

Neutrino physics
(theory & phenomenology)

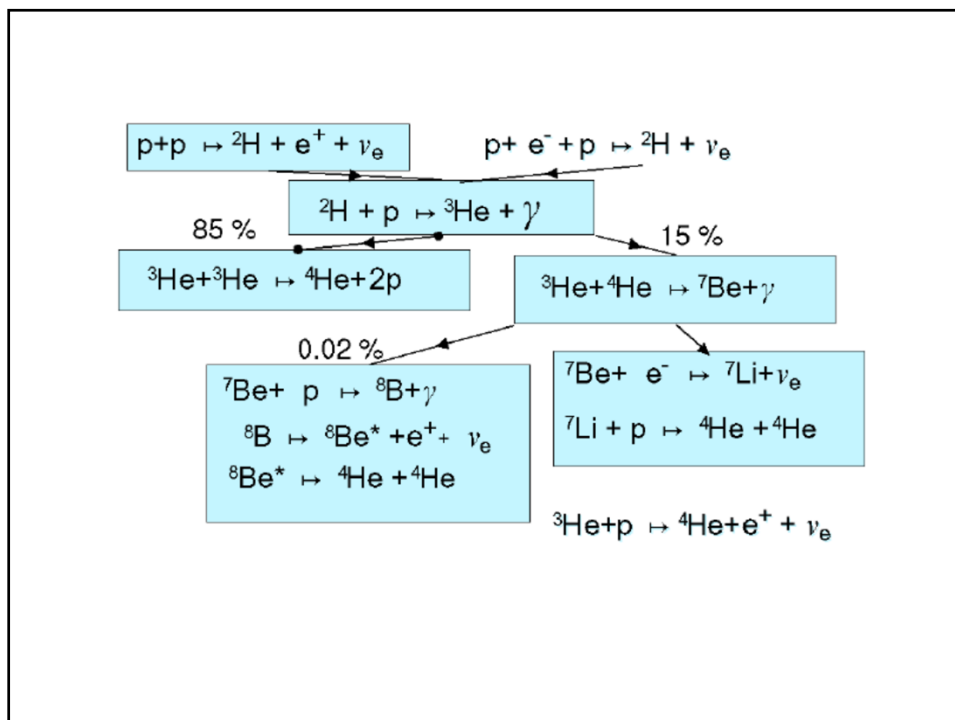
Gabriela Barenboim
U.Valencia and IFIC

Niels Bohr Institute, July 2023

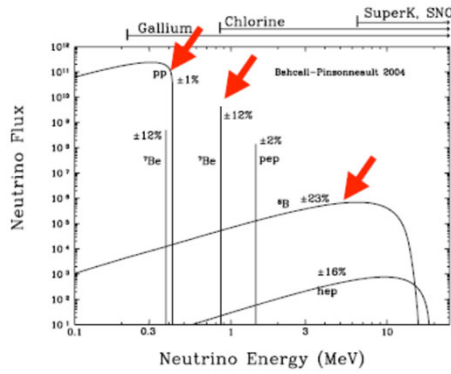
ν_{II}

HIDDe
 hunting Unobservable Dark sectors, Dark matter and Neutrinos

VNIVERSITAT ID VALÈNCIA
GENERALITAT VALENCIANA
IFIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
CSIC



Solar Spectrum:



$$p + p \rightarrow {}^2\text{H} + e^+ + \nu_e$$

$$\phi_{pp} = 5.94(1 \pm 0.01) \times 10^{10} \text{ cm}^{-2} \text{ sec}^{-1}$$

$${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$$

$$\phi_{{}^7\text{Be}} = 4.86(1 \pm 0.12) \times 10^9 \text{ cm}^{-2} \text{ sec}^{-1}$$

$${}^7\text{Be} + p \rightarrow {}^8\text{B} \rightarrow {}^8\text{Be}^* + e^- + \nu_e$$

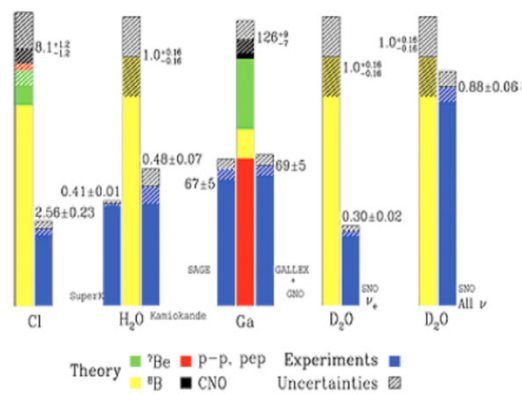
$$\phi_{{}^8\text{B}} = 5.82(1 \pm 0.23) \times 10^6 \text{ cm}^{-2} \text{ sec}^{-1}$$

Figure 1. The predicted solar neutrino energy spectrum. The figure shows the energy spectrum of solar neutrinos predicted by the BP04 solar model [22]. For continuum sources, the neutrino fluxes are given in number of neutrinos $\text{cm}^{-2} \text{s}^{-1} \text{MeV}^{-1}$ at the Earth's surface. For line sources, the units are number of neutrinos $\text{cm}^{-2} \text{s}^{-1}$. Total theoretical uncertainties taken from column 2 of table 1 are shown for each source. To avoid complication in the figure, we have omitted the difficult-to-detect CNO neutrino fluxes (see table 1).



Ray Davis & John Bahcall

Total Rates: Standard Model vs. Experiment
Bahcall-Serenelli 2005 [BS05(OP)]



Theory v Exp.

Neutrino Flavor Transitions!!!

8B

$\langle P_{ee} \rangle = \sin^2 \theta + f_1 \cos 2\theta_{\odot} \approx \sin^2 \theta_{\odot} = 0.31$

Wow!!! How did that happen???

energy dependence!!!

pp and 7Be

$f_1 \sim 69\%$
 $f_2 \sim 31\%$

$f_2 \sim 90\%$
 $f_1 \sim 10\%$

MSW

Coherent Forward Scattering:

Wolfenstein '78

MATTER EFFECTS
CHANGE THE NEUTRINO
MASSES AND MIXINGS

$m_i^2 (10^{-5} \text{ eV}^2)$ vs $\rho Y_e (E_\nu/10\text{MeV}) (\text{g}\cdot\text{cm}^{-3})$

▲ Mikheyev + Smirnov Resonance WIN '85

Neutrino Evolution:

$$-i \frac{\partial}{\partial t} \nu = H \nu$$

in the mass eigenstate basis

$$\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \text{ and } H = \begin{pmatrix} \sqrt{p^2 + m_1^2} & 0 \\ 0 & \sqrt{p^2 + m_2^2} \end{pmatrix}$$

$E = \sqrt{p^2 + m^2}$

$$H = \left(p + \frac{m_1^2 + m_2^2}{4p} \right) I + \frac{1}{4E} \begin{pmatrix} -\delta m^2 & 0 \\ 0 & \delta m^2 \end{pmatrix}$$

$$\delta m^2 = m_2^2 - m_1^2 > 0$$

in the flavor basis

$$\nu \rightarrow U \nu \text{ and } H \rightarrow U H U^\dagger$$

$$\text{where } \nu = \begin{pmatrix} \nu_e \\ \nu_\sigma \end{pmatrix} \text{ and } U = \begin{pmatrix} \cos \theta_\odot & \sin \theta_\odot \\ -\sin \theta_\odot & \cos \theta_\odot \end{pmatrix}$$

and therefore in flavor basis

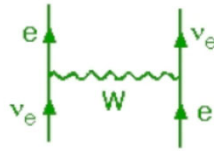
$$0 < \theta_\odot < \frac{\pi}{2}$$

$$H = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}$$

$$\text{i.e. } \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}_{mass} \Rightarrow \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}_{flavor}$$

Coherent Forward Scattering:

dimensions $[G_F N_e] = M^{-2} L^{-3} = M$

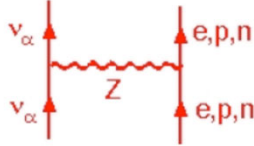


$$\pm \sqrt{2} G_F N_e \delta_{ee}$$

N_e is number density of electrons

+(-) for neutrinos (anti-neutrinos)

Wolfenstein '78



Same for all active flavors,
therefore overall phases

$$\begin{pmatrix} +\sqrt{2} G_F N_e & 0 \\ 0 & 0 \end{pmatrix} \rightarrow \frac{G_F N_e}{\sqrt{2}} I_2 + \frac{1}{2} \begin{pmatrix} +\sqrt{2} G_F N_e & 0 \\ 0 & -\sqrt{2} G_F N_e \end{pmatrix}$$

Including Matter Effects in the Flavor Basis:

$$H_{flavor} = \frac{1}{4E_\nu} \begin{pmatrix} -\delta m^2 \cos 2\theta_\odot + 2\sqrt{2} G_F N_e E_\nu & \delta m^2 \sin 2\theta_\odot \\ \delta m^2 \sin 2\theta_\odot & \delta m^2 \cos 2\theta_\odot - 2\sqrt{2} G_F N_e E_\nu \end{pmatrix}$$

Diagonalize by identifying with

$$H_{flavor} = \frac{1}{4E_\nu} \begin{pmatrix} -\delta m_N^2 \cos 2\theta_\odot^N & \delta m_N^2 \sin 2\theta_\odot^N \\ \delta m_N^2 \sin 2\theta_\odot^N & \delta m_N^2 \cos 2\theta_\odot^N \end{pmatrix}$$

Masses and Mixings in MATTER: δm_N^2 and θ_\odot^N

$$\begin{aligned} \delta m_N^2 \cos 2\theta_\odot^N &= \delta m^2 \cos 2\theta_\odot - 2\sqrt{2} G_F N_e E_\nu \\ \delta m_N^2 \sin 2\theta_\odot^N &= \delta m^2 \sin 2\theta_\odot \end{aligned}$$

Notice:

- (1) Possible zero when $\delta m^2 \cos 2\theta_\odot = 2\sqrt{2} G_F N_e E_\nu$
- (2) the invariance of the product $\delta m^2 \sin 2\theta_\odot$

ν_e disappearance in Looong Block of Lead:

$$1 - P(\nu_e \rightarrow \nu_e) = \sin^2 2\theta_{\odot}^N \sin^2 \Delta_N$$

$$\Delta_N = \frac{\delta m_N^2 L}{4E}$$

same form as vacuum

The Solution:

$$\delta m_N^2 = \sqrt{(\delta m^2 \cos 2\theta_{\odot} - 2\sqrt{2}G_F N_e E_{\nu})^2 + (\delta m^2 \sin 2\theta_{\odot})^2}$$

$$\sin^2 \theta_{\odot}^N = \frac{1}{2} \left(1 - \frac{(\delta m^2 \cos 2\theta_{\odot} - 2\sqrt{2}G_F N_e E_{\nu})}{\delta m_N^2} \right) \quad \theta_{\odot}^N > \theta_{\odot}$$

Quasi-Vacuum: $2\sqrt{2}G_F N_e E_{\nu} \ll \delta m^2 \cos 2\theta_{\odot}$ pp and ${}^7\text{Be}$

$$\delta m_N^2 = \delta m^2 \text{ and } \theta_{\odot}^N = \theta_{\odot}$$

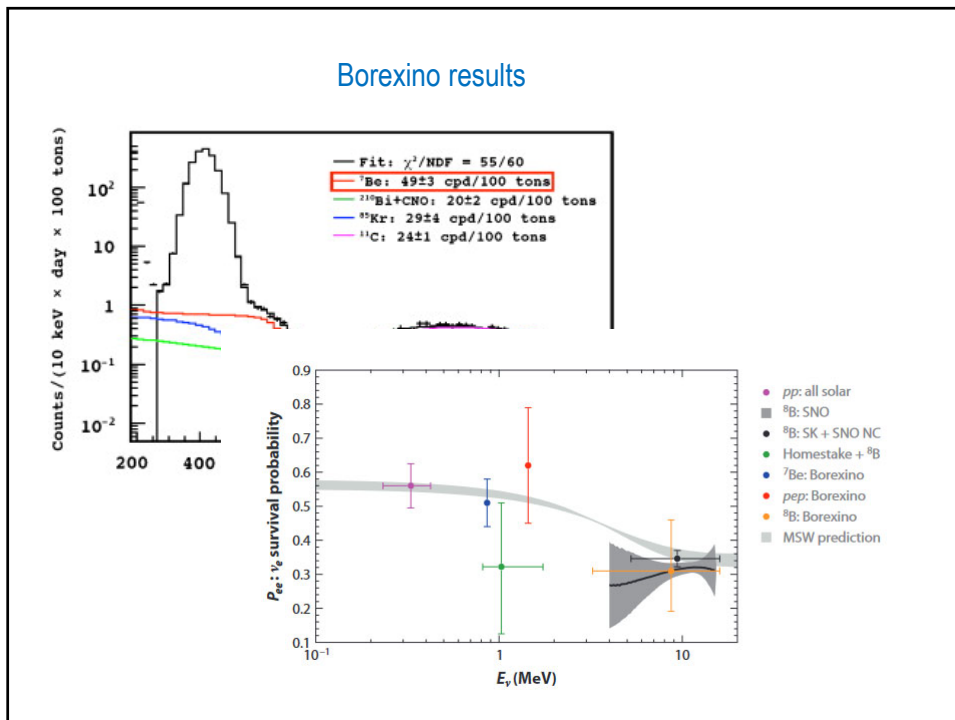
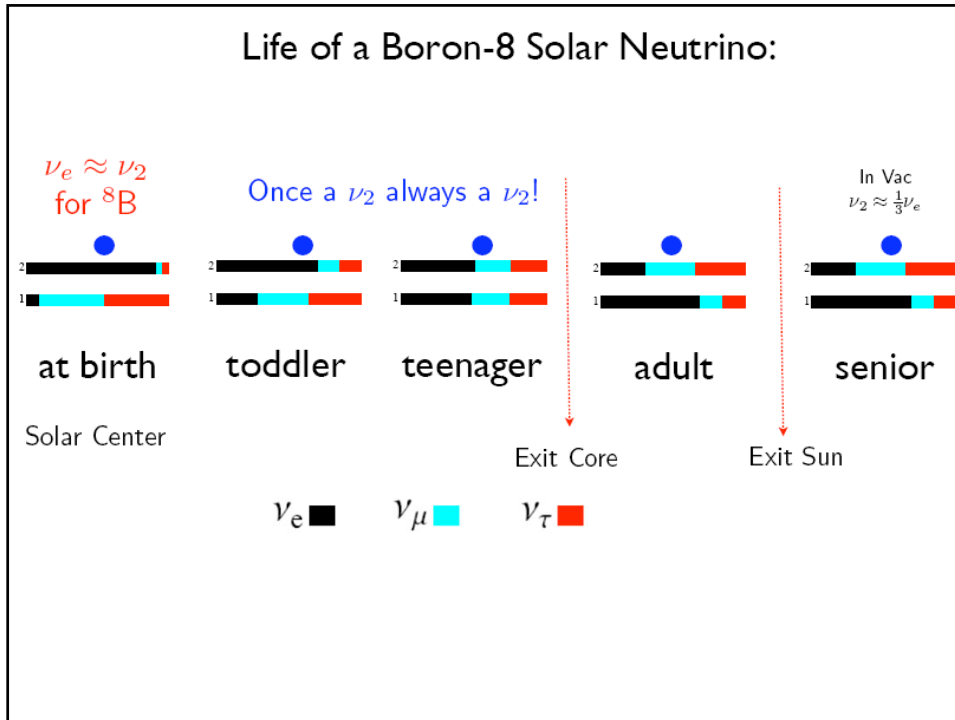
Resonance (Mikheyev + Smirnov '85): $2\sqrt{2}G_F N_e E_{\nu} = \delta m^2 \cos 2\theta_{\odot}$

$$\delta m_N^2 = \delta m^2 \sin 2\theta_{\odot} \text{ and } \theta_{\odot}^N = \pi/4$$

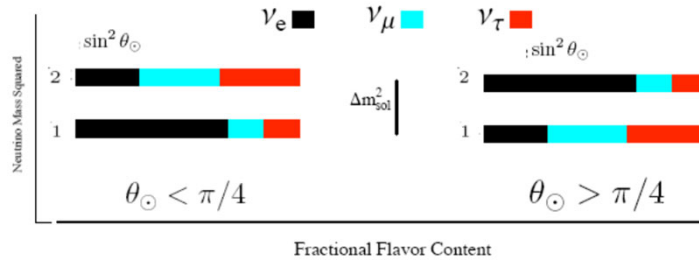
Matter Dominated: $2\sqrt{2}G_F N_e E_{\nu} \gg \delta m^2 \cos 2\theta_{\odot}$

$$\delta m_N^2 \rightarrow 2\sqrt{2}G_F N_e E_{\nu} \text{ and } \theta_{\odot}^N \rightarrow \pi/2$$

${}^8\text{B}$



Solar Pair Mass Hierarchy:



Who cares ?
SNO does !!!

for neutrino in matter
 $\theta_{\odot}^N > \theta_{\odot}$

$$\langle P_{ee} \rangle = \cos^2 \theta_{\odot}^N \cos^2 \theta_{\odot} + \sin^2 \theta_{\odot}^N \sin^2 \theta_{\odot} = \frac{1}{2} + \frac{1}{2} \cos 2\theta_{\odot}^N \cos 2\theta_{\odot}$$

if $\theta_{\odot} < \pi/4$
 $\langle P_{ee} \rangle \geq \sin^2 \theta_{\odot}$

if $\theta_{\odot} > \pi/4$
 $\langle P_{ee} \rangle \geq \frac{1}{2}(1 + \cos^2 2\theta_{\odot}) \geq \frac{1}{2}$

SNO: $\langle P_{ee} \rangle_{day} = 0.347 \pm 0.038$

**Solar Hierarchy
Determined !!!**

Day/Night Asymmetry:

$$\sin^2 \theta_{\odot} \rightarrow \sin^2 \theta_{\oplus} = \sin^2 \theta_{\odot} + \frac{1}{2} \sin^2 2\theta_{\odot} \left(\frac{A_{\oplus}}{\delta m_{\odot}^2} \right) \text{ in the earth.}$$

$A=2(D-N)/(D+N)$ expected to be few %

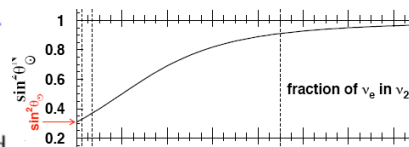
	Amplitude fit		separate D, N:
	Δm	Δm	$(D-N)/(D+N)/2$
SK-I	$-2.0 \pm 1.8 \pm 1.0\%$	$-1.9 \pm 1.7 \pm 1.0\%$	$-2.1 \pm 2.0 \pm 1.3\%$
SK-II	$-4.4 \pm 3.8 \pm 1.0\%$	$-4.4 \pm 3.6 \pm 1.0\%$	$-5.5 \pm 4.2 \pm 3.7\%$
SK-III	$-4.2 \pm 2.7 \pm 0.7\%$	$-3.8 \pm 2.6 \pm 0.7\%$	$-5.9 \pm 3.2 \pm 1.3\%$
SK-IV	$-3.6 \pm 1.6 \pm 0.6\%$	$-3.3 \pm 1.5 \pm 0.6\%$	$-4.9 \pm 1.8 \pm 1.4\%$
comb	$-3.3 \pm 1.0 \pm 0.5\%$	$-3.1 \pm 1.0 \pm 0.5\%$	$-4.1 \pm 1.2 \pm 0.8\%$
non-zero signif.	3.0σ	2.8σ	2.8σ

Spectral Distortion:

A characteristic of matter effects is that the Fraction of ν_2 is energy dependent .

Smaller at smaller E.

Implies an increase in P_{ee} near threshold.



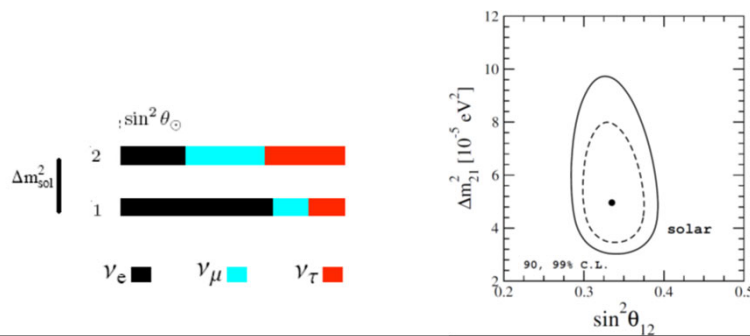
Summary:

The low energy pp and ${}^7\text{Be}$ Solar Neutrinos exit the sun as two thirds ν_1 and one third ν_2 due to (quasi-) vacuum oscillations.

$$f_1 = 65 \pm 2\%, f_2 = 35 \mp 2\% \text{ with } P_{ee} \approx 0.56$$

The high energy ${}^8\text{B}$ Solar Neutrinos exit the sun as "PURE" ν_2 mass eigenstates due to matter effects.

$$f_2 = 91 \pm 2\% \text{ and } f_1 = 9 \mp 2\% \text{ with } P_{ee} \approx 0.35.$$



Testing solar neutrino oscillations with reactors

$$1 - P(\nu_e \rightarrow \nu_e) = \sin^2 2\theta_{12} \sin^2 \Delta$$

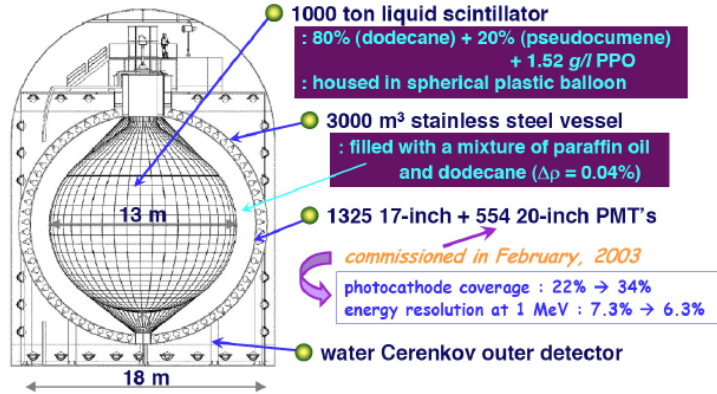
$$\Delta = \frac{10^{-5} \text{ eV}^2}{4E} L \quad 10^5 \text{ m} = 100 \text{ km}$$

1 MeV

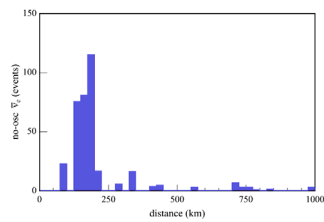
Reactor Neutrinos

KamLAND Detector

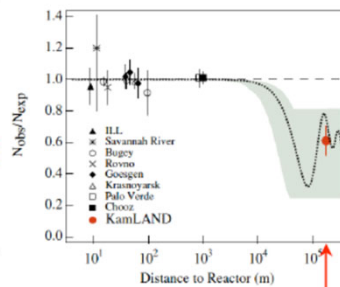
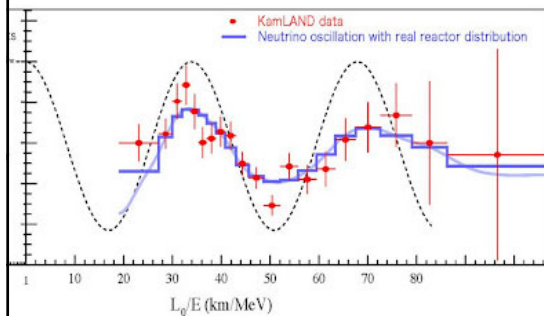
- detector location: old Kamiokande site
: 2700 m.w.e.



expected no-oscillation neutrino event rate at KamLAND



180 km



180 km

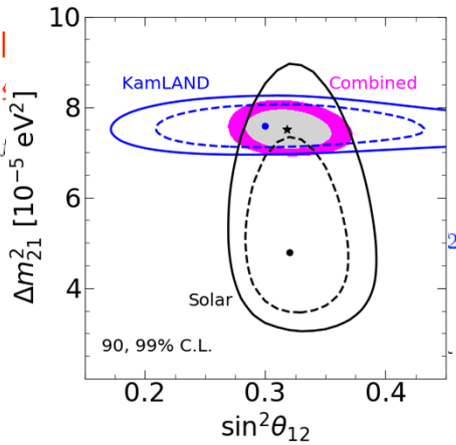
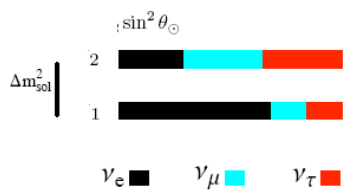
Summary:

The low energy pp and ⁷Be Solar Neutrinos exit the sun as two thirds ν_1 and one third ν_2 due to (quasi-) vacuum oscillations.

$$f_1 = 65 \pm 2\%, f_2 = 35 \mp 2\% \text{ with } P_{ee} \approx 0.56$$

The high energy ⁸B Sol "PURE" ν_2 mass eigenstate

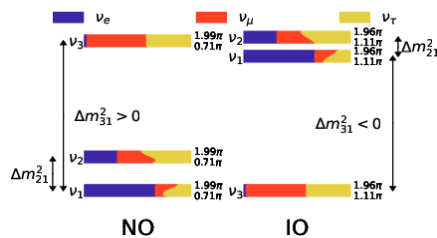
$$f_2 = 91 \pm 2\% \text{ and } j$$



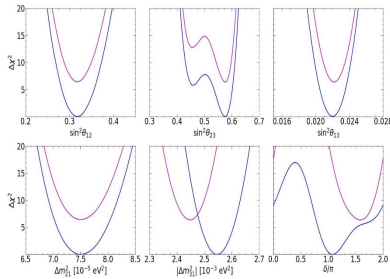
Three-neutrino oscillations

Neutrino mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



<https://globalfit.astroparticles.es/>



de Salas et al, JHEP 02 (2021) 071 [arXiv:2006.11237]

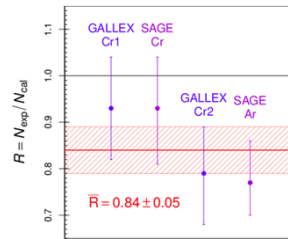
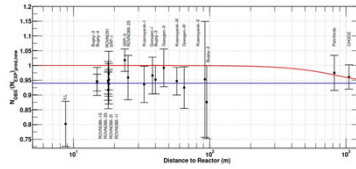
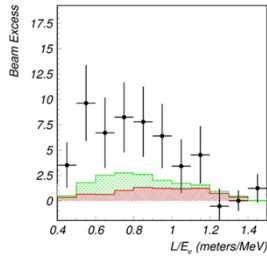
parameter	best fit $\pm 1\sigma$	2σ range	3σ range	Relative 1σ uncertainty
$\Delta m_{21}^2 [10^{-5} \text{eV}^2]$	$7.50^{+0.22}_{-0.20}$	7.12-7.93	6.94-8.14	2.7% PRECISION
$ \Delta m_{31}^2 [10^{-3} \text{eV}^2]$ (NO)	$2.55^{+0.02}_{-0.03}$	2.49-2.60	2.47-2.63	1.1% ORDERING?
$ \Delta m_{31}^2 [10^{-3} \text{eV}^2]$ (IO)	$2.45^{+0.02}_{-0.03}$	2.39-2.50	2.37-2.53	5.2% PRECISION
$\sin^2 \theta_{12}/10^{-1}$	3.18 ± 0.16	2.86-3.52	2.71-3.69	5.1% OCTANT?
$\sin^2 \theta_{23}/10^{-1}$ (NO)	5.74 ± 0.14	5.41-5.99	4.34-6.10	3.0% PRECISION
$\sin^2 \theta_{23}/10^{-1}$ (IO)	$5.78^{+0.10}_{-0.17}$	5.41-5.98	4.33-6.08	20% CPV?
$\sin^2 \theta_{13}/10^{-2}$ (NO)	$2.200^{+0.069}_{-0.062}$	2.069-2.337	2.000-2.405	9.0%
$\sin^2 \theta_{13}/10^{-2}$ (IO)	$2.225^{+0.064}_{-0.070}$	2.086-2.356	2.018-2.424	
δ/π (NO)	$1.08^{+0.13}_{-0.12}$	0.84-1.42	0.71-1.99	
δ/π (IO)	$1.58^{+0.15}_{-0.16}$	1.26-1.85	1.11-1.96	



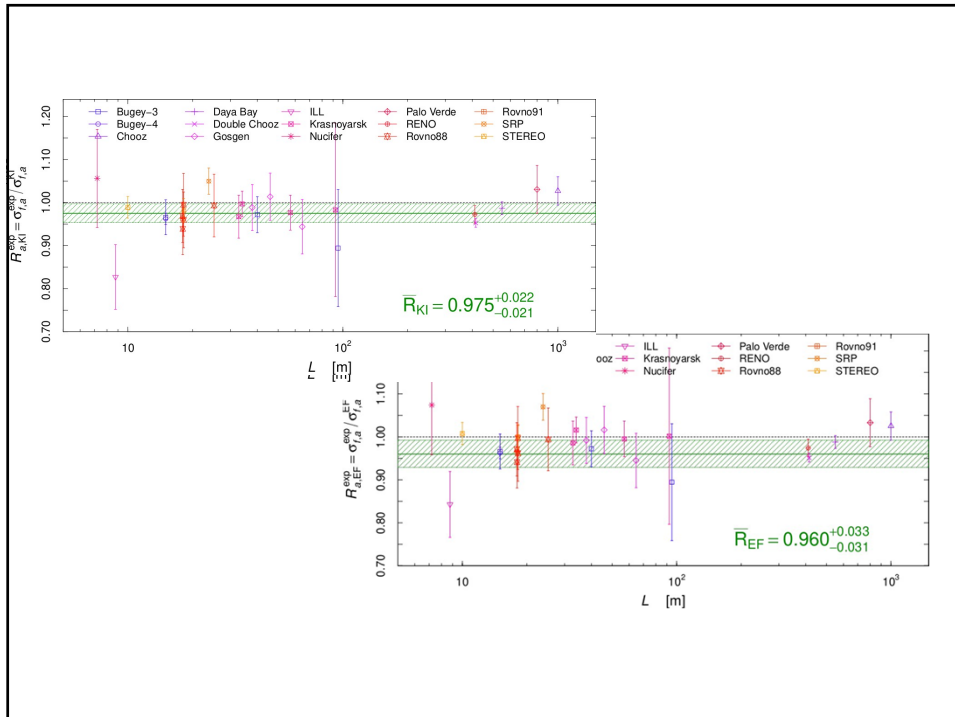
Parameter	Main contribution	Other contributions
θ_{12}	SOL	KamLAND
θ_{13}	REAC	ATM+LBL and SOL+KamLAND
θ_{23}	ATM+LBL	-
δ_{CP}	LBL	ATM
Δm_{21}^2	KamLAND	SOL
$ \Delta m_{31}^2 $	LBL+ATM+REAC	-
MO	LBL+REAC and ATM	-

SOL: Solar
 ATM: Armtopsheric neutrinos
 LBL: Long baseline accelerator experiments
 REAC: Short-baseline reactor experiments

Anomalies



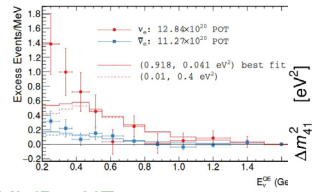
Need extra states !!!



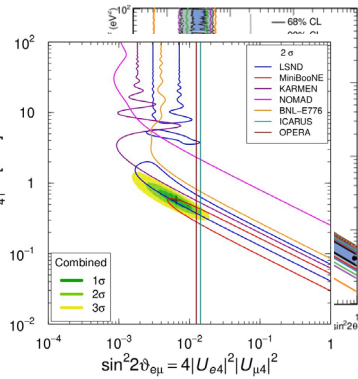
MiniBooNE

MiniBooNE was built to check the LSND results with a different baseline, but similar L/E

MiniBooNE has no near det



MiniBooNE sees an excess at $\sim 5\sigma$ at low energies



MicroBooNE

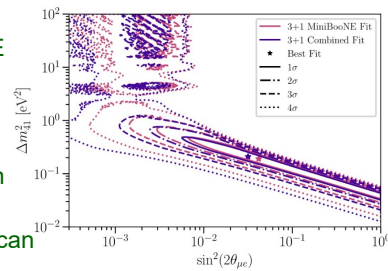
MicroBooNE was built to check the MiniBooNE results!

Looking for signals using several final state channels

The collaboration did not perform an oscillation analysis

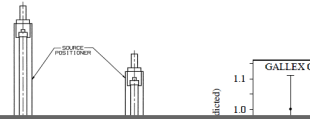
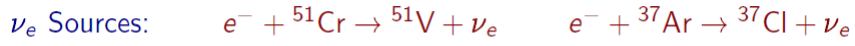
A combined analysis shows that MicroBooNE can not exclude the region of parameter space preferred by MiniBooNE

2201.01724



The Gallium Anomaly

Tests of the solar neutrino detectors **GALLEX** (Cr1, Cr2) and **SAGE** (Cr, Ar)



Deficit could be due to overestimate of $\sigma(\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-)$

Bahcall:

[Bahcall, PRC 56 (1997) 3391, hep-ph/9710491]

$$\sigma({}^{51}\text{Cr}) = 58.1 \times 10^{-46} \text{ cm}^2 \left(1_{-0.028}^{+0.036}\right)_{1\sigma} \implies R_{\text{Ga}} = 0.86 \pm 0.05$$

[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

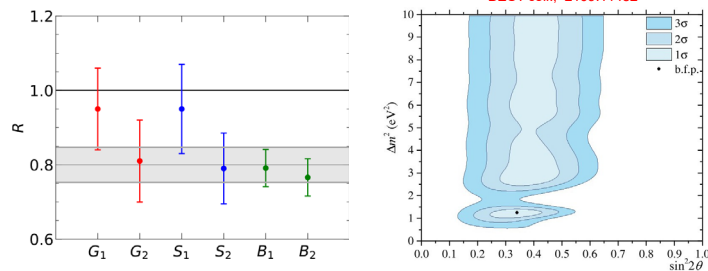
Haxton:

[Hata, Haxton, PLB 353 (1995) 422, nucl-th/9503017; Haxton, PLB 431 (1998) 110, nucl-th/9804011]

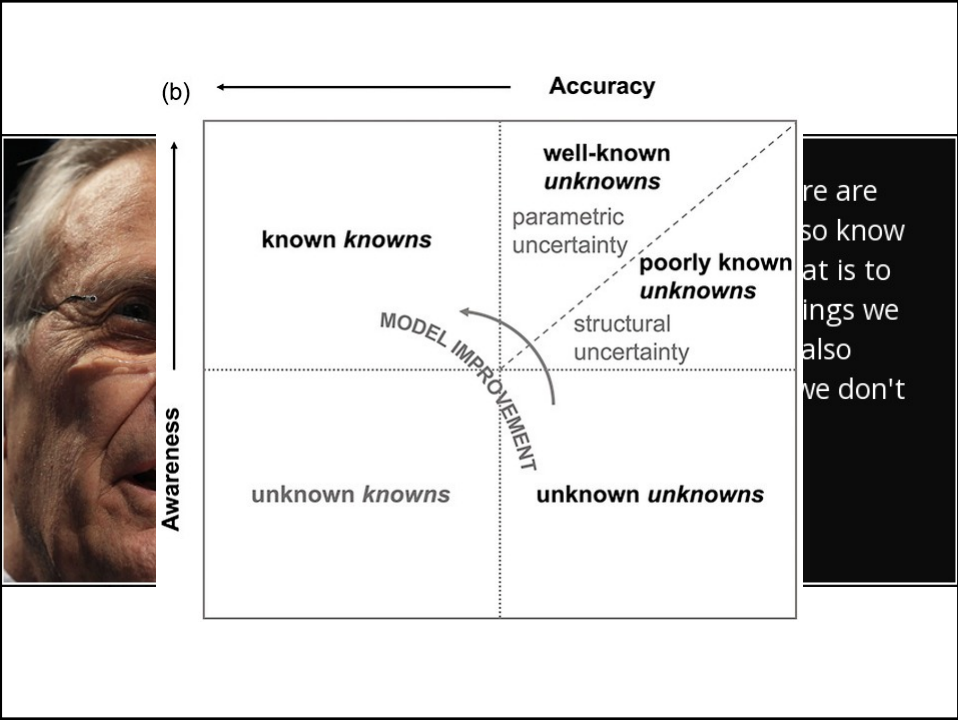
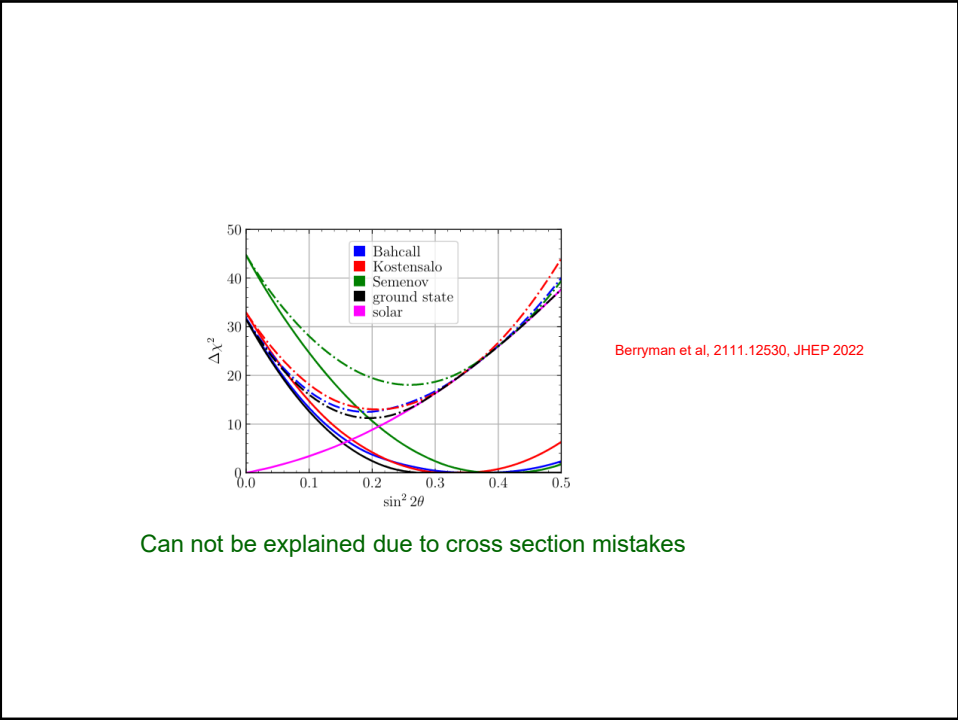
$$\sigma({}^{51}\text{Cr}) = 63.9 \times 10^{-46} \text{ cm}^2 (1 \pm 0.106)_{1\sigma} \implies R_{\text{Ga}} = 0.76_{-0.08}^{+0.09}$$

[SAGE, PRC 59 (1999) 2246, hep-ph/9803418]

The Gallium anomaly



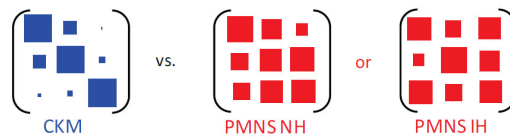
The Gallium anomaly is now at more than 5σ significance



The Known Unknowns

★ Next generation Long-Baseline experiments (such as DUNE) can address three of these questions:

- Are neutrinos Dirac or Majorana ?
 - Is there a connection to the GUT scale?
- Are there light sterile neutrino states ? Breaks 3-flavor paradigm
 - No clear theoretical guidance on mass scale, M , ...
- What is the neutrino mass hierarchy ?
 - An important question in flavor physics, e.g. CKM vs. PNMS



- Is CP violated in the leptonic sector ?
 - Are ν s key to understanding the matter-antimatter asymmetry?

We determined that $m(K_L) > m(K_S)$ by

- Passing kaons through matter (regenerator)
- Beating the unknown sign $[m(K_L) - m(K_S)]$ against the known sign[reg. ampl.]

We will determine the sign (Δm^2_{31}) by

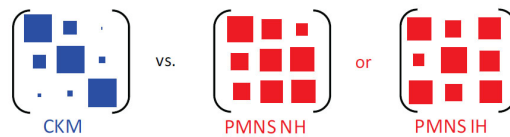
- Passing neutrinos through matter (Earth)
- Beating the unknown sign (Δm^2_{31}) against the known sign[forward $\nu_e e \rightarrow \nu_e e$ ampl]

$$L \approx \frac{2\pi}{G_F n_e} \approx 1.16 \cdot 10^4 \text{ km} \left(\frac{1.69 \cdot 10^{24} \text{ cm}^3}{n_e} \right)$$

The Known Unknowns

★ Next generation Long-Baseline experiments (such as DUNE) can address three of these questions:

- Are neutrinos Dirac or Majorana ?
 - Is there a connection to the GUT scale?
- Are there light sterile neutrino states ? Breaks 3-flavor paradigm
 - No clear theoretical guidance on mass scale, M , ...
- What is the neutrino mass hierarchy ?
 - An important question in flavor physics, e.g. CKM vs. PNMS



- Is CP violated in the leptonic sector ?
 - Are ν s key to understanding the matter-antimatter asymmetry?

In principle, it is straightforward

★ CPV \Rightarrow different oscillation rates for ν s and $\bar{\nu}$ s

$$P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 4s_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta \times \left[\sin\left(\frac{\Delta m_{21}^2 L}{4E}\right) \times \sin\left(\frac{\Delta m_{23}^2 L}{4E}\right) \times \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right) \right]$$

vacuum osc.

- ★ Requires $\{\theta_{12}, \theta_{13}, \theta_{23}\} \neq \{0, \pi\}$
 - now know that this is true, $\theta_{13} \approx 9^\circ$
 - but, despite hints, don't yet know "much" about δ
- ★ So "just" measure $P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$?
- ★ Not quite, there is a complication...

Neutrino Oscillations in Matter

- ★ Accounting for this potential term, gives a Hamiltonian that is **not diagonal** in the basis of the mass eigenstates

$$\mathcal{H} \begin{pmatrix} |v_1\rangle \\ |v_2\rangle \\ |v_3\rangle \end{pmatrix} = i \frac{d}{dt} \begin{pmatrix} |v_1\rangle \\ |v_2\rangle \\ |v_3\rangle \end{pmatrix} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \begin{pmatrix} |v_1\rangle \\ |v_2\rangle \\ |v_3\rangle \end{pmatrix} + V|v_e\rangle \leftarrow \text{ME}$$

- ★ Complicates the simple picture !!!!

$$P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) =$$

ME $\frac{16A}{\Delta m_{31}^2} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) c_{13}^2 s_{13}^2 s_{23}^2 (1 - 2s_{13}^2)$

ME $-\frac{2AL}{E} \sin \left(\frac{\Delta m_{31}^2 L}{4E} \right) c_{13}^2 s_{13}^2 s_{23}^2 (1 - 2s_{13}^2)$

CPV $-8 \frac{\Delta m_{21}^2 L}{2E} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) \sin \delta_{13} c_{13}^2 c_{23} s_{23} c_{12} s_{12}$

with $A = 2\sqrt{2}G_F n_e E = 7.6 \times 10^{-5} \text{eV}^2 \cdot \frac{\rho}{\text{g cm}^{-3}} \cdot \frac{E}{\text{GeV}}$

Experimental Strategy

EITHER:

- ★ Keep L small (~200 km): so that matter effects

- First oscillation maximum:

$$\frac{\Delta m_{31}^2 L}{4E} \sim \frac{\pi}{2} \Rightarrow E_\nu < 1 \text{ GeV}$$

- Want high flux at oscillation maximum

⇒ **Off-axis beam:** narrow range of neutrino energies

OR:

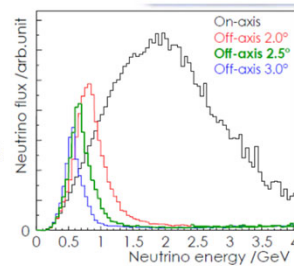
- ★ Make L large (>1000 km): measure the matter effects (i.e. MH)

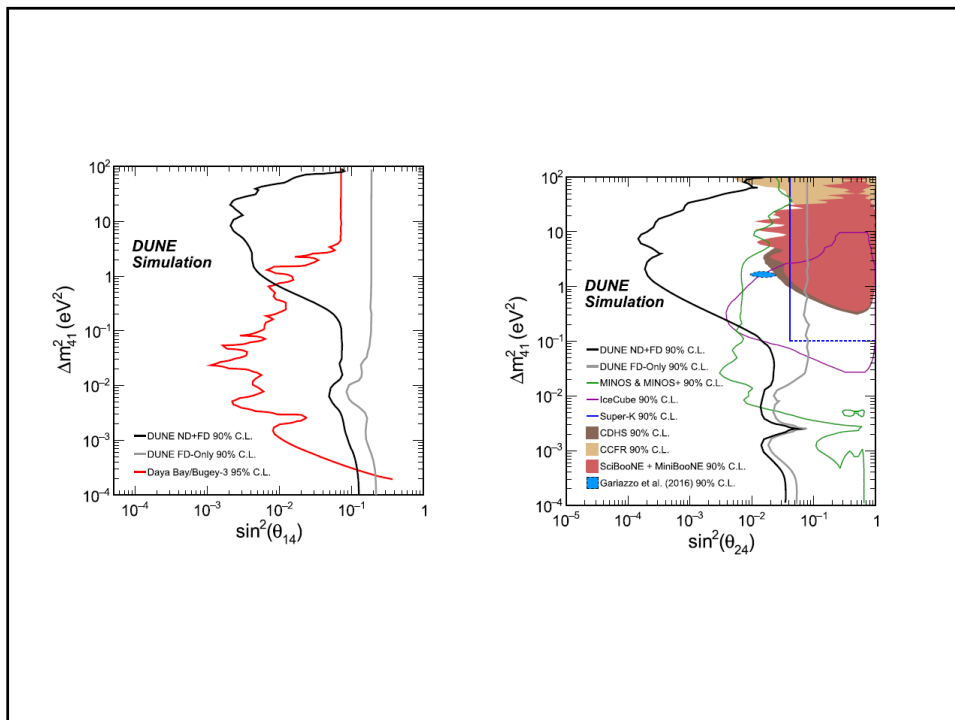
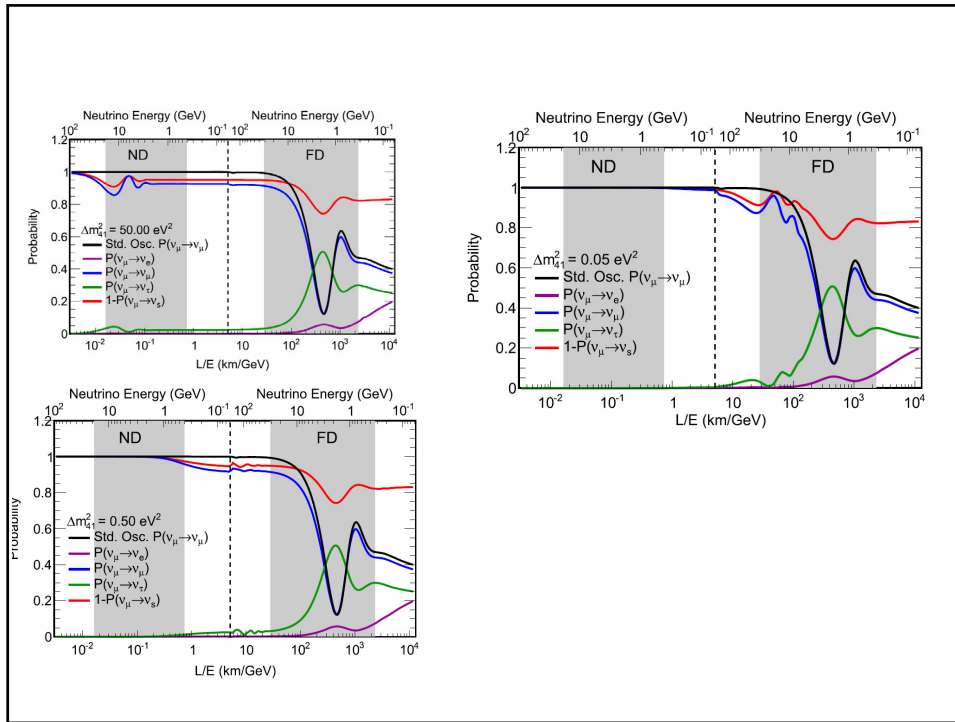
- First oscillation maximum:

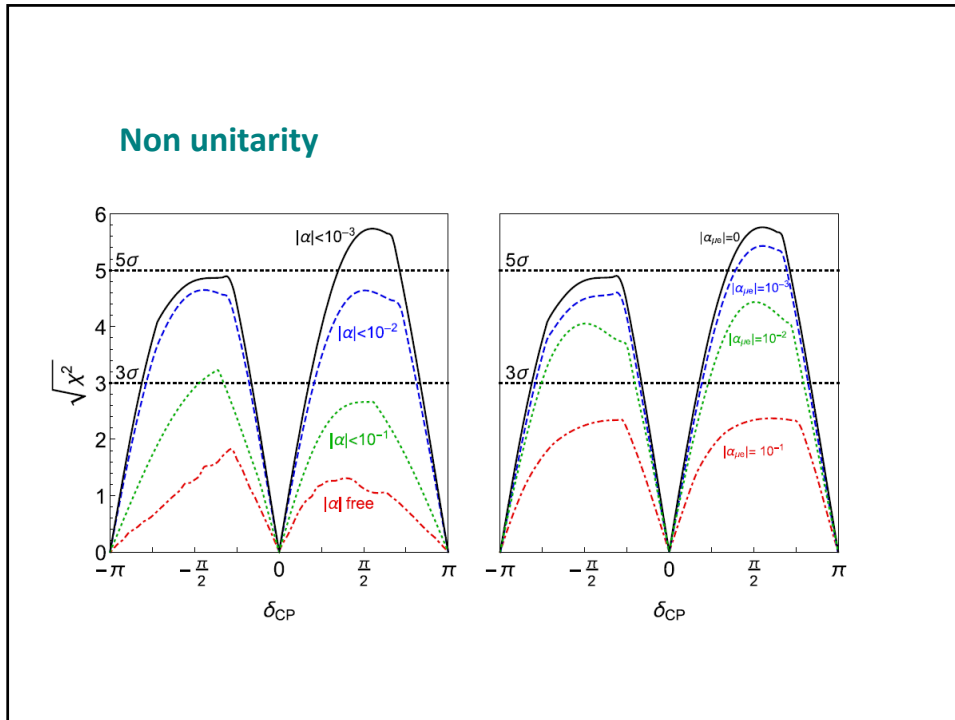
$$\frac{\Delta m_{31}^2 L}{4E} \sim \frac{\pi}{2} \Rightarrow E_\nu > 2 \text{ GeV}$$

- **Unfold CPV from Matter Effects through E dependence**

⇒ **On-axis beam:** wide range of neutrino energies







Non standard neutrino interactions

They can be described by effective four-fermion operators of the form

$$2\sqrt{2}G_F \varepsilon_{\alpha\beta} (\bar{\nu}_\beta \gamma^\mu P_L l_\alpha) (\bar{f} \gamma_\mu P_{L,R} f')$$

normalizing the operator with the Fermi constant

$$\varepsilon_{\alpha\beta} = \frac{M_W^2}{M_{NSI}^2}$$

NSNI can appear at every step. It is therefore necessary to break down the analysis in three stages

- the production process
- the time evolution
- the detection process

We are left “only” with neutral current NSNI

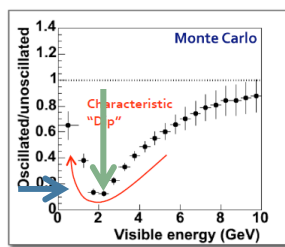
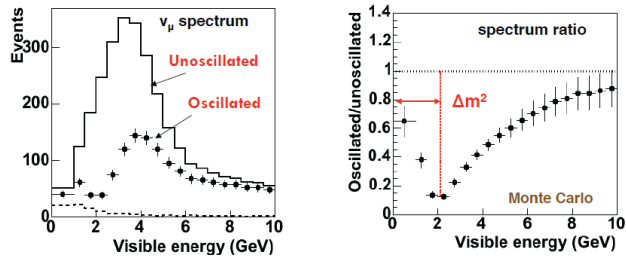
$$2\sqrt{2}G_F \varepsilon_{\alpha\beta} (\bar{\nu}_\beta \gamma^\mu P_L \nu_\alpha) (\bar{f} \gamma_\mu P_{L,R} f)$$

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + a \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$H = \frac{1}{2E} \left[U \begin{pmatrix} 0 & & \\ & \Delta m_{32}^2 & \\ & & \end{pmatrix} U^\dagger + a \begin{pmatrix} \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \right]$$

$$a \equiv 2\sqrt{2}G_F n_e E$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta) \sin^2(1.27 \Delta m^2 L / E)$$



$\epsilon_{\mu\tau}$ changes the disappearance probability at large energies
shifts the position of the minimum in energy

$$\Delta m^2$$

$\epsilon_{\tau\tau}$ modifies the disappearance probability near the first oscillation minimum, especially the depth of the minimum

$$\sin^2(2\theta_{23})$$