The Effective Field Theory Approaches To The Calculations Of Post-Newtonian Two-body Inspirals

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NBI Joint Theory Seminar

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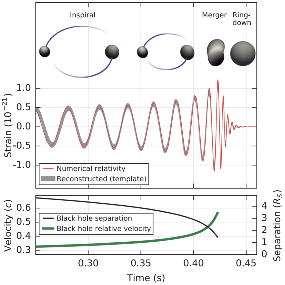
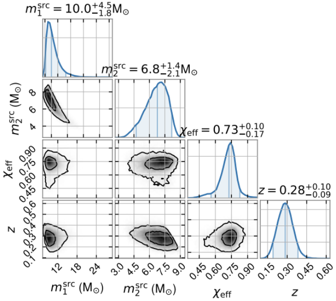
Overview of Post-Newtionian Binary Inspiral

Compact Binary Inspirals



GW150914, Abbott et al. PRL 116 (2016) 061102

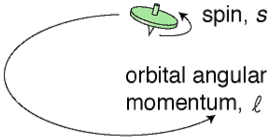


GW190718 160159 with a large positive effective spin at high confidence arXiv:2201.02252 [astro-ph.HE]

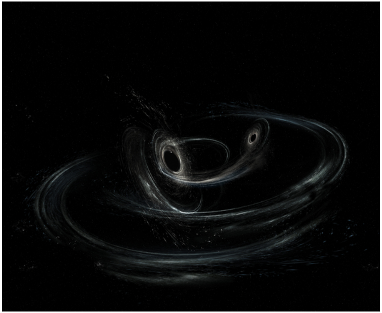
- The dynamical evolution of compact binaries has been the main cause of the gravitational waves(GW).
- The GWs produced by the inspiral, merger, and ring-down from the coalescence will carry vast amounts of information.
- During the inspiral phase the binary orbits slowly around each other and radiates GWs.

Spinning Kerr Black Holes

- Classical rotating angular momentum carried by the black holes. Total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$



- Spin can lead to significant corrections to the orbital motion of the compact binary, which results in the modulations on the gravitational-wave signal.



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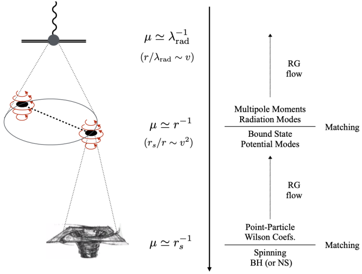
The Physical Picture and Scales in Inspiral

- The internal zone $r_s \sim 2GM/c^2$: the size of the black holes or neutron stars.
- The near / potential zone r: the binary orbital radius.
- The far / radiation zone $\lambda \sim \frac{r}{v}$: the typical wavelength for the gravitational waves.



The physical picture in the inspiral phase and the EFT scalings with $r_s \ll r \ll \lambda$

Post-Newtonian Expansions

The source with the total mass M is:

- slowly moving $\Rightarrow v/c \ll 1$,
- weak gravitational fields $\Rightarrow GM/rc^2 \ll 1$.

v is the typical binary orbital velocity

$$\text{Virial theorem } \frac{GM}{r} \sim v^2$$

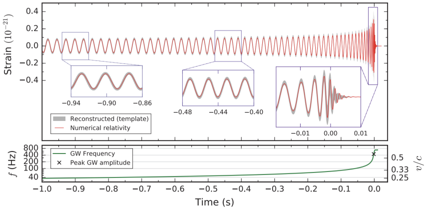
We refer the post-Newtonian terms with $\mathcal{O}\left(\frac{v^n}{c^n}\right)$ relative to the Newtonian acceleration in the equations of motion as $\frac{n}{2}$ PN.

$$\mathbf{a}^i = \mathbf{a}^i_{\text{Newtonian}} + \mathbf{a}^i_{1\text{PN}} + \mathbf{a}^i_{2\text{PN}} + \mathbf{a}^i_{\text{RR}} + ...,$$

where $\mathbf{a}^i_{\text{RR}}$ is the radiation reaction starting from 2.5PN.

Motivations in Higher Order Post-Newtonian Expansions

- During late inspirals v/c get considerably large
- Necessary to include high PN terms in the expansion approximations



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L. Blanchet, G. Faye and F. Larrouturou, "The quadrupole moment of compact binaries to the fourth post-Newtonian order: from source to canonical moment [arXiv:2204.11293]

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S. Marsat, A. Bohe, G. Faye and L. Blanchet, "Next-to-next-to-leading order spin-orbit effects in the equations of motion of compact binary systems," Class. Quant. Grav.[arXiv:1210.4143]

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Introduction To The Worldline EFT Framework

Non-Relativistic General Relativity and The Effective Field Theory Formalism

An action of the relativistic point particles coupled with gravity

$$S_{\text{eff}}[x^\mu, g_{\mu\nu}] = S_{EH}[g] + S_{pp}[x, g],$$

with the Einstein-Hilbert action

$$S_{EH} = -2m_{Pl}^2 \int \mathrm{d}^4x \sqrt{g} R(x),$$

where ${m_{pl}}^{-2} = 32\pi G_N$ and $R(x)$ is the Ricci scalar.

The point particle action

$$S_{pp} = -\sum_a m_a \int \mathrm{d}\tau_a + ...,$$

where $\mathrm{d}\tau_a^2 = g_{\mu\nu} x_a^\mu x_a^\nu$ and the parametrized worldline $x_a^\mu(\tau_a)$.

Potential Gravitons and Radiation Gravitons

Linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{m_{Pl}}.$$

Split the graviton into potential and radiation modes

$$h_{\mu\nu} = H_{\mu\nu} + \bar{h}_{\mu\nu}.$$

- Potential gravitons $H_{\mu\nu}$
  - scale as $\left(\partial_0 H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}, \partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu}\right)$
  - never on-shell
  - mediate the instantaneous exchanges between the point particles
- Radiation gravitons $\bar{h}_{\mu\nu}$
  - scale as $\left(\partial_\alpha \bar{h}_{\mu\nu} \sim \frac{v}{r} \bar{h}_{\mu\nu}\right)$
  - can appear on-shell
  - propagate to the observers and exist in the physics at long distance

Integrating Out The Potential Mode

The non-relativistic effective action integrating out the potential gravitons

$$e^{iS_{\rm NR}[x_a,\bar{h}]} = \int \mathcal{D}H e^{iS_{EH}[\bar{h}+H]} e^{iS_{pp}[x,\bar{h}+H]} e^{iS_{\rm GF}}.$$

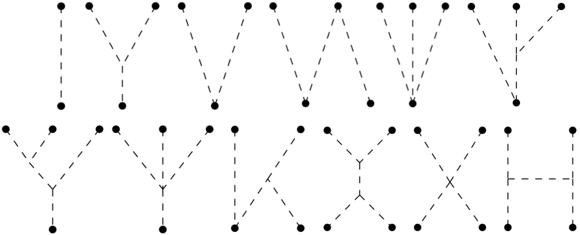
where $S_{\rm GF}[H,\bar{h}]$ is a gauge fixing term.

$$S_{\rm NR}[x_a,\bar{h}] = S_0[x_a] + S_1[x_a,\bar{h}] + \mathcal{O}(\bar{h}^2)$$

$S_0[x_a]$ → no radiation

$S_1[x_a,\bar{h}]$ → one radiation graviton $\bar{h}$

$$L = L_{\rm N} + L_{\rm 1PN} + L_{\rm 2PN} + L_{\rm 3PN} + ...$$



Topologies needed for the potential at NNLO order

Worldline couplings $L_{pp} = \sum_a \frac{m_a}{m_{Pl}} \Big[-\frac{1}{2} H^{00} - \frac{1}{2} \bar{h}^{00} - \frac{1}{2} H_{0i} \mathbf{v}_{ai} - \frac{1}{2} \bar{h}_{0i} \mathbf{v}_{ai} - \frac{1}{4} H^{00} \mathbf{v}_a^2 - \frac{1}{4} \bar{h}^{00} \mathbf{v}_a^2 - \frac{1}{2} H_{ij} \mathbf{v}_{ai} \mathbf{v}_{aj} - \frac{1}{2} \bar{h}_{ij} \mathbf{v}_{ai} \mathbf{v}_{aj} + \dots \Big]$

Potential graviton propagator $\langle H_{\mathbf{k}\mu\nu}(x^0) H_{\mathbf{q}\alpha\beta}(y^0) \rangle = -\frac{i}{\mathbf{k^2}} (2\pi)^3 \delta^3(\mathbf{k}+\mathbf{q}) \delta(x^0 - y^0) P_{\mu\nu;\alpha\beta}$

Three-graviton vertex $\langle H^{00}_{\mathbf{q_1}} H^{00}_{\mathbf{q_2}} H^{00}_{\mathbf{q_3}} \rangle = \left(\frac{i}{2m_{Pl}} \right) \left(\mathbf{q}_1^2 + \mathbf{q}_1 \cdot \mathbf{q}_2 + \mathbf{q}_2^2 \right).$

$$P_{\mu\nu;\alpha\beta} = \frac{1}{2} (\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \frac{2}{d-2} \eta_{\mu\nu}\eta_{\alpha\beta}).$$

Spinning Extended Objects

Anti-symmetric spin tensor $S^{\alpha\beta}$ describing the rotational degrees of freedom

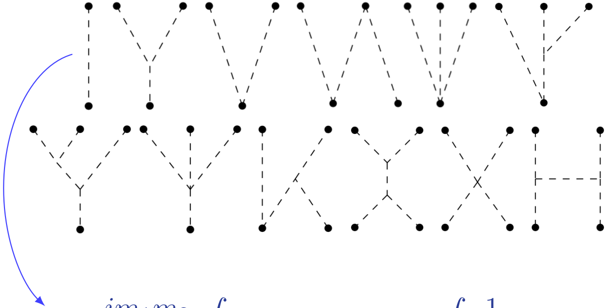
- is constrained by the covariant spin supplementary condition (SSC): $S^{\alpha\beta}p_\beta = 0$.
- projected onto the locally-flat frame $S^{ab} \equiv S^{\mu\nu} e^a_\mu e^b_\nu$, with the co-rotating tetrad e^a_μ satisfying $g^{\mu\nu} e^a_\mu e^b_\nu = \eta^{ab}$.
- S^{ab} obeys $\{S^{ab}, S^{cd}\} = \eta^{ac} S^{bd} + \eta^{bd} S^{ac} - \eta^{ad} S^{bc} - \eta^{bc} S^{ad}$.
- Spin four-vector is defined as $S^\mu_A \equiv \dfrac{1}{2m_A} \epsilon^\mu{}_{\nu\alpha\beta} S^{\alpha\beta}_A p^\nu_A$.

The point-particle worldline action is extended to $S_{\rm pp} \equiv \int_{-\infty}^{+\infty} d\tau \mathcal{R}$, where the Routhian $\mathcal{R}$ is given by

$$\mathcal{R} = -\frac{1}{2} \sum_{n=1,2} \Bigg(m_n \sqrt{g_{\mu\nu} v^\mu_n v^\nu_n} + \omega^{ab}_\mu S_{nab} v^\mu_n - \frac{C^{(n)}_{\rm ES^2}}{m_n} \frac{E_{ab} S^{ac}_n S_{nc}{}^b}{\sqrt{v^\mu_n v_{n\mu}}} + \frac{1}{m_n} R_{deab} S^{ab}_n S^{cd}_n v^e_n v_{nc} + \cdots \Bigg)$$

The last two terms account for the conservation of the SSC and finite-size effects to quadratic order in the spins.

From Feynman Diagrams To Potentials

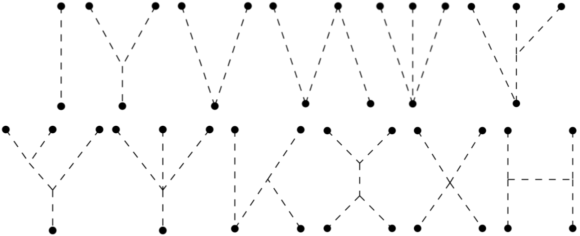


$$\text{diag } 1 = \frac{im_1 m_2}{8m_{Pl}^2} \int dt_1 dt_2 \delta\left(t_1 - t_2\right) \int_{\mathbf{k}} \frac{1}{\mathbf{k}^2} e^{-i\mathbf{k}\cdot(\mathbf{x}_1 - \mathbf{x}_2)}$$

$$= i \int dt \frac{G_N m_1 m_2}{|\mathbf{x}_1(t) - \mathbf{x}_2(t)|}$$

The exchange of a single H^{00} potential graviton between two particles leads to the Newtonian Potential.

Spinning Equations Of Motion



Same diagrams as non-spinning, but with spin insertions and finite size effects

The spin precession equations $\frac{dS^{ab}}{dt} = \{V, S^{ab}\} = 4S^{c\{a}\eta^{b\}d}\frac{\partial}{\partial S^{cd}}V$ describe the time evolution of the spin. At the leading order,

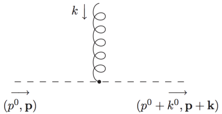
$$\frac{d\boldsymbol{S}_1}{dt} = \left(2 + \frac{3m_2}{2m_1}\right)\frac{\mu_m G_N}{r^2}(\hat{\boldsymbol{n}} \times \boldsymbol{v}) \times \boldsymbol{S}_1$$

where $\boldsymbol{S}_i = \frac{1}{2}\epsilon_{ijk}S^{jk}$ is the spin vector.

Comparing With Traditional PN method

Traditional Post-Newtionian Method	Worldline EFT Framework
Calculations in position space	Performed in momentum space
Iterative using metrics	Feynman Diagrams
Obtaining Hamiltionian first	Directly to Lagrangian and potential
Mixing PN orders	Clean power counting

Multipole Moments And Radiational Flux



$\mathbf{p} \sim 1/r$ and $\mathbf{k} \sim v/r$

The propagator of the outgoing potential graviton with momentum $\mathbf{p} + \mathbf{k}$ expanded in powers of v

$$\frac{1}{(\mathbf{p}+\mathbf{k})^2} = \frac{1}{\mathbf{p}^2}\left(1 - \frac{2\mathbf{p}\cdot\mathbf{k}}{\mathbf{p}^2} - \frac{\mathbf{k}^2}{\mathbf{p}^2} + \frac{4(\mathbf{p}\cdot\mathbf{k})^2}{\mathbf{p}^4} + \ldots\right),$$

$\Rightarrow$ Manifest power counting

Solution: Multipole Expansion

The source term of the single radiation graviton in the effective action is

$$S_1[x_a, \bar{h}] = -\frac{1}{2m_{Pl}} \int \mathrm{d}^4x\, T^{\mu\nu}(x)\bar{h}_{\mu\nu}(x)$$
$$= -\frac{1}{2m_{Pl}} \int \mathrm{d}t \sum_{n=0}^{\infty} \frac{1}{n!} \int \mathrm{d}^3\mathbf{x}\, T^{\mu\nu}(t, \mathbf{x}) x^N (\partial_N \bar{h}_{\mu\nu})(t, \mathbf{0})$$

where $x^N = (x^{i_1} x^{i_2} \ldots x^{i_n})$.

Gravitational Radiation And Multipole Expansion

Express the Taylor expanded source action in terms of manifestly gauge invariant operators ⇒ Riemann tensor $R_{\mu\nu\rho\sigma}$

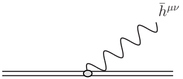
$$S_{\rm NR}[x_a, \bar{h}] = -\frac{1}{2m_{\rm Pl}} \int \mathrm{d}t \left[M\bar{h}_{00} + 2\mathbf{P}^i \bar{h}_{0i} + \mathbf{L}^i \epsilon_{ijk}\partial_j \bar{h}_{0k}\right] + \sum_{\ell=2} \left(\frac{1}{\ell!} I^L(t)\, \nabla_{L-2} E_{i_{\ell-1} i_\ell} - \frac{2\ell}{(\ell+1)!} J^L(t)\, \nabla_{L-2} B_{i_{\ell-1} i_\ell} \right).$$

mass type and current type multipole moments.

E_{ij} and B_{ij} are the electric and magnetic components of the Riemann tensor.

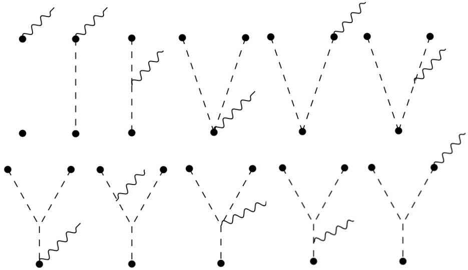
$$E_{ij} = \frac{1}{2m_{Pl}} \left(\partial_0 \partial_j \bar{h}_{0i} + \partial_0 \partial_i \bar{h}_{0j} - \partial_i \partial_j \bar{h}_{00} - \partial_0^2 \bar{h}_{ij} \right) + \mathcal{O}(\bar{h}^2)$$

$$B_{ij} = \frac{1}{2m_{Pl}} \epsilon_{imn} \left(\partial_0 \partial_n \bar{h}_{jm} + \partial_j \partial_m \bar{h}_{0n} \right) + \mathcal{O}(\bar{h}^2).$$

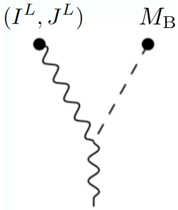


The effective action for the binary system is replaced by a series of multipole moments on the worldline interacting with the radiation fields.

Radiation In Feynman Diagrams



Topologies needed to match the multipole moments entering the radiated flux to NNLO



Tail coupling between the binarys mass monopole and other source moments.

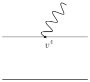
The dashed lines are potential modes sourced by the particles, while the wavy line represents the radiation field.

From Feynman Diagrams To Multipole Moments

From multipole expansion matching,

$$I^{ij} = \left[\int d^3\mathbf{x} T^{00}\mathbf{x}^i\mathbf{x}^j\right]_{\text{TF}} + \left[\int d^3\mathbf{x} T^{ll}\mathbf{x}^i\mathbf{x}^j\right]_{\text{TF}} - \frac{4}{3}\left[\int d^3\mathbf{x}\partial_0 T^{0l}\mathbf{x}^l\mathbf{x}^i\mathbf{x}^j\right]_{\text{TF}}$$
$$+ \frac{1}{6}\left[\int d^3\mathbf{x}\partial_0^2 T^{kl}\mathbf{x}^k\mathbf{x}^l\mathbf{x}^i\mathbf{x}^j\right]_{\text{TF}} + \frac{11}{42}\left[\int d^3\mathbf{x}\partial_0^2 T^{00}\mathbf{x}^2\mathbf{x}^i\mathbf{x}^j\right]_{\text{TF}} + \ldots$$

The pseudo stress-energy tensor $T^{\mu\nu}$ is read off from the gravitational amplitude $i\mathcal{A}_h(t,\mathbf{k}) = -\frac{i}{2m_{Pl}}\epsilon^*_{\mu\nu}(\mathbf{k},h)T^{\mu\nu}(t,\mathbf{k})$, which is in mixed Fourier space.



$$= -\frac{i}{2m_{\text{Pl}}}\epsilon^{00}\sum_a \frac{3}{8}m_a\mathbf{v}_a^4 e^{-i\mathbf{k}\cdot\mathbf{x}_a} \Rightarrow T^{00}(t,\mathbf{k}) = \sum_a \frac{3}{8}m_a\mathbf{v}_a^4 e^{-i\mathbf{k}\cdot\mathbf{x}_a}.$$

$$I^{ij} = \int d^3\mathbf{x} T^{00}(t,\mathbf{x})\left[\mathbf{x}^i\mathbf{x}^j\right]_{TF} = \sum_a \frac{3}{8}m_a\mathbf{v}_a^4\left[\mathbf{x}_a^i\mathbf{x}_a^j\right]_{TF}$$

which is a 2PN contribution piece.

Multipole Expansion In D-dimension

Multipole expansion at the level of the action with uniform power counting.

Needed d-dim multipole moments starting from 3PN Mass Quad, due to the IR divergence proportional to $\frac{1}{d-4}$ in $T^{\mu\nu}$ showing in 2-loop diagrams.

The new multipole moments are now organized in irreducible representations of the rotation group SO(N)

$$S[\bar{h}] = \sum_{\ell=2} \left(\frac{1}{\ell!} I^L(t) \nabla_{L-2} E_{i_{\ell-1} i_\ell} + J^{b|L}(t) \nabla_{L-2} B_{i_{\ell-1}|b i_\ell} + J^{ab|L} C_{a i_{\ell-1} b i_\ell} \right).$$

where $B_{i|jk} = R_{0ijk}$ and the $C_{ijk\ell}$ is the trace-free Weyl tensor.

Spin Power Countings In The Flux

The leading order nonspinning multipoles are given by

$$I^{ij} = \sum_a m_a \left[\boldsymbol{x}_a^i \boldsymbol{x}_a^j\right]_{\mathrm{TF}} + \mathcal{O}\left(v^2\right)$$

$$I^{ijk} = \sum_a m_a \left[\boldsymbol{x}_a^i \boldsymbol{x}_a^j \boldsymbol{x}_a^k\right]_{\mathrm{TF}} + \mathcal{O}\left(v^2\right),$$

$$J^{ij} = \sum_a m_a \left[\left(\boldsymbol{x}_a \times \boldsymbol{v}_a\right)^i \boldsymbol{x}_a^j\right]_{\mathrm{STF}} + \mathcal{O}\left(v^3\right).$$

For maximally rotating bodies, spin scales as $S = I_S \Omega \sim I_S \frac{v_{\mathrm{rot}}}{r_s} \sim m v_{rot} r_s \sim L v$.
The NLO spin-orbit and spin-spin potentials enter at 2.5PN and 3PN, respectively.

	$\mathcal{O}(\not{\boldsymbol{S}})$	$\mathcal{O}\left(\boldsymbol{S}_a\right)$	$\mathcal{O}\left(\boldsymbol{S}_a^2\right)$
K_ℓ^{00}	mr^ℓ	$mr^\ell v^3$	$mr^\ell v^4$
K_ℓ^{0i}	$mr^\ell v$	$mr^\ell v^2$	$mr^\ell v^5$
K_ℓ^{ij}	$mr^\ell v^2$	$mr^\ell v^3$	$mr^\ell v^6$

Spin-orbit effects in the energy flux at LO arise both from mass and current quadrupole radiation, and yield 1.5PN correction. Spin-spin contributions to the energy flux first appear at 2PN order, from the current quadrupole.

Power Loss And Physical Quantities

$$\text{Conserved energy } E = \int d^3\mathbf{x} T^{00}(x)$$

$$\text{Center of mass position } \mathbf{G} = \int d^3\mathbf{x} T^{00}(x)\,\mathbf{x}$$

$$\text{Linear momentum } \mathbf{P} = \int d^3\mathbf{x} T^{0l}_{1PN}(x)$$

$$\text{Angular momentum } \mathbf{L}^i = -\epsilon^{ilk} \int d^3\mathbf{x} T^{0l} \mathbf{x}^k$$

Total radiated power

$$P = -\frac{G}{5}\Big[I^{(3)}_{ij} I^{(3)}_{ij} - \frac{5}{189} I^{(4)}_{ijk} I^{(4)}_{ijk} + \frac{5}{9072} I^{(5)}_{ijkl} I^{(5)}_{ijkl} + \frac{16}{9} J^{(3)}_{ij} J^{(3)}_{ij} - \frac{5}{84} J^{(4)}_{ijk} J^{(4)}_{ijk} + \ldots\Big].$$

The evolution of the orbital frequency and the change in the orbital phase $\phi = \int \Omega(t)dt$ for the aligned-spin with circular orbits .

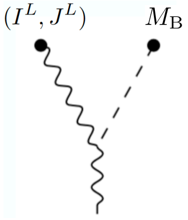
LISA	$10^6 M_\odot + 10 M_\odot$	$10^5 M_\odot + 10 M_\odot$	$10^4 M_\odot + 10^4 M_\odot$
3.5PN	$224615.\hat{S}_\ell + (47903.6 + 23951.8\kappa_+)\hat{S}_\ell^2 + 67352.3\hat{\Sigma}_\ell + (47902.6 - 23951.8\kappa_- + 23951.3\kappa_+)\hat{S}_\ell\hat{\Sigma}_\ell + (448.6 - 11975.6\kappa_- + 11975.6\kappa_+)\hat{\Sigma}_\ell^2$	$24002.2\hat{S}_\ell + (5119.3 + 2559.6\kappa_+)\hat{S}_\ell^2 + 7195.6\hat{\Sigma}_\ell + (5118.2 - 2559.6\kappa_- + 2559.1\kappa_+)\hat{S}_\ell\hat{\Sigma}_\ell + (47.5 - 1279.6\kappa_- + 1279.6\kappa_+)\hat{\Sigma}_\ell^2$	$4.7\hat{S}_\ell + (1.2 + 0.6\kappa_+)\hat{S}_\ell^2 - 0.6\kappa_-\hat{S}_\ell\hat{\Sigma}_\ell + (-0.3 + 0.2\kappa_+)\hat{\Sigma}_\ell^2$
4PN	$-164123.\hat{S}_\ell + (76034.4 - 8979.\kappa_- - 25119.6\kappa_+)\hat{S}_\ell^2 - 55624.3\hat{\Sigma}_\ell + (60639.1 + 16140.8\kappa_- - 16140.1\kappa_+)\hat{S}_\ell\hat{\Sigma}_\ell + (24160.6 + 8070.1\kappa_- - 8070.1\kappa_+)\hat{\Sigma}_\ell^2$	$-16758.5\hat{S}_\ell + (7764.3 - 916.7\kappa_- - 2565.\kappa_+)\hat{S}_\ell^2 - 5678.7\hat{\Sigma}_\ell + (6191.1 + 1648.5\kappa_- - 1647.8\kappa_+)\hat{S}_\ell\hat{\Sigma}_\ell + (2466.7 + 824.\kappa_- - 824.\kappa_+)\hat{\Sigma}_\ell^2$	$-2.8\hat{S}_\ell + (1.5 - 0.4\kappa_+)\hat{S}_\ell^2 + 0.4\kappa_-\hat{S}_\ell\hat{\Sigma}_\ell + (0.2 - 0.1\kappa_+)\hat{\Sigma}_\ell^2$
ET	$100\ M_\odot + 1.4\ M_\odot$	$10 M_\odot + 10 M_\odot$	$10 M_\odot + 1.4 M_\odot$
3.5PN	$163.2\hat{S}_\ell + (35.2 + 17.6\kappa_+)\hat{S}_\ell^2 + 47.3\hat{\Sigma}_\ell + (34.2 - 17.6\kappa_- + 17.1\kappa_+)\hat{S}_\ell\hat{\Sigma}_\ell + (-0.1 - 8.5\kappa_- + 8.5\kappa_+)\hat{\Sigma}_\ell^2$	$7.7\hat{S}_\ell + (2. + \kappa_+)\hat{S}_\ell^2 - \kappa_-\hat{S}_\ell\hat{\Sigma}_\ell + (-0.5 + 0.2\kappa_+)\hat{\Sigma}_\ell^2$	$20.5\hat{S}_\ell + (4.7 + 2.4\kappa_+)\hat{S}_\ell^2 + 4.4\hat{\Sigma}_\ell + (3.6 - 2.4\kappa_- + 1.8\kappa_+)\hat{S}_\ell\hat{\Sigma}_\ell + (-0.5 - 0.9\kappa_- + 0.9\kappa_+)\hat{\Sigma}_\ell^2$
4PN	$-119.9\hat{S}_\ell + (56 - 6.4\kappa_- - 18.4\kappa_+)\hat{S}_\ell^2 - 39.4\hat{\Sigma}_\ell + (43.5 + 12.2\kappa_- - 11.5\kappa_+)\hat{S}_\ell\hat{\Sigma}_\ell + (17.3 + 5.8\kappa_- - 5.8\kappa_+)\hat{\Sigma}_\ell^2$	$-6.1\hat{S}_\ell + (3.3 - \kappa_+)\hat{S}_\ell^2 + \kappa_-\hat{S}_\ell\hat{\Sigma}_\ell + (0.5 - 0.2\kappa_+)\hat{\Sigma}_\ell^2$	$-15.\hat{S}_\ell + (7.4 - 0.6\kappa_- - 2.4\kappa_+)\hat{S}_\ell^2 - 3.8\hat{\Sigma}_\ell + (4.5 + 1.9\kappa_- - 1.1\kappa_+)\hat{S}_\ell\hat{\Sigma}_\ell + (1.8 + 0.6\kappa_- - 0.7\kappa_+)\hat{\Sigma}_\ell^2$

Estimation of the number of GW cycles in future detectors bands operating at design-sensitivity.

Time Non-Locality

The tail effect: the non-local in time back-scattering of emitted gravitational waves, modifying the conservative dynamics of the system at the current time.

Memory effect: the non-linear self-interactions of the radiation field.



IR divergence: cancelation of IR singularities occurs to all order
UV divergence: counter-terms and the renormalization group flow

The coupling $M \times I_{ij} \times I_{ij}$ tails-of-memory, $(M \times I_{ij})^2$ tails-of-tails and $(M \times I_{ij})^3$ tails-of-tails-tails at 4PN.

Gravitational Radiation Reaction

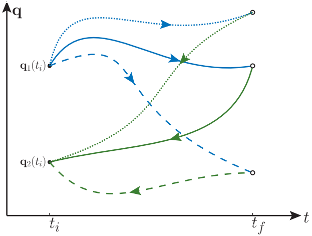
Classical Mechanics For Non-Conservative Systems

Dissipative dynamics such as radiation effects

- Only initial conditions specified
- Final conditions determined by the evolution

$\Rightarrow$ Hamilton's variation principle does not apply

$\Rightarrow$ Classical mechanics for non-conservative systems by introducing a double copy of the coordinate set.



System with coordinates $\mathbf{q}(\mathbf{t}) = \{q^I(t)\}_{I=1}^N$ and velocities $\dot{\mathbf{q}}(t) = \{\dot{q}^I(t)\}_{I=1}^N$:

$$q^I(t) \to \left(q_1^I(t), q_2^I(t)\right),$$

$$S[\mathbf{q}] = \int_{t_i}^{t_f} \mathrm{d}t L(\mathbf{q}, \dot{\mathbf{q}}, t) \to \int_{t_i}^{t_f} \mathrm{d}t \left[L(\mathbf{q}_1, \dot{\mathbf{q}}_1, t) - L(\mathbf{q}_2, \dot{\mathbf{q}}_2, t) + K(\mathbf{q}_1, \mathbf{q}_2, \dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2, t)\right].$$

The generalized non-conservative force

$$Q_I(q^J, \dot{q}^J, t) \equiv \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_-^I} - \frac{\partial L}{\partial q_-^I} = \left[\frac{\partial K}{\partial q_-^I} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial K}{\partial \dot{q}_-^I}\right]_{PL}.$$

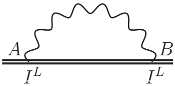
Physical Limit $q_+^I \equiv \dfrac{q_1^I + q_2^I}{2} \to q^I, \qquad q_-^I \equiv q_1^I - q_2^I \to 0.$

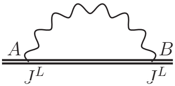
Gravitational Radiation Reaction

The effective action after integrating out potential modes and multipole expansion

$$S_{\rm eff}[x_a,\bar{h}] = \int {\rm d}t \left[\sum_{\ell=2} \left(\frac{1}{\ell!} I^L(t) \nabla_{L-2} E_{i_{\ell-1} i_\ell} - \frac{2\ell}{(\ell+1)!} J^L(t) \nabla_{L-2} B_{i_{\ell-1} i_\ell} \right) \right].$$

The radiation reaction force is calculated through integrating out E_{ij} and B_{ij}





A and B are history indices that label the doubled variables

The leading order mass quadrupole diagram leads to

$$\begin{aligned} iS_{\rm eff}^{2.5{\rm PN}} &= \frac{1}{2} \frac{i^2}{(2m_{Pl})^2} \int {\rm d}t \int {\rm d}t' \; I_A^{ij}(t) \langle E_{ij}^A(t) E_{kl}^B(t') \rangle I_B^{kl}(t') \\ &= \frac{G}{5} \int {\rm d}t \; I_-^{ij}(t) I_{ij+}^{(5)}(t). \end{aligned}$$

$$I_0^{ij} = \sum_a m_a [\mathbf{x}_a^i \mathbf{x}_a^j]_{\rm TF} \Rightarrow F_{ia}^{2.5{\rm PN}} = -\frac{2Gm_a}{5} \mathbf{x}_a^j I_{ij}^{(5)}$$

which is the 2.5PN radiation reaction Burke-Thorne force.

Conclusions and Outlooks

We have completed the knowledge of spin effects in the orbital phase evolution of compact binaries to NNLO in the PN expansion of GR and quadratic order in the spins, including both finite-size and tail effects. Our results will play a key role in elucidating the nature of compact objects through GW precision data.

Spin-induced finite-size effects can also help us reveal the existence of putative ultralight dark matter particles in nature, through GW signals from compact binary systems carrying a boson cloud. GW observations thus open new ways of probing physics beyond the Standard Model with current and future detectors like the Einstein Telescope.

We are currently building up the radiative flux to full 4PN order for non-spinning and the same loop-level for spin effects. The non-linearity including tail effects, memory effects and the mix of them will also be included.