

## Why $\nu$ 's ?

Neutrinos are so far the ONLY evidence beyond the SM

Neutrinos are rewarding

- Fermi: formulation of  $\beta$  decay theory
- Reines: detection electron antineutrinos
- Lederman, Schwartz and Steinberger:  $\nu_\mu$
- Davis and Koshiba: solar  $\nu$  interactions
- Kajita and McDonald:  $\nu$  oscillations

~~two~~  
2012 One major discovery ~~in~~ in particle physics

- A SM-like Higgs boson (ATLAS, CMS)  
The key to EWSB and a possible window to
- $\theta_{13} \sim 10^\circ$  (T2K, MINOS, Daya Bay, RENO)  
about as large as it could have been !  
The door to CP Violation in the leptonic sector

Is The Whole Universe made of—  
Electrons      Protons      Neutrons ?

NO!

Electrons      Protons      Neutrons  
are rareties!

For every one of them, the universe contains a  
billion neutrinos  $\nu$ !

Within each cubic centimeter of space  
~ 360 neutrinos from the Big Bang



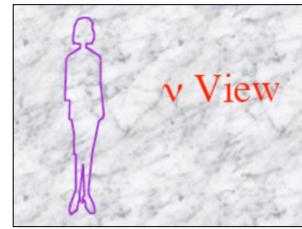
8 million  
neutrinos !!!!

Passing through each person on earth every second:  
One hundred trillion neutrinos from the sun.

The sun shines because of nuclear fusion in its core.

This fusion produces—

- Energy, including visible light
- Neutrinos
- The atoms more complicated than hydrogen



Almost all neutrinos zipping through us do nothing at all.

Typically, a solar neutrino would have to zip through 10,000,000,000,000,000 people before doing anything.

The probability that a particular solar neutrino will interact as it zips through one of us is  
 $1 / 10,000,000,000,000,000$ .

### Are Neutrinos Important to Our Lives?

If there were no vs, the sun and stars would not shine.

- No energy from the sun to keep us warm.
- No atoms more complicated than hydrogen.  
No carbon. No oxygen. No water.  
No earth. No moon. No us.

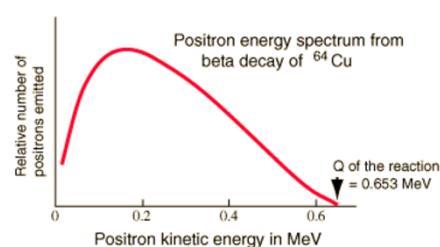
No vs is very **BAD** news.

Summer Schools (if existed) were VERY short .....



$$E_e = \frac{m_n^2 + m_e^2 - m_p^2}{2 m_n}$$

Studies of  $\beta$  decay revealed a continuous energy spectrum.



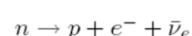
Another anomaly was the fact that the nuclear recoil was not in the direction opposite to the momentum of the electron.

The emission of another particle was a probable explanation of this behaviour, but searches found no evidence of either mass or charge.



*...desperate remedy to save the law of conservation of energy...*

Neutron Decay:



Fermi postulated a theory for  $\beta$  decay in terms of spinors

$$H_{ew} = \frac{G_F}{\sqrt{2}} \bar{\Psi}_p \gamma_\mu \Psi_n \bar{\Psi}_e \gamma^\mu \Psi_\nu$$

A Dirac field is described by a four component spinor

$$\begin{pmatrix} e_L \\ e_R \\ \hat{e}_L \\ \hat{e}_R \end{pmatrix}$$

### Standard Model of Particle Physics

Gauge Theory based on the group:

$$SU(3) \times SU(2) \times U(1)$$

$SU(3) \Rightarrow$  Quantum Chromodynamics

Strong Force (Quarks and Gluons)

$SU(2) \times U(1) \Rightarrow$  ElectroWeak Interactions broken to  $U_{EM}(1)$

by HIGGS

$$\underline{SU_L(2) \times U_Y(1) \Rightarrow U_{EM}(1) }$$

Force Carriers:  $W^\pm$ ,  $Z^0$  and  $\gamma$  masses: 80, 91 and 0 GeV

quark, SU(2) doublets:  $\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$

up-quark, SU(2) singlets:  $u_R, c_R, t_R$

down-quark, SU(2) singlets:  $d_R, s_R, b_R$

lepton, SU(2) doublets:  $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$

neutrino, SU(2) singlets: — — —

charge lepton, SU(2) singlets:  $e_R, \mu_R, \tau_R$

### Electron mass

comes from a term of the form

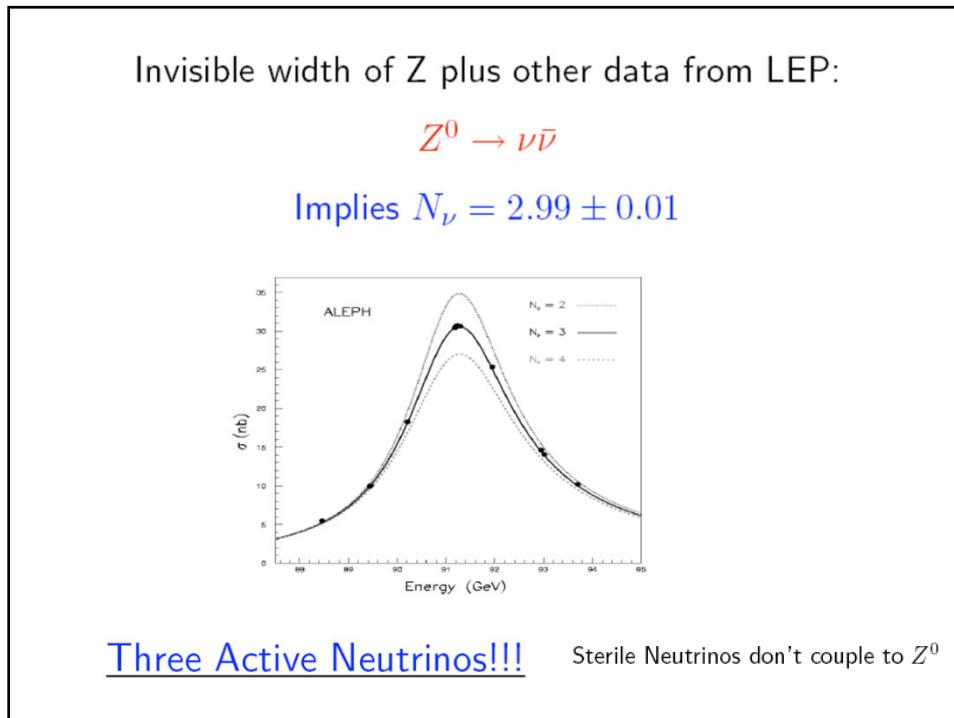
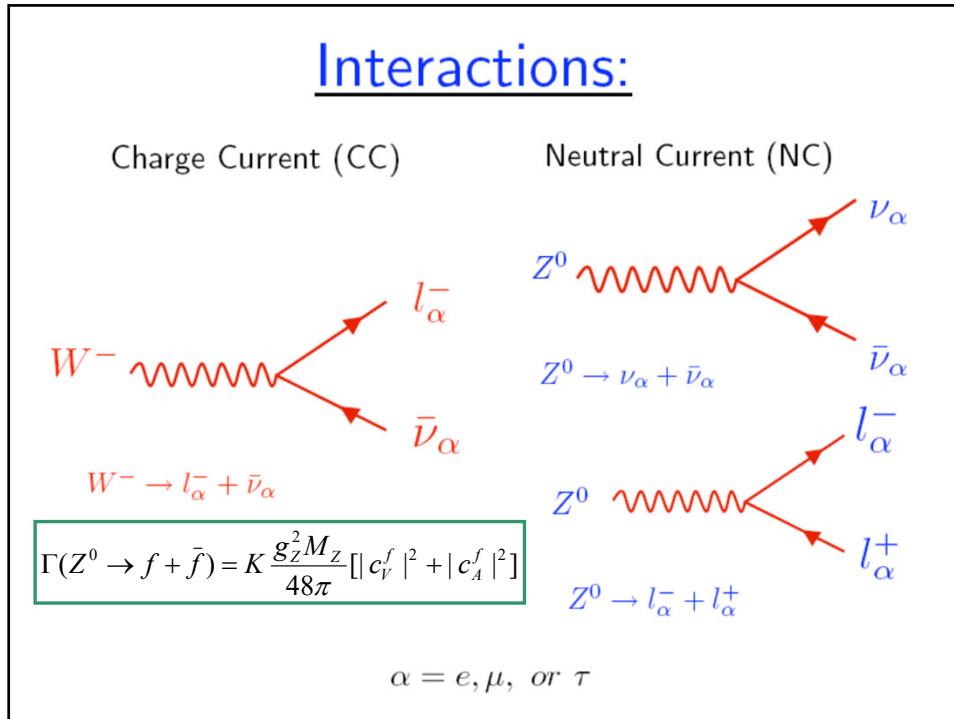
$$\bar{L}\phi e_R$$

Absence of  $\nu_R$

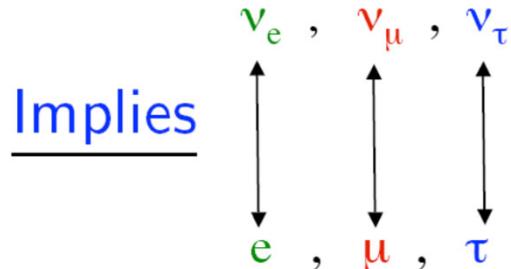
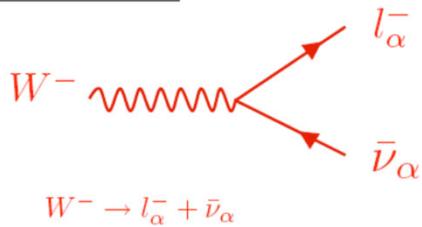
forbids such a mass term (dim 4)

for the Neutrino

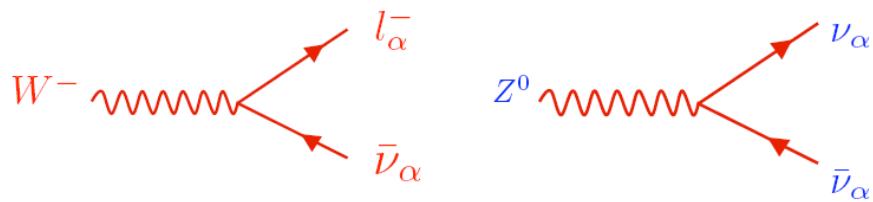
Therefore in the SM neutrinos are massless  
and hence travel at speed of light.



Note That



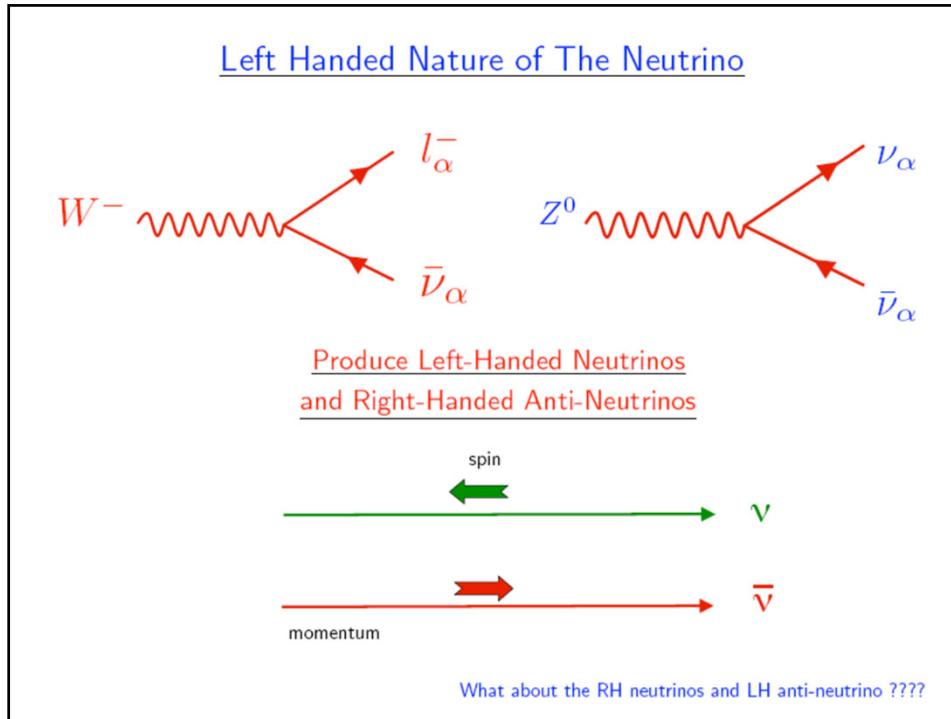
Standard Model



couplings conserve the Lepton Number L  
defined by—

$$L(v) = L(\ell^-) = -L(\bar{v}) = -L(\ell^+) = 1.$$

Actually  $L_e$ ,  $L_\mu$ , and  $L_\tau$   
separately



There exist three fundamental and discrete transformations in nature:

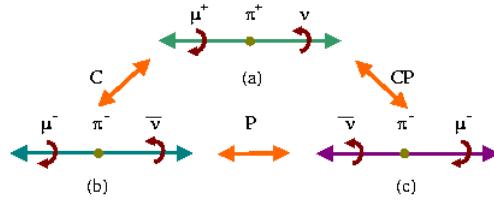
- Parity                       $\mathcal{P}$        $\vec{x} \rightarrow -\vec{x}$
- Time reversal               $\mathcal{T}$        $t \rightarrow -t$
- Charge conjugation         $\mathcal{C}$        $q \rightarrow -q$

$\mathcal{P}$ ,  $\mathcal{T}$  and  $\mathcal{C}$  are conserved in the classical theories of mechanics and electrodynamics!

$\mathcal{CPT} \leftrightarrow$  Lorentz invariance  $\oplus$  unitarity: is an essential building block of field theory

$\mathcal{CPT}$  : L particle  $\leftrightarrow$  R antiparticle

Neutrinos in the MSM are massless and exist only in two states: particle with negative helicity and antiparticle with positive one: Weyl fermion



$\mathcal{P}$ : L particle  $\leftrightarrow$  R particle

Parity violation is nowhere more obvious than in the neutrino sector: the reflection of a left-handed neutrino in a mirror is nothing !

### Summary of $\nu$ 's in SM:

Three flavors of massless neutrinos

$$W^- \rightarrow l_\alpha^- + \bar{\nu}_\alpha$$

$$W^+ \rightarrow l_\alpha^+ + \nu_\alpha$$

$$\alpha = e, \mu, \text{ or } \tau$$

Anti-neutrino,  $\bar{\nu}_\alpha$ , has +ve helicity, Right Handed

Neutrino,  $\nu_\alpha$ , has -ve helicity, Left Handed

$\nu_L$  and  $\bar{\nu}_R$  are CPT conjugates

massless implies helicity = chirality

## Beyond the SM

What if Neutrino have a MASS?

speed is less than c therefore time can pass  
and

Neutrinos can change character!!!

What are the stationary states?

How are they related to the interaction states?

## NEUTRINO OSCILLATIONS:

Two Flavors

flavor eigenstates  $\neq$  mass eigenstates

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

W's produce  $\nu_\mu$  and/or  $\nu_\tau$ 's

but  $\nu_1$  and  $\nu_2$  are the states  
that change by a phase over time, mass eigenstates.

$$|\nu_j\rangle \rightarrow e^{-ip_j \cdot x} |\nu_j\rangle \quad p_j^2 = m_j^2$$

$\alpha, \beta \dots$  flavor index       $i, j \dots$  mass index

Production:

$$|\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

Propagation:

$$\cos\theta e^{-ip_1 \cdot x}|\nu_1\rangle + \sin\theta e^{-ip_2 \cdot x}|\nu_2\rangle$$

Detection:

$$|\nu_1\rangle = \cos\theta|\nu_\mu\rangle - \sin\theta|\nu_\tau\rangle$$

$$|\nu_2\rangle = \sin\theta|\nu_\mu\rangle + \cos\theta|\nu_\tau\rangle$$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos\theta(e^{-ip_1 \cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2 \cdot x})\cos\theta|^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos\theta(e^{-ip_1 \cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2 \cdot x})\cos\theta|^2$$

$$\text{Same } E, \text{ therefore } p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$$

$$e^{-ip_j \cdot x} = e^{-iEt} e^{-ip_j L} \approx e^{-i(Et-EL)} e^{-im_j^2 L/2E}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2\theta \cos^2\theta |e^{-im_2^2 L/2E} - e^{-im_1^2 L/2E}|^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$\delta m^2 = m_2^2 - m_1^2 \text{ and } \frac{\delta m^2 L}{4E} \equiv \Delta \text{ kinematic phase:}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos \theta (e^{-ip_1 \cdot x}) (-\sin \theta) + \sin \theta (e^{-ip_2 \cdot x}) \cos \theta|^2$$

Same E, therefore  $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$

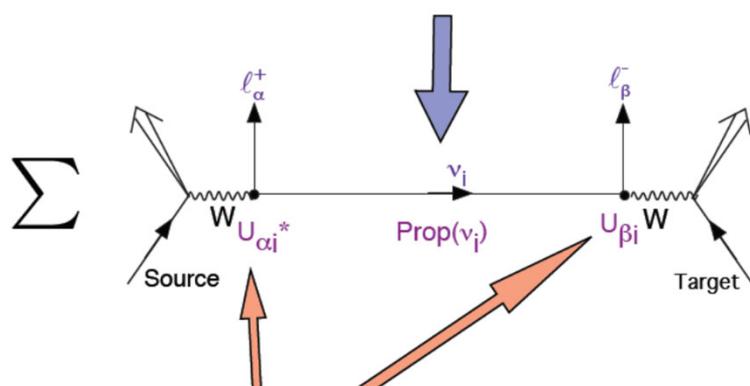
$$e^{-ip_j \cdot x} = e^{-iEt} e^{-ip_j L} \approx e^{-i(Et - EL)} e^{-im_j^2 L/2E}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \theta \cos^2 \theta |e^{-im_2^2 L/2E} - e^{-im_1^2 L/2E}|^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \left( \frac{\delta m^2 L}{4E} \frac{c^4}{hc} \right)$$

### Amplitude

$$e^{-im_j^2 L/2E}$$



$$U_{\alpha j} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

**Appearance:**

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

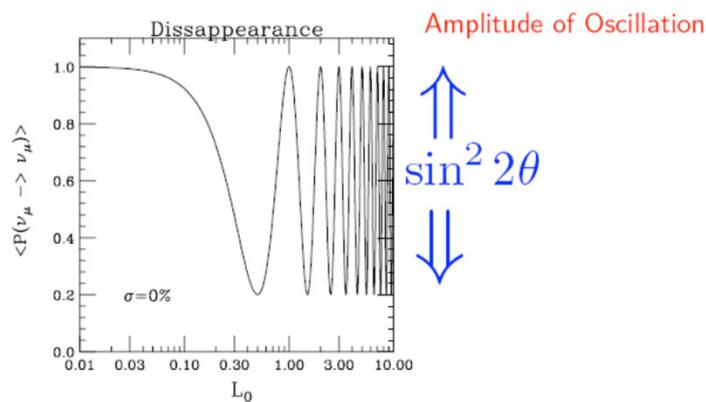
**Disappearance:**

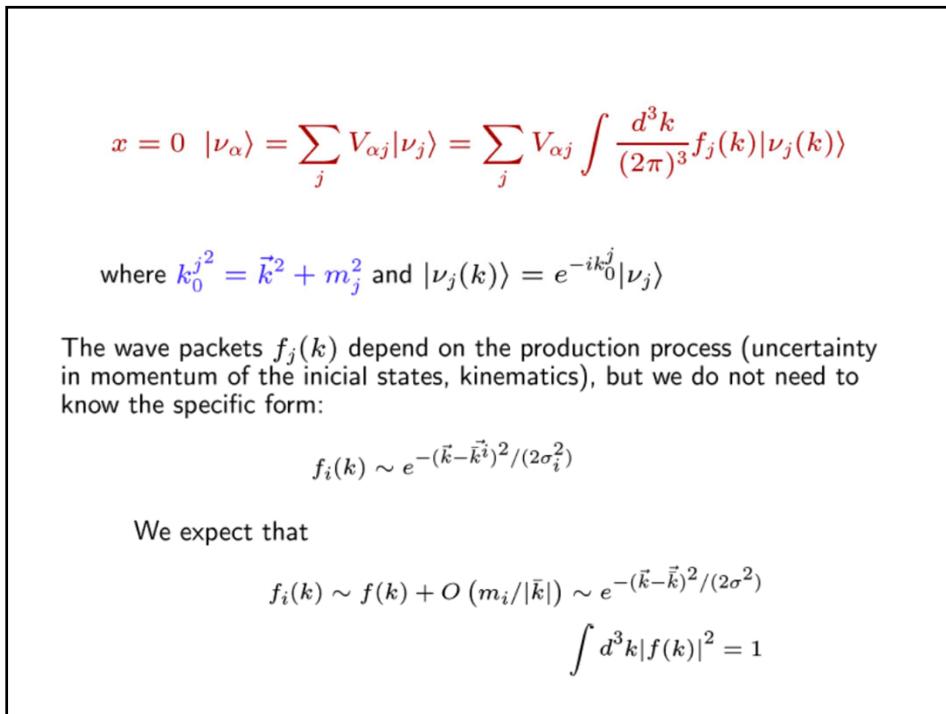
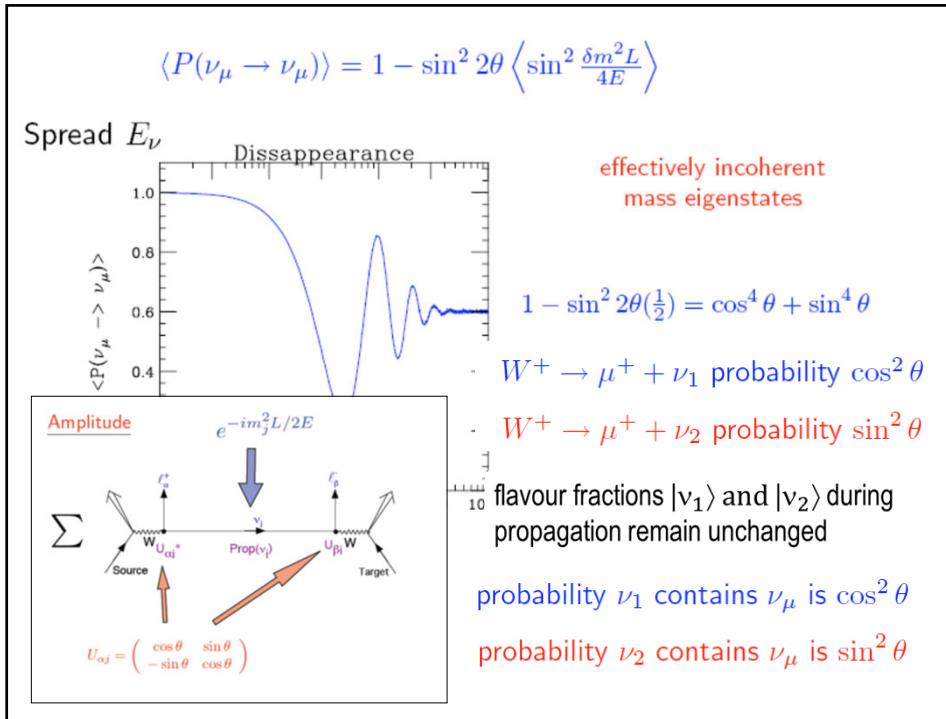
$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

Oscillation Length     $L_0 = 4\pi E / \delta m^2$

Fixed  $E_\nu$





We have a detector located at some distance down the beam line  $x = L$ :

$$x = L \quad |\nu_\alpha(x)\rangle = \sum_j \int \frac{d^3 k}{(2\pi)^2} e^{-i\sqrt{m_j^2 + \vec{k}^2}t} e^{i\vec{x}\cdot\vec{k}} f_j(k) V_{\alpha j} |\nu_j\rangle$$

where neutrino can interact producing a given flavour. The probability that the flavour we measured is  $\beta$  is:  $|\langle \nu_\beta | \nu_\alpha(x) \rangle|^2$ :

We can safely neglect terms of  $O(m_i/|\bar{k}|)$  except in the phase factor because they are enhanced by  $L$ :

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \sum_{i,j} V_{\beta j}^* V_{\alpha j} V_{\beta i} V_{\alpha i}^* \int d^3 k e^{-i\frac{\Delta_{ij}L}{2|k_x|}} |f(\vec{k})|^2 \\ &\simeq \sum_{i,j} V_{\beta j}^* V_{\alpha j} V_{\beta i} V_{\alpha i}^* e^{-i\frac{\Delta_{ij}L}{2|k_x|}} \end{aligned}$$

with  $\Delta_{ij} = m_j^2 - m_i^2$ .

There is an intrinsic limit to coherence, because  $\sigma \neq 0$ !

$$\left| \frac{\Delta_{ij}L_D}{2} \left( \frac{1}{|\bar{k}|} - \frac{1}{|\bar{k}| + \sigma} \right) \right| \sim 2\pi \Rightarrow L_D \sim L_{osc} \frac{|\bar{k}|}{\sigma}$$

If  $L \gg L_D$ :

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_i |V_{\alpha i} V_{\beta i}|^2 = 2 \cos^2 \theta \sin^2 \theta = \frac{1}{2} \sin^2 2\theta$$

In practice, the smearing in  $L$  and  $E_\nu$  produces the same effect when  $L \gg L_{osc}$ : averaged oscillations

$$\langle P(\nu_\alpha \rightarrow \nu_\beta) \rangle = \frac{1}{2} \sin^2 2\theta$$

Using the unitarity of the mixing matrix: ( $W_{\alpha\beta}^{jk} \equiv [V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k}]$ )

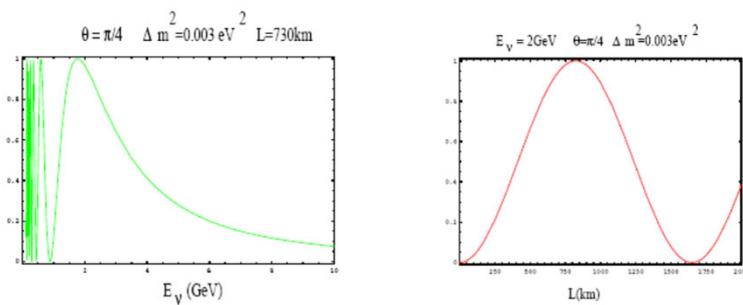
$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{k>j} \text{Re}[W_{\alpha\beta}^{jk}] \sin^2 \left( \frac{\Delta m_{jk}^2 L}{4E_\nu} \right)$$

$$\pm 2 \sum_{k>j} \text{Im}[W_{\alpha\beta}^{jk}] \sin \left( \frac{\Delta m_{jk}^2 L}{2E_\nu} \right)$$

For 2 families:  $V_{MNS} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right) \rightarrow \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance}$$



Oscillation probabilities show the expected **GIM** suppression of any flavour changing process: they vanish if the neutrinos are degenerate

## Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 =$$

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E\nu} \right) \rightarrow \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance}$$

## Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 =$$

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)$$

(appearance)  
(disappearance)

$$\left( 1.27 \frac{\Delta m^2 (eV^2) L(km)}{E(GeV)} \right)$$

L/E becomes crucial !!!

## Evidence for Flavor Change:

★★★ Atmospheric and Accelerator Neutrinos with  $L/E = 500 \text{ km/GeV}$

★★★ Solar and Reactor Neutrinos with  $L/E = 15 \text{ km/MeV}$

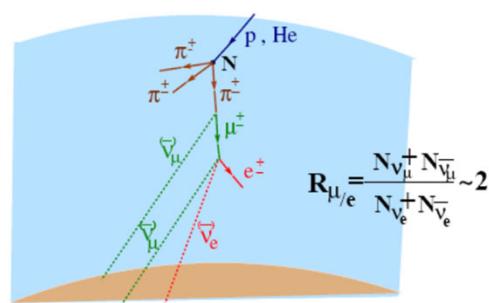
Neutrinos from Stopped muons  $L/E = 2m/\text{MeV}$  (Unconfirmed)

### Atmospheric neutrinos

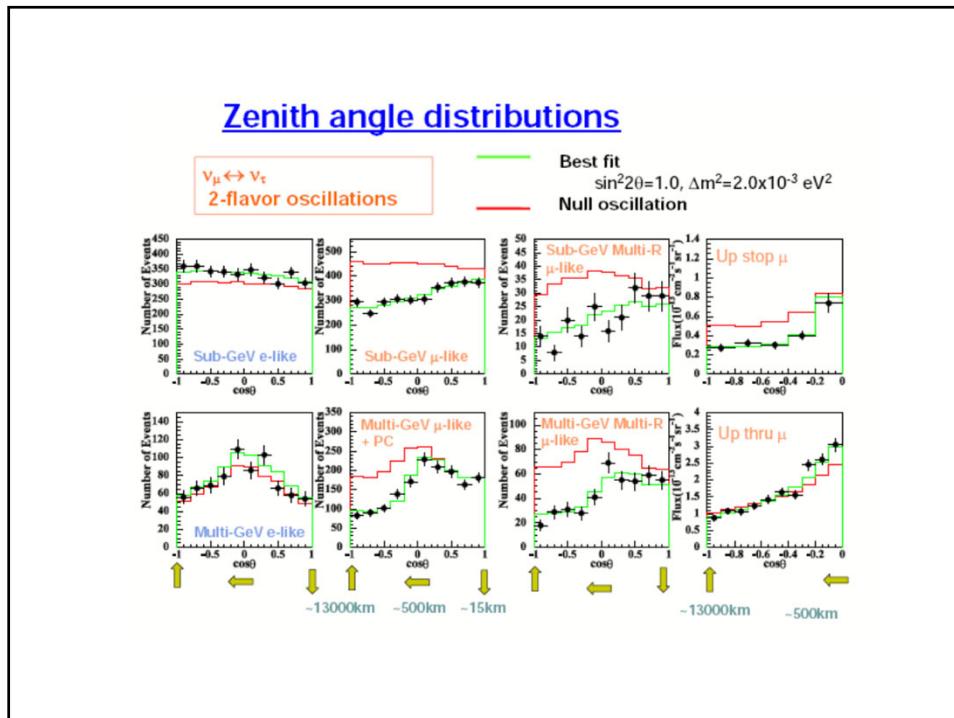
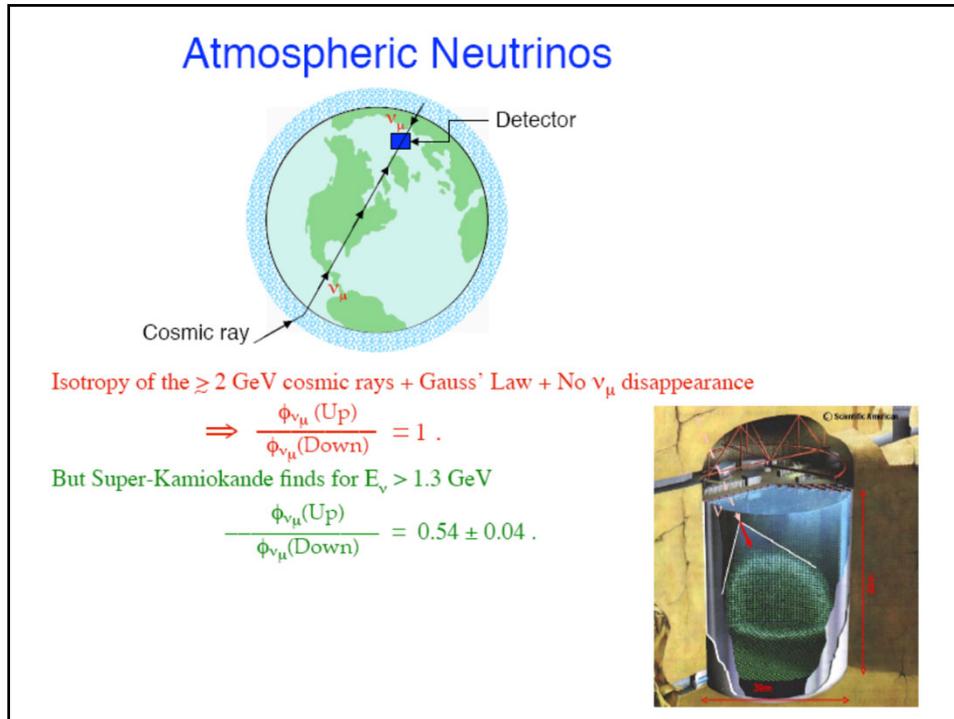
- Atmospheric neutrinos are produced by the interaction of *cosmic rays* ( $p, \text{He}, \dots$ ) with the Earth's atmosphere:

- 1  $A_{\text{cr}} + A_{\text{air}} \rightarrow \pi^\pm, K^\pm, K^0, \dots$
- 2  $\pi^\pm \rightarrow \mu^\pm + \nu_\mu,$
- 3  $\mu^\pm \rightarrow e^\pm + \nu_e + \bar{\nu}_\mu;$

- at the detector, some  $\nu$  interacts and produces a **charged lepton**, which is observed.



A deficit was observed in the ratio  $\mu/e$  events: **Soudan2, IMB, Kamiokande**



Half of the upward-going, long-distance-traveling  $\nu_\mu$  are disappearing.

Voluminous atmospheric neutrino data are well described by —

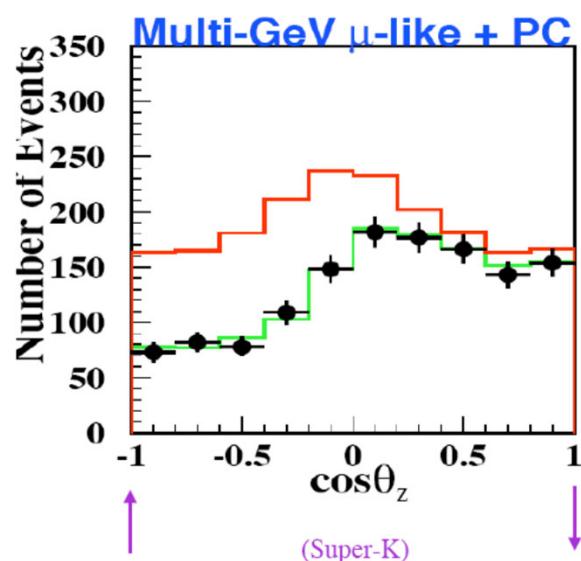
$$\nu_\mu \longrightarrow \nu_\tau$$

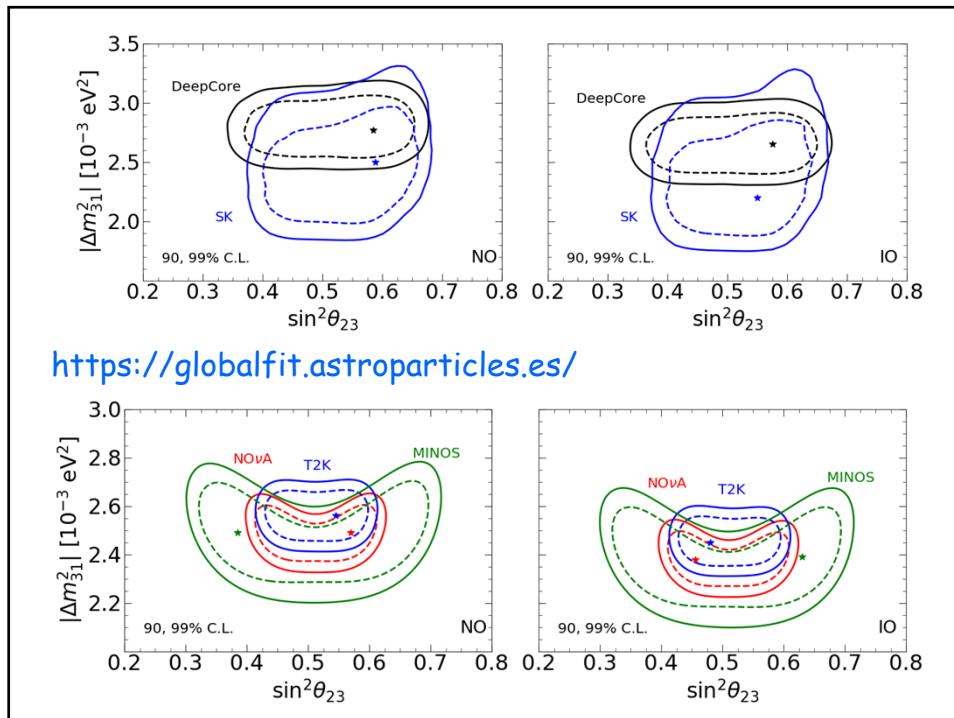
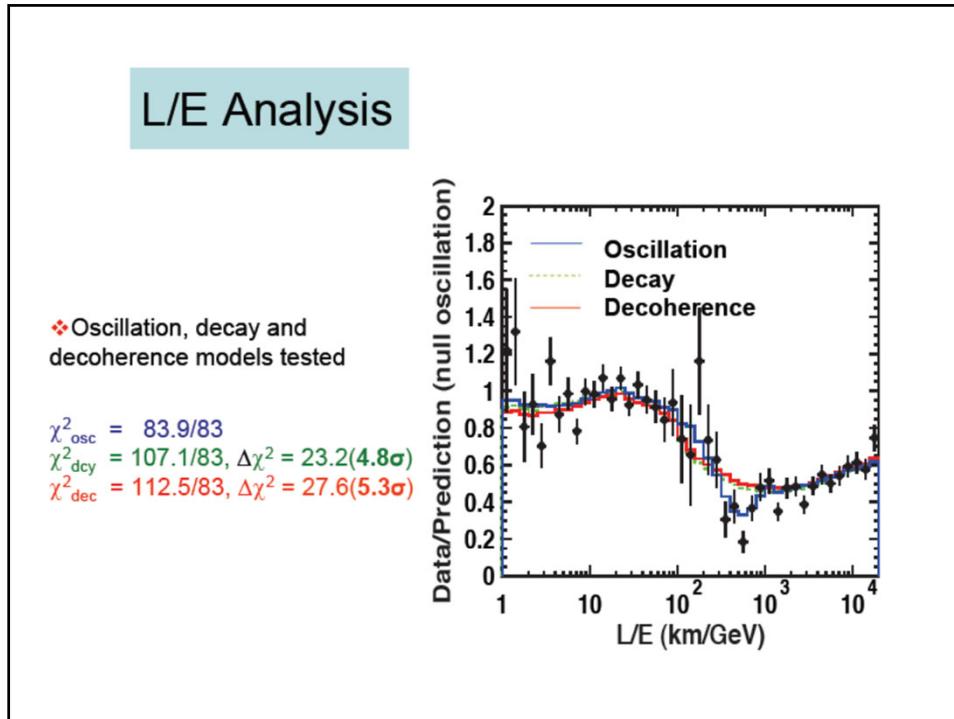
with —

$$\Delta m_{\text{atm}}^2 \cong 2.4 \cdot 10^{-3} \text{ eV}^2$$

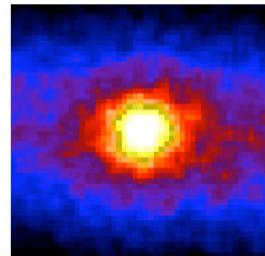
and —

$$\sin^2 2\theta_{\text{atm}} \cong 1$$





# Solar $\delta m^2$



Solar Engine:

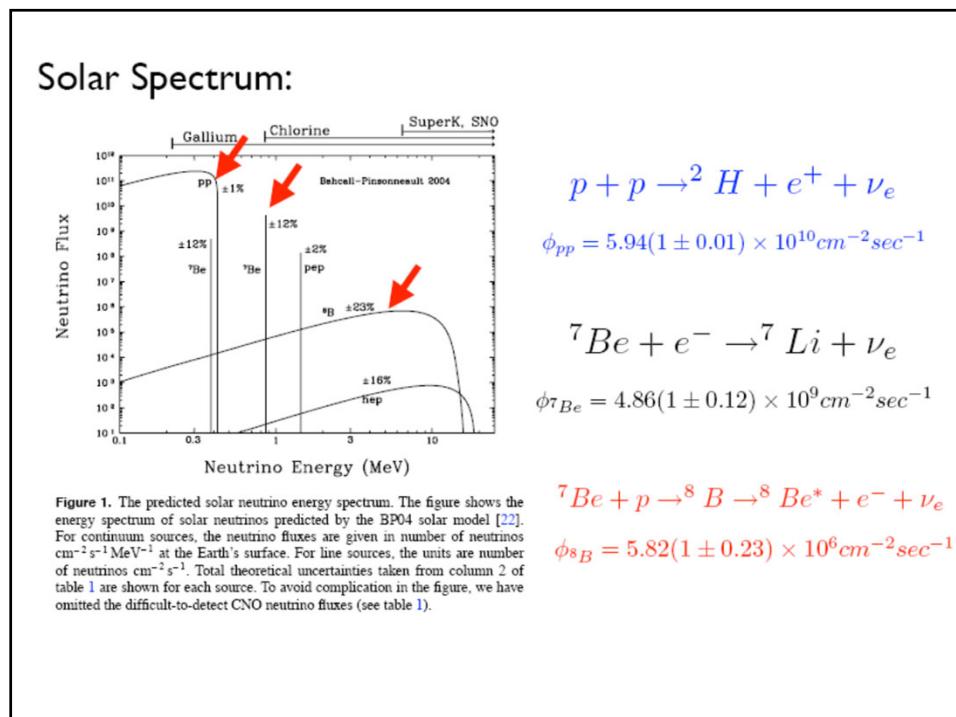
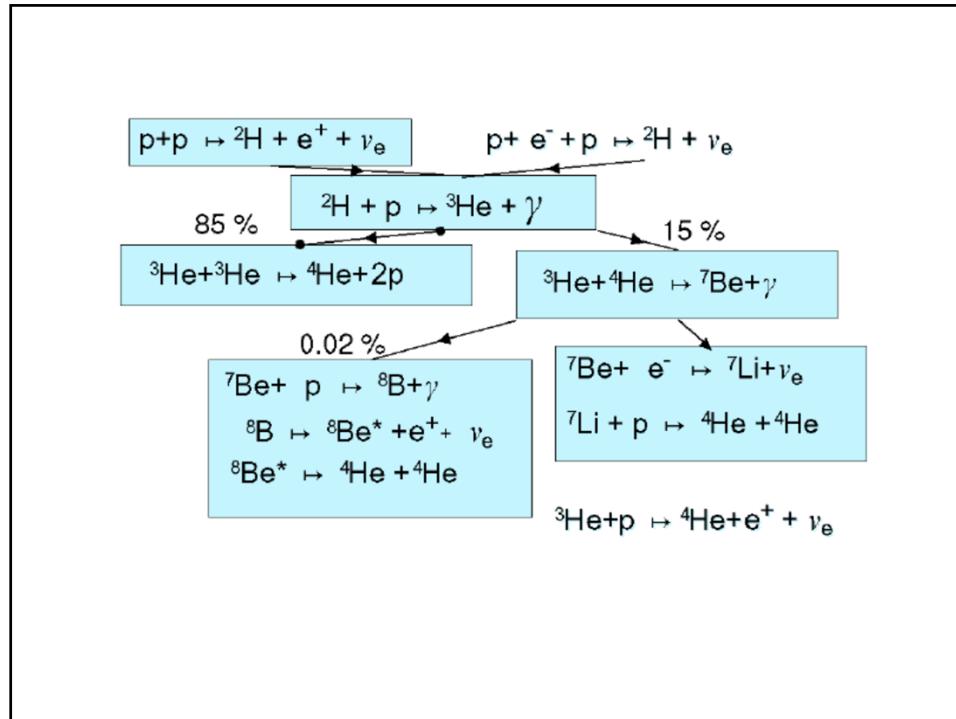


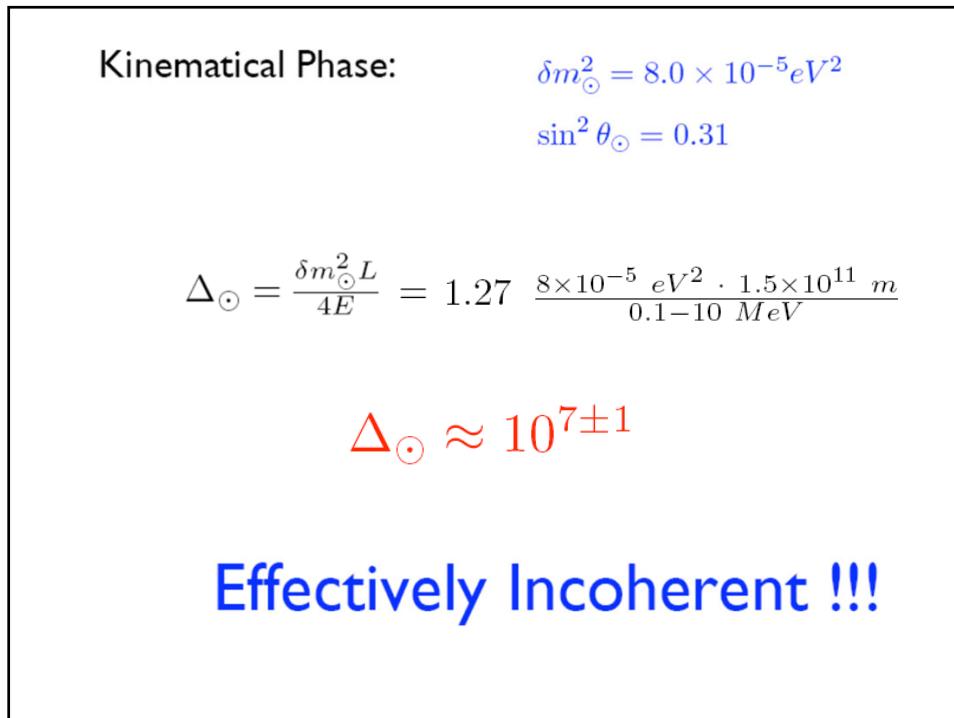
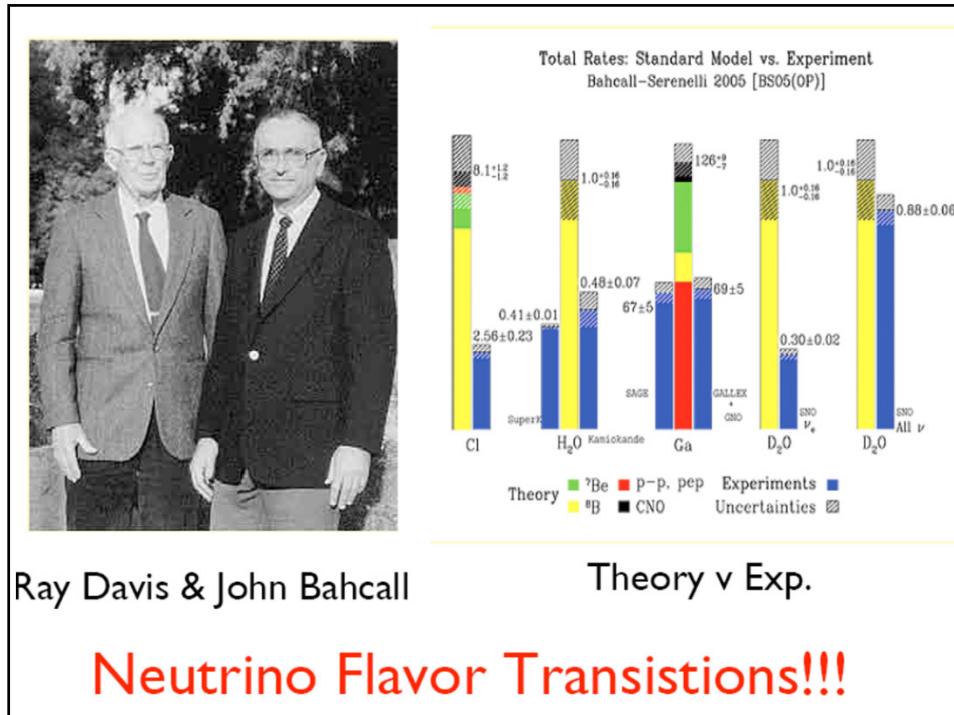
1  $\nu_e$  for every 13.4 MeV ( $= 2.1 \times 10^{-12}$  J)

$\mathcal{L}_\odot$  at earth's surface 0.13 watts/cm<sup>2</sup>

$$\phi_\nu = \frac{0.13}{2.1 \times 10^{-12}} = 6 \times 10^{10} / cm^2/sec$$

This corresponds to an average of 2  $\nu$ 's per cm<sup>3</sup>  
since they are going at speed c.



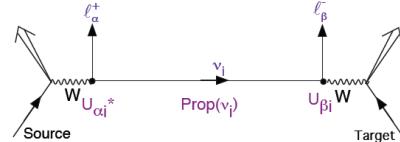


Vacuum  $\nu_e$  Survival Probability:

$$\langle P_{ee} \rangle = f_1 \cos^2 \theta_\odot + f_2 \sin^2 \theta_\odot$$

where  $f_1$  and  $f_2$  are the fraction of  $\nu_1$  and  $\nu_2$  at production.

In vacuum  $f_1 = \text{co} \sum$

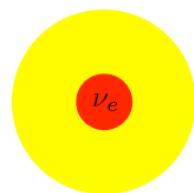


$$\langle P_{ee} \rangle = \cos^4 \theta_\odot + \sin^4 \theta_\odot = 1 - \frac{1}{2} \sin^2 2\theta_\odot$$

for pp and  ${}^7\text{Be}$  this is approximately THE ANSWER.

$$f_1 \sim 69\% \text{ and } f_2 \sim 31\% \text{ and } \langle P_{ee} \rangle \approx 0.6$$

pp and  ${}^7\text{Be}$



$$\begin{array}{ccc} \nu_1 & & \\ \nu_2 & \nu_1 & \nu_1 \\ \hline \nu_1 & \nu_1 & \nu_2 \\ \nu_1 & \nu_2 & \nu_1 \\ \hline & \nu_1 & \end{array} \quad \begin{array}{l} f_1 \sim 69\% \\ f_2 \sim 31\% \end{array}$$

$$\langle P_{ee} \rangle \approx 0.6$$

$$f_3 = \sin^2 \theta_{13} < 4\%$$

What about  $^8B$  ?

### SNO's CC/NC

CC:  $\nu_e + d \rightarrow e^- + p + p$

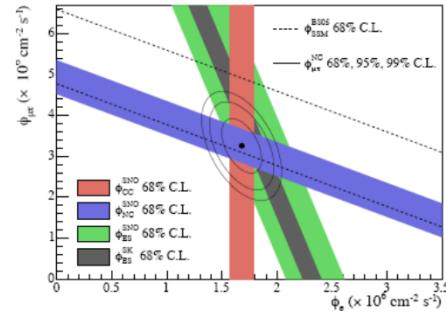
NC :  $\nu_x + d \rightarrow \nu_x + p + n$

ES:  $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$

$$\frac{CC}{NC} = \langle P_{ee} \rangle = f_1 \cos^2 \theta_\odot + f_2 \sin^2 \theta_\odot$$

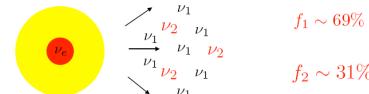
$$f_1 = \left( \frac{CC}{NC} - \sin^2 \theta_\odot \right) / \cos 2\theta_\odot$$

$$= (0.35 - 0.31) / 0.4 \approx 10$$



$^8B$

pp and  $^7Be$



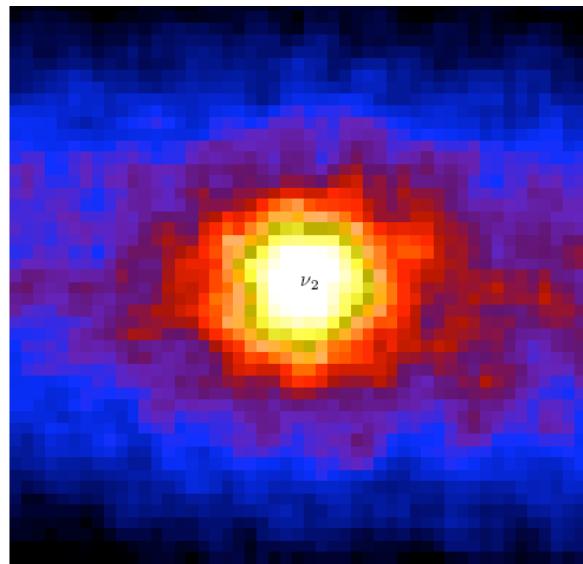
$f_2 \sim 90\%$

$f_1 \sim 10\%$

$$\langle P_{ee} \rangle = \sin^2 \theta + f_1 \cos 2\theta_\odot \approx \sin^2 \theta_\odot = 0.31$$

Wow!!! How did that happen???

energy dependence!!!



These are  $\nu_2$  Neutrinos !!!