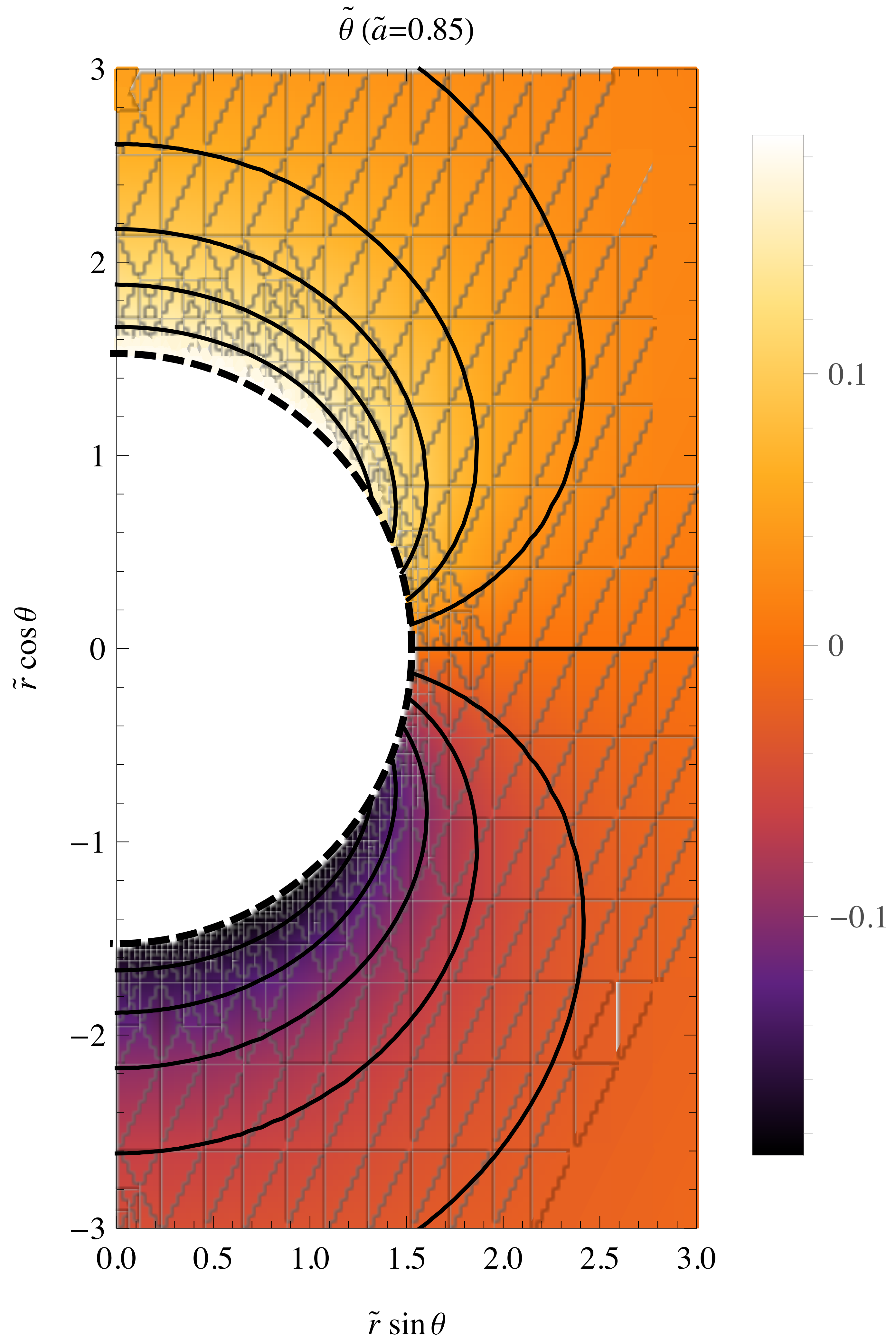
Quadratic gravity example: dCS

- Stationary BH solutions
- Post-Newtonian predictions (Yagi+ 2012, Shiralilou+ 2021)
- Binary black hole simulations (Okounkova+ 2019, Richards, Dima & Witek 2023)
- Strong constraints from NICER (Silva+ 2021)

$$\ell \lesssim 8.5\text{km}$$

- Slow-spin expansion for deform and ringdown (Cano+ 2020; Wagle+ 2021; Srivastava+ 2021)
- But parameter inference requires results at high spins



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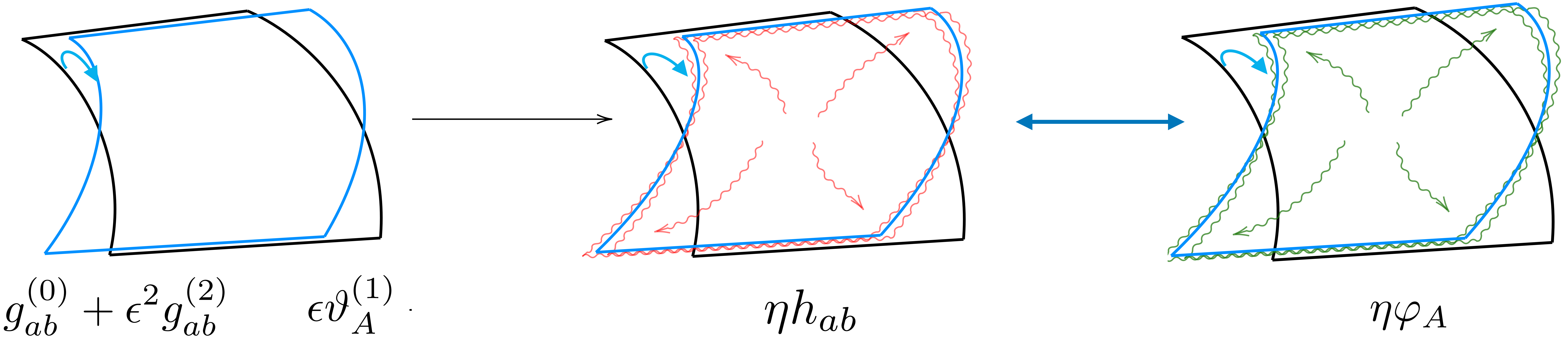
Perturbed black holes beyond Kerr

- To study ringdown add additional dynamical perturbations to all fields

$$\vartheta_A = \epsilon\vartheta_A^{(1)} + \eta\varphi_A + \ldots \qquad g_{ab} = g_{ab}^{(0)} + \epsilon^2 g_{ab}^{(2)} + \eta h_{ab} + \ldots$$



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- Coupled equations for perts

$$\begin{pmatrix} \mathcal{E}_{ab} + \epsilon^2 (\delta \mathcal{E}_{ab} - \delta T_{ab}^{\vartheta}) & \epsilon \mathcal{C}_{ab} \\ \epsilon \mathcal{F}_A & \mathcal{W}_A + \epsilon (\delta \mathcal{W}_A - \delta \rho_A) \end{pmatrix} \begin{pmatrix} h_{cd} \\ \varphi_B \end{pmatrix} = 0$$

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Modified Teukolsky equation

- Expand around preferred basis: partial decoupling

$$h_{ab} = h_{ab}^{(0)} + \epsilon^2 h_{ab}^{(2)} \qquad \varphi = 0 + \epsilon \varphi_A^{(1)}$$

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See Li+ arXiv: 2206.10652
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$$h_{ab}[\psi_s, \bar{\psi}_s]$$

Eigenvalue perturbations

- Now we have an eigenvalue perturbation problem

$$H^{(0)}|n\rangle = E_n^{(0)}|n\rangle \rightarrow (H^{(0)} + H^{(1)})|n\rangle = (E_n^{(0)} + E_n^{(1)})|n\rangle$$

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$$\langle n^{(0)}|H^{(0)}|n^{(1)}\rangle = E_n^{(0)}\langle n^{(0)}|n^{(1)}\rangle \longrightarrow E_n^{(1)} = \frac{\langle n^{(0)}|H^{(1)}|n^{(0)}\rangle}{\langle n^{(0)}|n^{(0)}\rangle}$$

Eigenvalue perturbations

- Need finite product where Teukolsky operator is self-adjoint

$$\langle \psi_\omega | \xi_\omega \rangle = c \qquad \langle \psi_\omega | \tilde{O}[\xi_\omega] \rangle = \langle \tilde{O}[\psi_\omega] | \xi_\omega \rangle$$

Mark, Yang, AZ, Chen, arXiv:1409.5800
AZ +, arXiv:1406.4206
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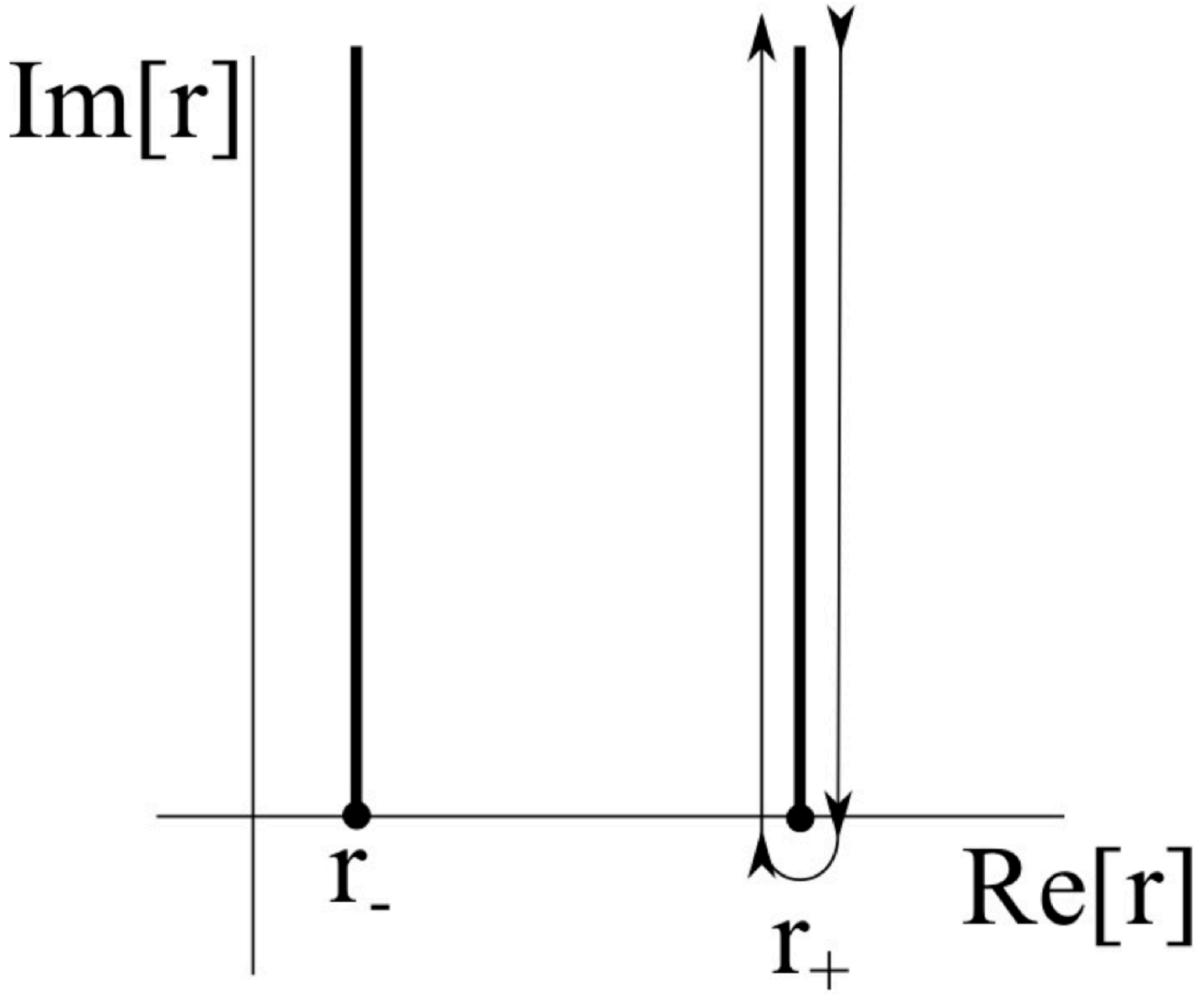

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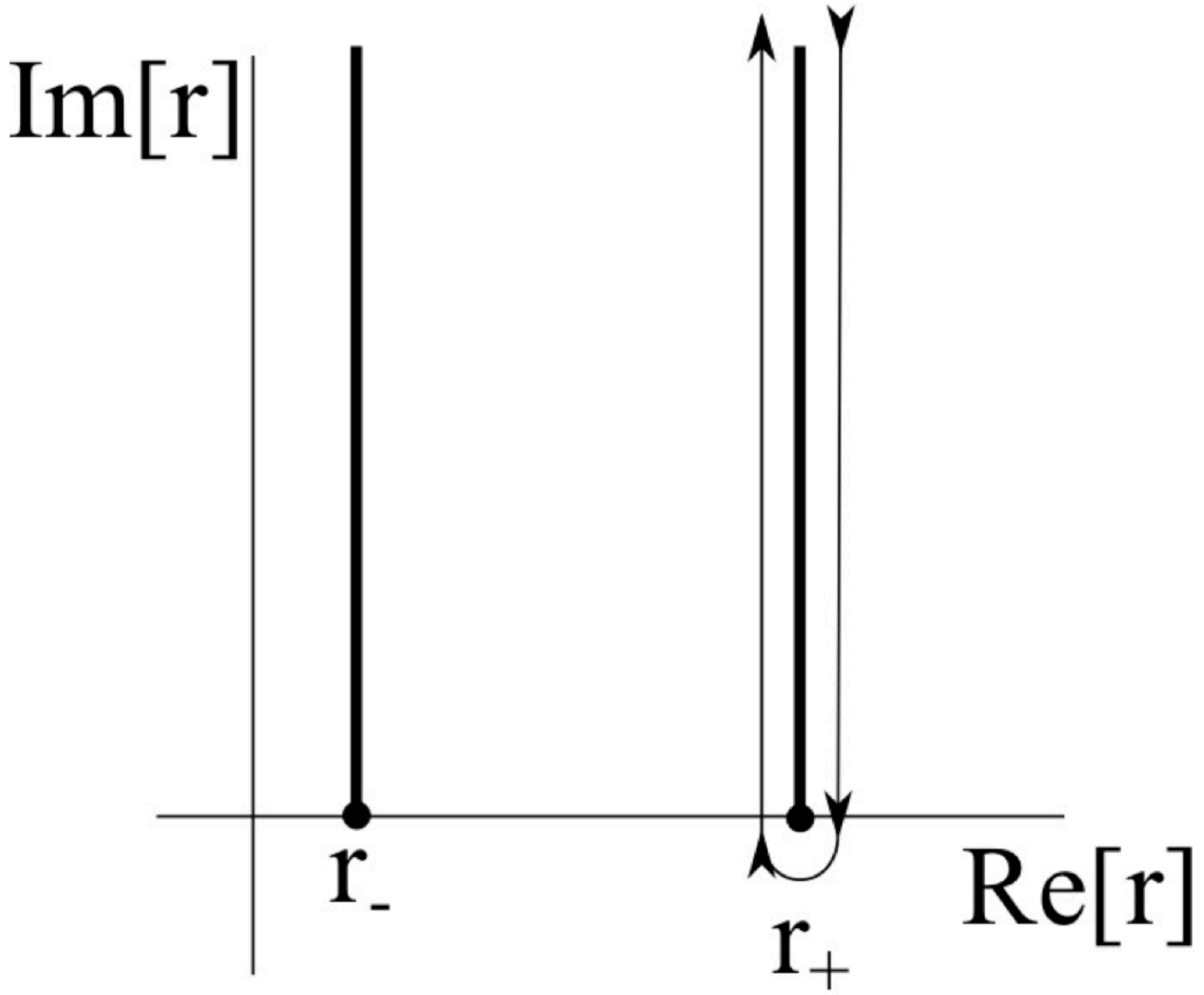

$$\omega^{(2)} \sim -\frac{\langle \psi_\omega^{(0)} | (\tilde{\mathcal{V}} + \tilde{\mathcal{C}}) | \psi_\omega^{(0)} \rangle}{\langle \psi_\omega^{(0)} | \partial_\omega \tilde{\mathcal{O}} | \psi_\omega^{(0)} \rangle}$$

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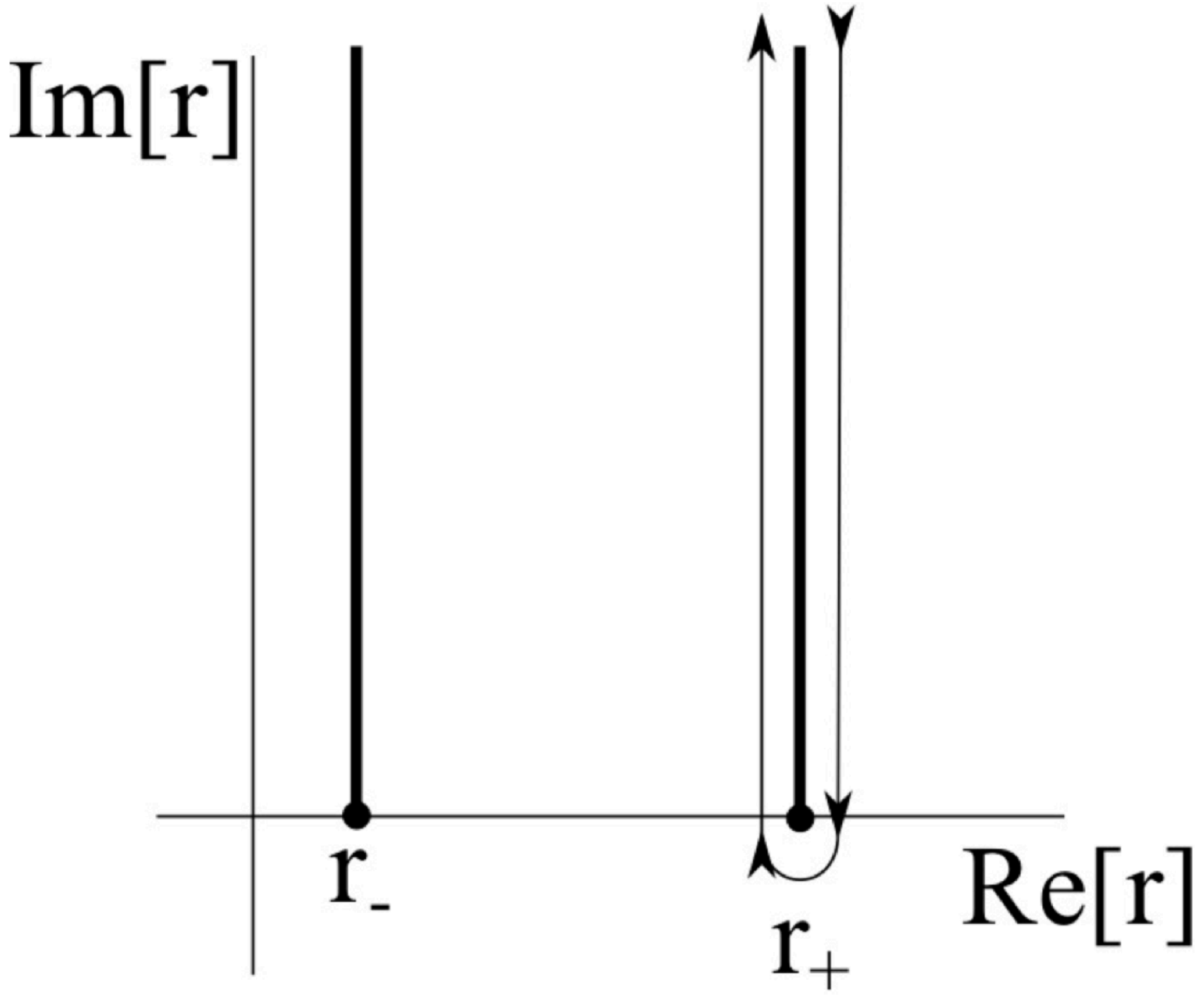

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Except CP symmetry requires degenerate pert theory, Li+ in prep

Mark, Yang, AZ, Chen, arXiv:1409.5800
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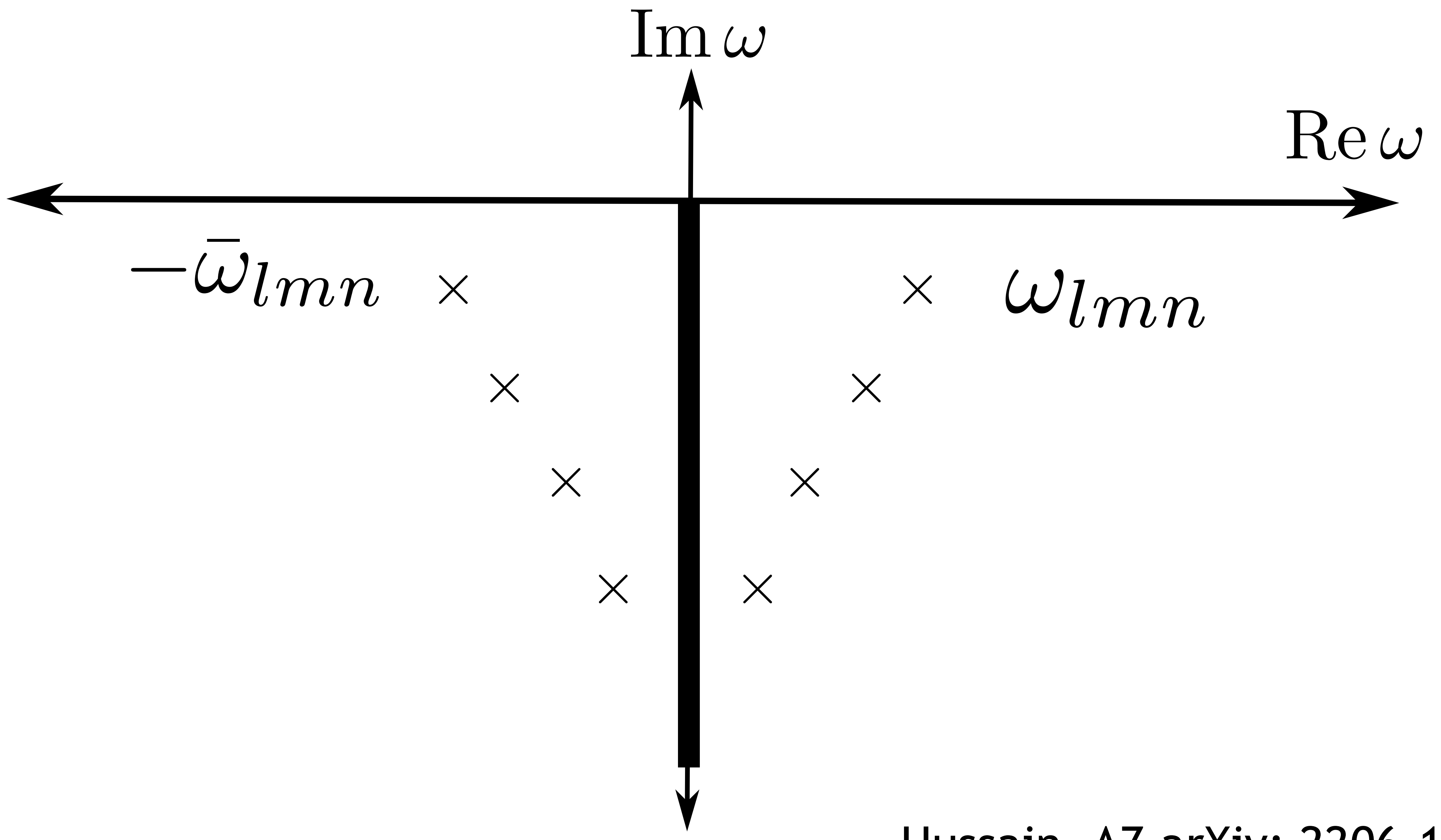
Breaking isospectrality

- One conceptual issue: metric reconstruction couples ψ_s and $\bar{\psi}_s$
- Couples two families of modes: ω_{lmn} and $-\bar{\omega}_{lmn}$
  - Equality of modes: even and odd parity modes have same spectrum (e.g. Nichols+ 2012)
- Really degenerate perturbation theory

$$\omega^{(2)}_{\text{even}} \neq \omega^{(2)}_{\text{odd}}$$



- Ongoing work on parity breaking: Li et al.

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Hussain, AZ arXiv: 2206.10653

Degenerate EVP

- Formally write metric reconstruction as

$$h^{(0)}_{ab} = \mathcal{K}_{ab}[\psi] + \bar{\mathcal{K}}_{ab}[\bar{\psi}] \qquad \mathcal{V}[h] = \mathcal{V}\mathcal{K}[\psi] + \mathcal{V}\bar{\mathcal{K}}[\bar{\psi}]$$

- Consider superposition of states that don't mix

$$\psi = \psi_+ + \alpha\psi_-$$

- Apply EVP approach

$$\omega^{(2)}_+ = -\frac{\langle\psi_+|(\mathcal{V}+\mathcal{C})\mathcal{K}|\psi_+\rangle + \alpha\langle\psi_+|(\mathcal{V}+\mathcal{C})\bar{\mathcal{K}}|\bar{\psi}_-\rangle}{\langle\psi_+|\partial_\omega\mathcal{O}|\psi_+\rangle}$$

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Hussain, AZ arXiv: 2206.10653

Roadmap

$$\mathcal{L}_\vartheta + \epsilon \mathcal{L}_{\text{int}}$$

Choose theory →

$$\vartheta_A^{(1)} \qquad g_{ab}^{(2)}$$

Stationary deformation

$$\psi_{slmn}^{(0)}$$

Choose mode →

$$h_{ab}^{(0)}[\psi_s, \bar{\psi}_s]$$

Reconstruct metric

$$\varphi_A^{(1)}[h]$$

Coupled dynamical fields →

$$\mathcal{O}[\psi_s] + \epsilon^2 \mathcal{V}[h] + \epsilon^2 \mathcal{C}[h]$$

Modified Teukolsky eqn

$$\omega_{lmn}^{(2)}$$

EVP

Ringdown inference

New physics?

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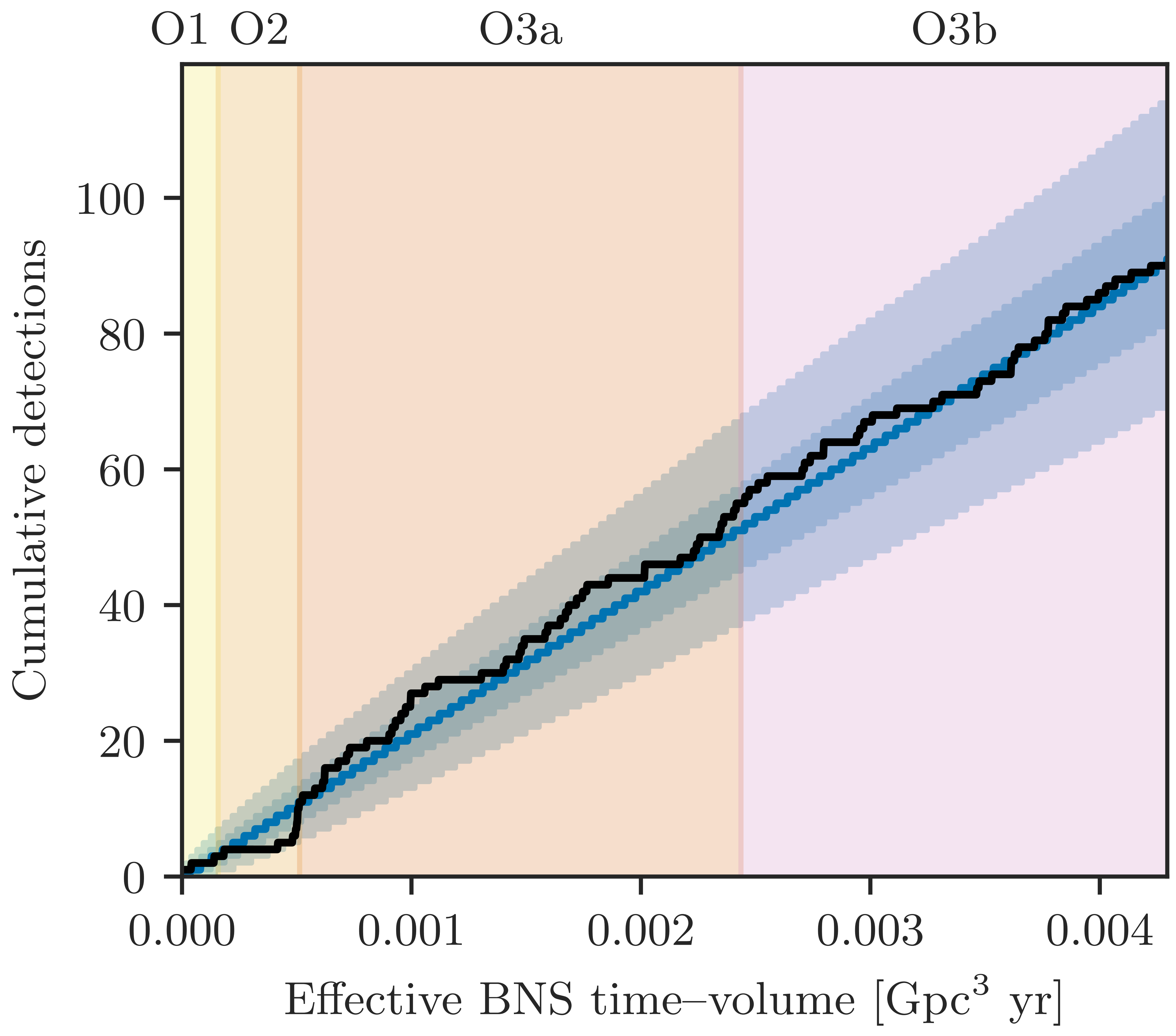
Looking ahead

- Predicting QNMs allow for multi-mode ringdown tests of Kerr
  - Derived modified Teukolsky eqn
  - EVP method: allows for high spins
  - Several challenges ahead in implementation
  - Many other methods to compare with
- Many detections in the coming years
  - Combine constraints
- 3rd gen and LISA: precision predictions needed



LVK arXiv:2111.03606

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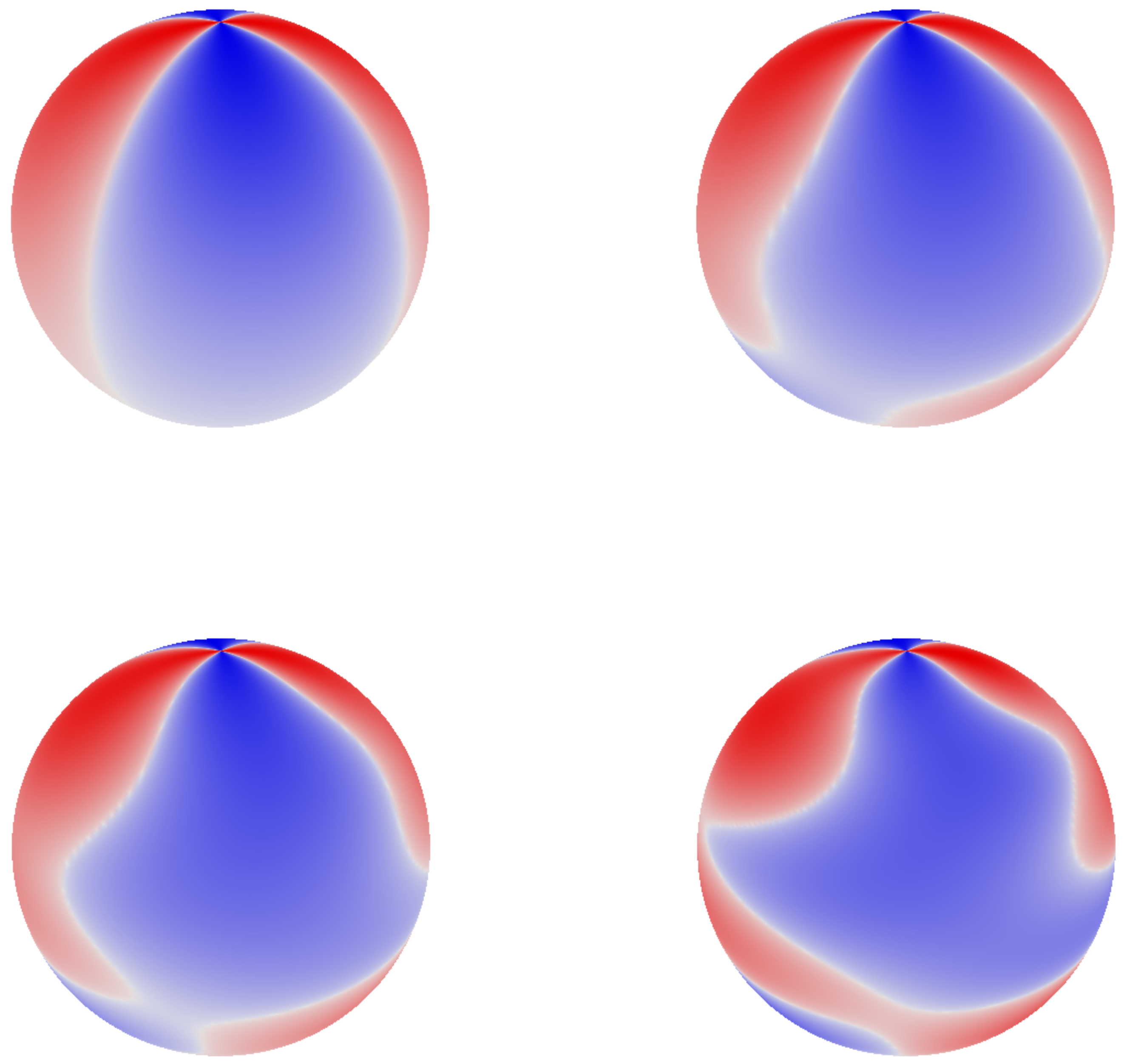
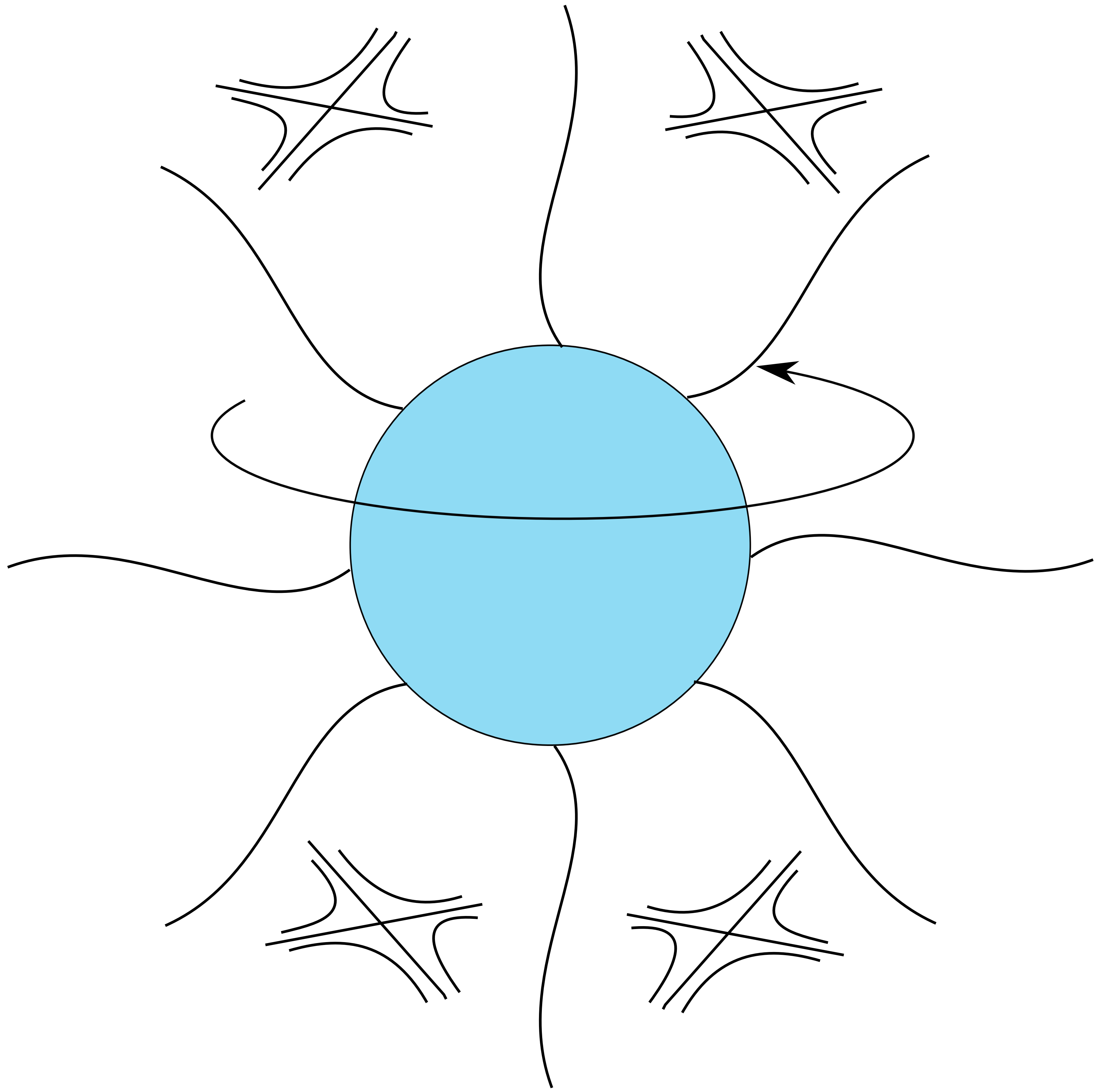
Extras

Eigenvalue perturbations



Transient "turbulence" of scalar perts

Yang, AZ, Lehner, arXiv:1402.4859



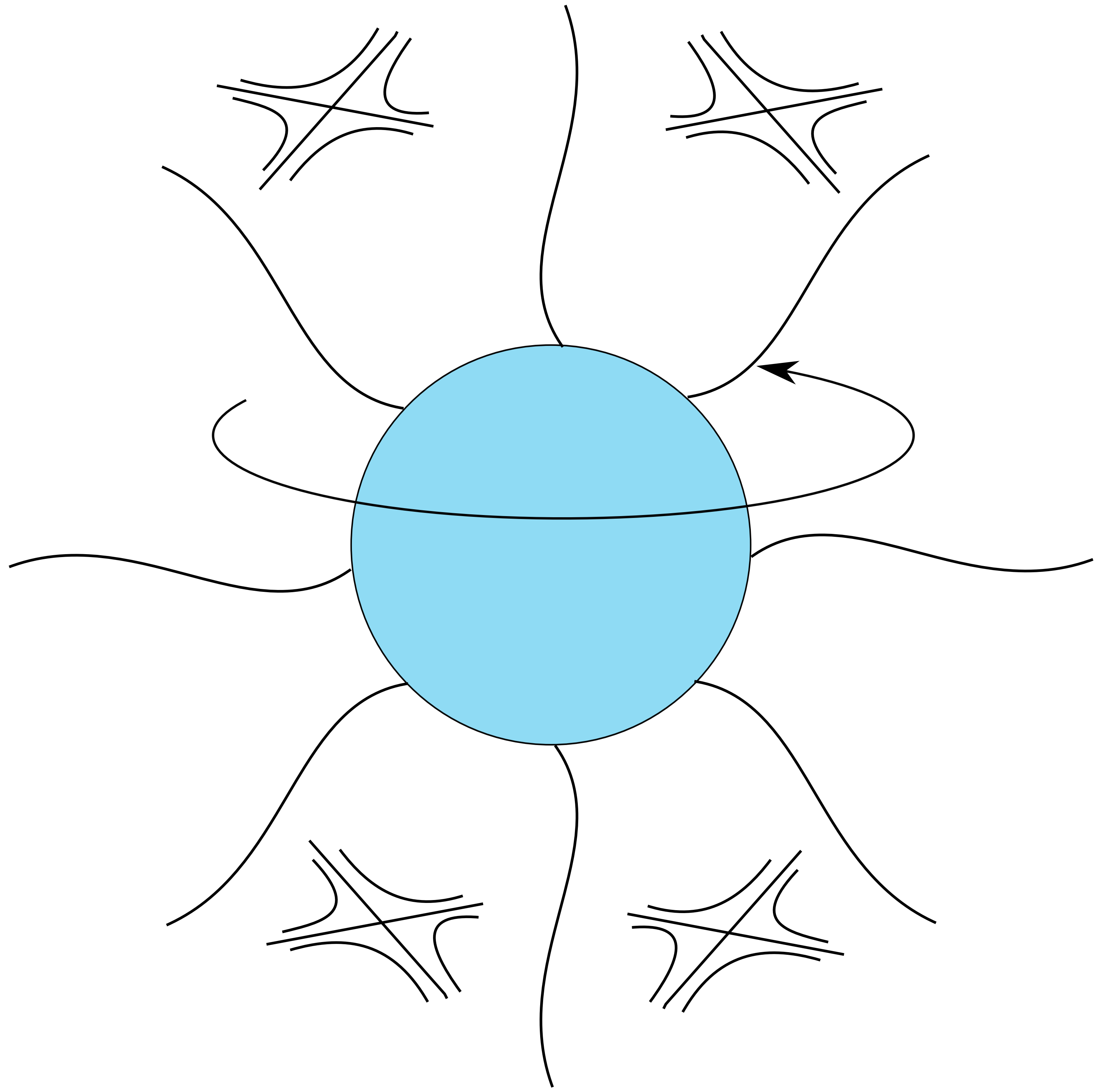
Weakly charged Kerr-Newman

Mark, Yang, AZ, Chen, arXiv:1409.5800

Example: weakly charged black holes

- Chandrasekhar: NP derivation

$$\begin{pmatrix} \tilde{\mathcal{O}}_2 + Q^2 \delta \tilde{\mathcal{O}}_2 & Q^2 \, \tilde{\mathcal{G}}_2 \\ Q^2 \, \tilde{\mathcal{G}}_1 & \tilde{\mathcal{O}} + Q^2 \delta \tilde{\mathcal{O}}_1 \end{pmatrix} \begin{pmatrix} \psi_{2\omega} \\ \psi_{1\omega} \end{pmatrix} = 0$$



Mark, Yang, AZ, Chen, arXiv:1409.5800

Example: weakly charged black holes

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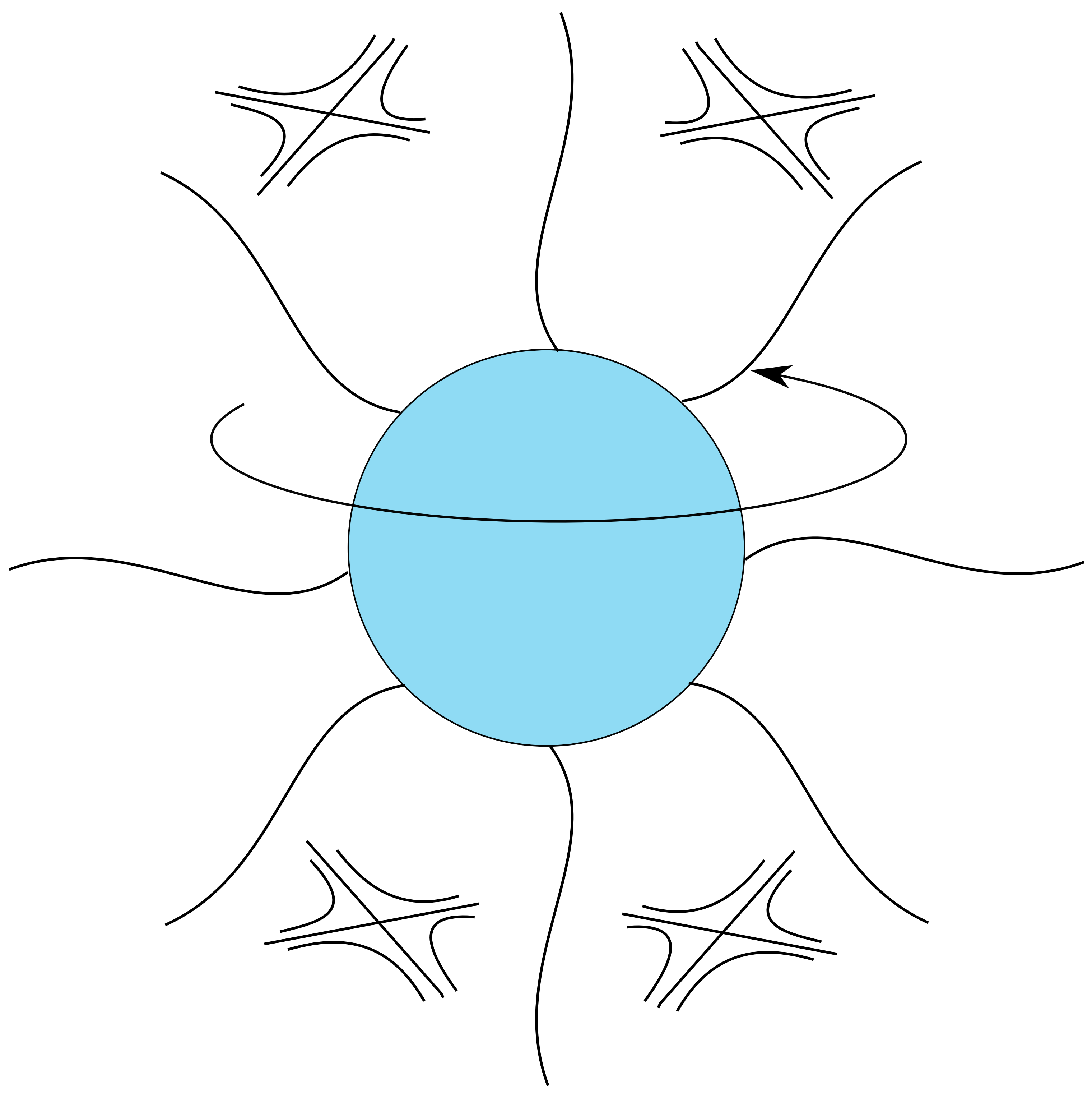

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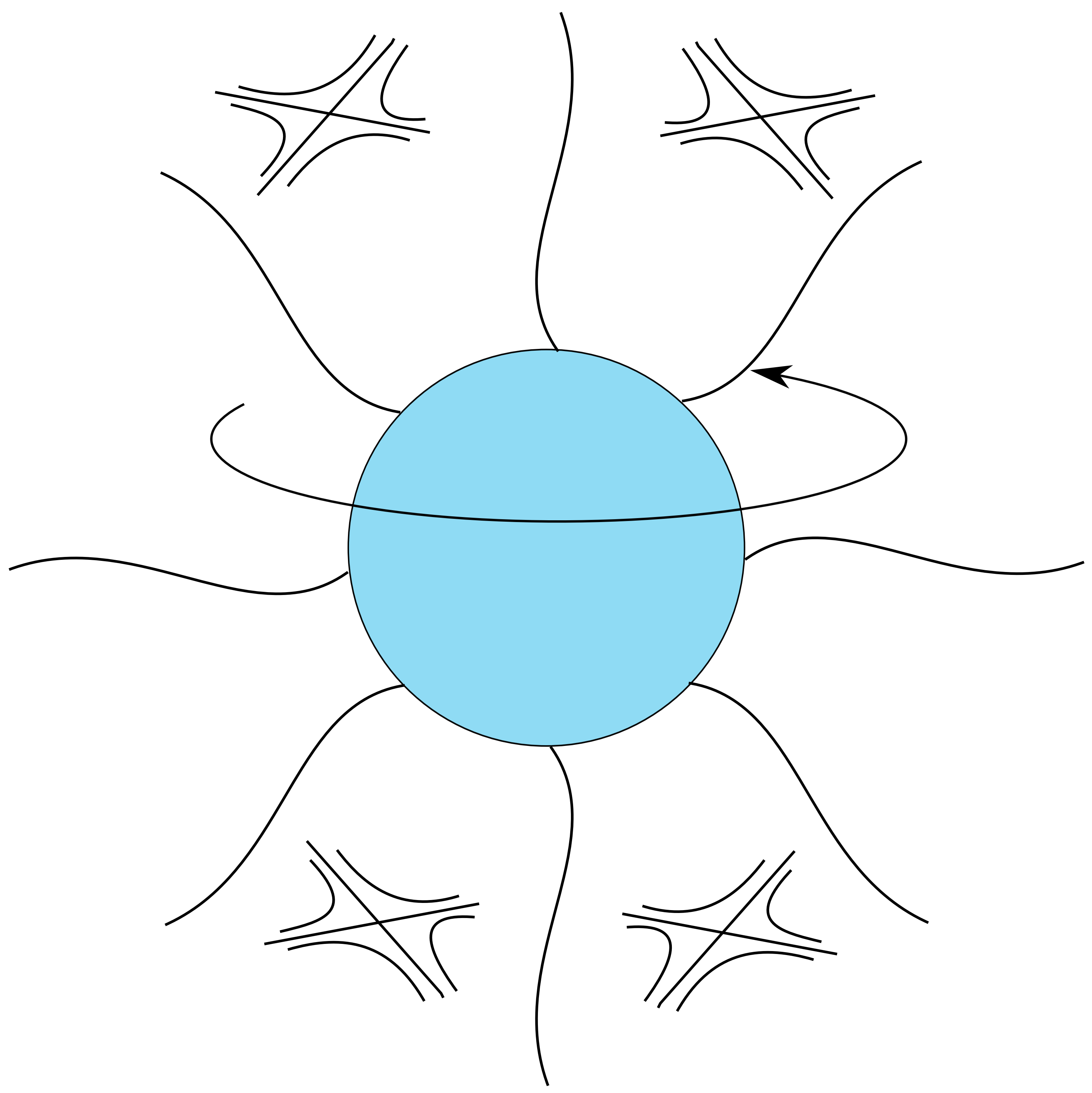

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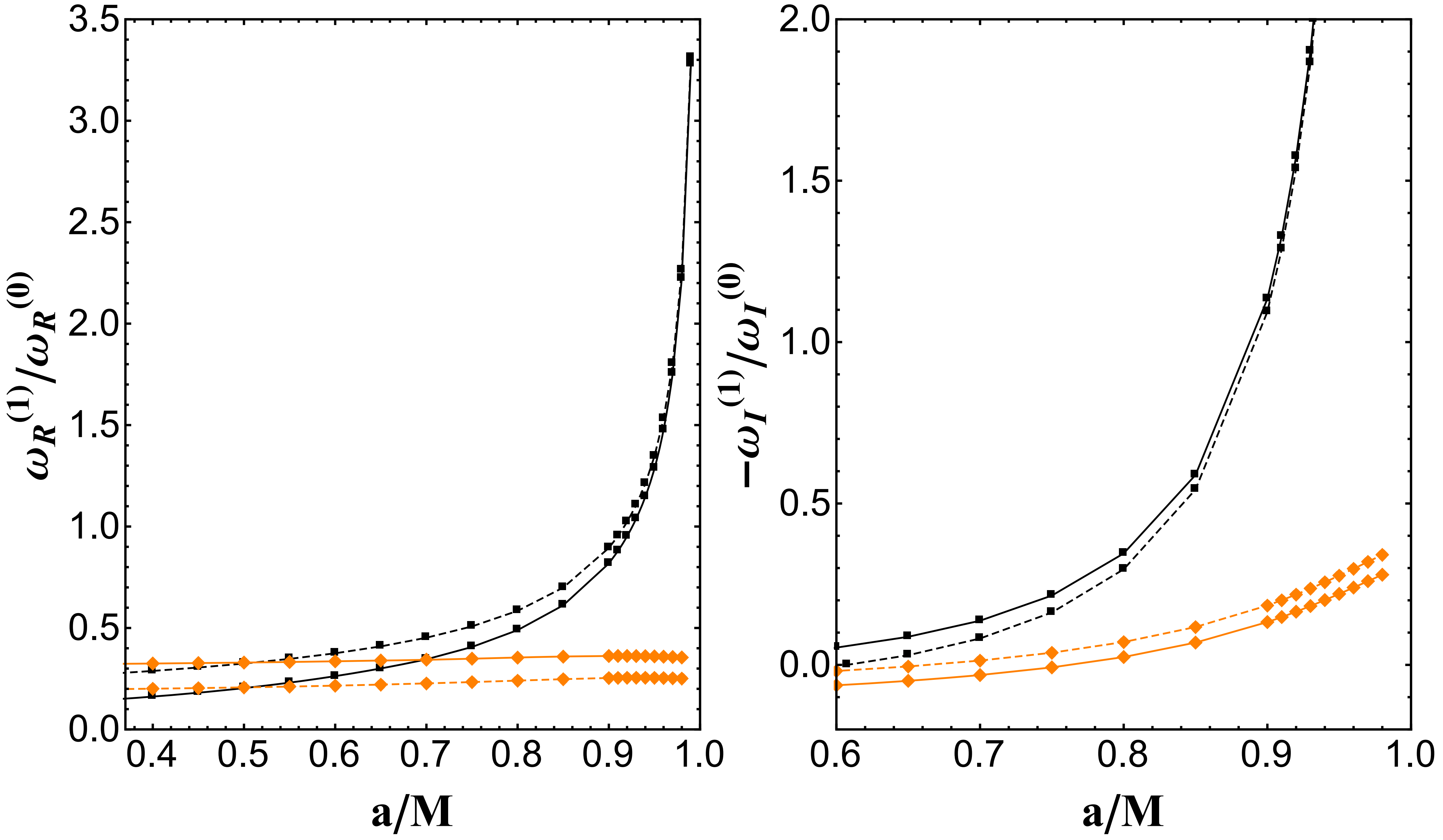

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$$\omega^{(2)} = -\frac{\langle \psi_{2\omega}^{(0)} | \delta\tilde{\mathcal{O}}_2 | \psi_{2\omega}^{(0)} \rangle}{\langle \psi_{2\omega}^{(0)} | \partial_\omega \tilde{\mathcal{O}}_2 | \psi_{2\omega}^{(0)} \rangle}$$

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Mark, Yang, AZ, Chen, arXiv:1409.5800

Example: weakly charged black holes

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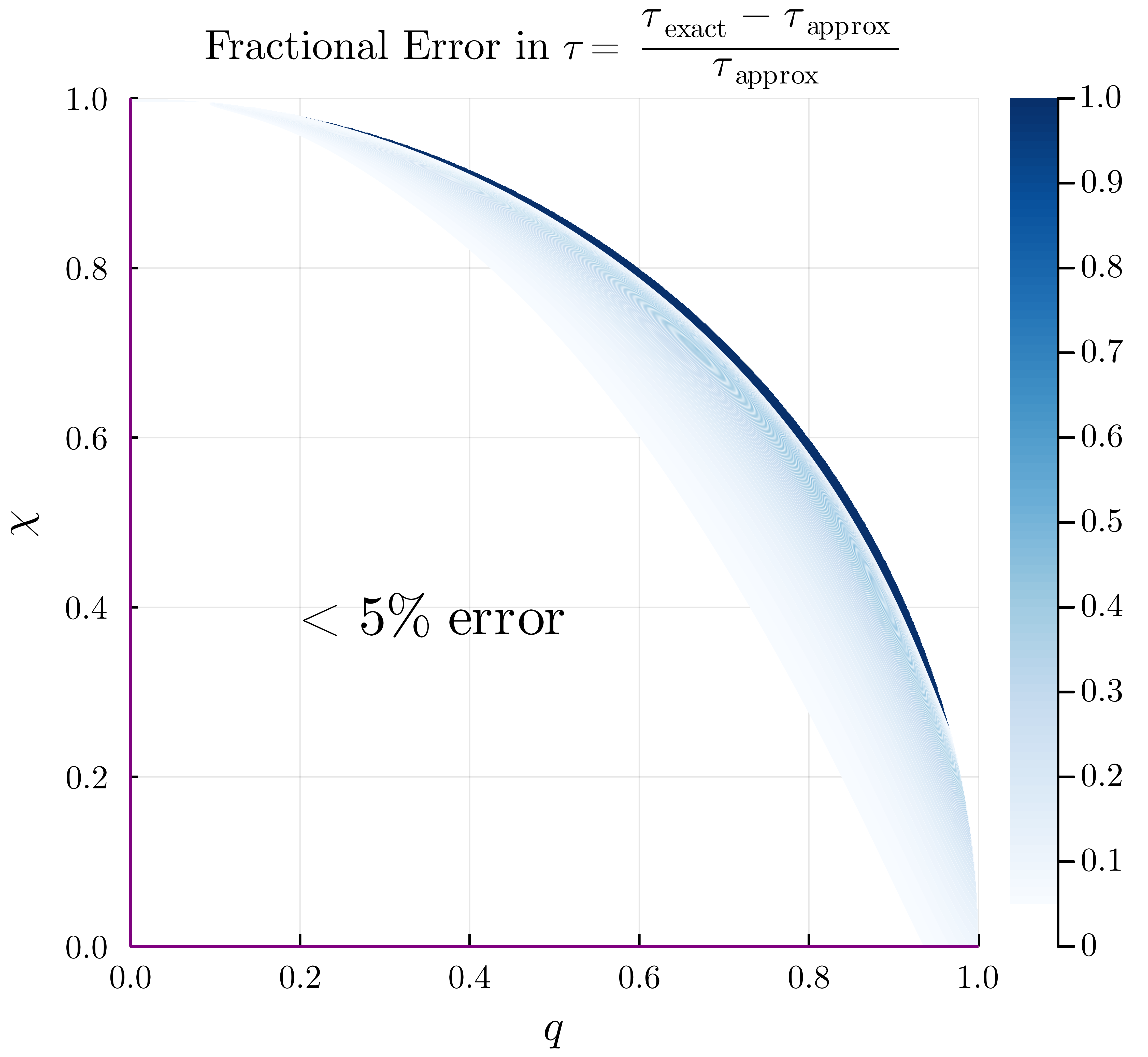

Dias, Godazgar, Santos, arXiv:1501.04625
Carullo+ arXiv:2109.13961

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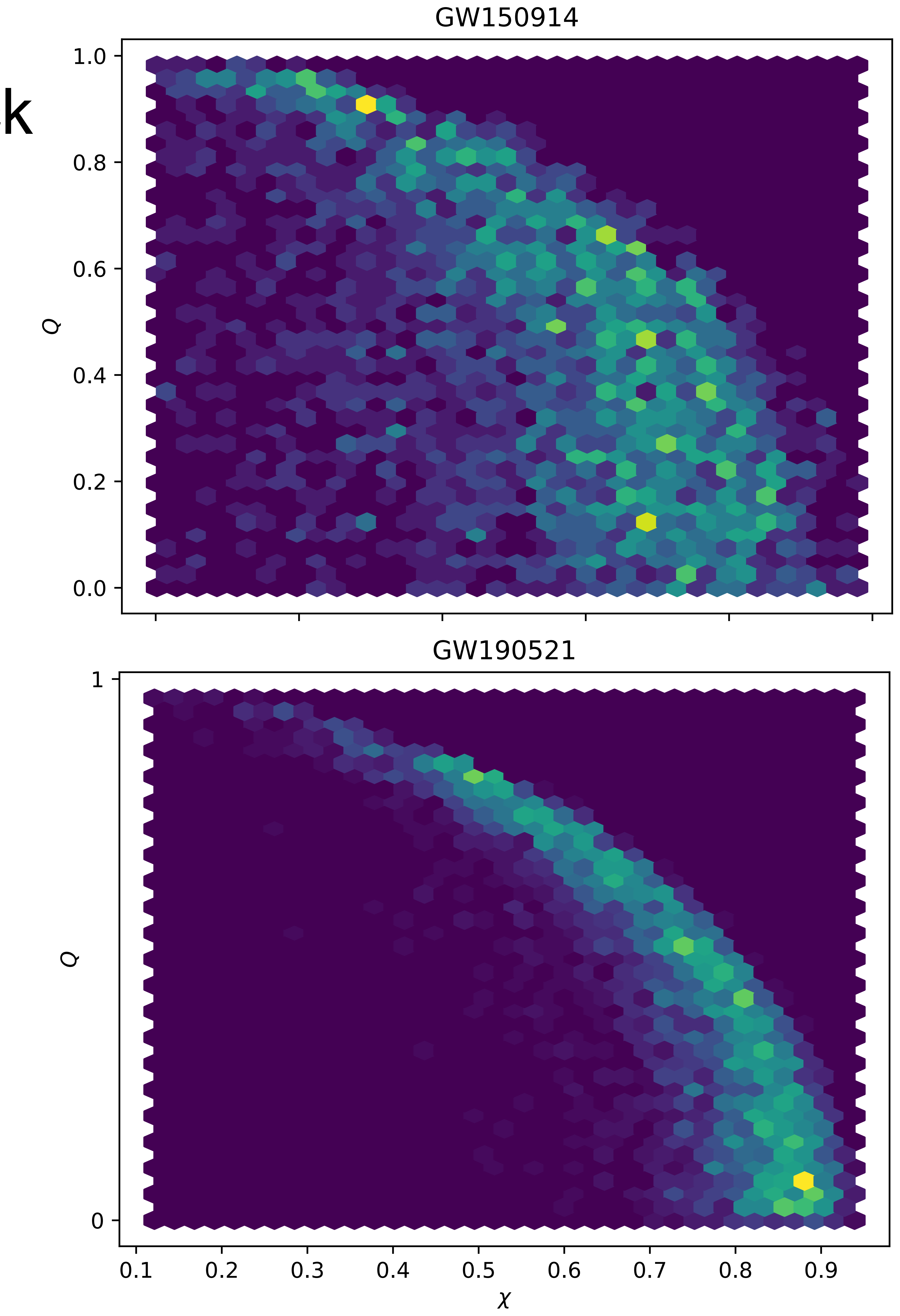
Combining events

- Beyond-GR parameter common to all events: stack likelihoods directly
- Beyond-GR parameter varies
  - Need population modeling (hierarchical modeling) to combine events
  - Modeling needs to account for degeneracies

$$p(\vec{\theta}) \rightarrow p(\vec{\theta}|\vec{\Lambda})p(\vec{\Lambda})$$

- Example: charged black holes
  - Use `ringdown` package (Isi, Farr)
  - Use multiple tones, infer M, χ, Q
  - Start from peak of full IMR waveform



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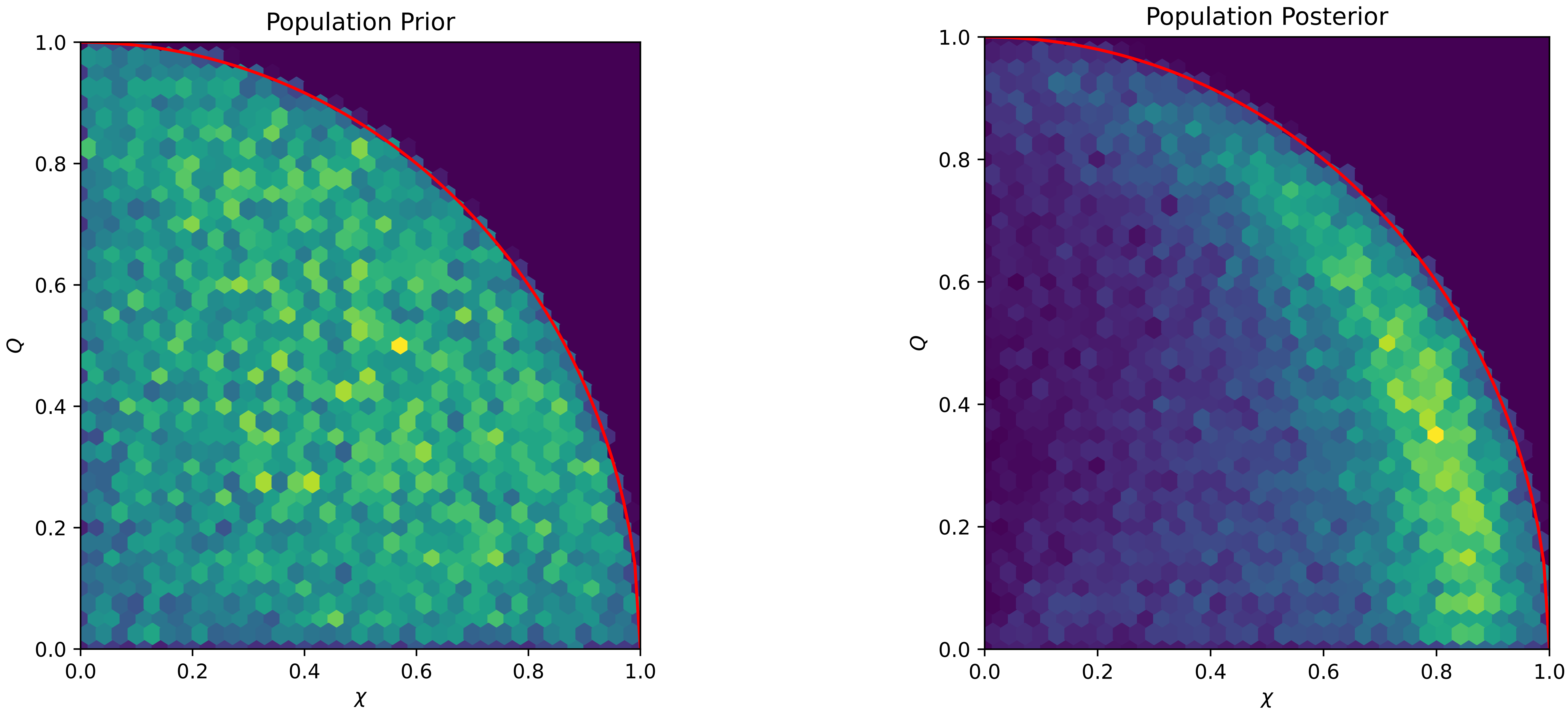
Hussain, Isi, AZ in prep
cf Carullo+ arXiv:2109.13961

Example: Charged BHs



Hussain, Isi, AZ in prep
c.f. Carullo+ arXiv: