Spin chains for $\mathcal{N}=2$ SCFTs

Elli Pomoni

NBI

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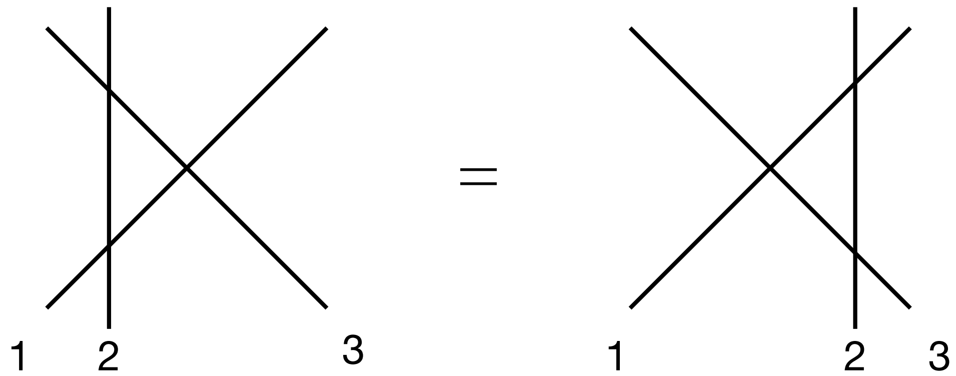
[2106.08449 Rabe, EP, Zoubos]   [2302.xxxxx Andriolo,Bertle,EP,Zhang,Zoubos]

Motivation

- $\mathcal{N}=4$ SYM is integrable in the planar limit.
- Is it the only* integrable theory in 4D?
- What happens when we have less supersymmetry?
- Can we do this in an organised way?

The past

- People believe that $\mathcal{N}=2$ theories are ***not integrable***.
- They do not obey the usual YBE. [1006.0015 Gadde, EP, Rastelli]
- Does this kill integrability? No!



$$S_{12}S_{13}S_{23} = S_{23}S_{13}S_{12}$$

Integrable models

- **Rational** (like XXX based on SU(2))
- **Trigonometric** (like XXZ based on $SU(2)_q$)
- **Elliptic** (like XYZ based on $SU(2)_{q,t}$)
- There are also **hyper-elliptic** examples (chiral potts model)

Elliptic models

* *Depending on the basis* we use, elliptic models do not have to obey the standard YBE but a modified, **dynamical YBE**.

[Felder 1994]

* In the "Baxter basis" (where the **usual YBE** is obeyed) there is **no highest weight state**.

* SCFTs have BPS operators which correspond to the highest weight states. They are naturally <u>**not**</u> **in the "Baxter basis"**.

[Drinfeld 1990]

Quasi-Hopf algebras

* More structure beyond elliptic models and the **dynamical YBE**.

* Drinfeld twist: **quasi-Hopf algebras**, **quasi-Hopf YBE**

$$R_{12}\Phi_{312}R_{13}\Phi^{-1}_{132}R_{23}\Phi_{123} = \Phi_{321}R_{23}\Phi^{-1}_{231}R_{13}\Phi_{213}R_{12}$$

* When the Drinfeld twist obeys the so called **shifted cocycle condition**, we get elliptic models and the **dynamical YBE**.

$\mathcal{N}=2$ SCFTs

✱ Lagrangian $\mathcal{N}=2$ SCFTs are classified. [Bhardwaj,Tachikawa 2013]

✱ Most of them can be obtain via ***orbifolding*** $\mathcal{N}=4$ SYM and then ***marginally deforming***.

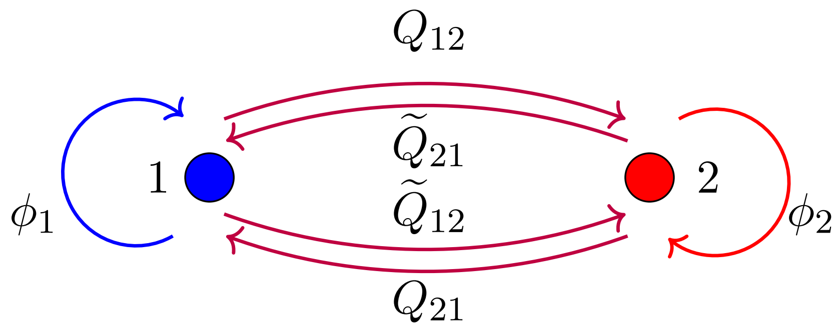
✱ We know the gravity duals for marginally deformed orbifolds.

✱ At the ***orbifold point*** (no marginal deform.) they are ***integrable***. [Beisert,Roiban 2005]

✱ Only understand how marginal deformations affect spin chains.

Our main example

The Z_2 quiver theory
SU(N)xSU(N)



Z_2 orbifold $\mathcal{N}=4$ SYM and then **marginally deform** away from the **orbifold point** ($g_1=g_2$)

Bifundamental

Adjoint

$$X = \begin{pmatrix} & Q_{12} \\ Q_{21} & \end{pmatrix}, \quad Y = \begin{pmatrix} & \tilde{Q}_{12} \\ \tilde{Q}_{21} & \end{pmatrix}, \quad Z = \begin{pmatrix} \phi_1 & \\ & \phi_2 \end{pmatrix}$$

Adjoint

- ✱ Enough to discover all novel features (dynamical, elliptic …).
- ✱ When $g_2 \longrightarrow 0$ gives $\mathcal{N}=2$ SCQCD in the Veneziano limit ($N_f=2N_c$).

The Plan of the talk

- The **(one-loop)** spin chains of $\mathcal{N}=2$ SCFTs are **dynamical**.
- $\mathcal{N}=2$ SCFTs enjoy a **quasi-Hopf symmetry** algebra.
- **R-matrix** in the *quantum plane limit* and the **Drinfeld twist**.
- Non-trivial dynamical twist/coproduct.
- The spectrum organises in quantum “SU(4)” multiplets.

(One-loop) dynamical spin chains

The Hilbert space

$\mathcal{N}=4$ SYM spin chain states: distribute on the lattice sites a “single letter” from the *unique* ultrashort **singleton multiplet**

$$V = \mathcal{D}^n \left(X,\, Y,\; Z,\, \bar{X},\, \bar{Y},\, \bar{Z},\, \lambda_\alpha^A,\, \bar{\lambda}_A^{\dot\alpha},\, \mathcal{F}_{\alpha\beta},\, \bar{\mathcal{F}}_{\dot\alpha\dot\beta} \right)$$

All single letters are in the adjoint representation of the color group.
The total space is $\otimes_\ell^L \mathcal{V}_\ell$.

$\mathcal{N}=2$ SCFTs spin chain states: *two distinct* ultrashort representations:

$$\mathcal{V} = \mathcal{D}^n \left(\phi\,,\, \lambda_\alpha^{\mathcal{I}}\,,\, \mathcal{F}_{\alpha\beta}\right)^a{}_b \quad , \quad \mathcal{H} = \mathcal{D}^n \left(Q^{\mathcal{I}}\,,\, \psi\,,\, \bar{\tilde{\psi}}\right)^a{}_i$$

In the adjoint and bifundamental representation of the color group G_1 x G_2 x …

The **color index structure** imposes **restrictions on the total space**! $\cancel{\otimes_\ell^L \mathcal{V}_\ell}$
Which up to recently we didn’t know how to efficiently account for.

$Q_{12}\, Q_{21}$ allowed, $Q_{12}\, Q_{12}$ not allowed, $\phi_1 Q_{12}$ allowed, $\phi_2 Q_{12}$ not allowed!

XY sector: an alternating spin chain

Every $\mathcal{N}=4$ SYM spin chain state $\quad |XYXYYX\cdots\rangle$

Gives two $\mathcal{N}=2$ spin chain states $\quad |Q_{12}\tilde{Q}_{21}Q_{12}\tilde{Q}_{21}\tilde{Q}_{12}Q_{21}\cdots\rangle$

$$\square_1\times\bar{\square}_2\;\square_2\times\bar{\square}_1\;\square_1\times\bar{\square}_2\;\square_2\times\bar{\square}_1\;\square_1\times\bar{\square}_2\;\square_2\times\bar{\square}_1$$

Which are Z_2 conjugate $\quad |Q_{21}\tilde{Q}_{12}Q_{21}\tilde{Q}_{12}\tilde{Q}_{21}Q_{12}\cdots\rangle$

$$\square_2\times\bar{\square}_1\;\square_1\times\bar{\square}_2\;\square_2\times\bar{\square}_1\;\square_1\times\bar{\square}_2\;\square_2\times\bar{\square}_1\;\square_1\times\bar{\square}_2$$

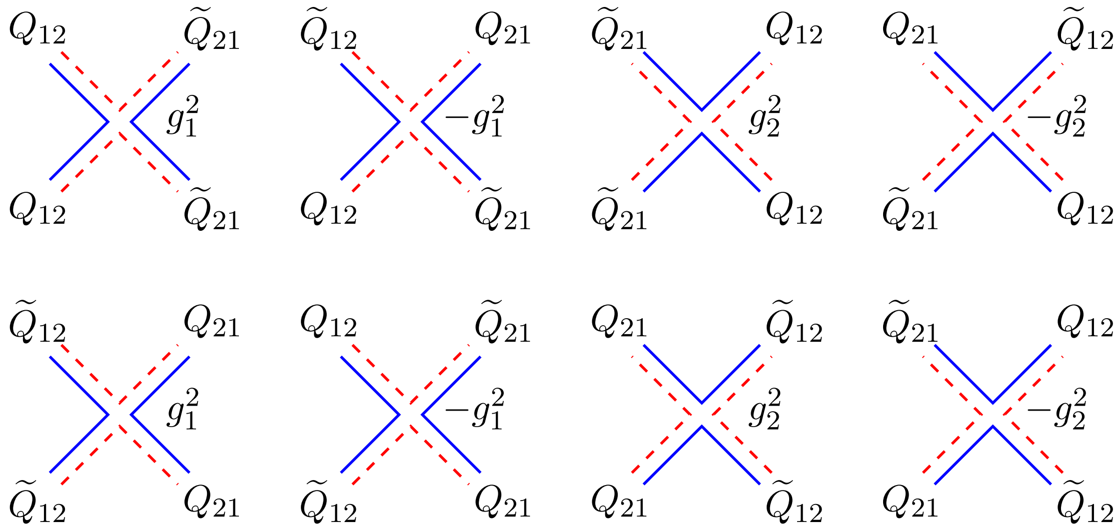
(k states for a rank k orbifold)

To identify which of the two states we have, it is enough to **specify** the gauge group of **the first color index**. This can be done by labelling

$$|XYXYYX\cdots\rangle_{i=1,2}$$

The XY sector Hamiltonian

The one-loop Hamiltonian:



$$\mathcal{H}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa^{-1} & -\kappa^{-1} & 0 \\ 0 & -\kappa^{-1} & \kappa^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix}_{i=1,2}$$

$$\mathcal{H}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa & -\kappa & 0 \\ 0 & -\kappa & \kappa & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$H_{\ell,\ell+1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa^{-1} & -\kappa^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\kappa^{-1} & \kappa^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa & -\kappa & 0 \\ 0 & 0 & 0 & 0 & 0 & -\kappa & \kappa & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Q_{12}Q_{21} \\ Q_{12}\tilde{Q}_{21} \\ \tilde{Q}_{12}Q_{21} \\ \tilde{Q}_{12}\tilde{Q}_{21} \\ Q_{21}Q_{12} \\ Q_{21}\tilde{Q}_{12} \\ \tilde{Q}_{21}Q_{12} \\ \tilde{Q}_{21}\tilde{Q}_{12} \end{pmatrix}$$

Two alternating XXX Hamiltonians with different overall coefficients κ and 1/κ

$$\kappa = \frac{g_2}{g_1}$$

Dynamical XXX

XZ sector: dynamical spin chain

Every $\mathcal{N}=4$ SYM spin chain state $$|XZXZZX\cdots\rangle$$

Gives two $\mathcal{N}=2$ spin chain states $$|\underset{\square_1\times\overline{\square}_2}{Q_{12}}\underset{\square_2\times\overline{\square}_2}{\phi_2}\underset{\square_2\times\overline{\square}_1}{Q_{21}}\underset{\square_1\times\overline{\square}_1}{\phi_1}\underset{\square_1\times\overline{\square}_1}{\phi_1}\underset{\square_1\times\overline{\square}_2}{Q_{12}}\cdots\rangle$$

Which are Z_2 conjugate $$|\underset{\square_2\times\overline{\square}_1}{Q_{21}}\underset{\square_1\times\overline{\square}_1}{\phi_1}\underset{\square_1\times\overline{\square}_2}{Q_{12}}\underset{\square_2\times\overline{\square}_2}{\phi_2}\underset{\square_2\times\overline{\square}_2}{\phi_2}\underset{\square_2\times\overline{\square}_1}{Q_{21}}\cdots\rangle$$

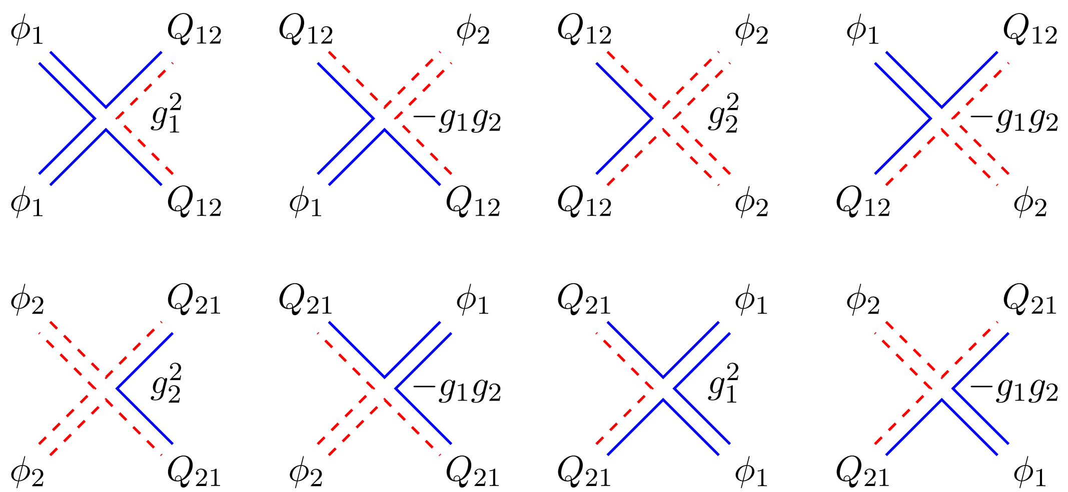
(k states for a rank k orbifold)

We **specify** the gauge group of **the first color index** and identify which of the two states we have. This can be done by labelling

$$|XZXZZX\cdots\rangle_{i=1,2}$$

The XZ sector Hamiltonian

The one-loop Hamiltonian:



$$\mathcal{H}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa & -1 & 0 \\ 0 & -1 & \kappa^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{H}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa^{-1} & -1 & 0 \\ 0 & -1 & \kappa & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} XX \\ XZ \\ ZX \\ ZZ \end{pmatrix}_{i=1,2}$$

$$H_{i,i+1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & \kappa^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa^{-1} & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & \kappa & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} Q_{12}Q_{21} \\ Q_{12}\phi_2 \\ \phi_1 Q_{12} \\ \phi_1\phi_1 \\ Q_{21}Q_{12} \\ Q_{21}\phi_1 \\ \phi_2 Q_{21} \\ \phi_2\phi_2 \end{pmatrix}$$

$$\kappa = \frac{g_2}{g_1}$$

Two Temperley-Lieb Hamiltonians with two different deformation parameters κ and 1/κ

Dynamical Temperley-Lieb

Elliptic from dynamical

Old idea: **elliptic** R matrices from **trigonometric** ones by ‘**averaging**’.

[Jimbo, Konno, Opake, Shiraishi 1997]

✱ Originally discovered ellipticity via explicit coordinate Bethe ansatz (Q-vaccuum)

[Rabe, EP, Zoubos 2021]

✱ New computation: 3-body coordinate Bethe ansatz (ϕ-vaccuum)

[Bozkurt, EP in preparation]

At the level of the R matrix, the *dynamical parameter* of Felder keeps track of the *color indices*!

Our model is more intricate than Felder: when we ***cross a Z*** (field in the adjoint rep) we ***don’t*** want to ***shift the dynamical parameter.***

It is a dilute RSOS/CSOS model.

The holomorphic SU(3) sector is captured by a **dynamical 15-vertex** model which is specified by the **adjacency graph**, which is the dual to the **brane-tiling diagram** of the quiver theory.

Quasi-Hopf symmetry

The Manin quantum plane

$$qxy = yx$$

Can be obtain from an R-matrix:

$$\boxed{\lambda x^b x^a = R^{ab}_{jl} x^j x^l}$$

$$R = q^{-\frac{1}{2}} \begin{pmatrix} q & 0 & 0 & 0 \\ 0 & 1 & q - q^{-1} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & q \end{pmatrix}$$

The quantum plane is invariant under the transformations $x'^i = \mathrm{t}^i_{\ j} x^j$.

They obey $SU(2)_q$ which is obtained using the Rtt relations:

$$\boxed{R^{i\ k}_{\ a\ b} t^a_{\ j} t^b_{\ l} = t^k_{\ b} t^i_{\ a} R^{a\ b}_{\ j\ l}}$$

$$\mathrm{t}^1_{\ 2}\mathrm{t}^2_{\ 1} = \mathrm{t}^2_{\ 1}\mathrm{t}^1_{\ 2}\ ,\ \mathrm{t}^1_{\ 1}\mathrm{t}^2_{\ 2} - \mathrm{t}^2_{\ 2}\mathrm{t}^1_{\ 1} = (q^{-1} - q)\mathrm{t}^1_{\ 2}\mathrm{t}^2_{\ 1}$$

$$\mathrm{t}^1_{\ 1}\mathrm{t}^1_{\ 2} = q^{-1}\mathrm{t}^1_{\ 2}\mathrm{t}^1_{\ 1}\ ,\ \mathrm{t}^1_{\ 1}\mathrm{t}^2_{\ 1} = q^{-1}\mathrm{t}^2_{\ 1}\mathrm{t}^1_{\ 1}\ ,\ \mathrm{t}^1_{\ 2}\mathrm{t}^2_{\ 2} = q^{-1}\mathrm{t}^2_{\ 2}\mathrm{t}^1_{\ 2}\ ,\ \mathrm{t}^2_{\ 1}\mathrm{t}^2_{\ 2} = q^{-1}\mathrm{t}^2_{\ 2}\mathrm{t}^2_{\ 1}$$

3D quantum planes classified

[Ewen,Ogievetsky1994]

Parameterise using two tensors E_{ijk} and F_{ijk}:

$$E^{\alpha}_{ij} x^i x^j = 0 \qquad u_i u_j F^{ij}_{\alpha} = 0$$

Quantum plane   Quantum co-plane

The R-matrix is given by:

$$\boxed{\hat{R}^{ij}_{kl} = \delta^i_k \delta^j_\ell - c\, E_{klm} F^{ijm}}$$

$$\hat{R} = P\ R$$

Using this R-matrix we get back the right quantum plane relations and through the Rtt relations we can write down the quantum algebra.

The R-matrix encodes symmetries of the quantum plane.

[Roiban2004] [Berenstein,Cherkis2004] … [Månsson,Zoubos2008] … [Dlamini,Zoubos2016&19]

Leigh–Strassler theory

$$\phi^1\phi^2 = q\phi^2\phi^1 - h(\phi^3)^2$$
$$\phi^2\phi^3 = q\phi^3\phi^2 - h(\phi^1)^2$$
$$\phi^3\phi^1 = q\phi^1\phi^3 - h(\phi^2)^2$$

3D Quantum plane

$$\mathcal{W}_{\mathcal{N}=4} = g\mathrm{Tr}\left\{\Phi^1[\Phi^2,\Phi^3]\right\} = \frac{g}{3}\epsilon_{ijk}\mathrm{Tr}\left\{\Phi^i\Phi^j\Phi^k\right\}$$

$$\downarrow$$

$$\mathcal{W}_{LS} = \frac{1}{3}E_{ijk}\mathrm{Tr}\{\Phi^i\Phi^j\Phi^k\}$$

The quantum co-plane: hermitian conjugate: $F^{ijk} = \overline{E}_{ijk}$

$$E_{123} = E_{231} = E_{312} = \frac{1}{d}\,,$$
$$E_{321} = E_{213} = E_{132} = -\frac{q}{d}\,,$$
$$E_{111} = E_{222} = E_{333} = \frac{h}{d}\,,$$

The **Hamiltonian** is obtained by: $H^{jk}_{mn} = E_{mna}F^{ajk}$

The **R-matrix**: $\hat{R}^{ij}_{kl} = \delta^i_k\delta^j_\ell - c\,E_{klm}F^{ijm}$

$$d^2 = \frac{1+\bar{q}q+\bar{h}h}{2}$$

$$R = \frac{1}{2d^2}\begin{pmatrix}
1+q\bar{q}-h\bar{h} & 0 & 0 & 0 & 0 & -2\bar{h} & 0 & 2\bar{h}q & 0\\
0 & 2\bar{q} & 0 & 1-q\bar{q}+h\bar{h} & 0 & 0 & 0 & 0 & 2h\bar{q}\\
0 & 0 & 2q & 0 & -2h & 0 & q\bar{q}+h\bar{h}-1 & 0 & 0\\
0 & q\bar{q}+h\bar{h}-1 & 0 & 2q & 0 & 0 & 0 & 0 & -2h\\
0 & 0 & 2\bar{h}q & 0 & 1+q\bar{q}-h\bar{h} & 0 & -2\bar{h} & 0 & 0\\
2h\bar{q} & 0 & 0 & 0 & 0 & 2\bar{q} & 0 & 1-q\bar{q}+h\bar{h} & 0\\
0 & 0 & 1-q\bar{q}+h\bar{h} & 0 & 2h\bar{q} & 0 & 2\bar{q} & 0 & 0\\
-2h & 0 & 0 & 0 & 0 & q\bar{q}+h\bar{h}-1 & 0 & 2q & 0\\
0 & -2\bar{h} & 0 & 2\bar{h}q & 0 & 0 & 0 & 0 & 1+q\bar{q}-h\bar{h}
\end{pmatrix}$$

The **Lagrangian is invariant** under the transformations $\Phi^i \to \mathrm{t}^i{}_j\Phi^j$
which form a **quantum version of SU(3)** defined by the Rtt relations.

AdS point of view

Gravity dual reason why we have a quantum algebra:

NSNS B-field turned on the C^3 (transverse to the D3).

When there is a B-field the open strings ending on the D3 branes see a non-commutative geometry. *Open strings see a quantum plane*!

[Seiberg,Witten1999]

✱ For the Leigh–Strassler background [Kulaxizi 2006]

✱ Marginally deformed orbifolds also have a B-field on the orbifolded $C^2 \subset C^3$ (transverse to the D3) allowing us to go away from the orbifold point:

$$\frac{1}{g_1^2} + \frac{1}{g_2^2} = \frac{1}{2\pi g_s} \qquad \frac{g_1^2}{g_2^2} = \frac{\beta}{1-\beta} \text{ with } \beta = \int_{S^2} B_{NS}$$

[Gadde, EP, Rastelli 2009]

The Z_2 quiver quantum group

$$\mathcal{W}_{\mathcal{N}=4} = g\mathrm{Tr}\left\{\Phi^1[\Phi^2,\Phi^3]\right\} = \frac{g}{3}\epsilon_{ijk}\mathrm{Tr}\left\{\Phi^i\Phi^j\Phi^k\right\}$$

$$\downarrow$$

$$\mathcal{W} = E^{(1)}_{ijk}\mathrm{Tr}_1\left(X^iX^jX^k\right) + E^{(2)}_{ijk}\mathrm{Tr}_2\left(X^iX^jX^k\right)$$

There are **two** copies (**images**) of the quantum plane:

XY: $g_1 Q_{12}\widetilde{Q}_{21} = g_1\widetilde{Q}_{12}Q_{21} \quad , \quad g_2 Q_{21}\widetilde{Q}_{12} = g_2\widetilde{Q}_{21}Q_{12}$

XZ: $\phi_2 Q_{21} = \frac{1}{\kappa}Q_{21}\phi_1 \quad , \quad \phi_1 Q_{12} = \kappa Q_{12}\phi_2$

YZ: $\phi_2\widetilde{Q}_{21} = \frac{1}{\kappa}\widetilde{Q}_{21}\phi_1 \quad , \quad \phi_1\widetilde{Q}_{12} = \kappa\widetilde{Q}_{12}\phi_2$

$$E^{(1)}_{123} = g_1 \,, E^{(1)}_{231} = g_2 \,, E^{(1)}_{312} = g_1 \,, E^{(1)}_{132} = -g_2 \,, E^{(1)}_{321} = -g_1 \,, E^{(1)}_{213} = -g_1$$

$$E^{(2)}_{123} = g_2 \,, E^{(2)}_{231} = g_1 \,, E^{(2)}_{312} = g_2 \,, E^{(2)}_{132} = -g_1 \,, E^{(2)}_{321} = -g_2 \,, E^{(2)}_{213} = -g_2$$

(k images for a rank k orbifold)

$$\boxed{\hat{R}^{ij}_{kl} = \delta^i_k\delta^j_\ell - c\,E_{klm}F^{ijm}}$$

The Z_2 quiver quantum group

<u>XY sector the R ∝ 1</u>: the SU(2) that rotates X and Y is **unbroken** (*indeed true*)

<u>XZ nontrivial R</u>: the SU(2) that rotates X and Z is <u>naively **broken**</u> (***not lost but <u>upgraded to quantum</u>***)

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2\kappa}{\kappa^2+1} & -\frac{\kappa^2-1}{\kappa^2+1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\kappa^2-1}{\kappa^2+1} & \frac{2\kappa}{\kappa^2+1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2\kappa}{\kappa^2+1} & \frac{\kappa^2-1}{\kappa^2+1} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\kappa^2-1}{\kappa^2+1} & \frac{2\kappa}{\kappa^2+1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

8 x 8 because of the two copies!

The Rtt relations define the quantum group $SU(2)_\kappa$

Two copies $SU(2)_\kappa$ *and* $SU(2)_{1/\kappa}$!

For the XZ sector the R-matrix is generated by a ***Drinfeld-twist***:

$$R = F_{21} F_{12}^{-1} = (F_{12})^{-2}$$

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & -\beta & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{with:} \quad \alpha = \frac{\kappa+1}{\sqrt{2}\sqrt{1+\kappa^2}}, \quad \beta = \frac{\kappa-1}{\sqrt{2}\sqrt{1+\kappa^2}}$$

A quasi-Hopf symmetry algebra!

Understanding the twist/coproduct

The Z_2 quiver has extra symmetry

- Change of basis to a simpler twist, where the coproduct takes the form

$$\Delta R^I{}_{\hat{I}} = K_{I\hat{I}} \otimes R^I{}_{\hat{I}} + R^I{}_{\hat{I}} \otimes K_{\hat{I}I}$$

[Ihry 2008]
[Garus 2017]

and is easily generalised to **<u>any number of sites</u>**.

- The **superpotential** is **<u>invariant</u>** under the **quantum** "**$SU(3)_\kappa$**" symmetry.

- The $SU(3)_\kappa$ has in it the XZ "$SU(2)_\kappa$" (as well as the YZ "$SU(2)_\kappa$").

- The action of the generators of the $SU(3)_\kappa$ (and the **full $SU(4)_\kappa$**) on fields is inherited from $\mathcal{N}=4$ SYM (orbifold point). But now ***groupoid***!

- The spectrum organises in **multiplets of the quantum** "**$SU(4)_\kappa$**".

[2302.xxxxx Andriolo,Bertle,EP,Zhang,Zoubos]

The Z_2 quiver has extra symmetry

Example of an **SU(2)$_\kappa$ multiplet**

$$\boxed{Q_{12}Q_{21}\dots Q_{21}}$$

$$\Delta\sigma^+ \qquad \Delta\sigma^-$$

$$\boxed{\kappa^{-\frac{1}{L}}Q_{12}\dots Q_{12}\phi_2 + \kappa^{\frac{L-1}{L}}Q_{12}\dots Q_{21}\phi_1 Q_{12} + \dots + \kappa^{\frac{L-1}{L}}\phi_1 Q_{12}\dots Q_{12}}$$

$$\Delta\sigma^+ \qquad \Delta\sigma^-$$

$$\boxed{\dots}$$

$$\Delta\sigma^+ \qquad \Delta\sigma^-$$

$$\boxed{\kappa^{\frac{1}{L}-\frac{L}{2}}Q_{12}\phi_2\dots\phi_2 + \dots + \kappa^{\frac{1}{L}+\frac{L}{2}-1}\phi_1\dots\phi_1 Q_{12}}$$

$$\Delta\sigma^+ \qquad \Delta\sigma^-$$

$$\boxed{\phi_1\dots\phi_1}$$

This SU(2) was known to be broken!

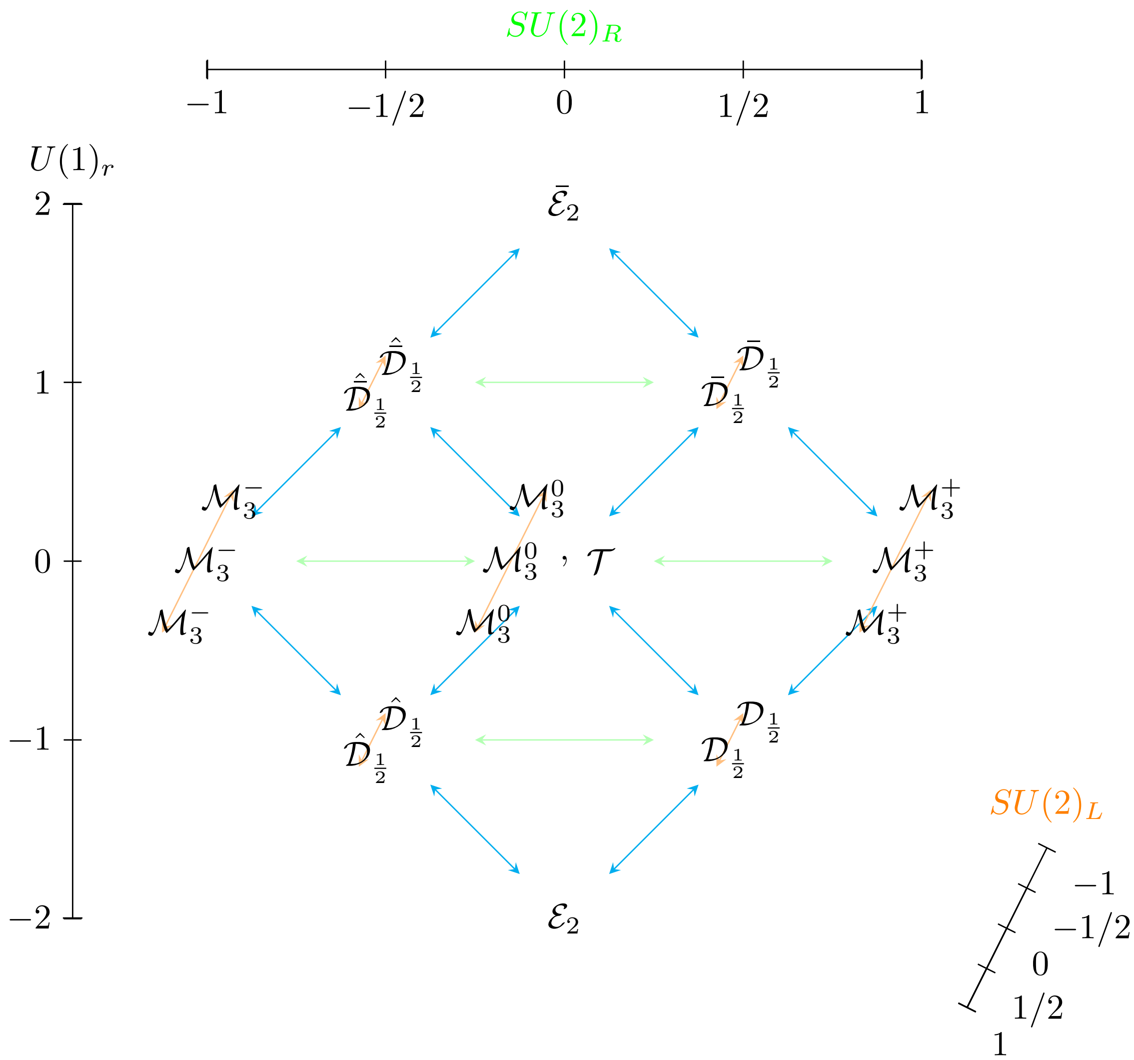
From the point of view of $\mathcal{N}=2$ representation theory, these operators are unrelated!

The **kappa symmetrised** operators (eigenvectors) we can also obtain by direct diagonalisation of the one-loop Hamiltonian.

[2302.xxxxx Andriolo,Bertle,EP,Zhang,Zoubos]

The Z_2 quiver has extra symmetry

The stress energy tensor multiplet of $\mathcal{N}=4$ breaks into $\mathcal{N}=2$ multiplets. Nonetheless, the **naively broken generators** get **upgraded to quantum generators**. With the new coproduct allow us to have one multiplet!



The D multiplets drop after you trace.

[2302.xxxxx Andriolo,Bertle,EP,Zhang,Zoubos]

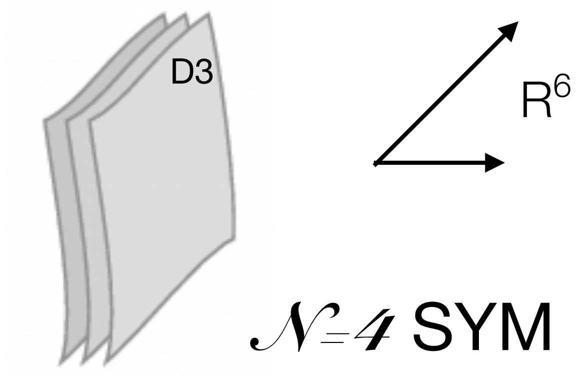
Conjecture

- The $\mathcal{N}<4$ theories which can be obtained via ***orbifolding***, ***orientifolding***, … the mother $\mathcal{N}=4$ SYM theory, enjoy a **quantum deformation** of the **"SU(4)"** R-symmetry of the mother theory and possibly a quantum PSU(2,2|4).

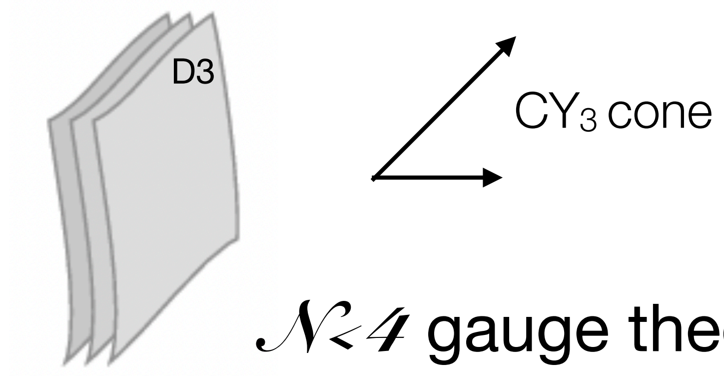
- The **naively broken** generators of SU(4) → U($\mathcal{N}$) get **upgraded** to *quantum generators*.

… any susy breaking that is due to R-symmetry breaking.

3D q-planes and $\mathcal{N} < 4$ SCFTs



$\mathcal{N}=4$ SYM



$\mathcal{N}<4$ gauge theories

✱ $\mathcal{N} < 4$ theories: **F-terms:** define (complex 3D) **quantum planes**.

✱ Can read off the **R-matrix** at the *quantum plane* (*Braid) limit.*

✱ The Rtt relations: give us the **quantum group**.

✱ From R-matrix: **Drinfeld twist** and **the quasi-Hopf** symmetry **algebra**.

✱ Translate to a "dynamical twist". Then it is easy to write the coproduct for any number of sites, it is possible to check the invariance of the Lagrangian and look for the quantum "SU(4)" multiplets.

Where we currently are

- ✱ Studying a large class of $\mathcal{N}=2$ SCFTs. (A-type orbifolds)

- ✱ More multiplets, also non-BPS, 6x6=1+15+20', 6x6x6= 50+10+…

- ✱ The SU(3) scalar sector as a **dynamical 15-vertex model**.

- ✱ Elliptic nature via an explicit 2-body coordinate Bethe ansatz (Q-vacuum).

$$E_1(p;\kappa) = \frac{1}{\kappa} + \kappa \pm \frac{1}{\kappa}\sqrt{(1+\kappa^2)^2 - 4\kappa^2 \sin^2 p}$$

- ✱ Explicit 3-body coordinate Bethe ansatz (ϕ-vacuum).
  The **Yang operator** (seems to obey a type of YBE)

[in preparation with Deniz Bozkurt]

Where are we going?

- ✱ How to go from 2- to 3-sites: Derive the co-cycle condition.

- ✱ This will formally answer the Integrability question.

- ✱ Boost operator program for DYBE! (First the alternating XY!)
  [with de Leeuw, Retore, Zoubos]

- ✱ All-loops S-matrix and hopefully integrability.

- ✱ Detailed study of more $\mathcal{N}=2$ SCFTs and $\mathcal{N}=1$ SCFTs.

- ✱ The SU(3) scalar sector as a **dynamical 15/19-vertex model**.

Outlook

* Study **the gravity dual** of marginally deformed orbifolds!

* "4D Chern-Simons" approach [1709.09993 Costello, Witten, Yamazaki]

* Generalize [2104.08263 Gaberdiel, Gopakumar] [2206.08795 Gaberdiel, Galvagno]

  To the quantum string Dual to Free $\mathcal{N}=2$ SCFTs.

Thanks!