Towards a weak-field hybrid 2PA waveform model

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About the model

1. **What do I mean by a “weak-field hybrid” model?**

2. **Will the model to be accurate?**

3. Modular and can assimilate continual improvement. Implemented in the Fast EMRI Waveforms (FEW) package.

4. Will* include approximated features such as orbital resonances.

Why?

- An updated generic EMRI model beyond 0PA suitable for LISA preparatory work. (See Ollie Burke's talk tomorrow morning).

- Approximate the 2PA truncation error on 1PA waveforms.

$$\Phi = \frac{1}{\epsilon}\Phi^{(0)} + \Phi^{(1)} + \epsilon\Phi^{(2)} + \mathcal{O}(\epsilon^2)$$

- Develop methods of mapping between generic self-force and PN results.

Basic Model Overview*

***For now**

PA order	0PA	1PA	2PA
Inspiral Composite	1SF + 1SF-5PN-e10	1SF + 2PN	2PN

Why 2PN? Generic ✓ Spin-Spin ✓ Analytic ✓

[Wardell et al 2023]
[Isoyama et al 2022]
[Klein 2021]

Inspiral Mapping

$$\dot{\Omega}_A = F_A(\Omega_A, \chi_i, M, \nu)$$

Want to map between fixed values of the frequencies.

$$\Omega_A(\mathcal{P}_B, \chi_i, M, \nu) = \sum_{n=0}^{N} \nu^n \Omega_A^{(n)}(\mathcal{P}_B, \chi_i, M) + \mathcal{O}(\nu^{N+1})$$

$$\mathcal{P}_B = \sum_{n=0}^{N} \nu^n \mathcal{P}_B^{(n)} + \mathcal{O}(\nu^{N+1}) \quad \Longrightarrow \quad \Omega_A^{(1)} + \mathcal{P}_B^{(1)} \partial_{\mathcal{P}_B} \Omega_A^{(0)} = 0$$

...

Piecing together

Similar composite technique as described yesterday in separate talks by Adam Pound and Lorenzo Kulcher.

$$\dot{\Omega}_A = \sum_{n=0}^{N} \nu^{n+1} F_A^{(n)}(\Omega_A, \chi_i, M) + \mathcal{O}(\nu^{N+1})$$

e.g.

$$F^{(1)} = F_{\rm SF}^{(1)} + F_{\rm 2PN}^{(1)} - (\text{2PN common terms})$$

Generic 2PN waveforms

→ **Step 1: Solve the conservative problem.**

- **Based off [Klein 2021]; model suitable for bayesian PE for stellar mass binary LISA sources.**
- **Eccentricity expansion and multi-scale approach.**

Step 2: Evolve quasi-adiabatically.

The 2PN precession equations

$$D\hat{\boldsymbol{L}} = -y^6\left(\boldsymbol{\Omega}_1 + \boldsymbol{\Omega}_2\right),$$

$$D\boldsymbol{\chi}_1 = \frac{\mu_2}{\mu_1} y^5 \boldsymbol{\Omega}_1,$$

$$D\boldsymbol{\chi}_2 = \frac{\mu_1}{\mu_2} y^5 \boldsymbol{\Omega}_2,$$

7 conserved Qs,
3 angular momenta,
2 dynamical variables.

- [Barker + O'Connell 1975]
- Orbit-averaged over radial librations (difference is higher PN order). [Racine 2008]

$$\boldsymbol{\Omega}_i = \frac{\mu_i}{2}\left[\mu_i + 3\left(1 - y\hat{\boldsymbol{L}}\cdot\boldsymbol{s}\right)\right]\hat{\boldsymbol{L}}\times\boldsymbol{\chi}_i + \frac{1}{2}y\nu\boldsymbol{\chi}_j\times\boldsymbol{\chi}_i,$$

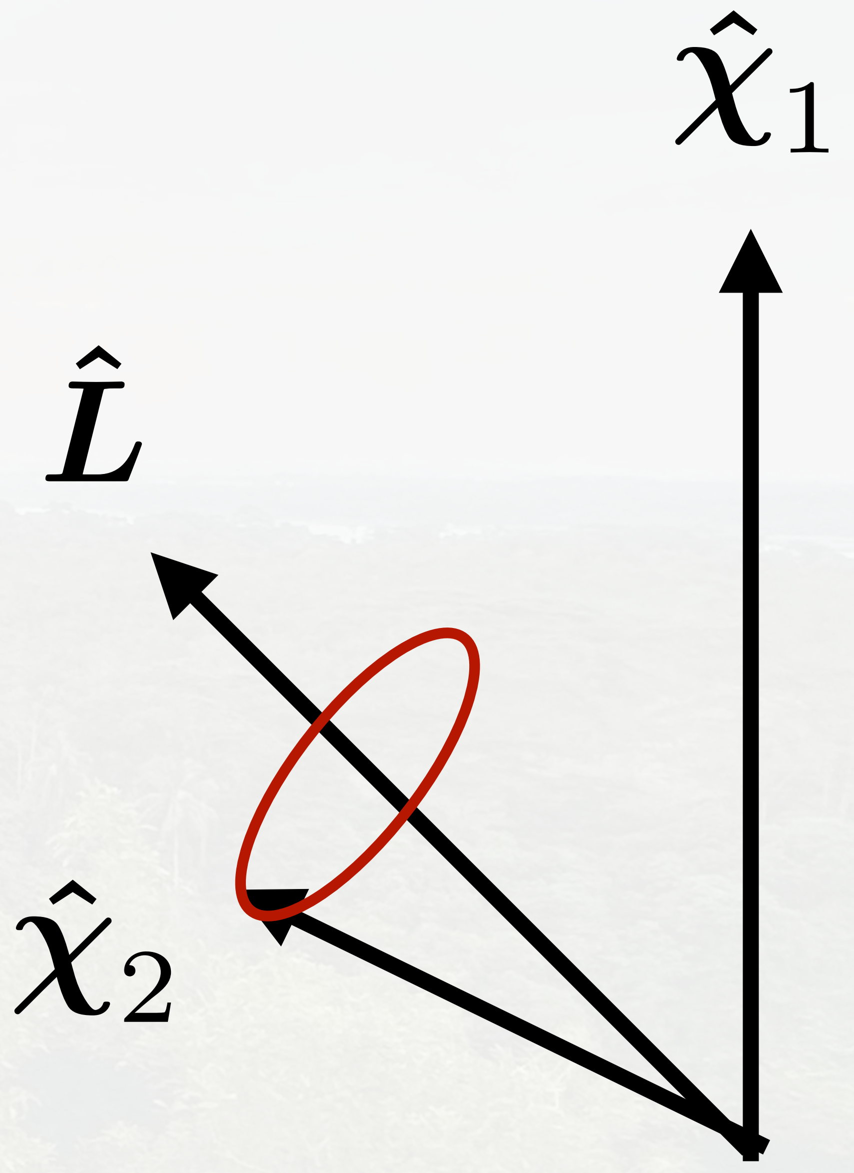
Solving the 2PN precession problem I

Co-precessing frame - only 1 ‘master’ dynamical variable.

$$(Dx)^2 = -\omega_x^2(Ax^3 + Bx^2 + Cx + D)$$

$$x = x_1 + (x_2 - x_1)\mathrm{sn}^2\left(\omega_x\sqrt{A(x_1 - x_3)}, \frac{x_1 - x_2}{x_1 - x_3}\right)$$

As in [Chatziioannou et al 2017, Klein 2021]



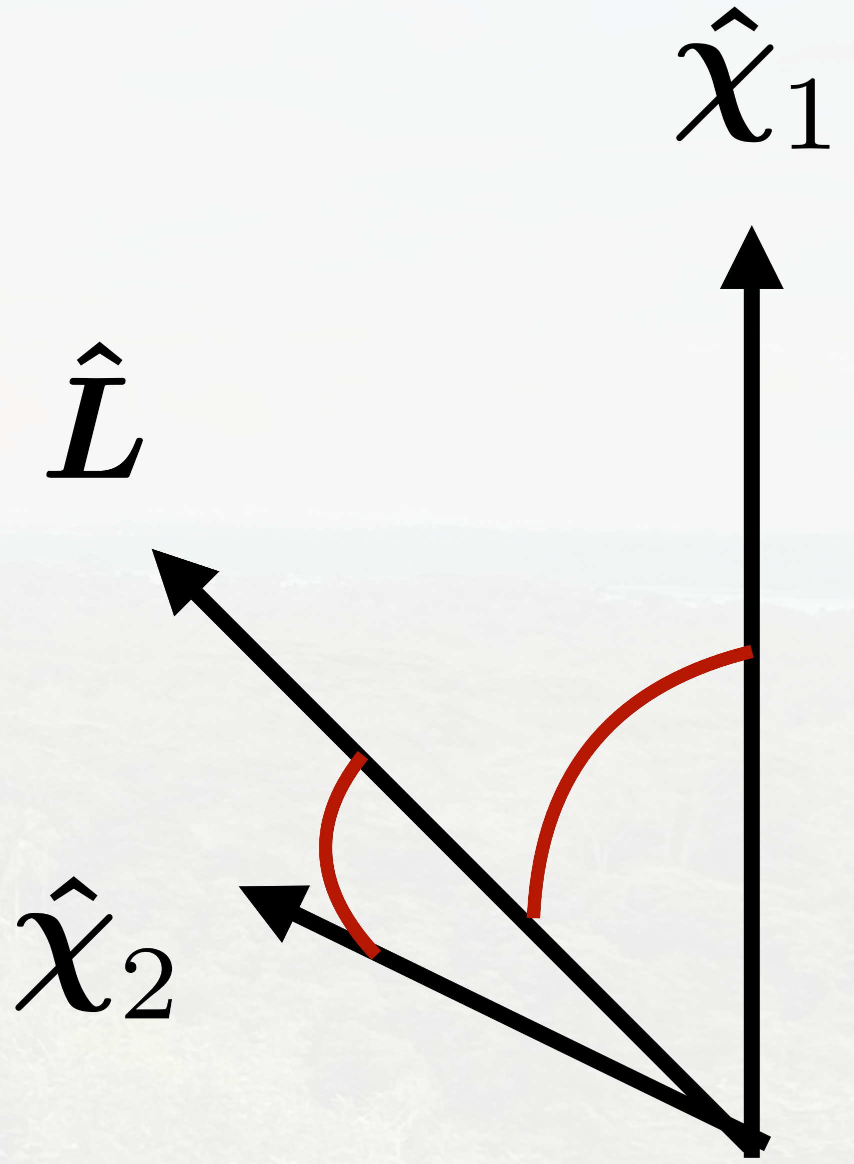
Our approach: pick a dummy quantity such that:

$$(Dx)^2 = -\omega_x^2(Ax^3 + Bx^2 + Cx + \not{D})$$

Solving the 2PN precession problem II

Intuition from the MPD equations:

$$\boldsymbol{\chi}_2 = \chi_2 \cos(\theta_s)\hat{\boldsymbol{L}} + \chi_2 \sin(\theta_s)\left(\cos\psi_s \hat{\boldsymbol{x}}_L + \sin\psi_s \hat{\boldsymbol{y}}_L\right)$$



2PN conserved quantities imply:

$$\cos\theta_\chi = X_\chi + \delta\mu x,$$
$$\cos\theta_N = X_N - y\mu_2\chi_2 x,$$
$$\cos\theta_s = X_s + y\mu_1\chi_1 x,$$

2PN precession equations and vector geometry:

$$\mathcal{D}x = \omega_x \sin\theta_N \sin\theta_s \sin\psi_s$$

$$\downarrow$$

$$(\mathcal{D}x)^2 = -\omega_x^2 x\left(Ax^2 + Bx + C\right)$$

Solution:

$$\cos\theta_\chi = X_\chi + (\delta\mu r_2)\mathrm{sn}^2(\frac{1}{2}\bar{\omega}_x\lambda, m),$$
$$\cos\theta_N = X_N - \mu_2\chi_2(yr_2)\mathrm{sn}^2(\frac{1}{2}\bar{\omega}_x\lambda, m),$$
$$\cos\theta_s = X_s + \mu_1\chi_1(yr_2)\mathrm{sn}^2(\frac{1}{2}\bar{\omega}_x\lambda, m).$$

Future Work/Scope

- Finish piecing together the model!
- A follow up parameter inference study.
- Replace adiabatic term with full generic 1SF results once available.
- Extend the 2PN 2PA part to include BH-NS binaries?