Detection and modelling of eccentric intermediate mass ratio in-spirals

Chandra Kant Mishra
IIT Madras, Chennai (India)

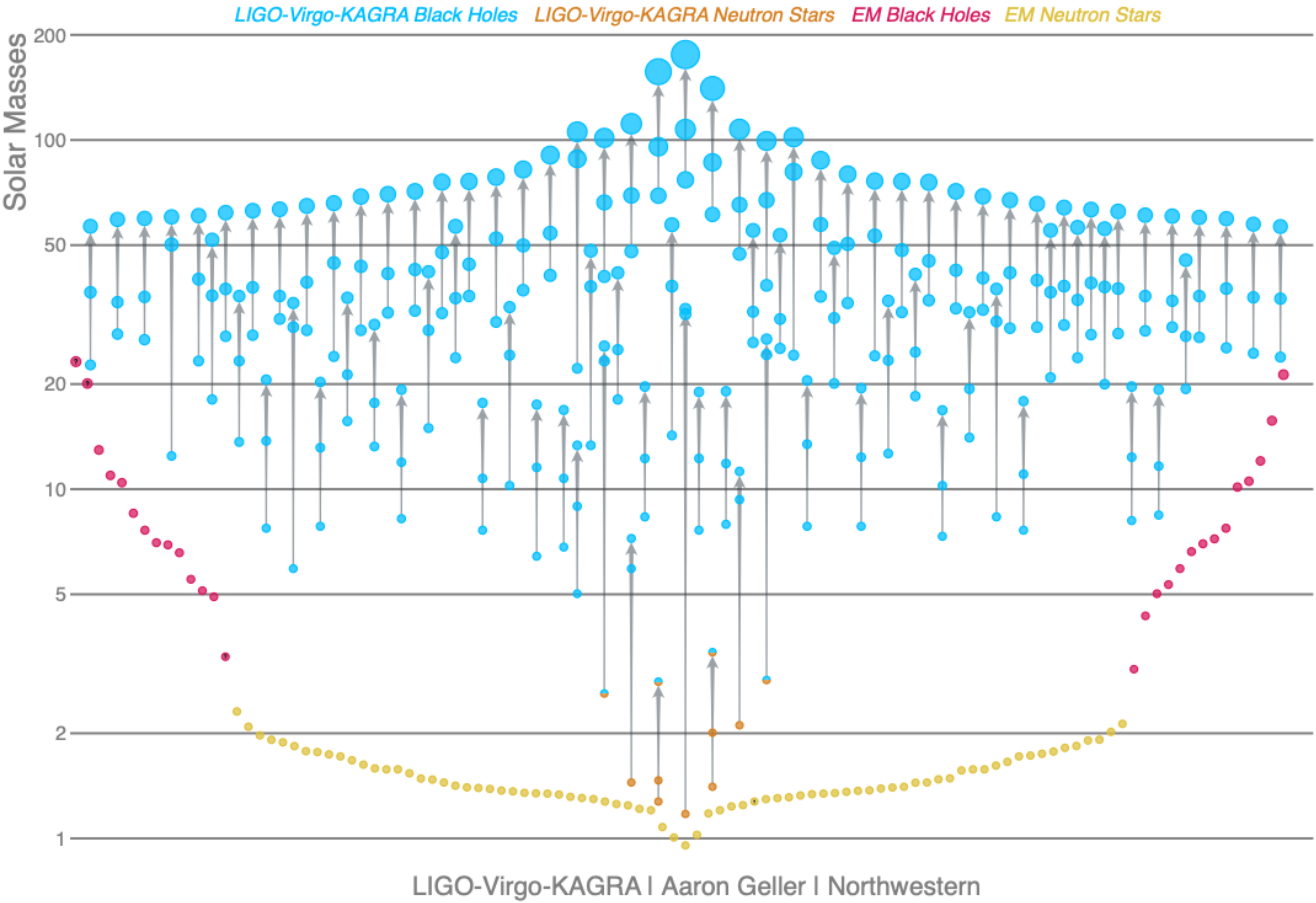
With

Ryuichi Fujita, M Laxman & Estuti Shukla

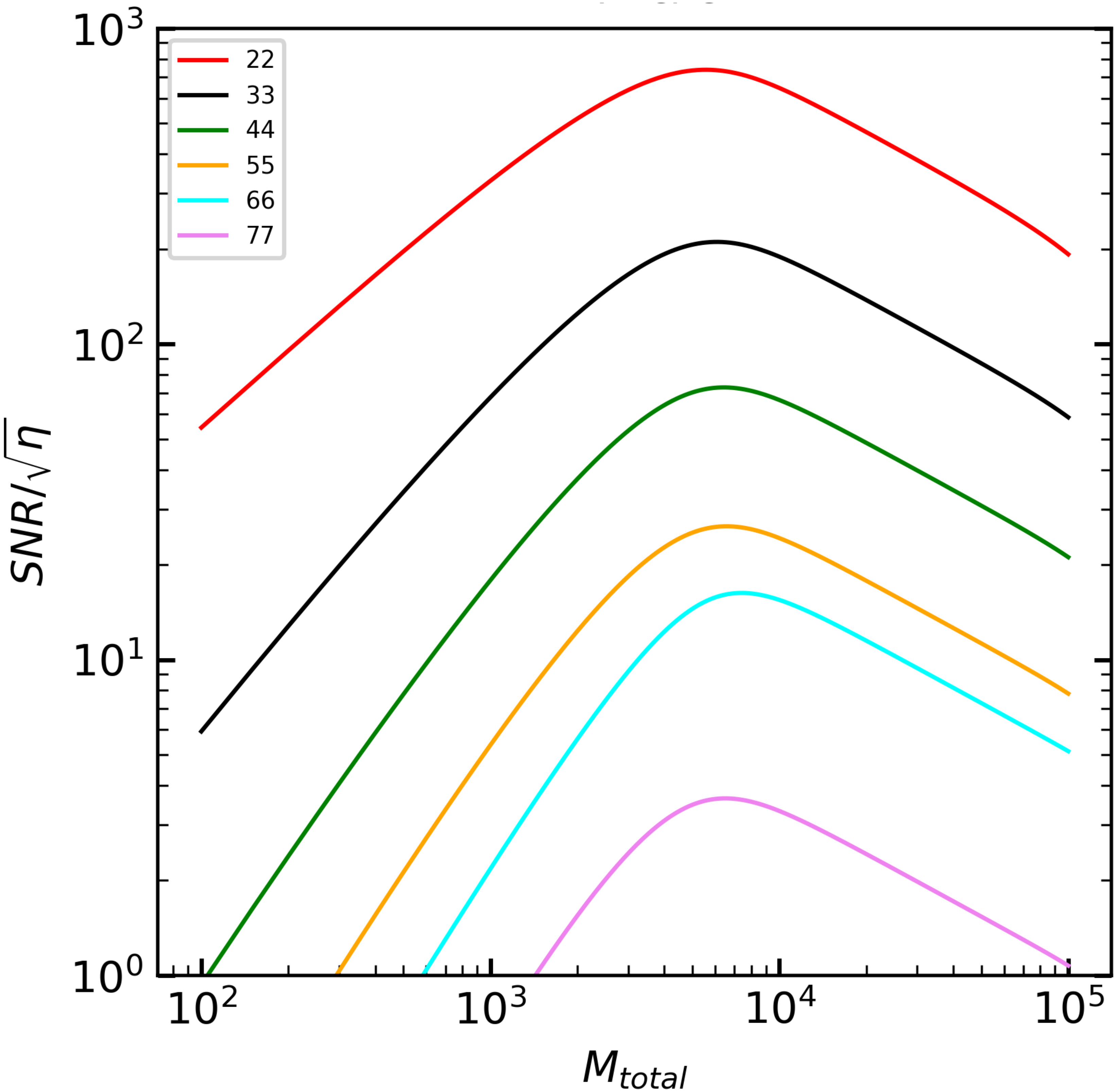
@

26th Capra meeting, Niels Bohr Institute

Masses in the Stellar Graveyard



Detectability of a circular binary with at least one IMBH merging in DECIGO-B band



$$\langle \rho^2 \rangle = \frac{3}{20} \sum_n \int_{\mathrm{f_{low}}}^{\mathrm{f_{up}}} \frac{h_{c,n}^2(f_n)}{f_n^2 S_h(f_n)} \, df$$

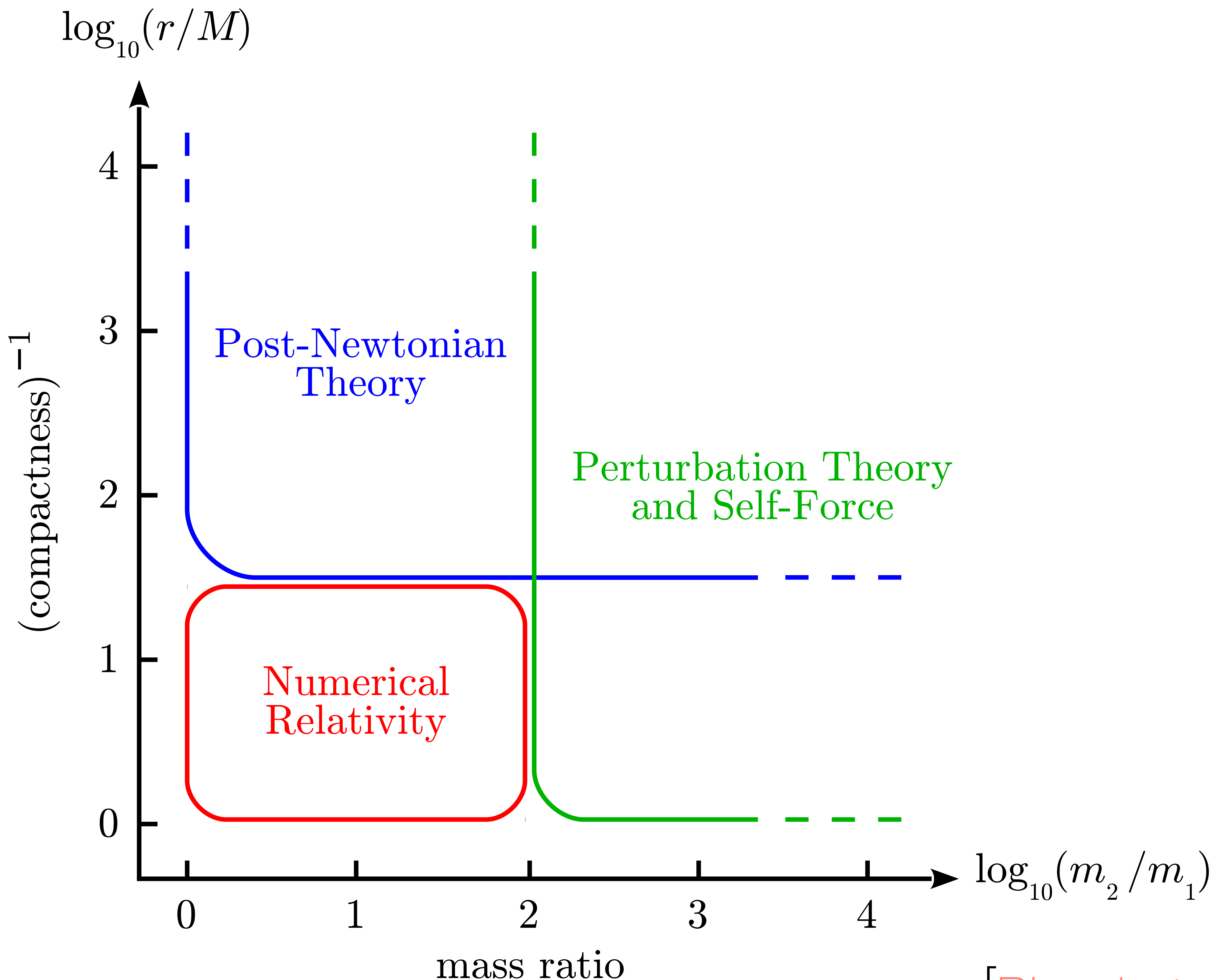
$$h_{c,n} = (\pi d)^{-1} \sqrt{2\dot{E}_n / \dot{f}_n}$$

Scaling

$$1/\sqrt{\eta}$$
$$= 2 \, (\eta = 0.25)$$
$$\sim 3 \, (\eta = 0.1)$$
$$\sim 10 \, (\eta = 0.01)$$

[in prep.]

Approximation schemes



[Blanchet, LLR (2014)]

Outline

- PN and BHP solutions for ICBs in circular orbits
- Eccentric corrections and hybridisation
- Model systematics : impact of mass-ratio, eccentricity and higher order BHP terms

Approximation schemes

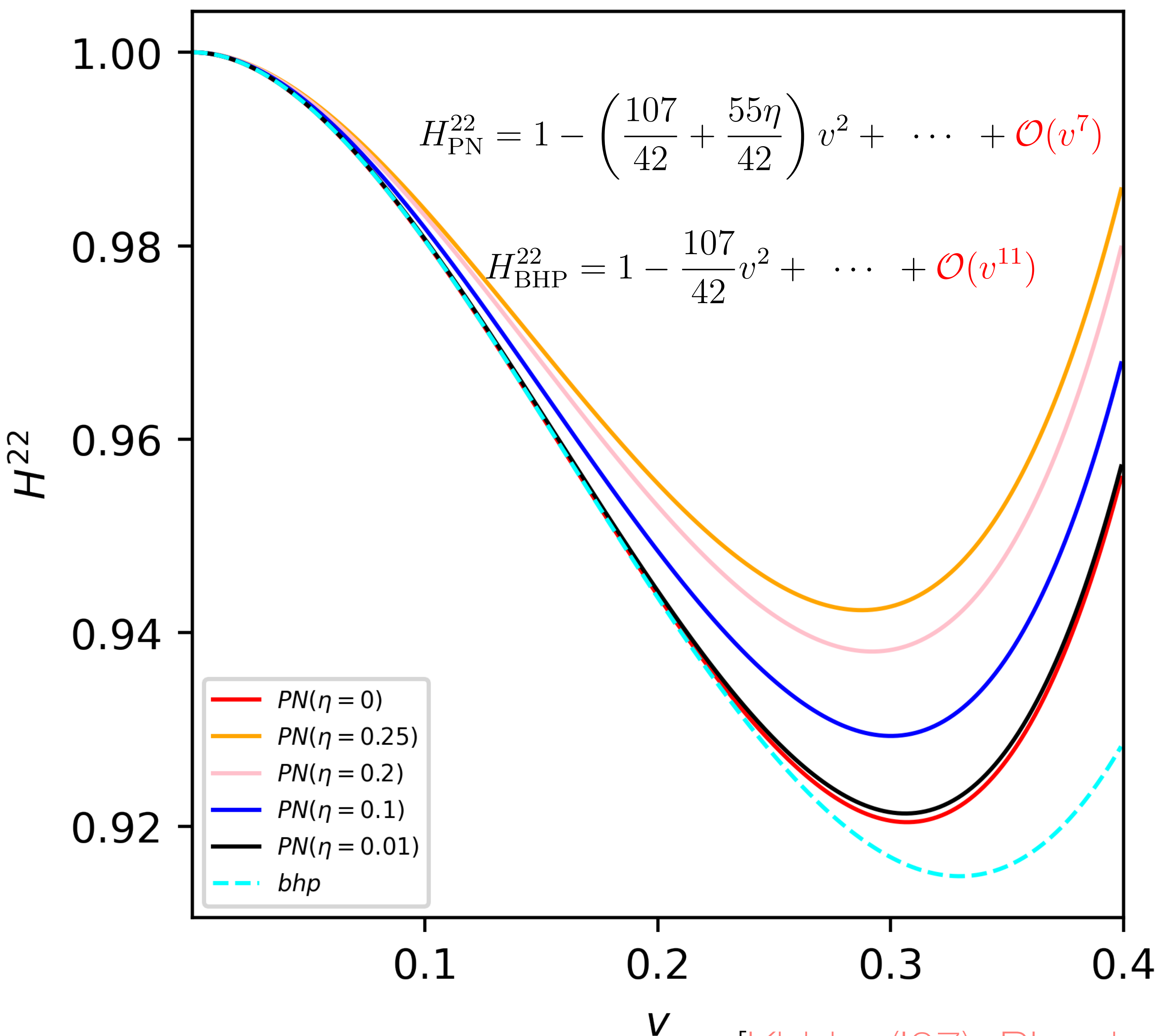
- Taylor series expansions in small parameters such as internal source velocities
- Point particles, multipolar expansions etc.
- High order computations (nontrivial due to IR divergences)

Detector	LIGO/Virgo		ET		LISA	
Masses ($M_\odot$)	1.4×1.4	10×10	1.4×1.4	500×500	$10^5 \times 10^5$	$10^7 \times 10^7$
PN order	cumulative number of cycles					
Newtonian	2 562.599	95.502	744 401.36	37.90	28 095.39	9.534
1PN	143.453	17.879	4 433.85	9.60	618.31	3.386
1.5PN	−94.817	−20.797	−1 005.78	−12.63	−265.70	−5.181
2PN	5.811	2.124	23.94	1.44	11.35	0.677
2.5PN	−8.105	−4.604	−17.01	−3.42	−12.47	−1.821
3PN	1.858	1.731	2.69	1.43	2.59	0.876
3.5PN	−0.627	−0.689	−0.93	−0.59	−0.91	−0.383
4PN	−0.107	−0.064	−0.12	−0.04	−0.12	−0.013
4.5PN	0.098	0.118	0.14	0.10	0.14	0.065

[Order counting terminology : nPN := v^{2n}]

[Blanchet+, arXiv:2304.11185]

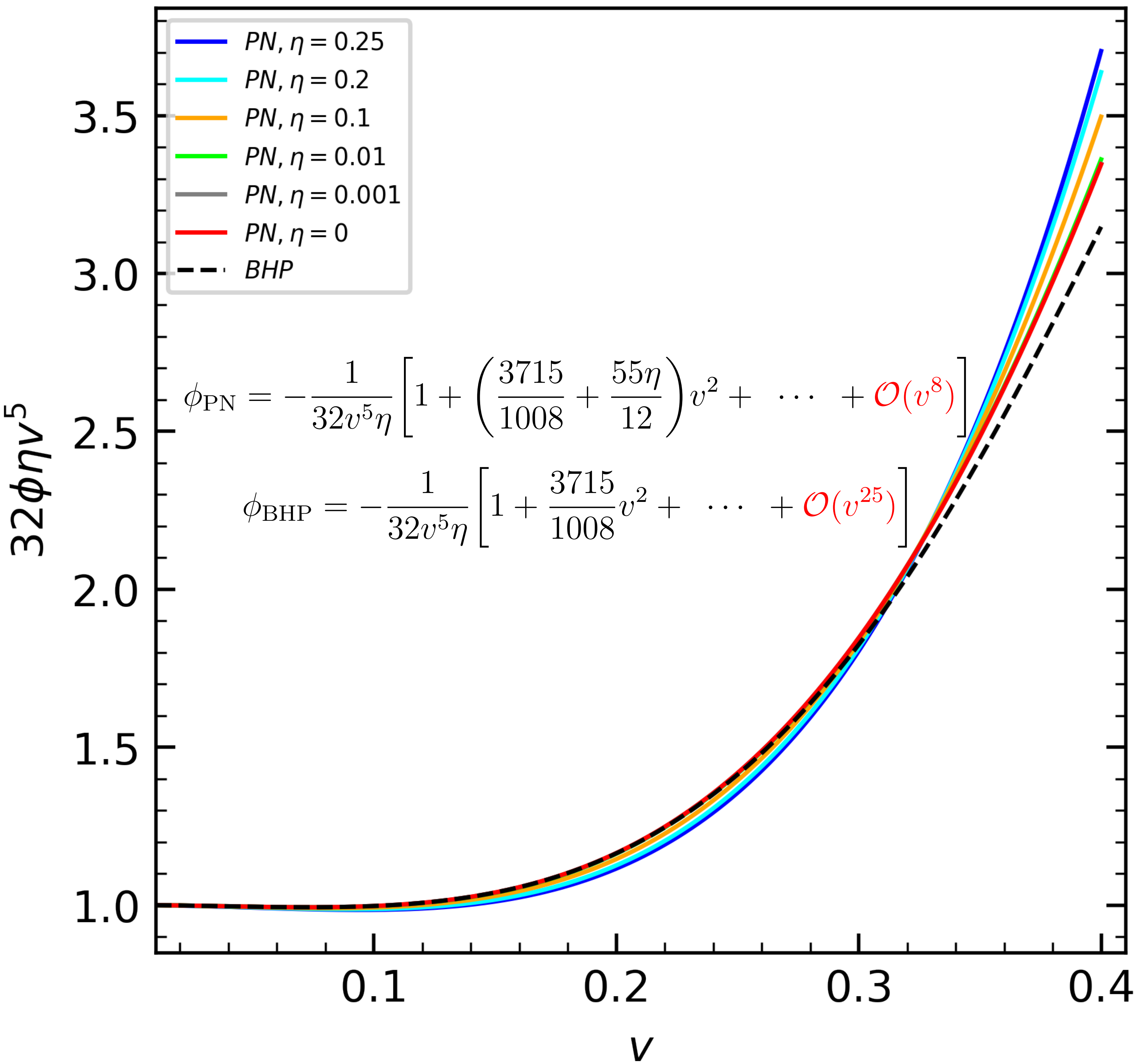
PN-BHP comparisons



[$O(v^{2n})$: terms beyond nPN and ignored]

[Kidder ('07), Blanchet + ('08)]
[Fujita-Iyer ('10)]

PN-BHP comparisons



Scaling

$$1/(32\eta v^5)|_{v=0.4}$$
$$\sim 10\,(\eta = 0.25)$$
$$\sim 30\,(\eta = 0.1)$$
$$\sim 300\,(\eta = 0.01)$$

[Blanchet + ('02)]
[Fujita ('12)]

Eccentric Case

PN-Phase

$$\phi_{\rm PN} = -\frac{1}{32v^5\eta}\left\{1 + \left(\frac{3715}{1008} + \frac{55\eta}{12}\right)v^2 + \cdots + \mathcal{O}(v^8)\right.$$
$$- \frac{785}{272}e_0^2\left(\frac{v_0}{v}\right)^{19/3}\left[1 + \left(\frac{6955261}{2215584} + \frac{436441\eta}{79128}\right)v^2 + \left(\frac{2833}{1008} - \frac{197\eta}{36}\right)v_0^2 + \cdots + \mathcal{O}(v^7)\right]$$
$$\left. + \mathcal{O}(e_0^4)\right\}$$

PN-BHP-Phase

$$\phi_{\rm PN-BHP} = -\frac{1}{32v^5\eta}\left\{1 + \left(\frac{3715}{1008} + \frac{55\eta}{12}\right)v^2 + \cdots + \frac{2554404624135128353}{2214468081745920}v^8 + \cdots + \mathcal{O}(v^{25})\right.$$
$$- \frac{785}{272}e_0^2\left(\frac{v_0}{v}\right)^{19/3}\left[1 + \left(\frac{6955261}{2215584} + \frac{436441\eta}{79128}\right)v^2 + \left(\frac{2833}{1008} - \frac{197\eta}{36}\right)v_0^2 + \cdots\right.$$
$$\left. + (\cdots)\,v^8 + (\cdots)\,v^7v_0 + \cdots + (\cdots)\,vv_0^7 + (\cdots)\,v_0^8 + \cdots + \mathcal{O}(v^{11})\right]$$
$$\left. + \cdots + \mathcal{O}(e_0^{10})\right\}$$

[Sago-Fujita ('15), Moore+ ('16), Fujita-Shibata ('20)]
[In prep.]

Impact of mass ratio on # GW cycles : PN contribution

- System : M=1000 Msun, q=200
- DECIGO-B : f_{low}=0.01Hz, f_{up} = f_{Iso}
- PN phase model : 3.5PN (circular) 3PN (Eccentric)
- BHP phase model : 12PN (circular) & 5PN (Eccentric)

PN Order ↓	$(\Delta N_{cycle}^{PN(e_0^{10})}$ - $\Delta N_{cycle}^{BHP(e_0^{10})})$	$(\Delta N_{cycle}^{PN(e_0^{10})}$ - $\Delta N_{cycle}^{BHP(e_0^{10})})$	$(\Delta N_{cycle}^{PN(e_0^{10})}$ - $\Delta N_{cycle}^{BHP(e_0^{10})})$	$(\Delta N_{cycle}^{PN(e_0^{10})}$ - $\Delta N_{cycle}^{BHP(e_0^{10})})$
$e_0 \rightarrow$	$e_0 = 0$	$e_0 = 0.1$	$e_0 = 0.2$	$e_0 = 0.3$
0 PN	0	0	0	0
1 PN	293.963	-23316.496 $\times 10^{-5}$	-2.479	-9.671
1.5 PN	0	0	0	0
2 PN	4.351	-5023.803$\times 10^{-5}$	-0.223	-0.541
2.5 PN	0.515	-229.227 $\times 10^{-5}$	-1028.718$\times 10^{-5}$	-2400.073$\times 10^{-5}$
3 PN	2.917	-279.143$\times 10^{-5}$	-1105.646$\times 10^{-5}$	-2393.290$\times 10^{-5}$
3.5 PN	-0.2	-	-	-
Total cycles accumulated	301.746	-0.288	-2.724	-10.260

[In prep.]

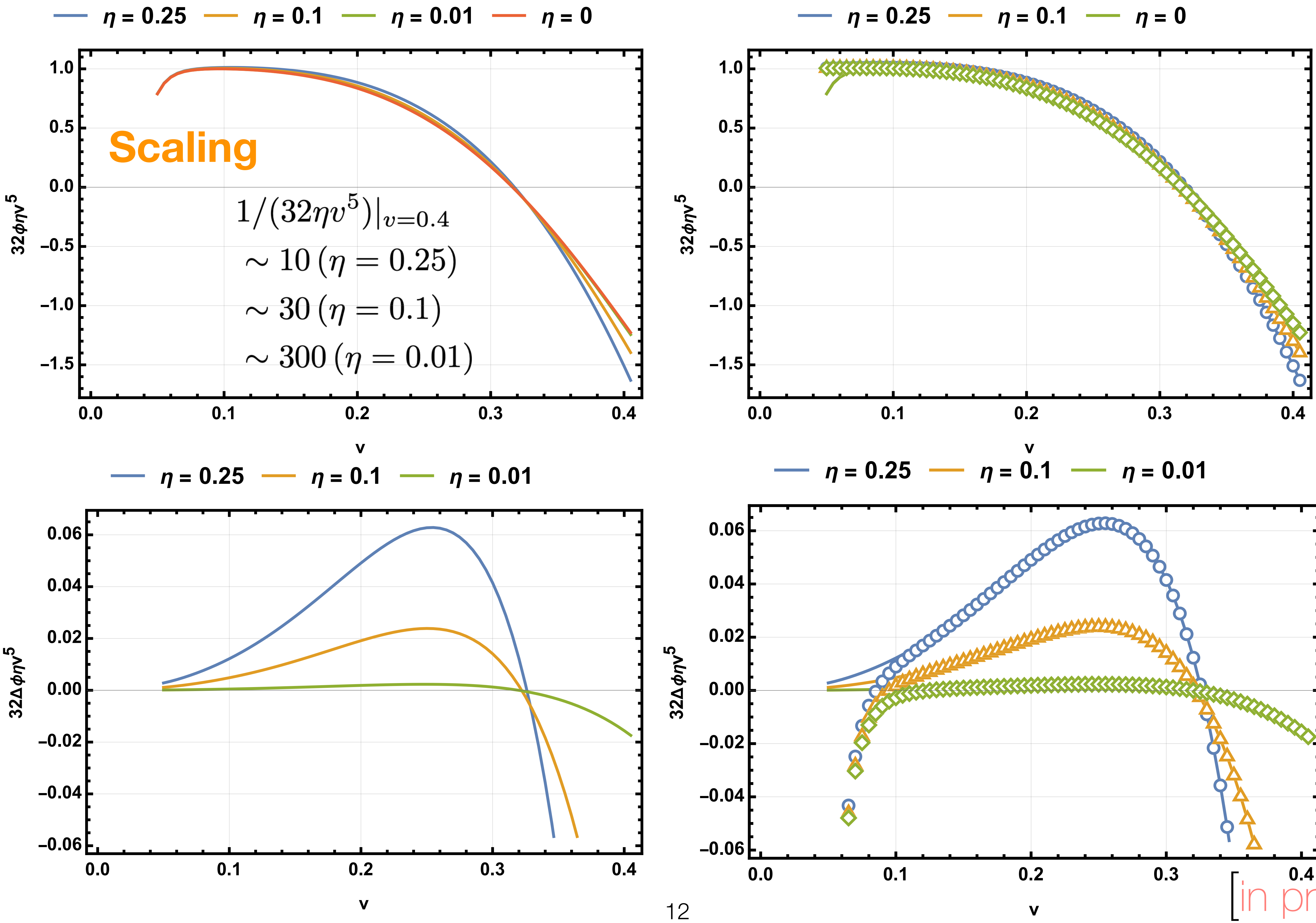
Impact of higher order terms in BHP theory on # GW cycles

PN Order ↓	$\Delta N_{cycle}^{BHP(e_0^{10})}$	$\Delta N_{cycle}^{BHP(e_0^{10})}$	$\Delta N_{cycle}^{BHP(e_0^{10})}$	$\Delta N_{cycle}^{BHP(e_0^{10})}$
$e_0 \rightarrow$	$e_0 = 0$	$e_0 = 0.1$	$e_0 = 0.2$	$e_0 = 0.3$
3.5 PN	-	-350.27×10^{-4}	-140.11×10^{-3}	-315.245×10^{-3}
4 PN	-63.011	423.11×10^{-4}	169.24×10^{-3}	380.80×10^{-3}
4.5 PN	47.008	-12.82×10^{-4}	-5.13×10^{-3}	-11.54×10^{-3}
5 PN	-51.388	13.39×10^{-4}	5.36×10^{-3}	12.05×10^{-3}
5.5PN-12PN	26.49	-	-	-
Total cycles accumulated	-40.90	73.41×10^{-4}	29.36×10^{-3}	66.07×10^{-3}

Likely to be more relevant for higher mass ratios

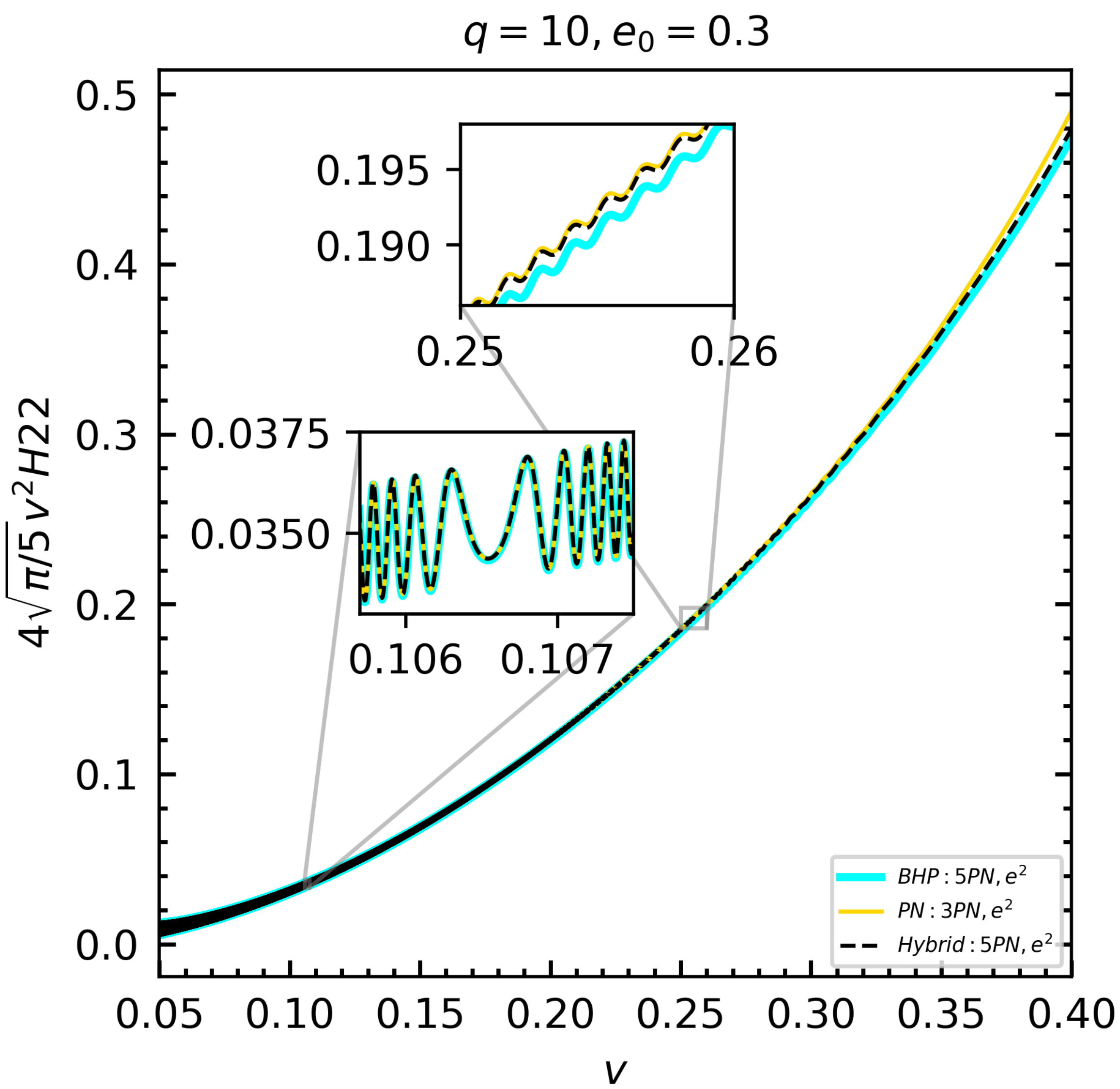
[In prep.]

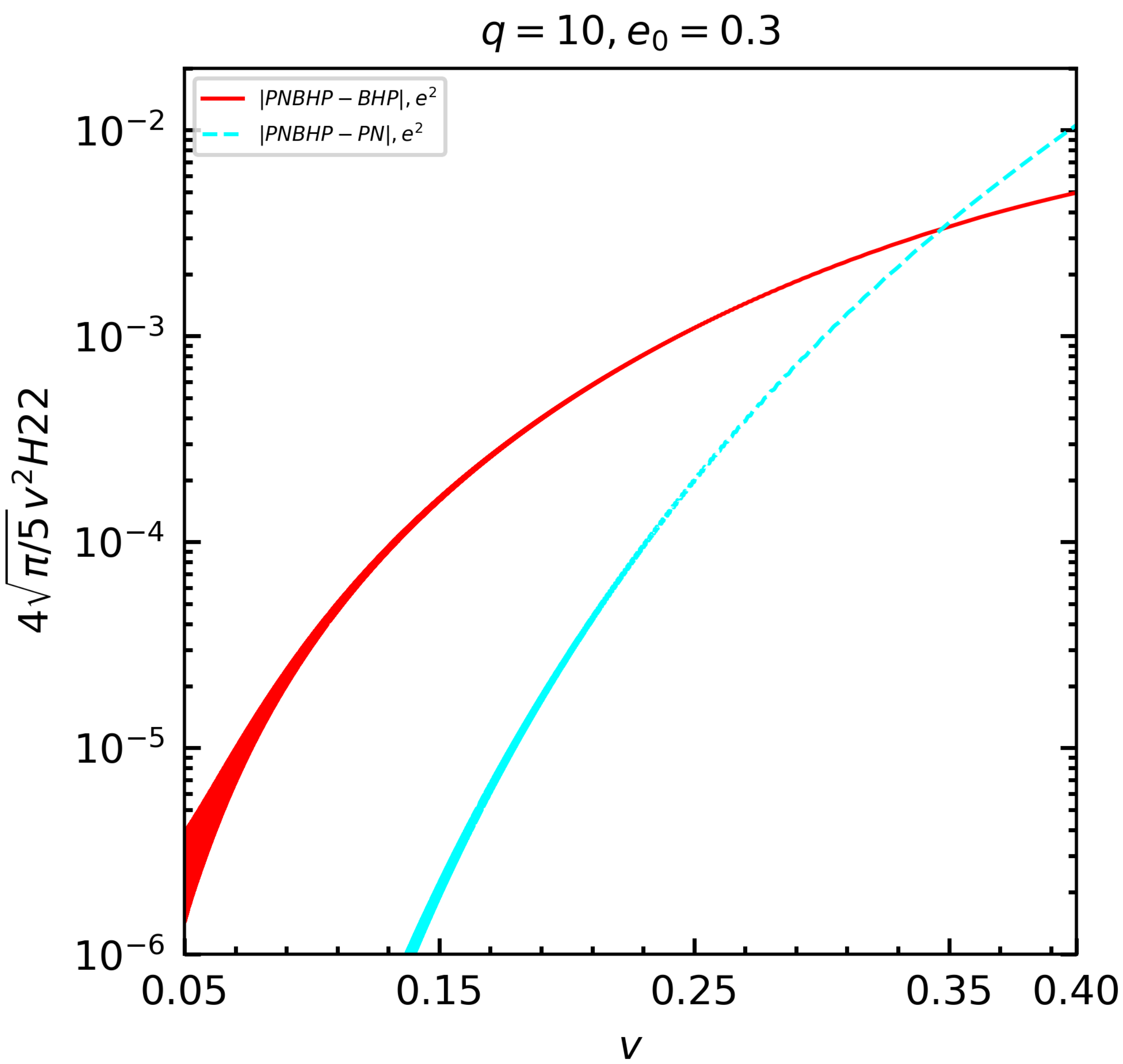
phase comparisons (e~0.3)



[in prep.]

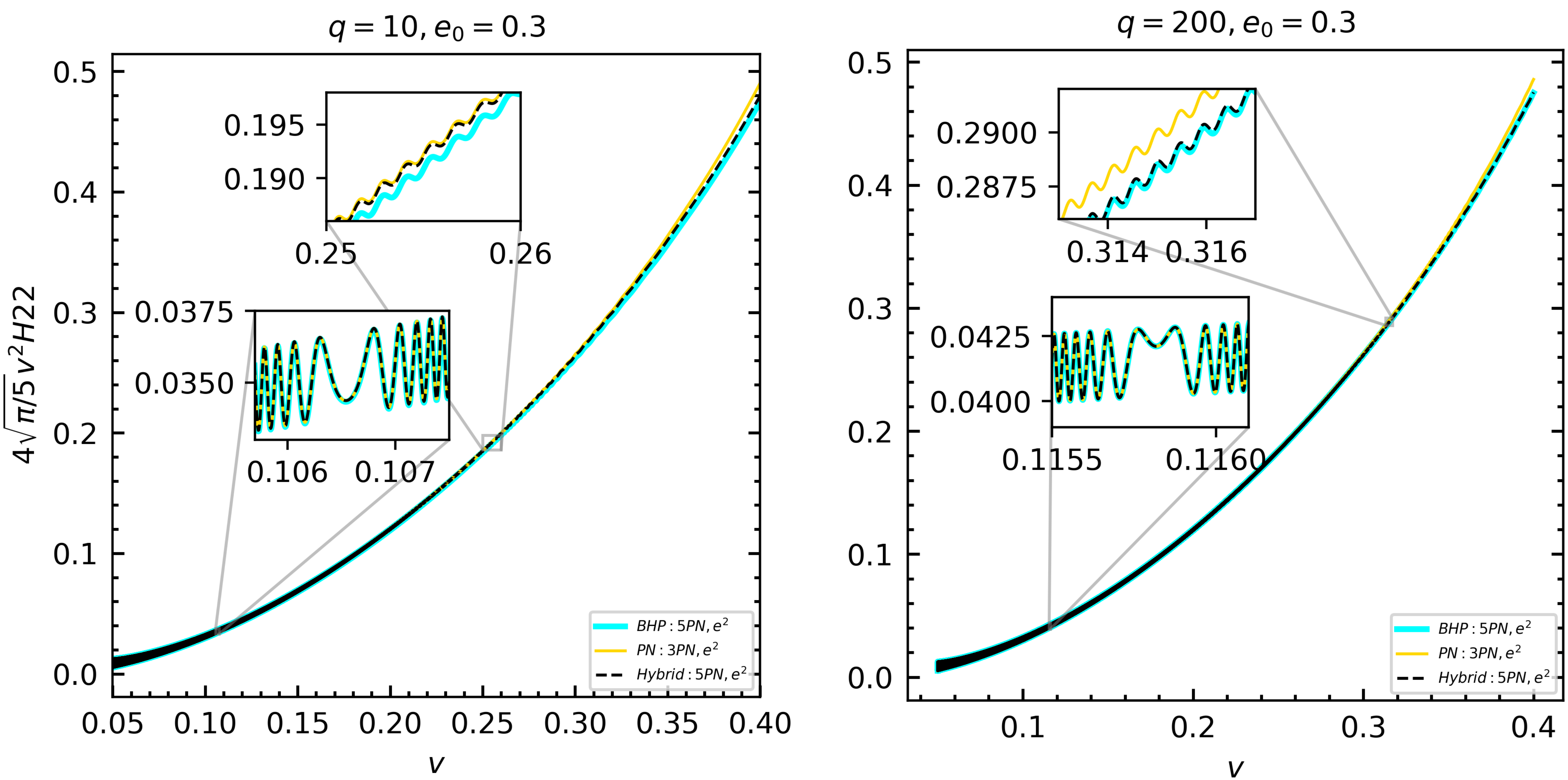
22 mode amp comparisons





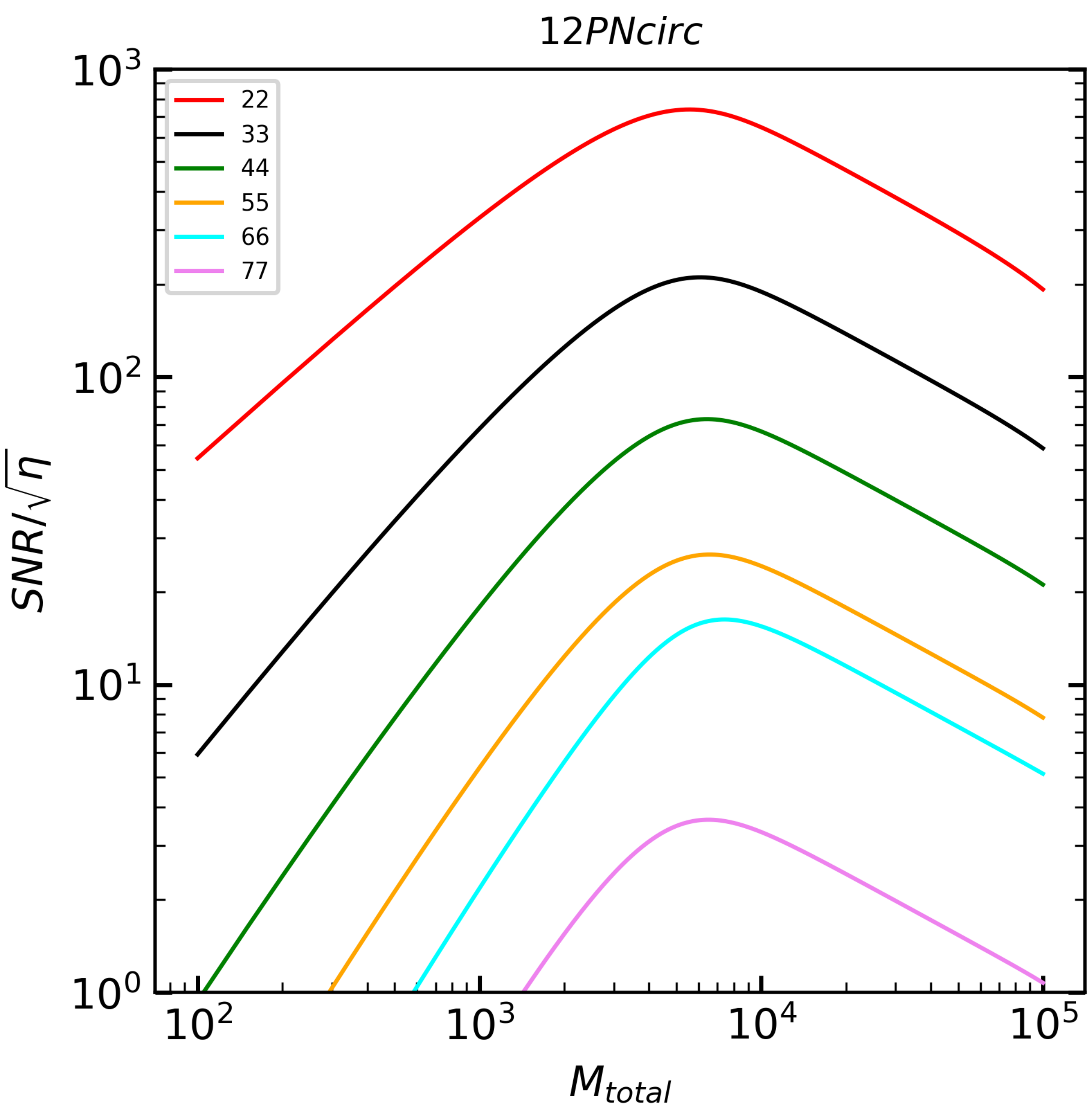
[in prep.]

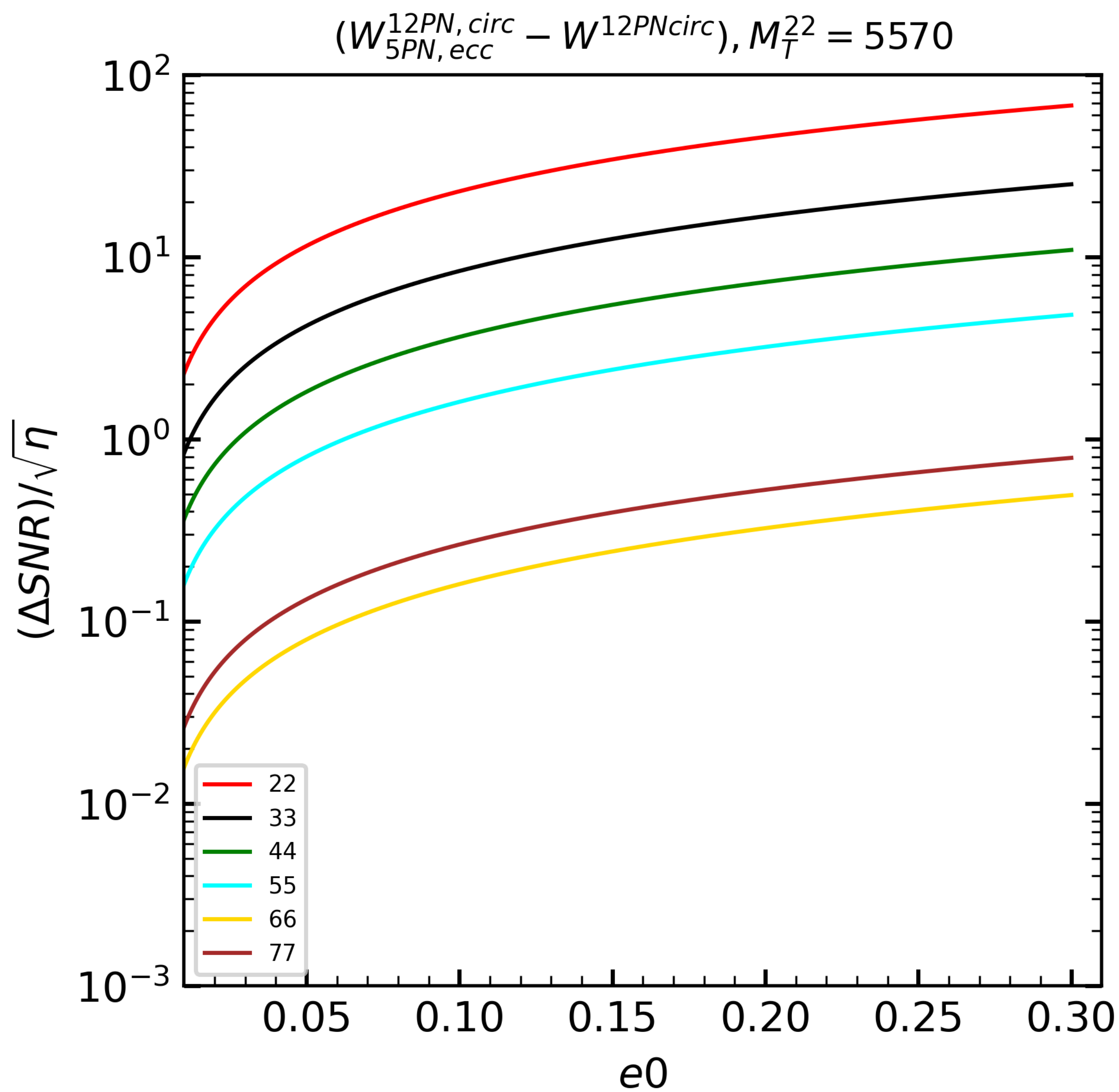
22 mode comparisons



[in prep.]

Impact of eccentricity on SNR





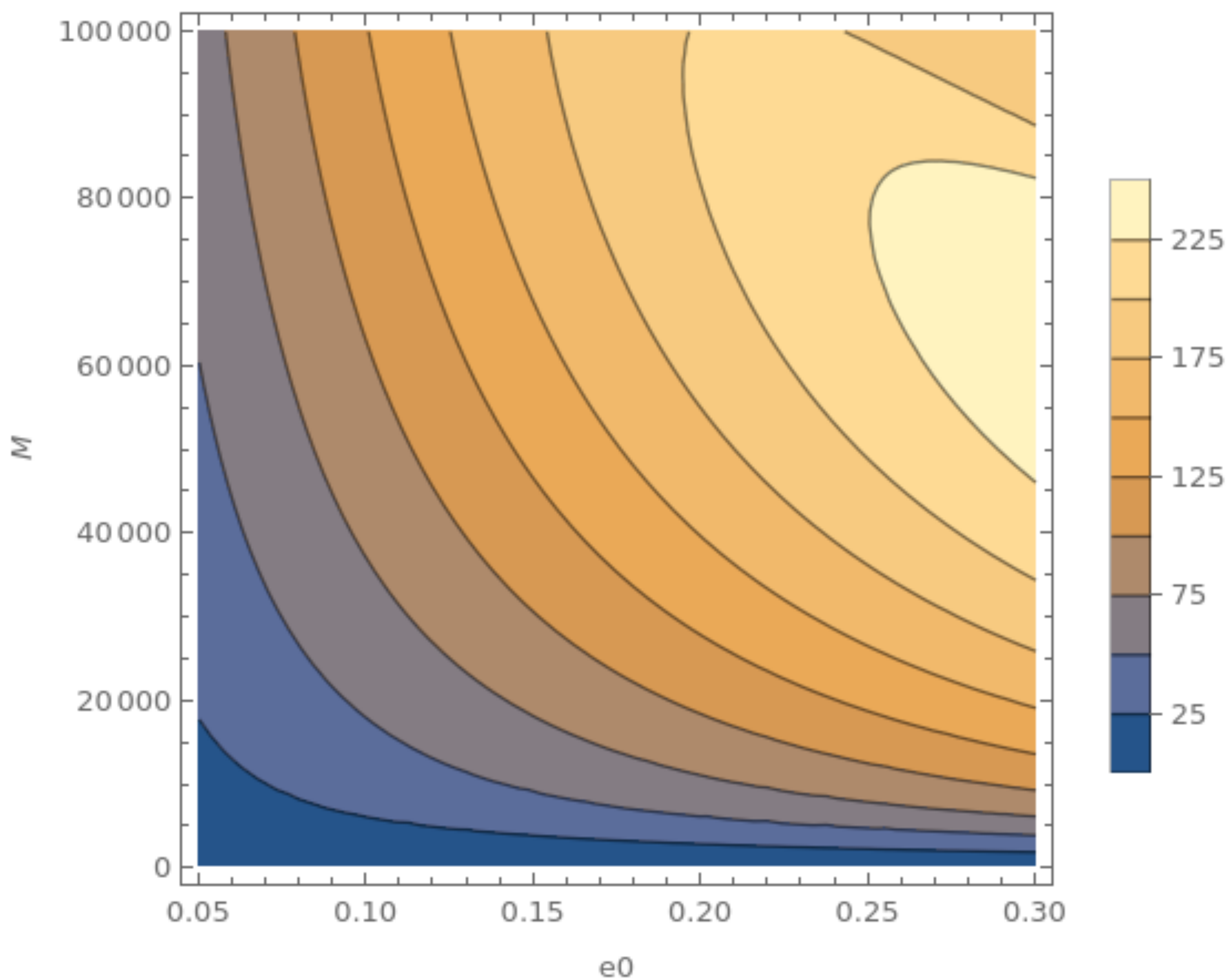
[in prep.]

Summary

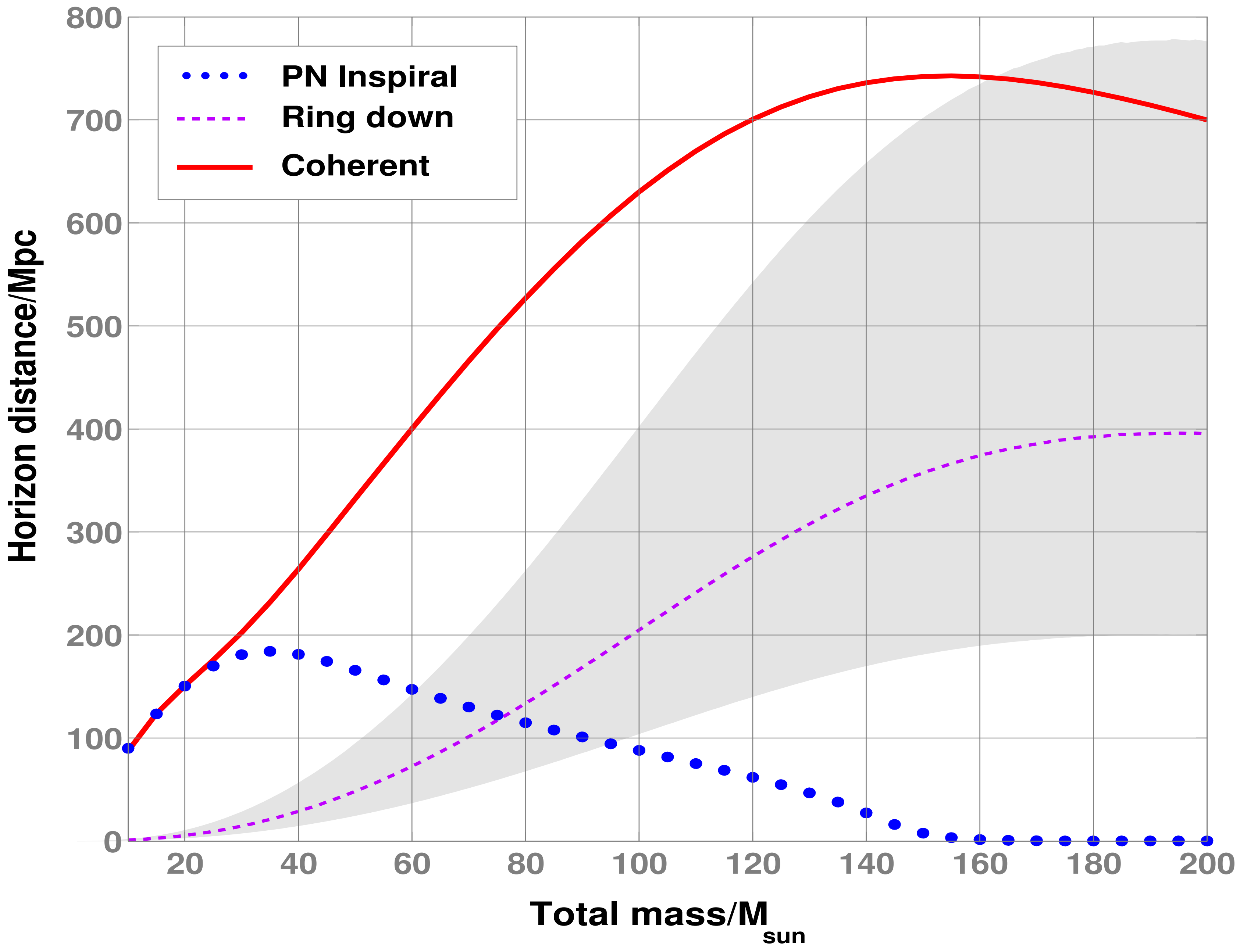
- Compact binaries involving at least one IMBH and a stellar or supermassive BH can be classified as IMRIs and should be detectable by detectors in space and on ground.
- Analytical solutions from perturbative approaches in GR may be combined to obtain models that may help detect and extract properties of these sources.
- Based on simple SNR estimates, once may be able to infer the presence of higher modes and/or eccentric nature of the binary.

additional slides

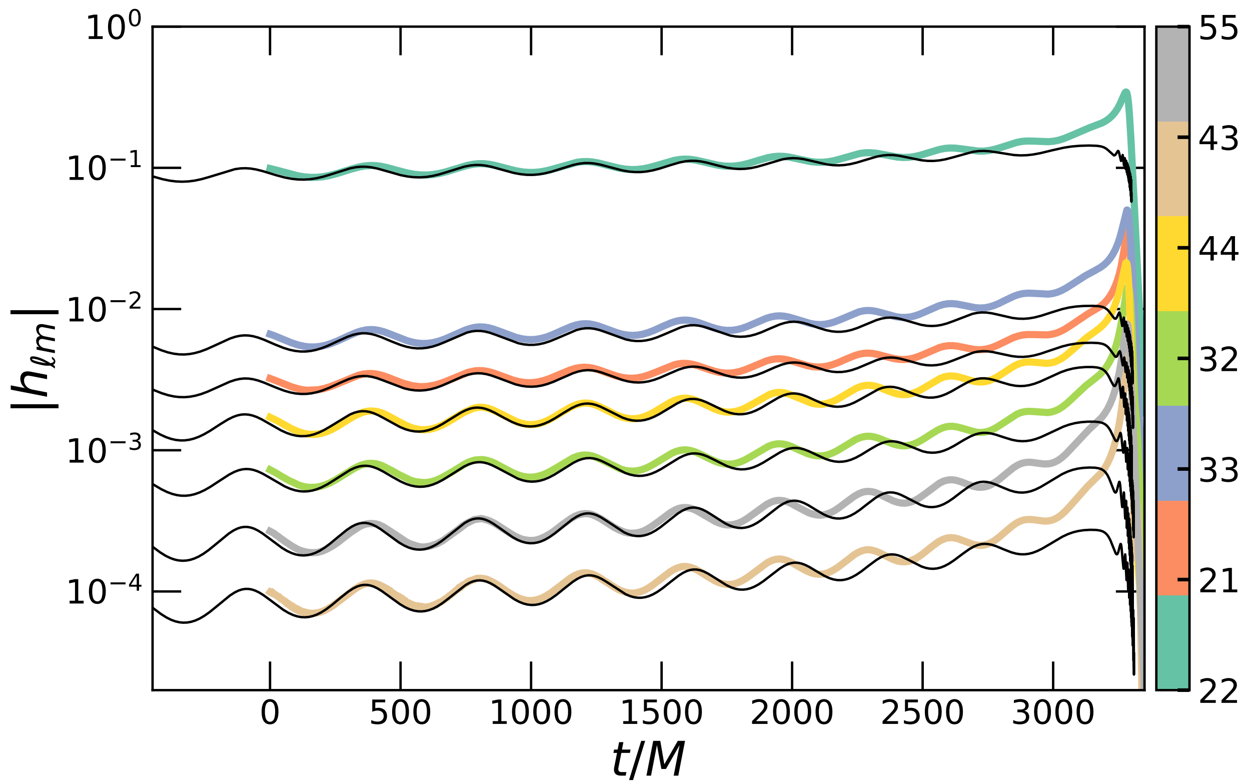
Gain in eccentricity

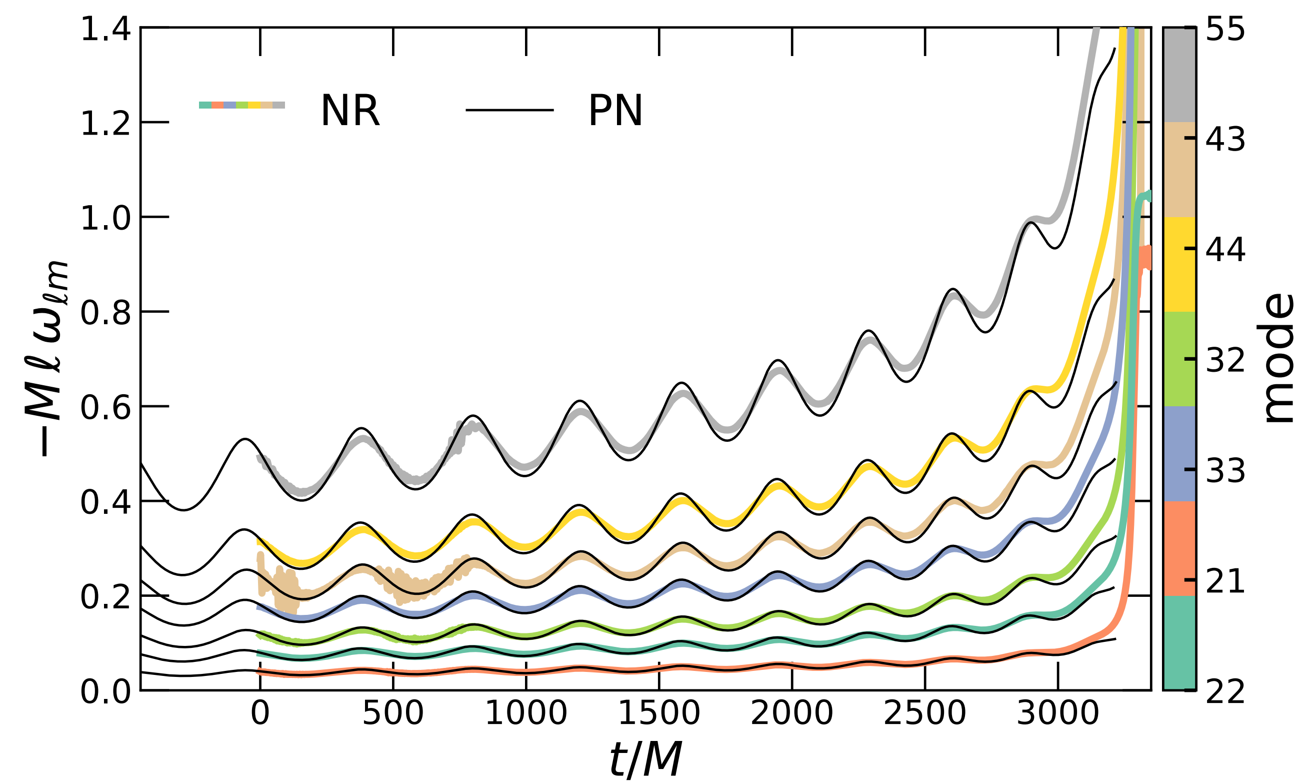


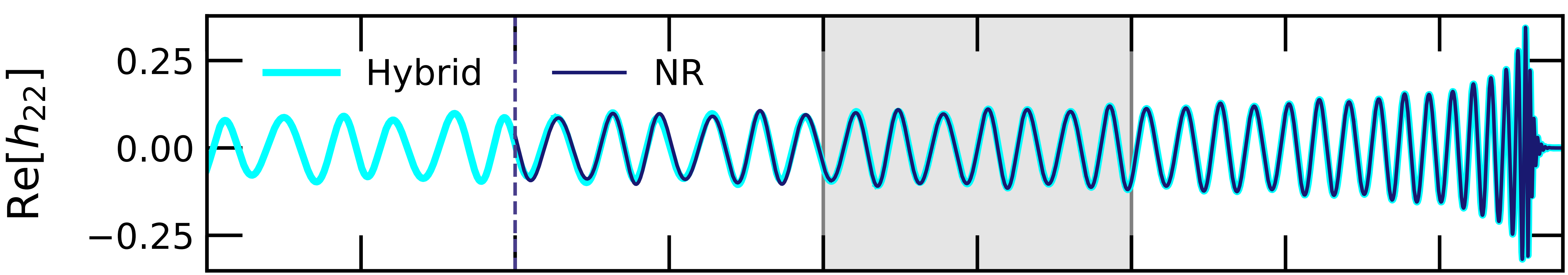
Coherent inputs



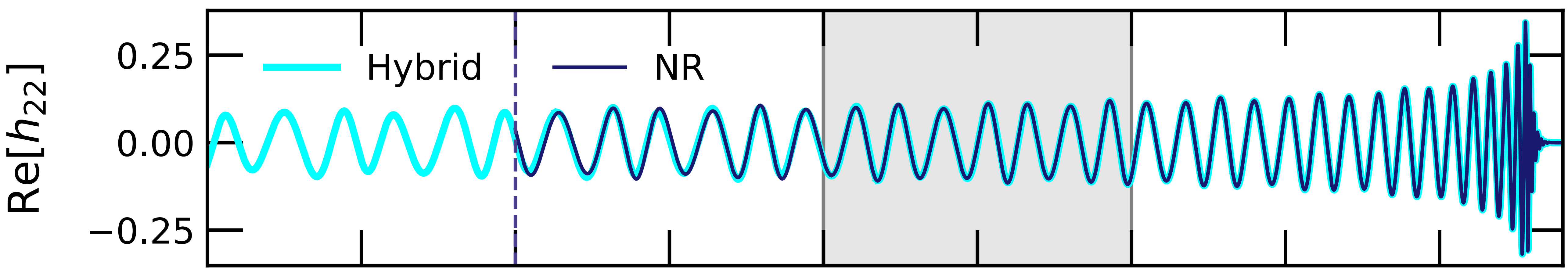
PN-NR comparisons







Construction of Target WF



[Chattaraj + (2022)]

$$\delta = \min_{t_0,\varphi_0,\psi} \int_{t_\mathrm{i}}^{t_\mathrm{f}} dt \sum_{\ell,m} \left| h_{\ell m}^{\mathrm{NR}}(t - t_0) e^{i(m\varphi_0 + \psi)} - h_{\ell m}^{\mathrm{PN}}(t) \right|$$

$$h_{\ell m}^{\mathrm{hyb}}(t) \equiv \tau(t)\, h_{\ell m}^{\mathrm{NR}}(t - t_0')\, e^{i(m\varphi_0' + \psi')} + (1 - \tau(t))\, h_{\ell m}^{\mathrm{PN}}(t)$$

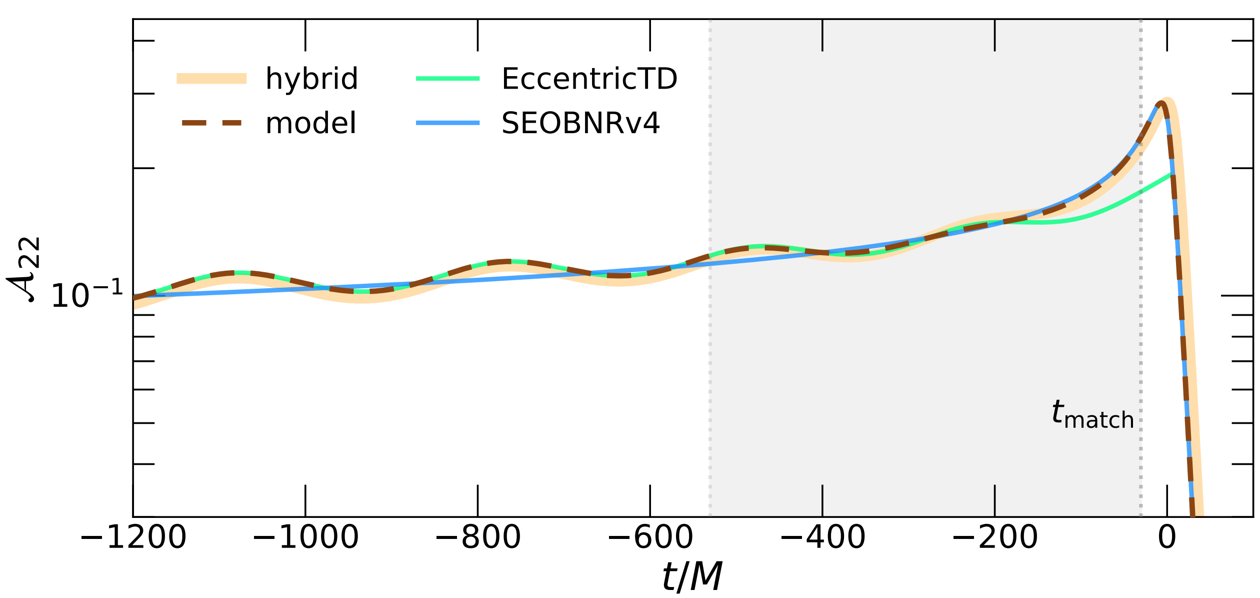
$$\tau(t) \equiv \begin{cases} 0 & \text{if } t < t_\mathrm{i} \\ \frac{t - t_\mathrm{i}}{t_\mathrm{f} - t_\mathrm{i}} & \text{if } t_\mathrm{i} \le t < t_\mathrm{f} \\ 1 & \text{if } t_\mathrm{f} \le t. \end{cases}$$

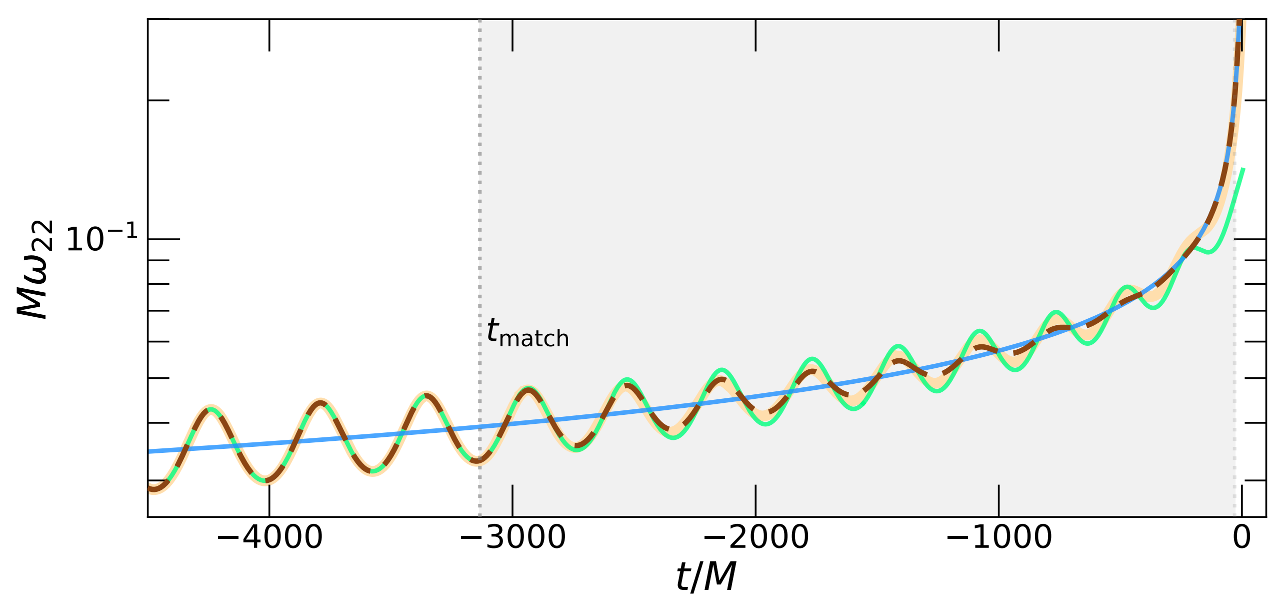
A quadrupole mode model

$$\mathcal{A}_{22}^{\text{model}}(t) \equiv \tau_{\text{a}}(t)\,\mathcal{A}_{22}^{\text{IMR}}(t) + (1-\tau_{\text{a}}(t))\,\mathcal{A}_{22}^{\text{inspiral}}(t)$$

$$\omega_{22}^{\text{model}}(t) \equiv \tau_{\text{a}}(t)\,\omega_{22}^{\text{IMR}}(t) + (1-\tau_{\text{a}}(t))\,\omega_{22}^{\text{inspiral}}(t)$$

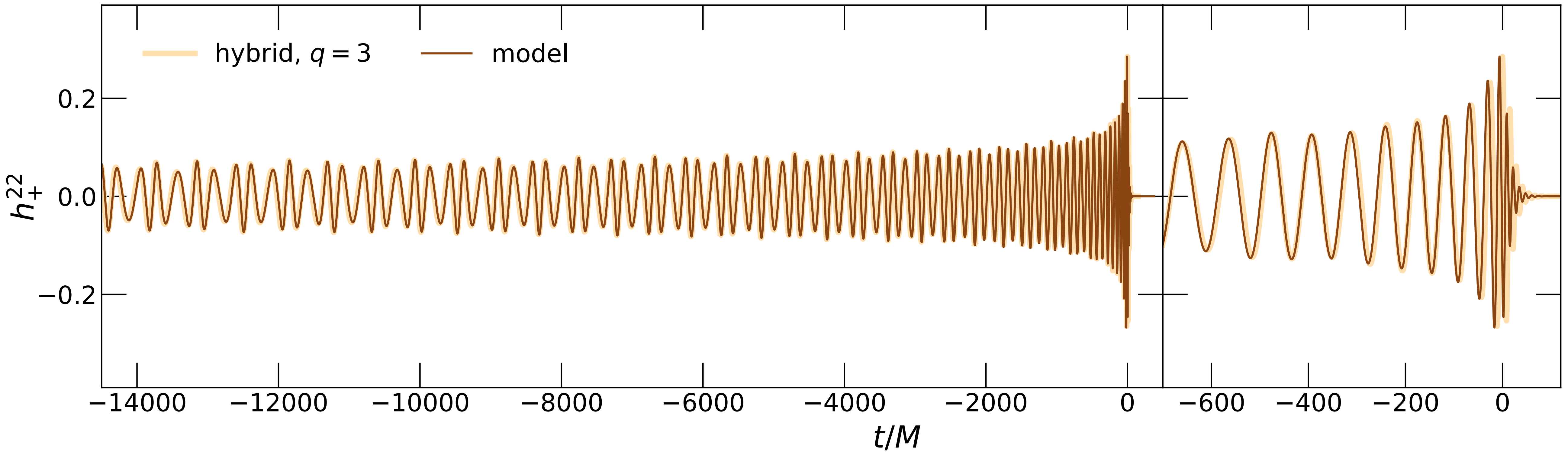
$$\tau_{\text{a}}(t) \equiv \begin{cases} 0 & \text{if } t < t_{\text{i}} \\ \frac{t-t_{\text{i}}}{t_{\text{f}}-t_{\text{i}}} & \text{if } t_{\text{i}} \leq t < t_{\text{f}} \\ 1 & \text{if } t_{\text{f}} \leq t. \end{cases}$$





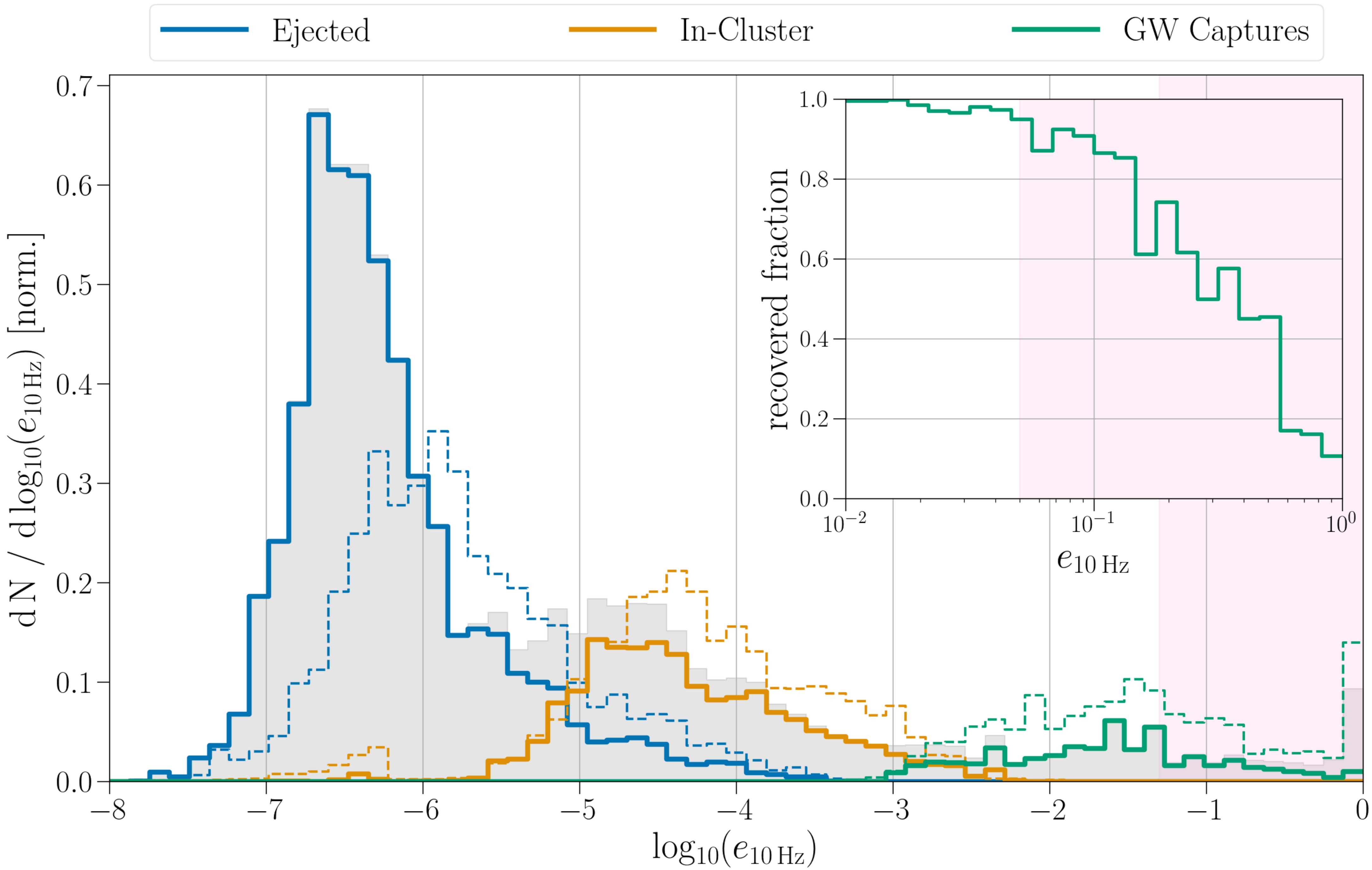
[Chattaraj + (2022)]

IMR construction



[Chattaraj + (2022)]

Eccentric Population



[Zevin et al., ApJL, 921 L43 (2021)]