Reducing the computational cost of EMRI waveforms using machine learning

Christian Chapman-Bird, Christopher Berry, Graham Woan
Institute for Gravitational Research, University of Glasgow

4th July 2023, CAPRA 26

Chapman-Bird *et al.* (2023) arXiv:2212.06166

Outline

- Context: the computational cost of extreme mass ratio inspirals (EMRIs)
- The machine learning tools used in this work
- Two methods for addressing the high cost of EMRI waveforms:
  1. **Reducing waveform computations:** the signal-to-noise ratio (SNR) function
  2. **Reducing waveform generation cost:** mode selection in waveform generation

Technical challenges

Complicated systems; expensive to simulate

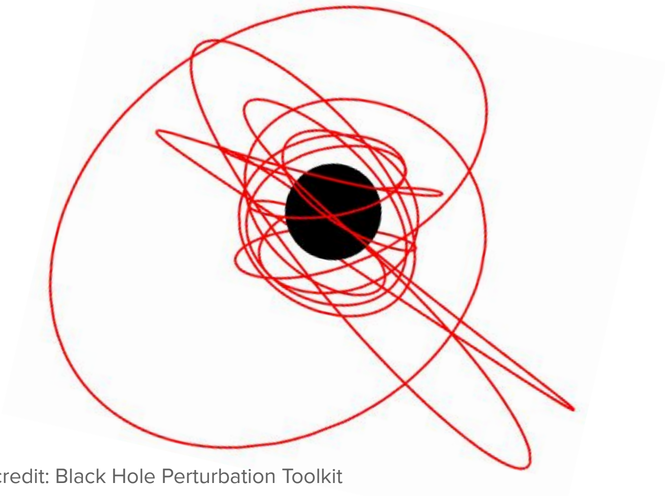


Image credit: Black Hole Perturbation Toolkit

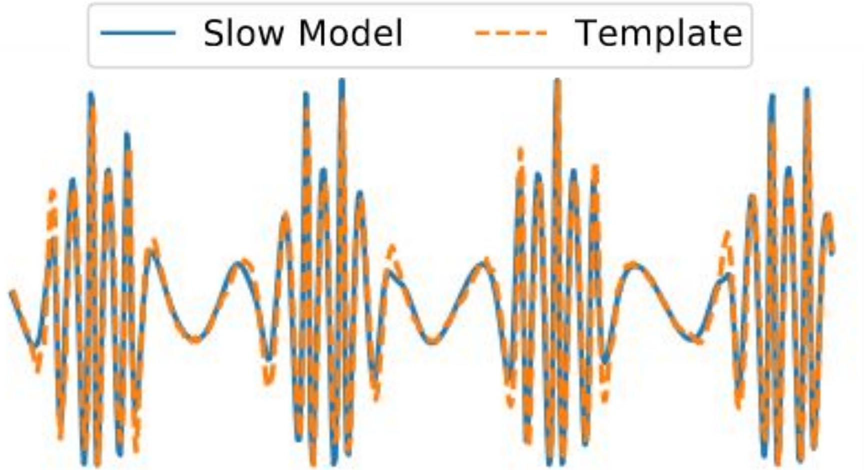
High precision means sources are hard to detect

Image credit: Walter Books

Need to improve either the speed of evaluating EMRI waveforms or develop new search techniques for finding them in the data.

The need for speed

- Adiabatic inspirals
- Take 10^{-2}-10^{-1} s (parameter dependent)
- Valid for eccentric inspirals of a spin-zero CO into a spin-zero MBH



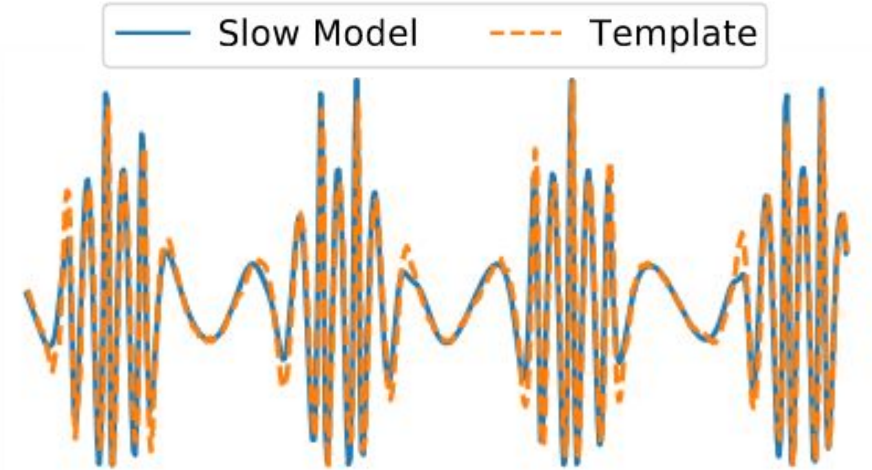
Katz *et al.* (2021) arXiv:2104.04582

`FastEMRIWaveforms`

Waveforms **must be extended** to include higher-order self force contributions in the generic Kerr regime - **eccentric, inclined inspirals with spins.**

The need for speed

- Adiabatic inspirals
- Take 10^{-2}-10^{-1} s (parameter dependent)
- Valid for eccentric inspirals of a spin-zero CO into a spin-zero MBH



Katz *et al.* (2021) arXiv:2104.04582

FastEMRIWaveforms

Waveforms **must be extended** to include higher-order self force contributions in the generic Kerr regime - **eccentric, inclined inspirals with spins.**

We want to keep the wall-time per waveform as low as possible, or avoid needing to compute it entirely where we can.

Machine learning (ML)

Well-suited to learning processes that are:

Computationally costly to perform

Image credit: https://www.fractionalciso.ca/be-like-netflix-not-reddit-saas-disaster-response/

Repeated a large number of times

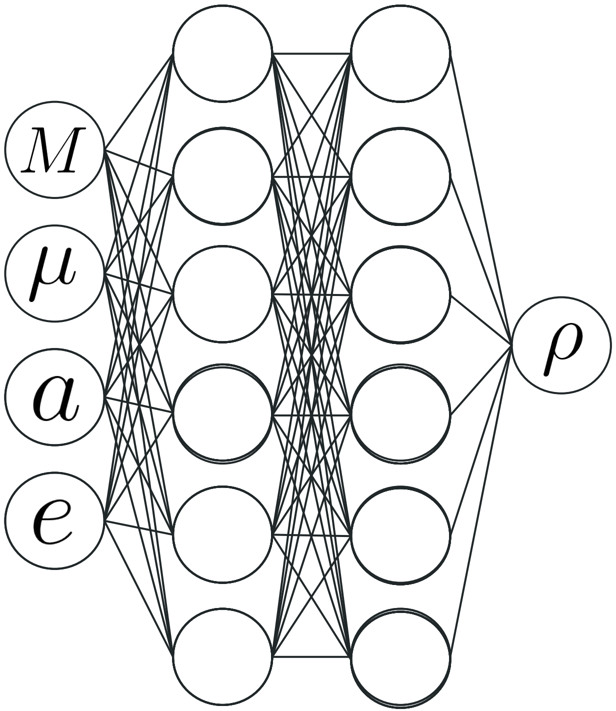
Image credit: Spotify

Many problems in EMRI data analysis fulfill both of these criteria.

Function approximation

Multi-layer perceptrons (MLPs): layers of neurons which feed inputs forward via weighted sums.

- Universal function approximators
  - Excellent classifiers, even for high-dimensional data
- Fit to a specific problem by tuning the weights given an example dataset (**training**)
- Well-optimised (highly efficient with GPUs)



Report No. 85-460-1

THE PERCEPTRON

A PERCEIVING AND RECOGNIZING AUTOMATON

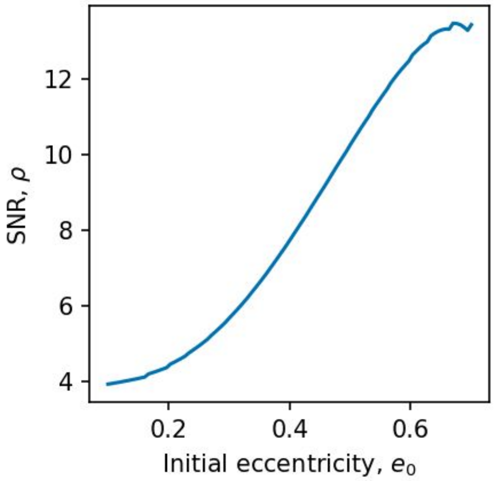
(PROJECT PARA)

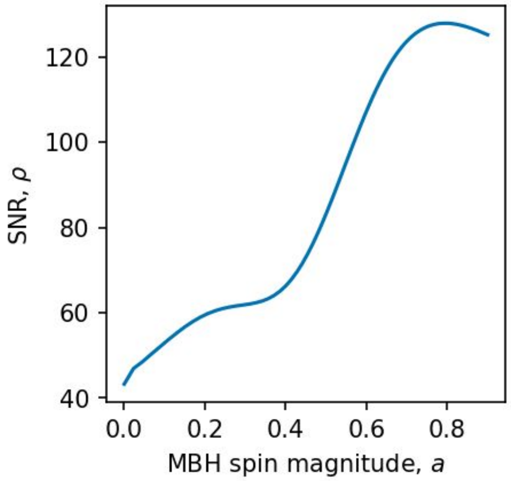
January, 1957

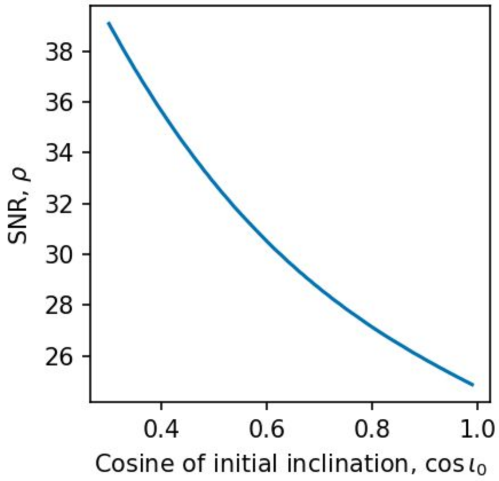
Learning the EMRI SNR function

Motivations

1. Population studies require us to evaluate the detectability of **millions of EMRIs** to correct for selection bias: **infeasible to perform directly**.
2. Searching for EMRI signals is expensive due to the large parameter space, but vast regions correspond to low-SNR signals that cannot be found: inefficient.



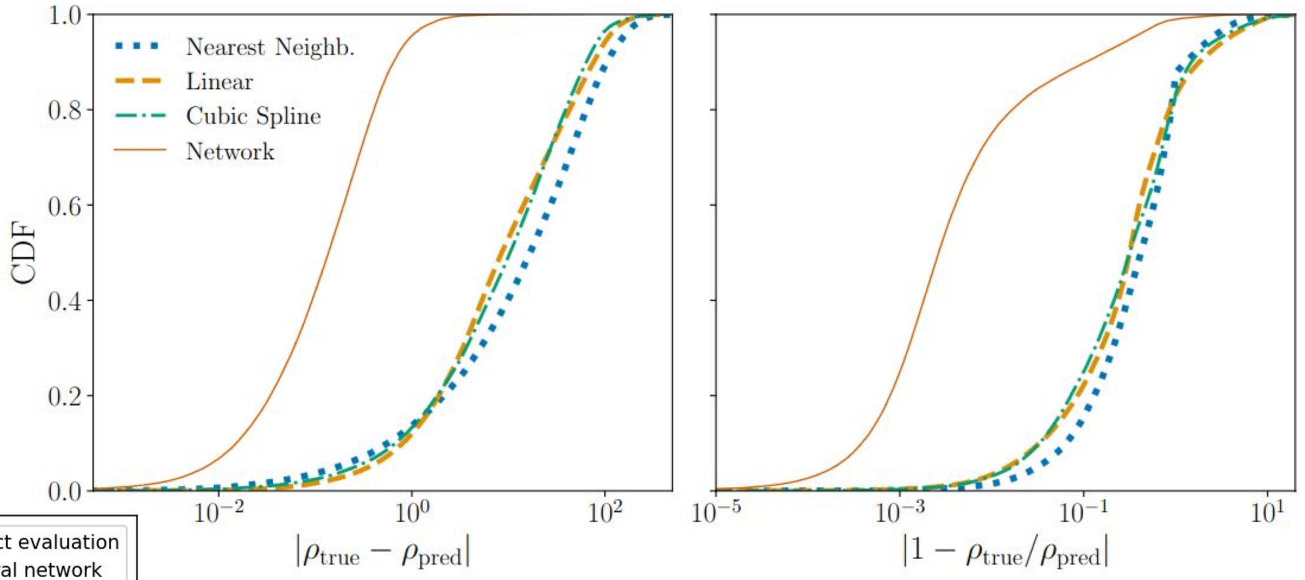


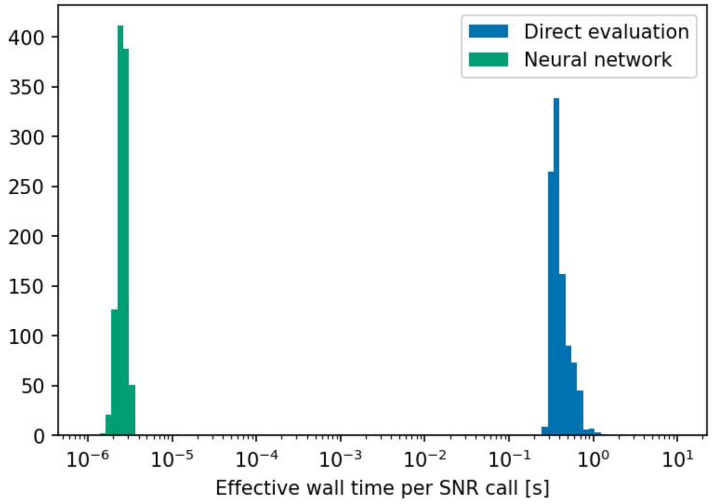


Results

Chapman-Bird *et al.* (2023)
arXiv:2212.06166

poplar





Vectorised evaluation over batches yields extremely low per-SNR cost, especially on GPUs.

Evaluation on a single set of EMRI parameters takes roughly **10^{-3} s**.

Neural waveform mode selection

Mode phase evolution

Mode amplitudes

$$h = \frac{\mu}{d_L} \sum_{lmkn} A_{lmkn}(t) S_{lmkn}(t, \theta) e^{im\phi} e^{-i\Phi_{mkn}(t)}$$

Sum over mode indices

Spin-weighted spheroidal harmonics

$$h = \frac{\mu}{d_L} \sum_{lmkn} A_{lmkn}(t) S_{lmkn}(t, \theta) e^{im\phi} e^{-i\Phi_{mkn}(t)}$$

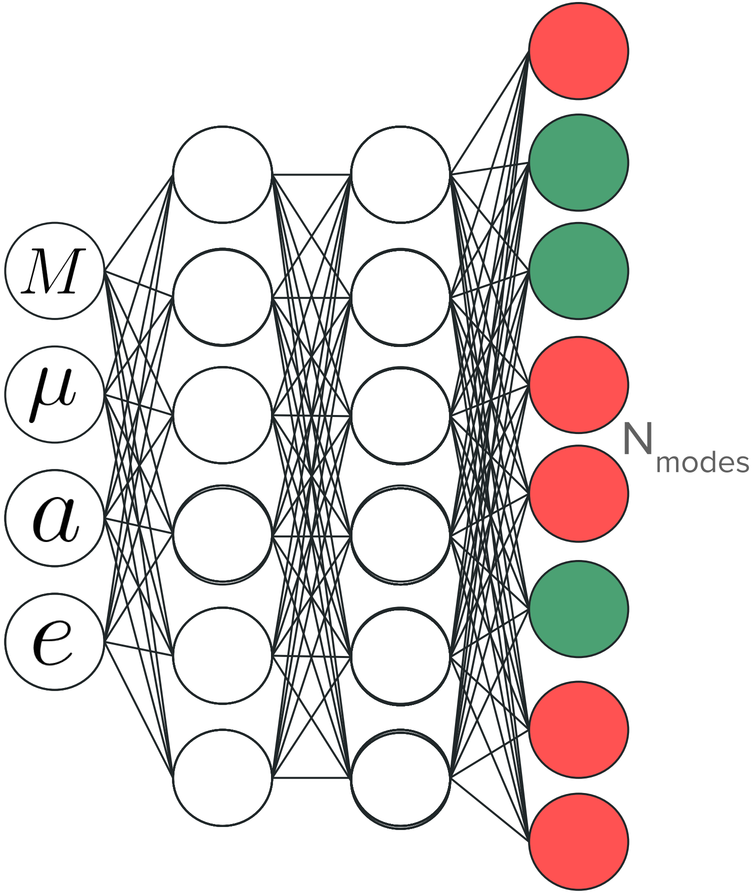
$S_{lmkn}(\theta, \phi)$ $\Rightarrow$ Mode selection $\Leftarrow$ $A_{lmkn}(t)$

This is an expensive operation - the bottleneck in generic Kerr.

Mode selection is a multi-label classification problem

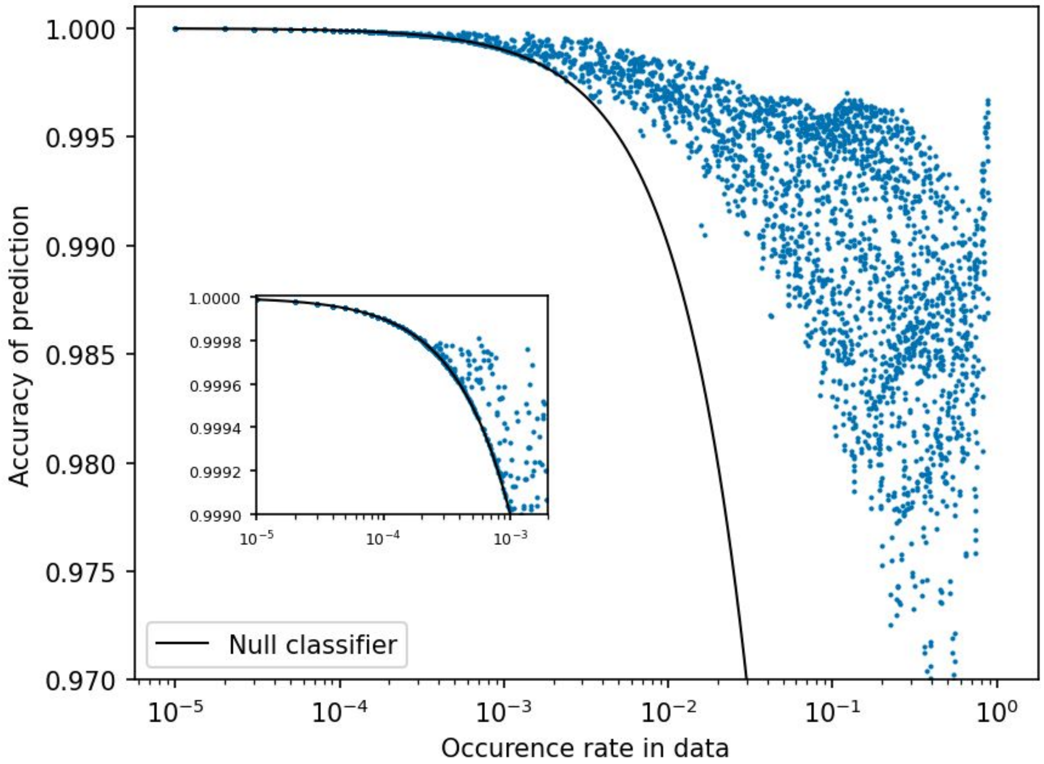
For a given set of EMRI parameters, we want to make N_{modes} classifications.

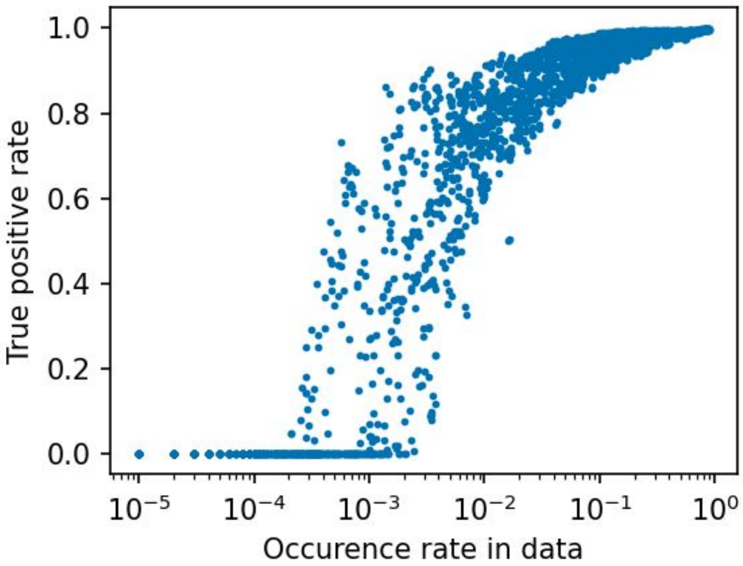
We build an MLP with N_{modes} outputs.



Accuracy

Chapman-Bird *et al.* (in prep)

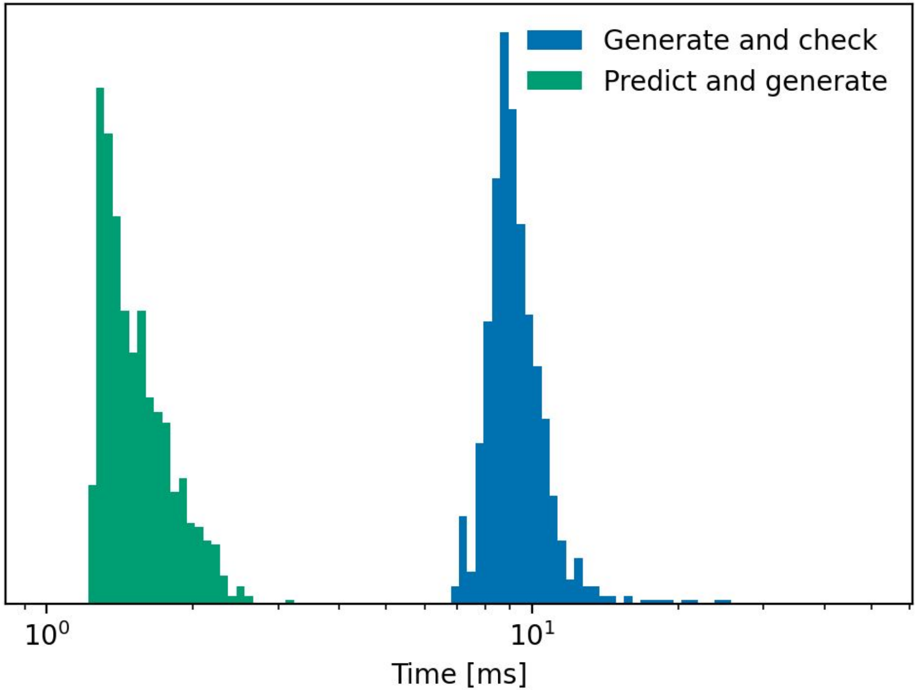


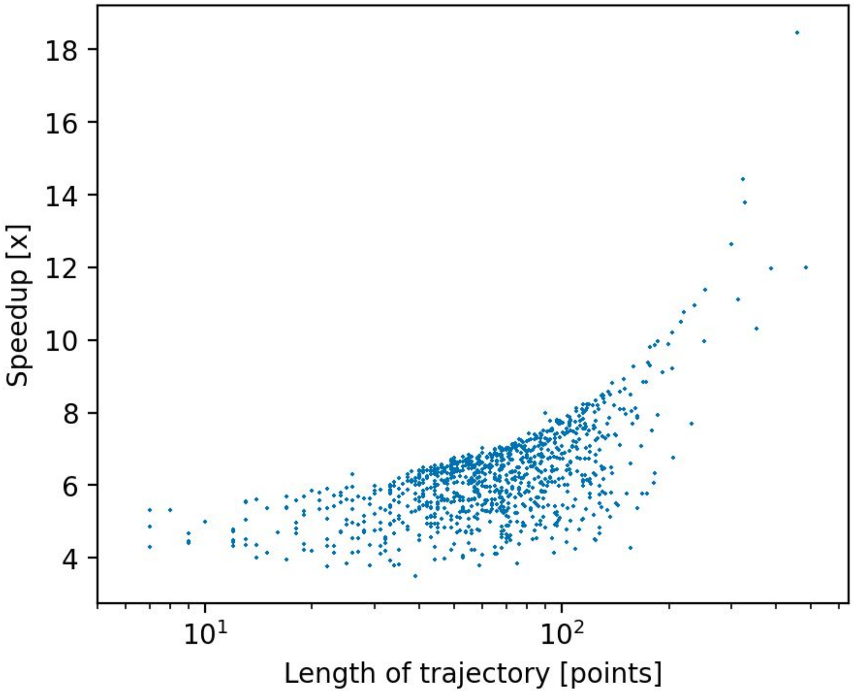


Excellent performance for common modes, but tends to neglect weaker (less common) modes.

Cost analysis

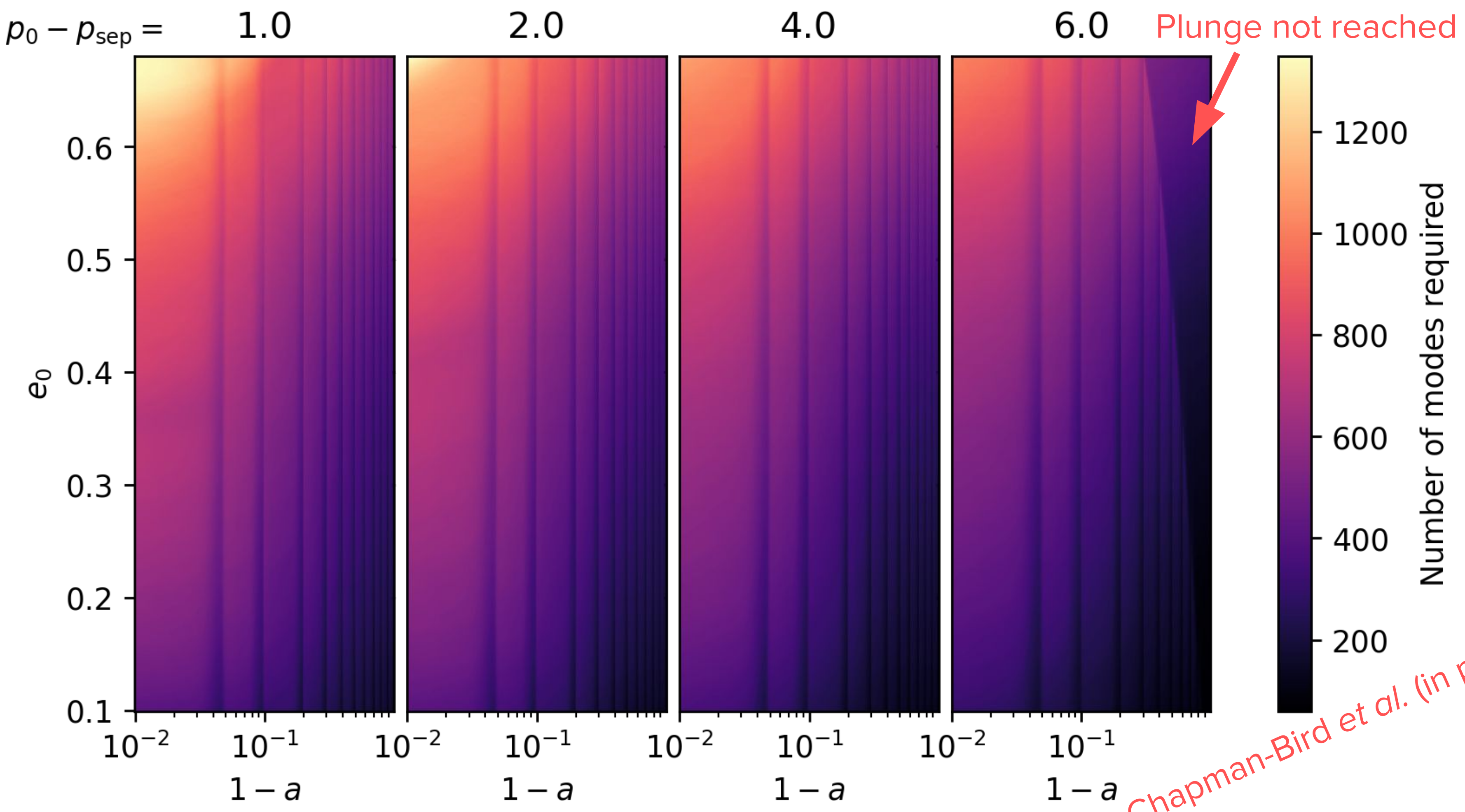
Chapman-Bird *et al.* (in prep)





Wall-time per waveform: **10-20%** decrease

Expect significant further speedups for generic Kerr waveforms (more modes)



Chapman-Bird *et al.* (in prep)

Conclusions

- Learning the EMRI SNR function enables **large-scale population detectability estimates**, and provides a means of optimising signal detection
- Neural mode selection reduces the cost of existing waveform models, and is **readily extendable to generic Kerr waveforms** when the time comes

As more complex and expensive physics is built into waveform models,

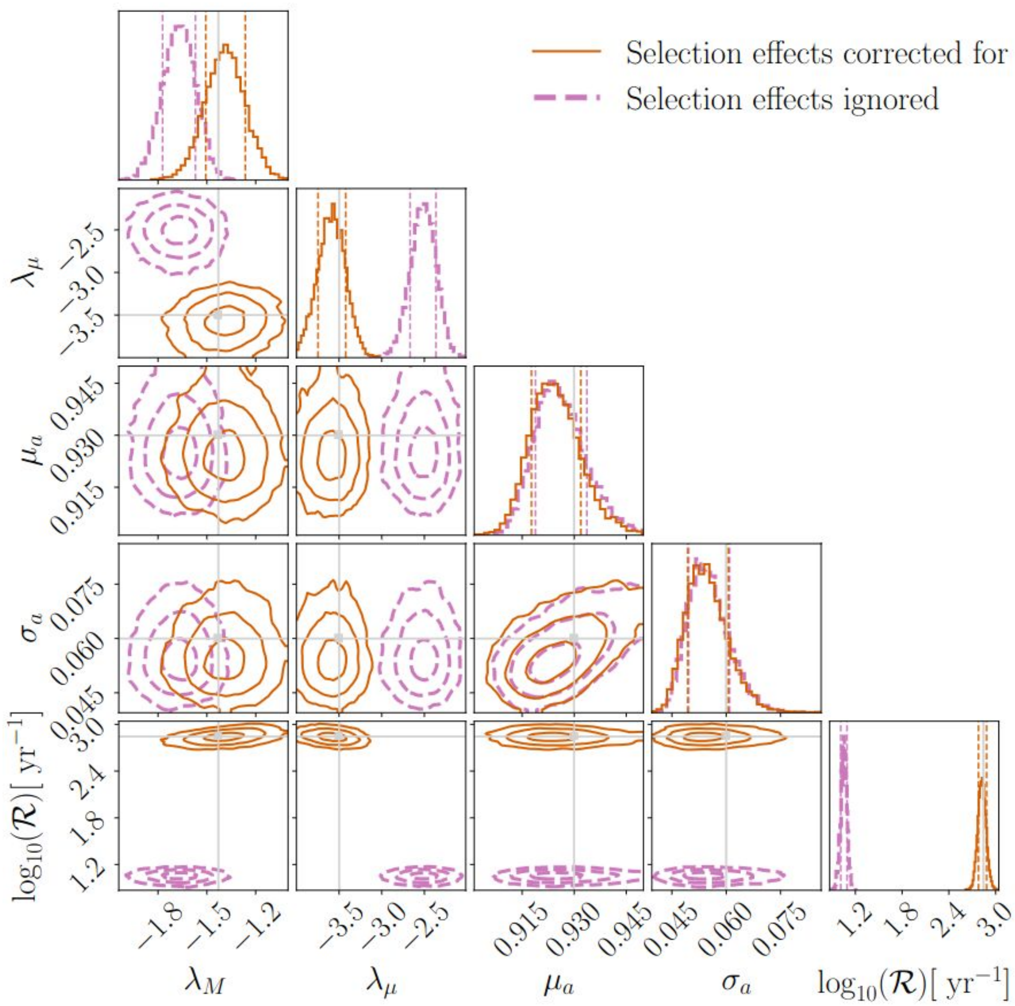
- The efficiency of these methods will improve
- there will be **further opportunities** to incorporate ML methods

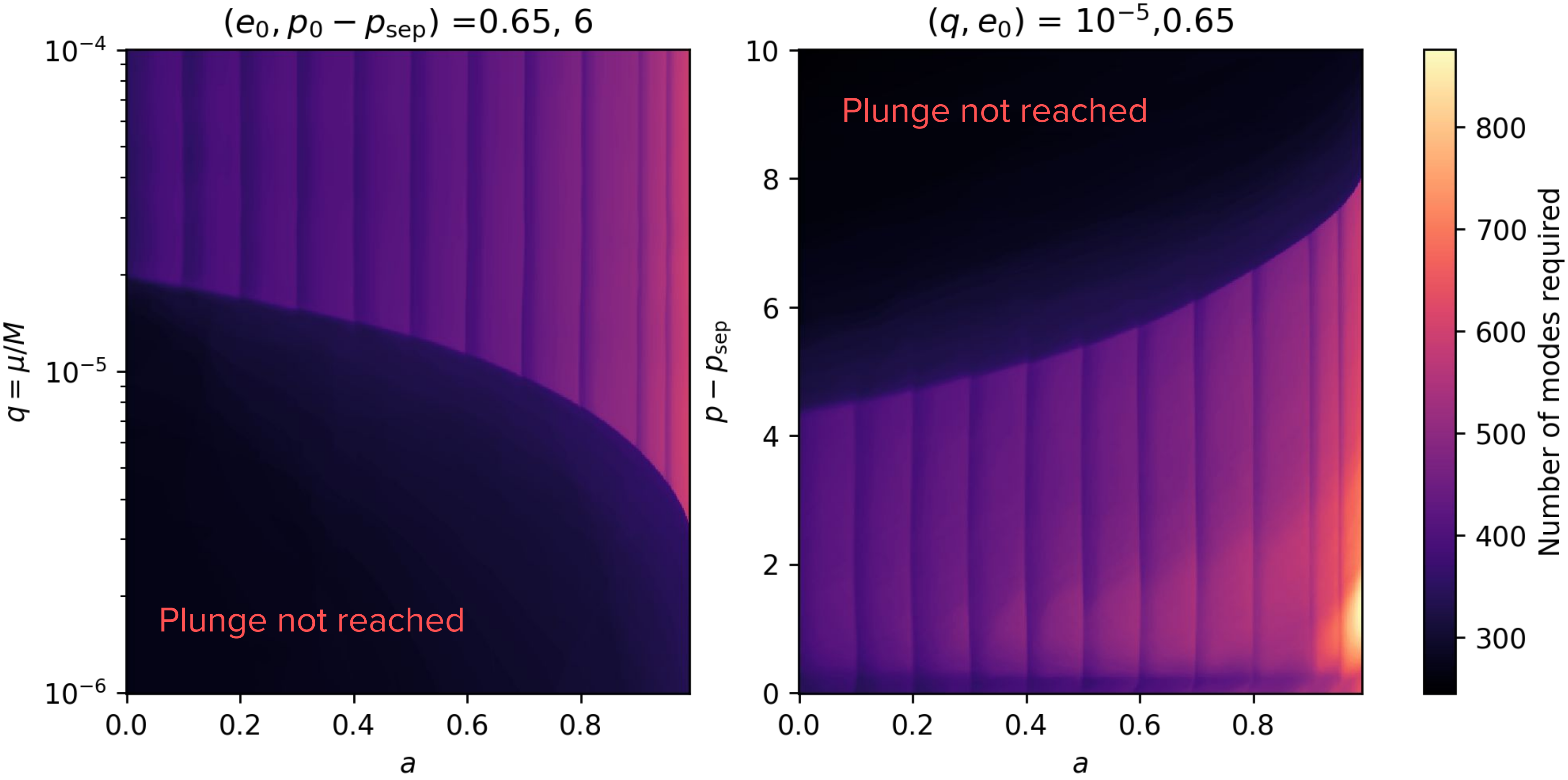
poplar

c.chapman-bird.1@research.gla.ac.uk

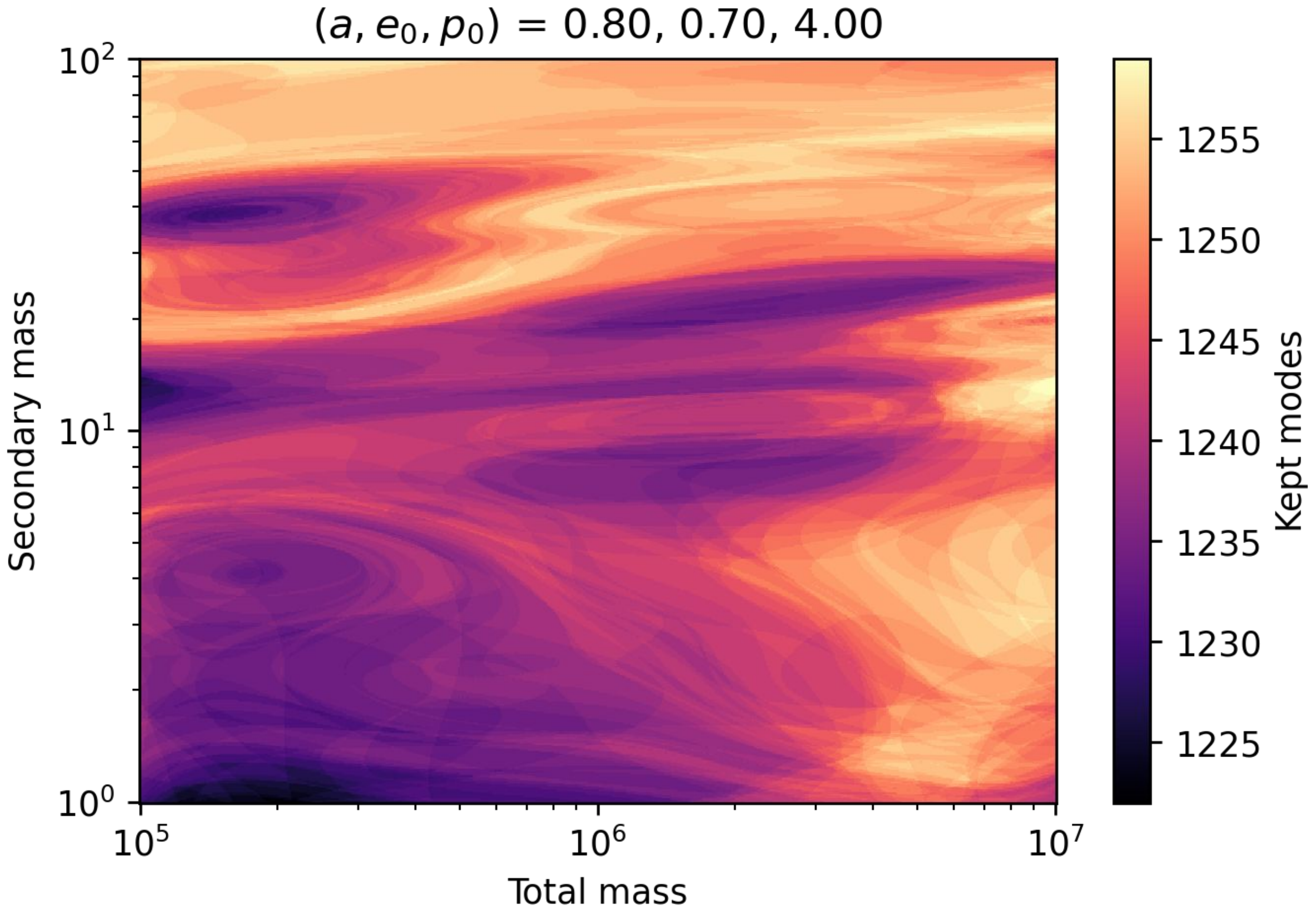
Chapman-Bird *et al.* (2023) arXiv:2212.06166

Bonus slides


Selection effects corrected for
Selection effects ignored
λ_μ
μ_a
σ_a
$\log_{10}(\mathcal{R})[\,\mathrm{yr}^{-1}]$
λ_M
λ_μ
μ_a
σ_a
$\log_{10}(\mathcal{R})[\,\mathrm{yr}^{-1}]$

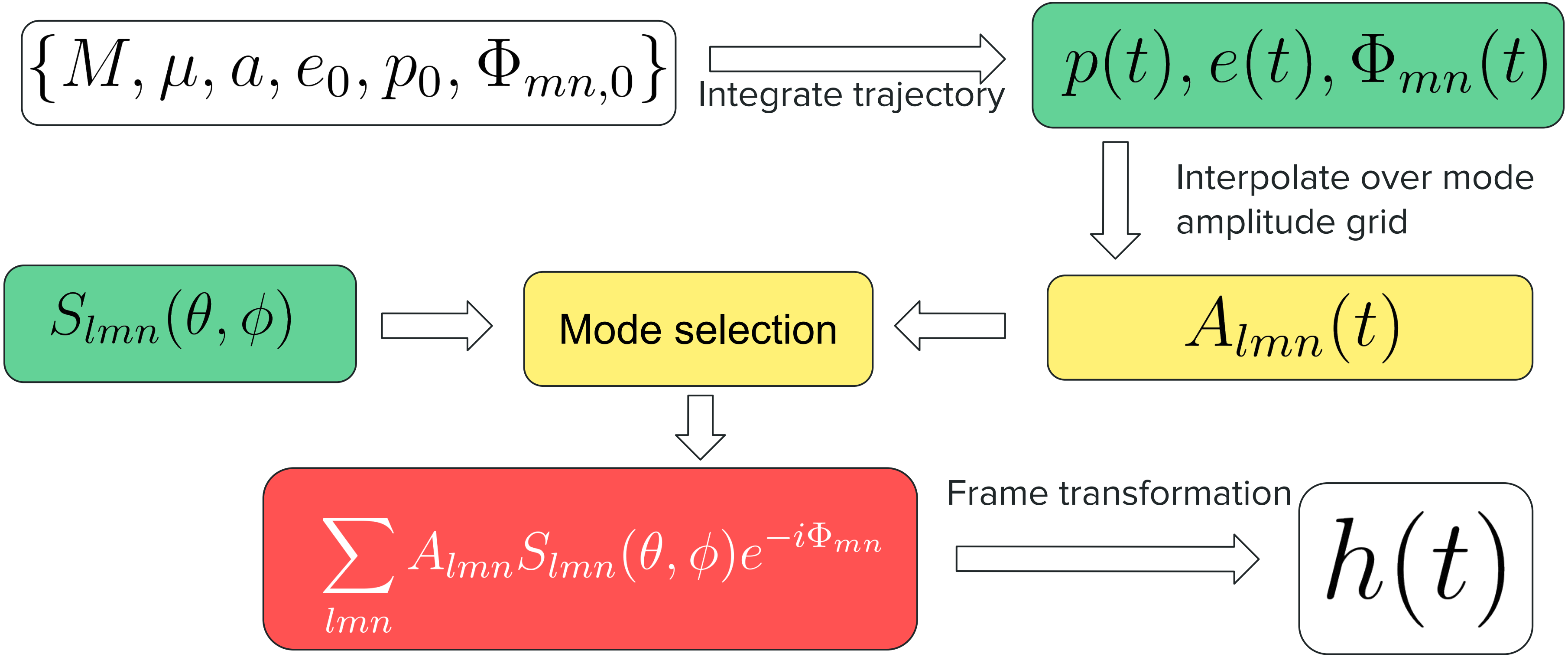


$(e_0, p_0 - p_{\rm sep})$ =0.65, 6
10^{-4}
10^{-5}
10^{-6}
$q = \mu/M$
Plunge not reached
0.0
0.2
0.4
0.6
0.8
a
(q, e_0) = 10^{-5},0.65
10
8
6
4
2
0
$p - p_{\rm sep}$
Plunge not reached
0.0
0.2
0.4
0.6
0.8
a
800
700
600
500
400
300
Number of modes required

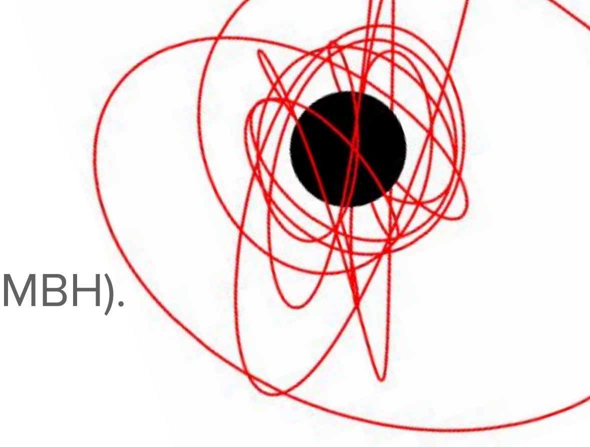


(a, e_0, p_0) = 0.80, 0.70, 4.00
Secondary mass
Total mass
Kept modes
10^2
10^1
10^0
10^5
10^6
10^7
1255
1250
1245
1240
1235
1230
1225

Building an (equatorial) EMRI waveform



Extreme mass ratio inspirals (EMRIs)



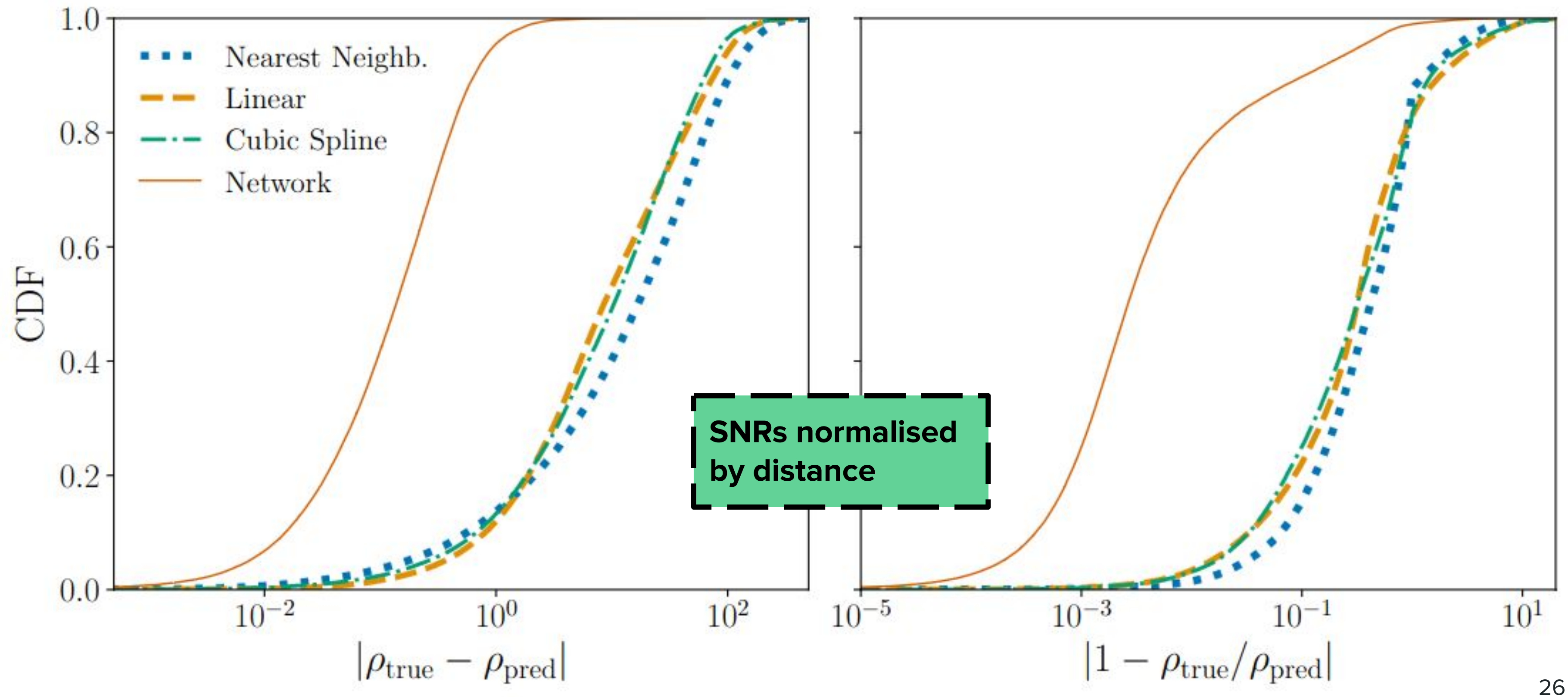
Inspiral of a compact object (CO) into a massive black hole (MBH).

A highly promising source of gravitational waves:

- Observable by detectors such as the Laser Interferometer Space Antenna (LISA) with high signal-to-noise ratio (SNR)
- System parameters can be measured to *extreme* precision
- Enable stringent tests of general relativity in the Kerr regime
- Significant numbers of EMRIs expected to be observed: population studies!
    - Sources well-localised at redshifts up to $z \sim 6$ - excellent dark sirens for cosmology

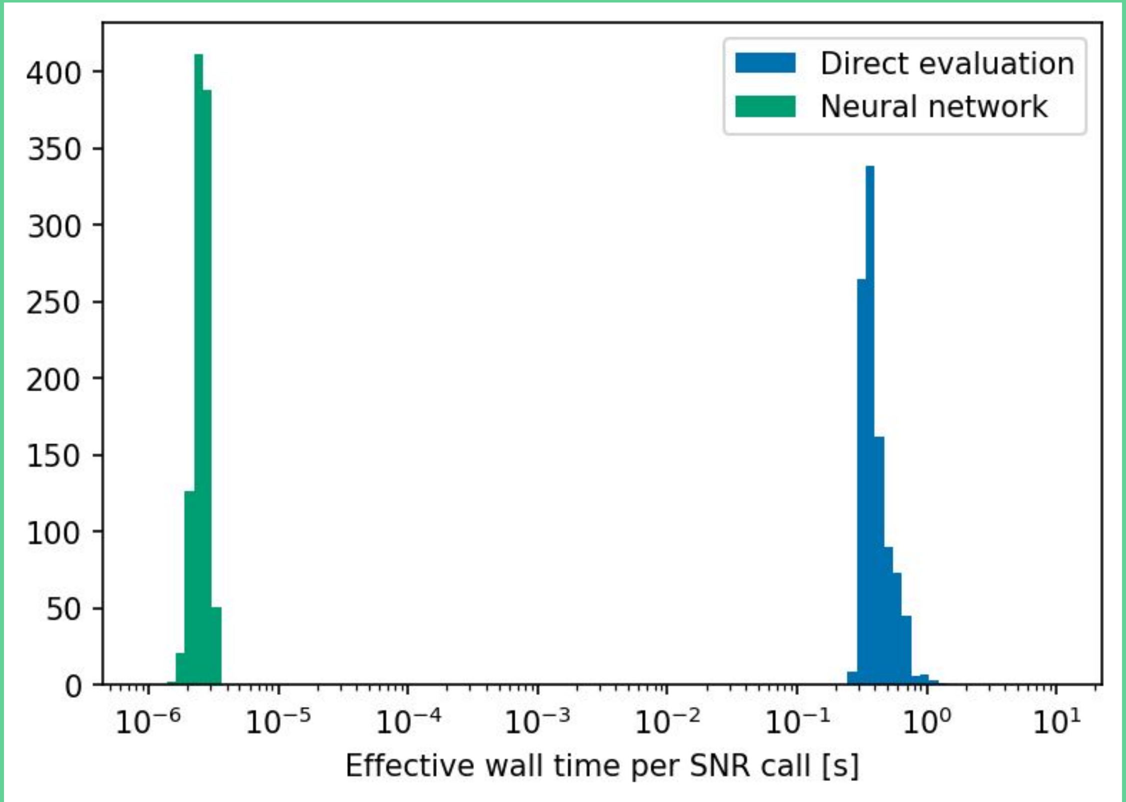
Image credit: Black Hole Perturbation Toolkit

Accuracy: 10^5 data points



Cost analysis

poplar



Vectorised evaluation over batches yields extremely low per-SNR cost, especially on GPUs (shown).

Evaluation on a single set of EMRI parameters takes roughly **10^{-3} s**.