# A Quantum Mechanics for Magnetic Horizons

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- Corrected spectrum might have a gap at $M_{\mathrm{b}}$ [Maldacena and Susskind 1996], or different $T$ scaling [Iliesiu and Turiaci 2020].

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- Since $M_{\text{gap}} \ll 1/r_0$, near-extremal spectrum can be studied by dimensionally reducing on $S^2_{r_0}$, obtaining a 2d dilaton-gravity coupled to gauge fields.
- Near extremality, reduces to a Schwarzian theory with coefficient depending on the charge/chemical potential [Iliesiu and Turiaci 2020]. Same computation for near-BPS black holes in [Heydeman et al. 2020], etc...

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- Also need SUSY to describe near-BPS black holes [Fu, Gaiotto, Maldacena and Sachdev 2016] (only $\mathrm{U}(1)_R$). $\mathrm{U}(1)_F \times \mathrm{U}(1)_R$ and SUSY [Heydeman, Turiaci, Zhao 2022].

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- Can we be more systematic? Embed in higher-dimensional AdS/CFT

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- Asymptotics of black hole $\implies$ dual theory is put on $S^2 \times \mathbb{R}$ with a topological twist on $S^2$.
- Plan: reduce dual theory on $S^2$ with topological twist and find the QM.

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- Background of the reduction coincides with that of the topologically twisted index [Benini and Zaffaroni 2015]

$$\mathcal{I}(y) = \sum_{\{\mathfrak{m}\}} \oint \prod_{i=1}^{N} \frac{du^i}{2\pi} \mathcal{Z}_{\mathfrak{m}}(y, u)\,, \quad \mathcal{Z}_{\mathfrak{m}}(y, u) = e^{\mathfrak{m} \cdot V'(u) + \Omega(u)}\,.$$

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  Each $\mathcal{Z}_{\mathfrak{m}}$ = Witten index of a gauged QM [Hori, Kim and Yi 2014]
- $\mathcal{I}$ at large $N$ reproduces the entropy of BPS black holes [Benini, Khachatryan and Milan 2017].
- Strategy: compute $\mathcal{I}$ at large $N$ via saddle point in $\mathfrak{m}$; isolate the $\hat{\mathfrak{m}}$ that dominates $\mathcal{I}$ and reproduces BPS black hole entropy.

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$$\mathcal{I} = \oint \prod_{i=1}^{N} \frac{du^i}{2\pi} \frac{e^{\Omega(u)}}{1 - e^{V'(u)}} = \sum_{\hat{u}|V'(\hat{u})=0} \frac{e^{\Omega(\hat{u})}}{i^N V''(\hat{u})} \approx \sum_{\hat{u}|V'(\hat{u})=0} e^{\Omega(\hat{u})} .$$

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- (II) $\mathfrak{m}_i \to \mathfrak{m}(t)$, Saddle point in $\mathfrak{m}$ and $u$. $\mathfrak{m}$: $V'(\hat{u}) = 0$, $u$: $\hat{\mathfrak{m}} \cdot V''(\hat{u}) + \Omega'(\hat{u}) = 0$ fixes $\hat{\mathfrak{m}}$ in terms of $\hat{u}$ and vice versa. Result same as above.

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- (III) Isolate term for $\hat{\mathfrak{m}}$ in (II) and perform saddle point in $u$

$$\mathcal{Z}_{\hat{\mathfrak{m}}} = \oint \prod_{i=1}^{N} \frac{du^i}{2\pi} e^{\hat{\mathfrak{m}} \cdot V'(u) + \Omega(u)} = \sum_{\hat{u}|V'(\hat{u})=0} \frac{e^{\Omega(\hat{u})}}{\sqrt{J}} \approx \sum_{\hat{u}|V'(\hat{u})=0} e^{\Omega(\hat{u})}$$

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- Global symmetry $\mathrm{SU}(3)_F \times \mathrm{U}(1)_R$. Magnetic charges $\mathfrak{n}_{a=1,2,3}$, chemical potentials $\Delta_{a=1,2,3}$, $\sum_a \mathfrak{n}_a = -2$, $\sum_a \Delta_a \in 2\pi\mathbb{Z}$. Saddle point flux:

$$\hat{\mathfrak{m}}(t) = \left(\frac{N}{9kG^2}\right)^{\frac{1}{3}} f_+ \, t \,, \quad \rho(t) \equiv \frac{1}{N}\frac{di}{dt} = \frac{3}{4}\left(1 - t^2\right) \,, \quad t \in [-1, 1] \,,$$

$$G = \sum_{a=1}^{3} g_+(\Delta_a) \,, \quad f_+ \equiv -\sum_{a=1}^{3}(1 + \mathfrak{n}_a)\Big(g'_+(\Delta_a) - g'_+(0)\Big) \,,$$

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- This result is also found in [Hosseini and Zaffaroni 2022].

# Background of the reduction

- Topological twist (global magnetic fluxes) breaks 3d $\mathcal{N} = 2 \to 1$ complex supercharge $Q$, generating 1d $\mathcal{N} = 2$. Also generically breaks $\mathrm{SU}(3)_F \times \mathrm{U}(1)_R \to \mathrm{U}(1)_F^2 \times \mathrm{U}(1)_R$

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- For gauge fluxes $\hat{\mathfrak{m}}$ to preserve $Q$, also turn on the backgrounds

$$\sigma = -\frac{\hat{\mathfrak{m}}}{2m_k R^2}\,, \quad A_t = \frac{\hat{\mathfrak{m}}}{2m_k R^2}\,, \quad m_k \equiv \frac{k\, e_{3\mathrm{d}}^2}{2\pi}\,.$$

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- Add SYM action as regulator, taking $e_{3\mathrm{d}}^2 \to \infty$ before $R \to 0$. Full Lagrangian:

$$\mathcal{L} = k\mathcal{L}_{\mathrm{CS}} + \frac{1}{e_{3\mathrm{d}}^2}\mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_{\Phi,\mathrm{kin}} + \mathcal{L}_{\mathrm{superpot}} \,.$$

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- Unusual fact: background is not a saddle point of $\mathcal{L}_{\mathrm{CS}}$.

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- Does not affect $s$-closed observables, eg. Ward identities can be derived.

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In this case, expansion around background:

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots\,, \quad \mathcal{L}^{(1)} = \mathrm{Tr}\left(\frac{k\mathfrak{m}}{4\pi R^2}\left(A_t + \sigma\right)\right)$$

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- One might try the temporal gauge $A_t + \sigma = 0$. SUSY is manifest but there are $\infty$ towers of light modes.

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- Why is this allowed? Using the fact that $\mathcal{Q}\Psi_{\text{gf}}$ has $n_g = -1, -2$, easy to show that $\langle \mathcal{O}_{n_g \leq 0} \rangle_\delta = \langle \mathcal{O}_{n_g \leq 0} \rangle_s$, observables are not affected.

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- Further include $\mathcal{Q}\Psi_{\text{gf}}$ in addition to $s\Psi_{\text{gf}}$, i.e. $\delta\Psi_{\text{gf}}$ in total, $\delta \equiv s + \mathcal{Q}$. $\mathcal{Q} \equiv Q + \overline{Q}$ on physical fields, acts non-trivially on $b$, $c$.
- Why is this allowed? Using the fact that $\mathcal{Q}\Psi_{\text{gf}}$ has $n_g = -1, -2$, easy to show that $\langle \mathcal{O}_{n_g \leq 0} \rangle_\delta = \langle \mathcal{O}_{n_g \leq 0} \rangle_s$, observables are not affected.

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- New gauge-fixed action is $\delta$-closed since $\delta\mathcal{L} = 0$ and $\delta^2 =$ time-translation + residual gauge transformations.
- Can redefine $A_t' + \sigma' = A_t + \sigma + \frac{1}{2}\{c, c\}$ (still hermitian) so that $\delta(A_t' + \sigma') = 0$. Now

$$\delta\mathcal{L}'^{(1)} = \delta\,\mathrm{Tr}\left[\frac{k\mathfrak{m}}{4\pi R^2}(A_t' + \sigma')\right] = 0$$

and $\mathcal{L}'^{(2)}$ is invariant under linearized $\delta \implies$ spectrum is supersymmetric.

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- Each chiral has a linear kinetic term for its boson and the fermion is auxiliary.

$$\int d\theta d\bar{\theta}\, \overline{\Phi} e^V \Phi \overset{\text{WZ}}{=} i\overline{\phi} D_t^+ \phi + \overline{\psi}\psi\,, \quad D_t^+ \equiv \partial_t - i(A_t + \sigma)$$

Has not been considered before in [Hori ..., Fu et al., Heydeman et al.] New possibilities for building SUSY SYK-like models.

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Appears in the standard kinetic term

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- For each Fermi, specify holomorphic $J(\Phi)$ in dual representation of gauge and flavour symmetries, and $R(J) = -R(\mathcal{Y}) + 1$

$$J^{ji}_{C,-m} = 0\,, \quad J^{ji}_{a,-m} \sim \epsilon^{abc} \sum_{k,m'} C\begin{pmatrix} q^b_{jk} & q^c_{ki} & |q^a_{ij}|-1 \\ m' & -m-m' & -m \end{pmatrix} \Phi^{jk}_{b,m'}\, \Phi^{ki}_{c,-m-m'}$$

  Contributes as $\int d\theta \mathcal{Y} J + \text{c.c.}$

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- This is the first explicit derivation of the vector 1-loop determinant of the topologically twisted index (the temporal gauge will be easier for this purpose).
- $\implies$ BPS states of the QM account for the entropy of BPS black holes.

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$$E_C = J_C = 0\,, \quad E_a \sim \Phi_a \sum_{m \ge 1} \Xi^m\,, \quad J_a \sim \epsilon^{abc} \Phi_b \Phi_c \sum_{n \ge 0} \Xi^n$$

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- We have only kept $m = 1$, $n = 0$ but other terms cannot be ruled out. Not suppressed by EFT/power counting since $[\Xi] = 0$ classically. Possible that $\Xi$ gains a positive anomalous dimension.

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- Perform a similar reduction on $\Sigma_{\mathfrak{g}\geq 1}$ or for ABJM.