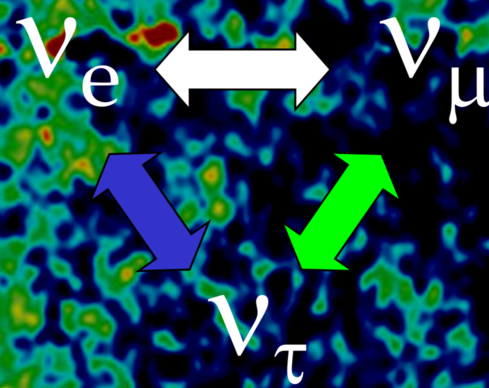
Neutrino Cosmology



STEEN HANNESTAD
NBI, july 2023

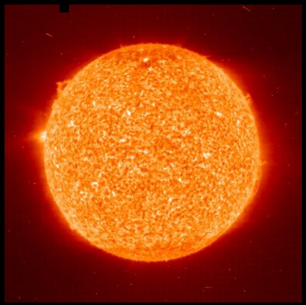
Lecture 1 (today): Neutrino cosmology

- ★ Basic cosmology – the Friedmann equation
- ★ Neutrino thermodynamics and decoupling in the early universe
- ★ Bounds on massive neutrinos
- ★ Neutrinos and BBN - Theory vs. Observations
- ★ Bounds on the number of neutrino species

Lectures 2 and 3:

- ★ Structure formation in the universe
- ★ Absolute value of neutrino masses
- ★ Future observational probes
- ★ Sterile neutrinos and cosmology

Where do Neutrinos Appear in Nature?

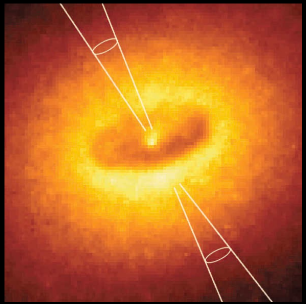


✓ Nuclear Reactors

Sun ✓

Supernovae
(Stellar Collapse)
SN 1987A ✓

✓ Particle Accelerators



✓ Earth Atmosphere
(Cosmic Rays)

Astrophysical
Accelerators ✓

✓ Earth Crust
(Natural
Radioactivity)

Big Bang
(Today 330 ν/cm^3)
Indirect Evidence

Friedmann-Robertson-Walker Cosmology

Line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Reduces to

$$ds^2 = -dt^2 + a^2(t)\left\{dr^2 + S_k^2(r)\,d\Omega^2\right\}$$

in a homogeneous and isotropic universe

$$S_k^2(r) = \begin{cases} \sin^2(r) & k = 1 \\ r^2 & k = 0 \\ \sinh^2(r) & k = -1 \end{cases}$$

a(t): Scale factor, only dynamical variable

The Einstein equation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

is a combination of 10 coupled differential equations since the involved tensors are explicitly symmetric

The *tt* component reduces to an evolution equation for *a(t)* (The Friedmann equation) in a homogeneous and isotropic universe

$$\boxed{H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{k}{a^2} = \frac{8\pi G\rho_{TOT}}{3} - \frac{k}{a^2}}$$

THE TOTAL ENERGY DENSITY THEN BY DEFINITION INCLUDES NON-RELATIVISTIC MATTER, RADIATION AND THE COSMOLOGICAL CONSTANT

$$\boxed{\rho_{TOT} = \rho_{MATTER} + \rho_{RADIATION} + \rho_{\Lambda}}$$

HOWEVER, THESE TYPES OF ENERGY BEHAVE COMPLETELY DIFFERENTLY AS A FUNCTION OF TIME AND SCALE FACTOR

$$\boxed{\begin{aligned} &\rho_{MATTER} \propto n \times m \propto a(t)^{-3} \\ &\rho_{RADIATION} \propto n \times \lambda^{-1} \propto a(t)^{-4} \\ &\rho_{\Lambda} \propto \Lambda \propto \text{Constant} \end{aligned}}$$

FROM THE ABOVE EQUATION

$$\rho_{\mathrm{RADIATION}} \sim a^{-4}(t)$$

AND THE FACT THAT

$$\rho_{\mathrm{RADIATION}} \sim T^{4}$$

IT CAN BE SEEN THAT THE EFFECTIVE 'TEMPERATURE' OF RADIATION SCALES AS

$$T \propto a^{-1}$$

A DEFINITION:

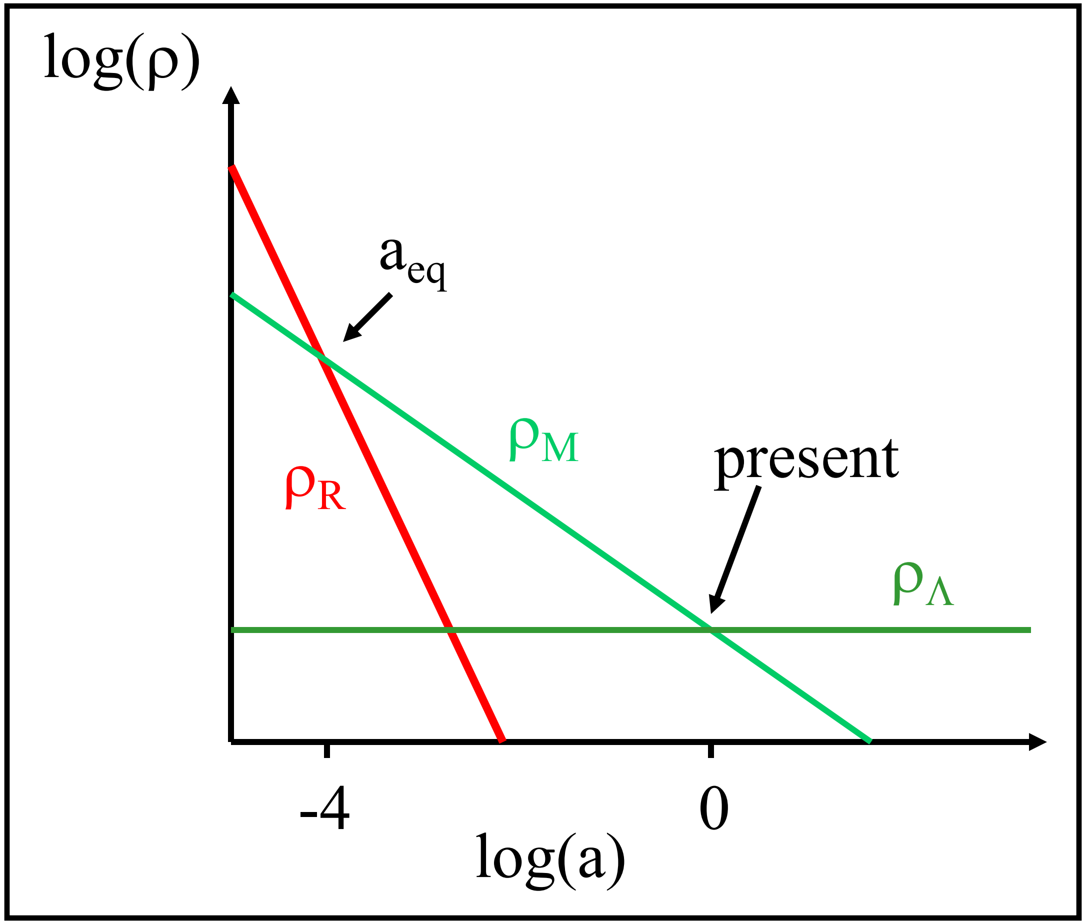
A QUANTITY WHICH IS CONSISTENTLY USED IS THE REDSHIFT, DEFINED AS

$$1+z \equiv \frac{a_0}{a}$$

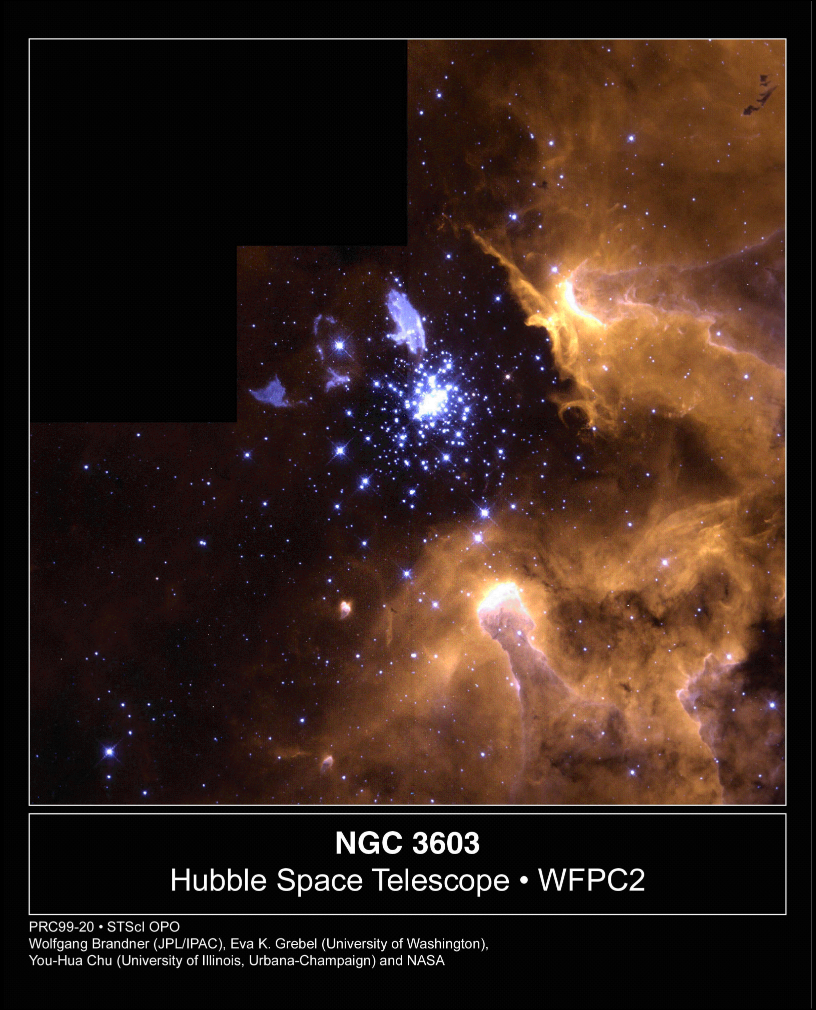
FROM THE SCALING OF PHOTON ENERGY IT CAN IMMEDIATELY BE SEEN THAT THE OBSERVED WAVELENGTH OF A PHOTON IS RELATED TO THE SCALE FACTOR OF THE UNIVERSE WHEN IT WAS EMITTED

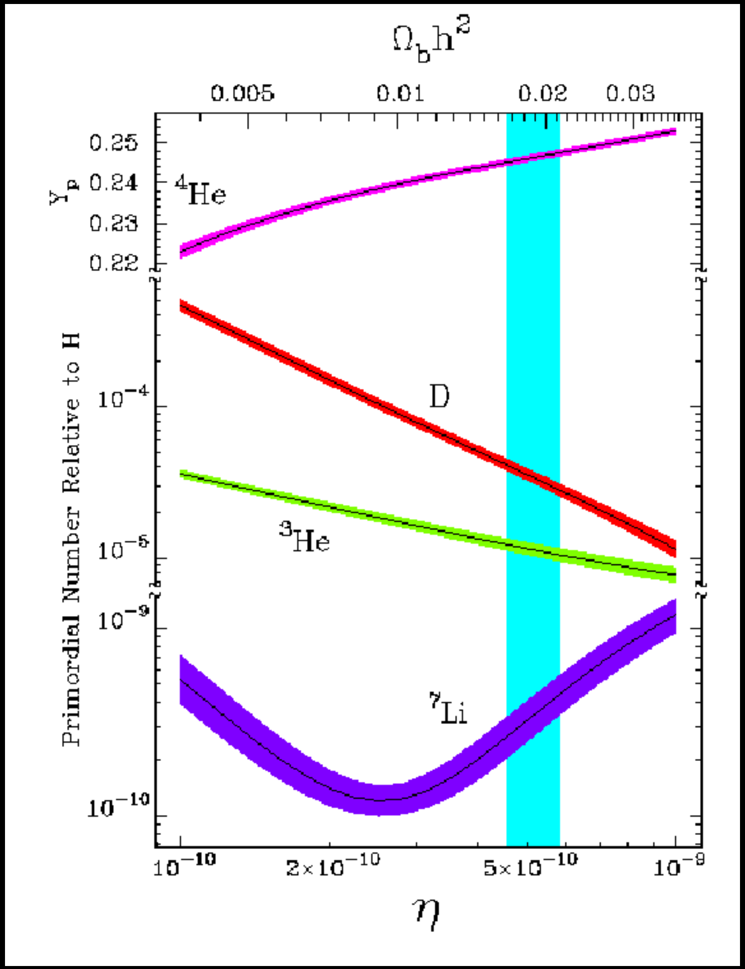
$$\frac{\lambda_{\mathrm{OBSERVED}}}{\lambda_{\mathrm{EMITTED}}} = 1+z$$

EVOLUTION OF ENERGY DENSITY WITH SCALE FACTOR



NEUTRINO THERMODYNAMICS AND BIG BANG NUCLEOSYNTHESIS





Thermodynamics in the early universe

In equilibrium, distribution functions have the form

$$f_{EQ} = \frac{1}{\exp((E-\mu)/T \pm 1)} \quad , \quad E = \sqrt{p^2 + m^2}$$

When $m \sim T$ particles disappear because of Boltzmann supression

$$f \rightarrow f_{MB} = e^{-(m-\mu)/T} e^{-p^2/2mT}$$

Decoupled particles: If particles are decoupled from other species their comoving number density is conserved. The momentum redshifts as $p \sim 1/a$

The entropy density of a species with MB statistics is given by

$$s = -\int f \ln f \, d^3 p \quad , \quad f = e^{-(E-\mu)/T}$$

In equilibrium, $\mu(X) = -\mu(\bar{X})$

(true if processes like $X\bar{X} \leftrightarrow \gamma\gamma$ occur rapidly)

This means that entropy is maximised when

$$\mu(X) = -\mu(\bar{X}) = 0$$

In equilibrium neutrinos and anti-neutrinos are equal in number!
However, the neutrino lepton number is not nearly as well constrained observationally as the baryon number

It is possible that $\dfrac{n_\nu}{n_\gamma} >> \dfrac{n_B}{n_\gamma} \sim 10^{-10}$

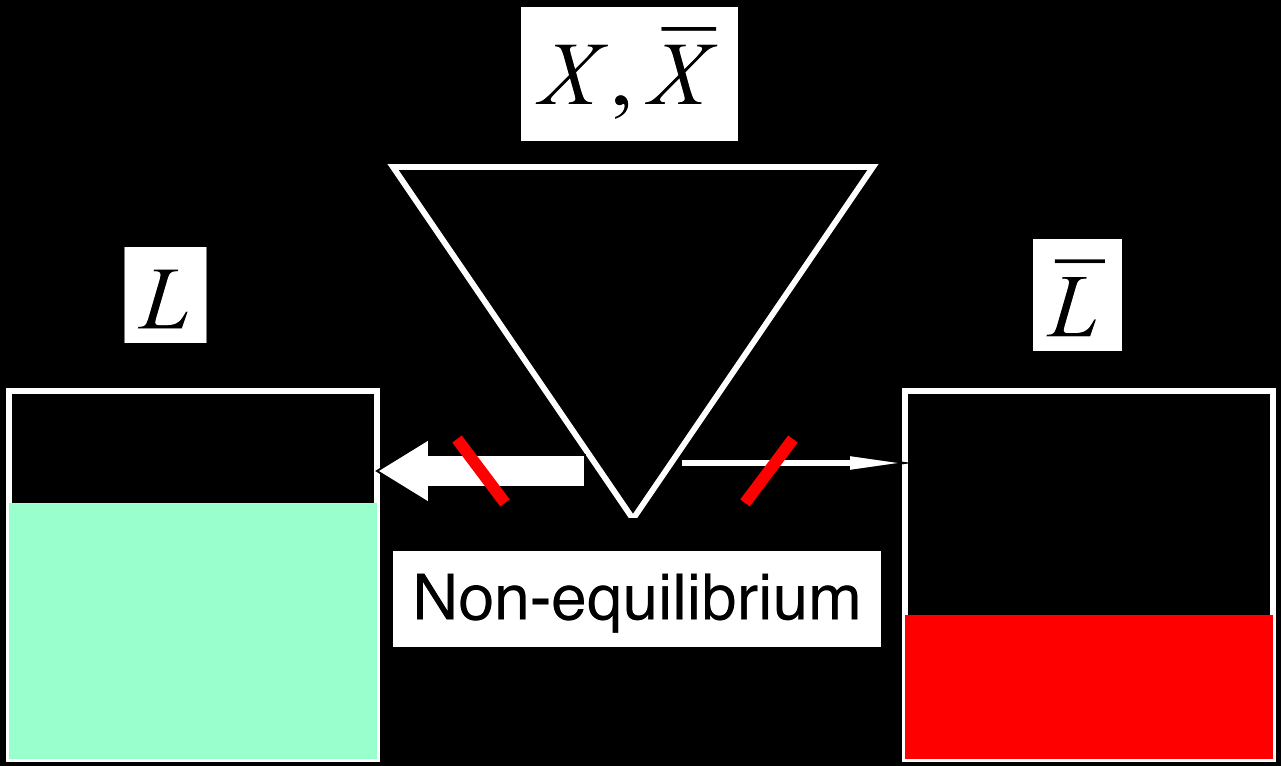
A small note on how to generate asymmetry (for leptons or baryons)

Conditions for lepto (baryo) genesis (Sakharov conditions)

L-violation: Processes that can break lepton number (e.g. $X \rightarrow ll$)

CP-violation: Asymmetry between particles and antiparticles e.g. $X \rightarrow ll$ has other rate than $\bar{X} \rightarrow \bar{l}\bar{l}$

Non-equilibrium thermodynamics: In equilibrium $n_L = n_{\bar{L}}$ always applies.


$X, \overline{X}$
L
$\overline{L}$
Non-equilibrium

Thermal evolution of standard, radiation dominated cosmology

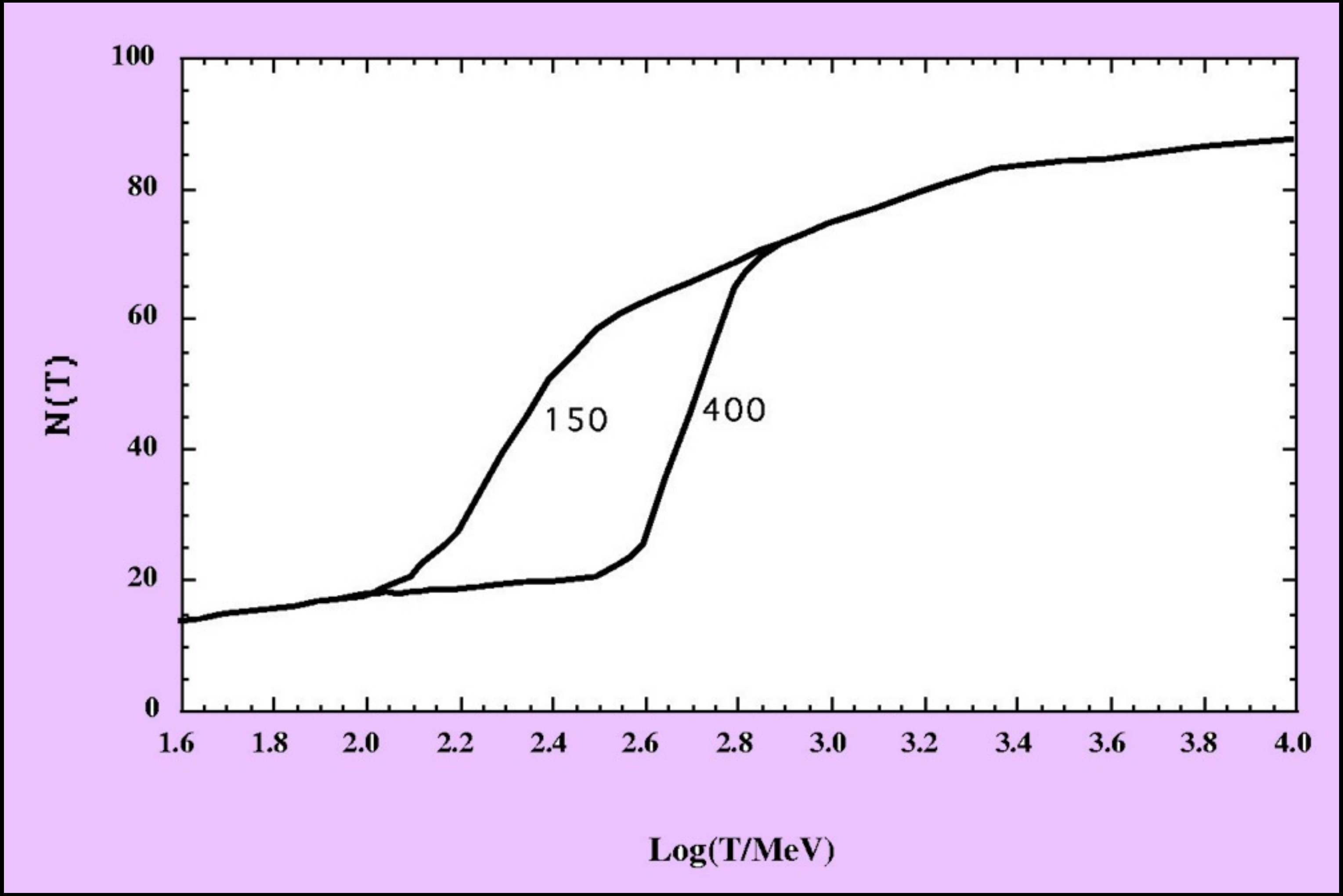
$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3}$$

$$\rho_B = \frac{\pi^2}{30} g T^4 \quad \text{for bosons}$$

$$\rho_F = \frac{7}{8}\frac{\pi^2}{30} g T^4 \quad \text{for fermions}$$

Total energy density

$$\rho_{TOT} = (\sum g_B + \frac{7}{8}\sum g_F)\frac{\pi^2}{30}T^4 = \frac{\pi^2}{30} g_* T^4$$



Temperature evolution of g_*

Temperature	New Particles	$4N(T)$
$T < m_e$	γ's + ν's	29
$m_e < T < m_\mu$	$e^\pm$	43
$m_\mu < T < m_\pi$	$\mu^\pm$	57
$m_\pi < T < Tc^*$	π's	69
$T_c < T < m_{\mathrm{strange}}$	- π's + $u, \bar{u}, d, \bar{d}$ + gluons	205
$m_s < T < m_{charm}$	$s, \bar{s}$	247
$m_c < T < m_\tau$	$c, \bar{c}$	289
$m_\tau < T < m_{bottom}$	$\tau^\pm$	303
$m_b < T < m_{W,Z}$	$b, \bar{b}$	345
$m_{W,Z} < T < m_{top}$	$W^\pm, Z$	381
$m_t < T < m_{Higgs}$	$t, \bar{t}$	423
$M_H < T$	H^o	427

In a radiation dominated universe the time-temperature relation is then of the form

$$t = \frac{1}{2} H^{-1} = \left(\frac{3}{32\pi G\rho} \right)^{1/2} \Rightarrow t_s \approx 2.4 g_*^{-1/2} T_{MeV}^{-2}$$

The number and energy density for a given species, *X*, is given by the Boltzmann equation

$$\boxed{\frac{\partial f_X}{\partial t} + pH\frac{\partial f_X}{\partial p} = C_e[f_X] + C_i[f_X]}$$

$C_e[f]$: *Elastic collisions*, conserves particle number but energy exchange possible (e.g. $X + i \to X + i$) [scattering equilibrium]

$C_i[f]$: *Inelastic collisions*, changes particle number (e.g. $X + \bar{X} \to i + \bar{i}$) [chemical equilibrium]

Usually, $C_e[f] >> C_i[f]$ so that one can assume that elastic scattering equilibrium always holds.

If this is true, then the form of f is always Fermi-Dirac or Bose-Einstein, but with a possible chemical potential.

Particle decoupling

The inelastic reaction rate per particle for species *X* is

$$\Gamma_{\text{int}} = \int C_i\left[f_X\right] \frac{d^3 p_X}{(2\pi)^3} = n_X \langle \sigma v \rangle$$

In general, a species decouples from chemical equlibrium when

$$\Gamma_{\text{int}} \approx H$$

$$H \approx 2N(T)^{1/2} \frac{T^2}{m_{Pl}}$$

The prime example is the decoupling of light neutrinos ($m < T_D$)

$$\Gamma_{weak} = n\langle \sigma v \rangle \approx T^3 G_F^2 T^2 \Rightarrow T_D \approx 1\,\text{MeV}$$

After neutrino decoupling electron-positron annihilation takes place (at T~m_e/3)

Entropy is conserved because of equilibrium in the e^+- e^-- γ plasma and therefore

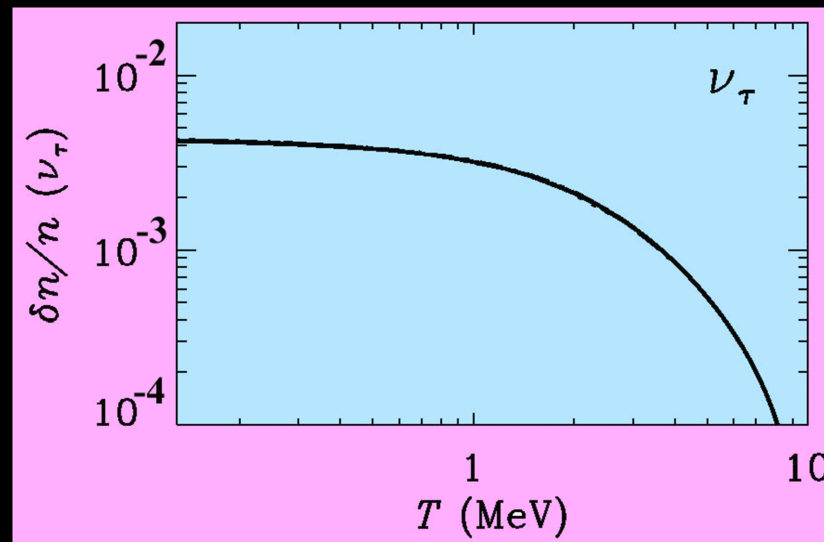
$$s_i = s_f \;\Rightarrow\; (2+4\frac{7}{8})T_i^3 = 2T_f^3 \;\Rightarrow\; \frac{T_f}{T_i} = \left(\frac{11}{4}\right)^{1/3}$$

The neutrino temperature is unchanged by this because they are decoupled and therefore

$$T_\nu = (4/11)^{1/3} T_\gamma \approx 0.71 T_\gamma \quad \text{(after annihilation)}$$

There are small corrections to this because neutrinos still interact slightly when electrons and positrons annihilate (neutrino heating)

	ρ / ρ_{ν_0}
ν_e	1.0083
$\nu_{\mu,\tau}$	1.0041



Additional small effect from finite temperature QED effect ($O(\alpha)$) ~ 1%

In total the neutrino energy density gets a correction of

$$\frac{\delta\rho_\nu}{\rho_{\nu_0}} \approx 0.04$$

Dicus et al. ’82, Lopez & Turner ’99
+ several other papers

Upper limit on the mass of light neutrinos:

For light neutrinos, $m << T_{\mathrm{dec}}$, the present day density is

$$\Omega_\nu h^2 = 3 \times \frac{3}{4} \times \left(\frac{T_\nu}{T_\gamma}\right)^3 n_\gamma m_\nu / \rho_c \approx \frac{m_\nu}{30\ \mathrm{eV}}$$

Assuming that the three active species have the same mass

A conservative limit ($\Omega_\nu h^2 < 1$) on the neutrino mass is then

$$m_\nu \leq 30\ \mathrm{eV}$$

For any of the three active neutrino species

Mass limits on very massive neutrinos ($m >> T_D$):

If MB statistics is used one finds (as long as elastic scattering equilibrium holds) by integrating the Boltzmann equation

$$\dot{n} + 3Hn = -\langle \sigma v \rangle_{\text{inelastic}} (n^2 - n_{eq}^2)$$

This is the standard equation used for WIMP annihilation (e.g. massive neutrinos, neutralinos, etc)

This equation is usually not analytically solvable (Riccati equation), but is trivial to solve numerically.

For a particle with standard weak interactions one finds that the species decouples from *chemical* equilibrium when

$$\frac{m}{T_D} \approx 20$$

As long as a species is close to equilibrium

$$\Gamma = n\langle\sigma v\rangle \approx n_{eq}\langle\sigma v\rangle$$

with

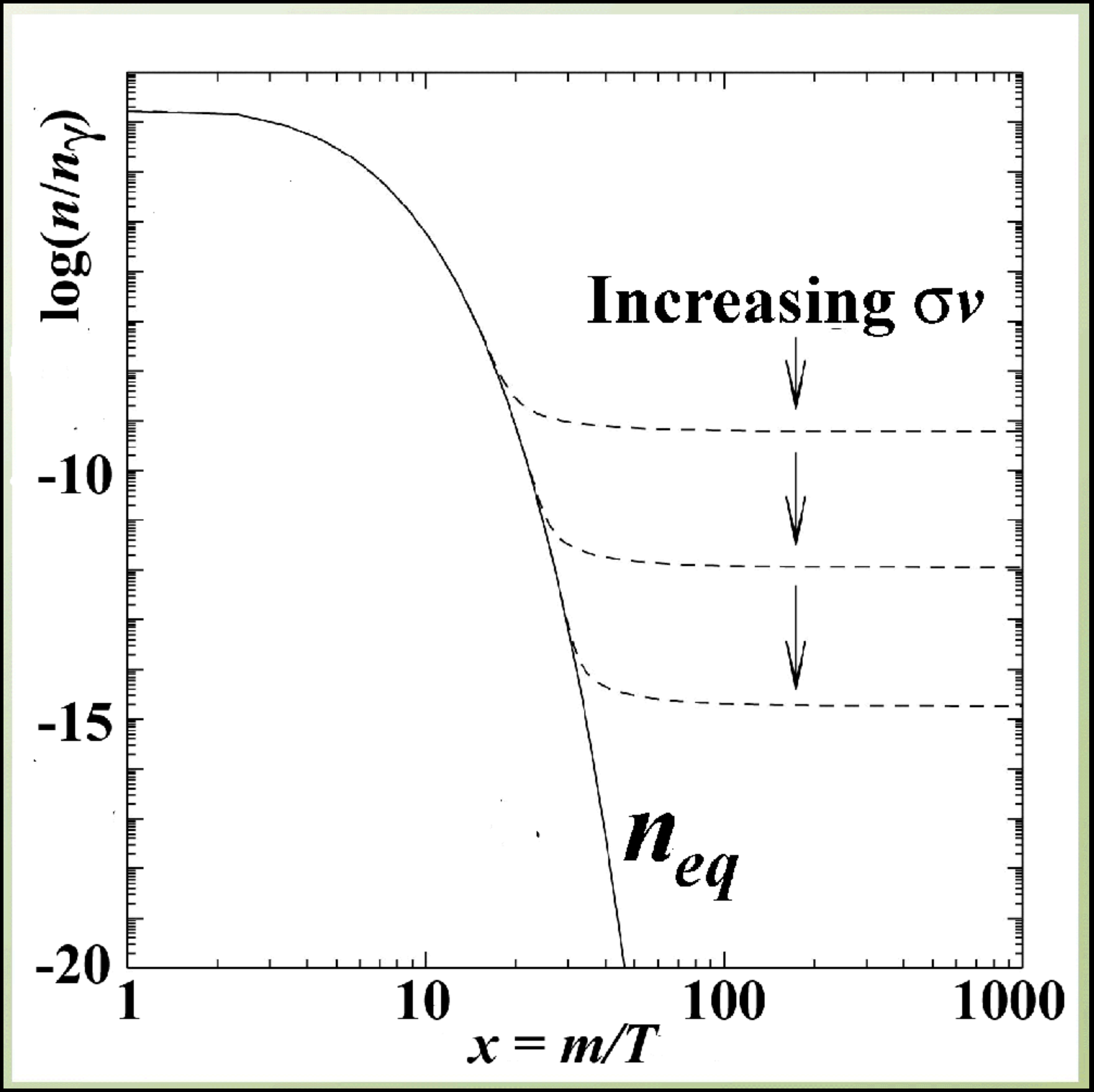
$$n_{eq} = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

Comparing this to the Hubble rate

$$H \approx 2N(T)^{1/2}\,\frac{T^2}{m_{Pl}}$$

yields

$$\Gamma \sim H \;\Rightarrow n_0 / n_\gamma \propto \langle\sigma v\rangle^{-1}$$



log(n/nγ)
-10
-15
-20
1
10
100
1000
x = m/T
Increasing σv
n_eq

Dirac neutrinos

$$\boxed{\langle \sigma v \rangle_D \approx \frac{G_F^2 m^2}{2\pi}}$$

Majorana neutrinos

$$\boxed{\langle \sigma v \rangle_M \approx \frac{G_F^2 m^2}{2\pi} \frac{m_i^2}{m^2} = \frac{G_F^2 m_i^2}{2\pi}}$$

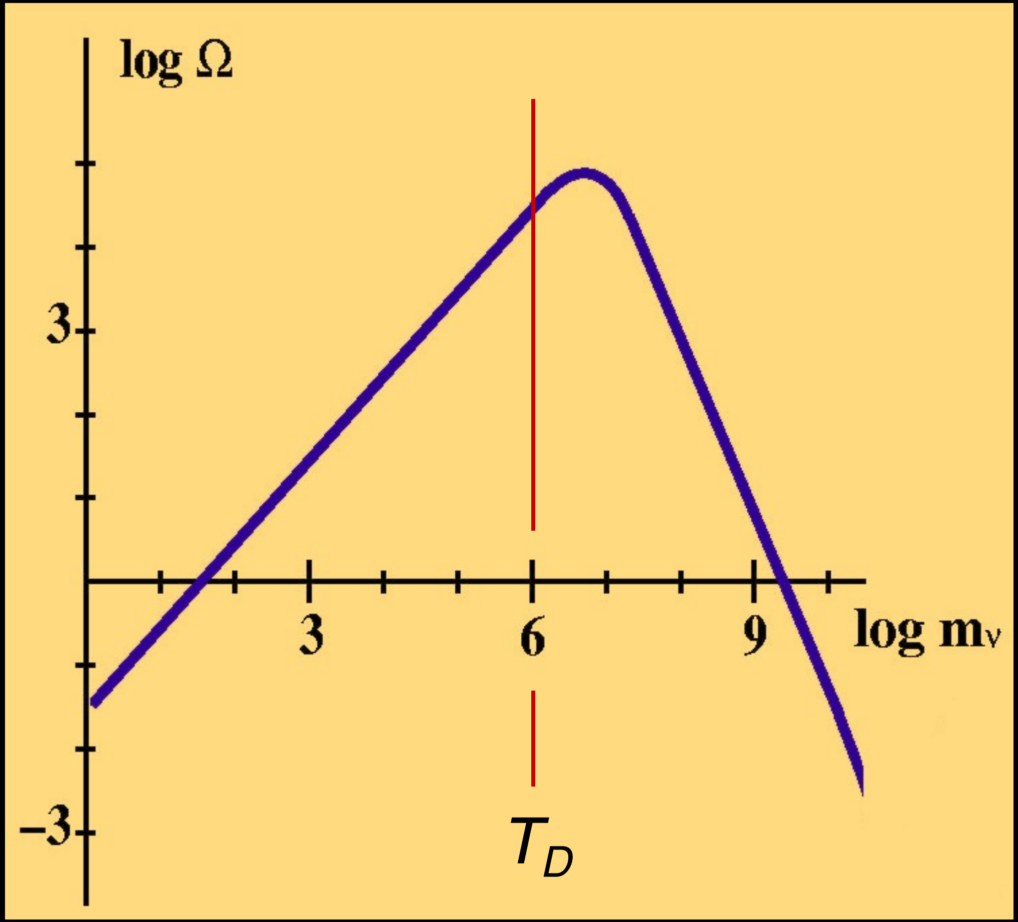
m_i is the mass of the annihilation product (usually the most massive final state available)

This means that

$$\boxed{\Omega_\nu h^2 \propto \begin{cases} m^{-2} & \text{for Dirac} \\ m_i^{-2} & \text{for Majorana} \end{cases}}$$

This leads to a lower bound (the Lee-Weinberg bound) on very massive neutrinos

$$m_\nu > \begin{cases} 4\,\text{GeV} & \text{(Dirac)} \\ 12\,\text{GeV} & \text{(Majorana)} \end{cases}$$



Because of the stringent bound from LEP on neutrinos lighter than about 45 GeV

$$\boxed{N_\nu = 2.984 \pm 0.008}$$

This bound is mainly of academic interest. However, the same argument applies to any WIMP, such as the neutralino (very similar to Majorana neutrino).

BIG BANG NUCLEOSYNTHESIS

The baryon number left after baryogenesis is usually expressed in terms of the parameter η

$$\eta \equiv \left.\frac{n_B}{n_\gamma}\right|_{t=t_0}$$

According to observations $\eta \sim 10^{-10}$ and therefore the parameter

$$\eta_{10} \equiv 10^{10} \times \eta$$

is often used

From η the present baryon density can be found as

$$\Omega_b h^2 = 0.0037\,\eta_{10}$$

Immediately after the quark-hadron transition almost all baryons are in pions. However, when the temperature has dropped to a few MeV (T << m_π) only neutrons and protons are left

In thermal equilibrium

$$\frac{n_n}{n_p} = \exp(-\Delta m / T) \quad , \quad \Delta m = 1.293\,\text{MeV}$$

However, this ratio is dependent on weak interaction equilibrium

n-p changing reactions

$$\nu_e + n \leftrightarrow e^- + p$$
$$e^+ + n \leftrightarrow \bar{\nu}_e + p$$
$$n \leftrightarrow e^- + p + \bar{\nu}_e$$

Interaction rate (the generic weak interaction rate)

$$\Gamma_{n-p} = n\langle \sigma v \rangle \approx T^3 G_F^2 T^2 \Rightarrow T_{freeze} \approx 1\,\text{MeV}$$

After that, neutrons decay freely with a lifetime of

$$\tau_n = 886 \pm 0.8\ s$$

However, before complete decay neutrons are bound in nuclei.

Nucleosynthesis should intuitively start when
$T \sim E_b$ (D) ~ 2.2 MeV via the reaction

$$\boxed{p + n \leftrightarrow D + \gamma}$$

However, because of the high entropy it does not.
Instead the nucleosynthesis starting point can be found from the condition $\boxed{\Gamma_{production}(D) = \Gamma_{destruction}(D)}$

$$\left.\begin{aligned} \Gamma_{production} &\approx n_B \langle \sigma v \rangle \\ \Gamma_{destruction} &\approx n_\gamma \langle \sigma v \rangle e^{-E_b/T} \end{aligned}\right\} \Rightarrow T_{BBN} \approx -\frac{E_b}{\ln(\eta)} \approx 0.2\,\mathrm{MeV}$$

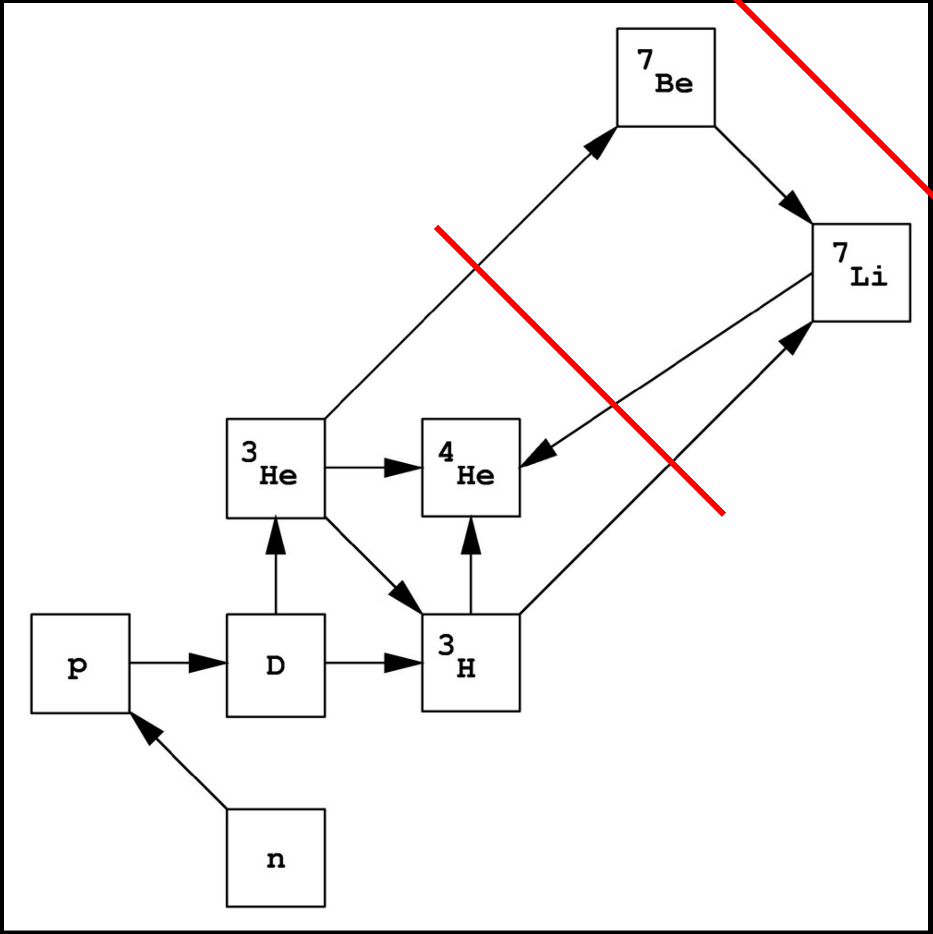
Since $t(T_{\mathrm{BBN}}) \sim 50$ s $<< \tau_n$ only few neutrons have time to decay

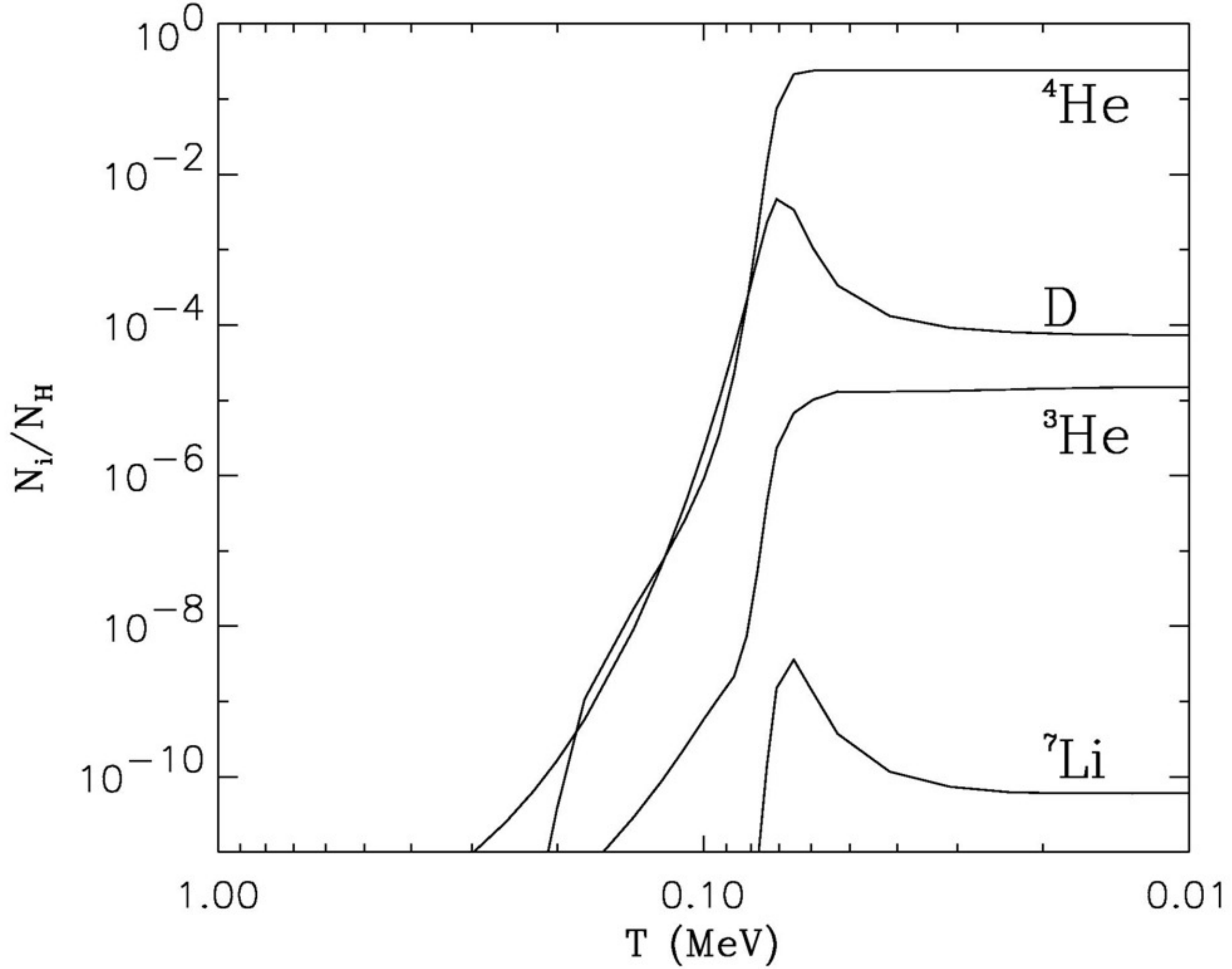
At this temperature nucleosynthesis proceeds via the reaction network

The mass gaps at $A = 5$ and 8 lead to small production of mass numbers 6 and 7, and almost no production of mass numbers above 8

The gap at $A = 5$ can be spanned by the reactions

$$^{3}He(^{4}He,\gamma)^{7}Be$$
$$T(^{4}He,\gamma)^{7}Li$$

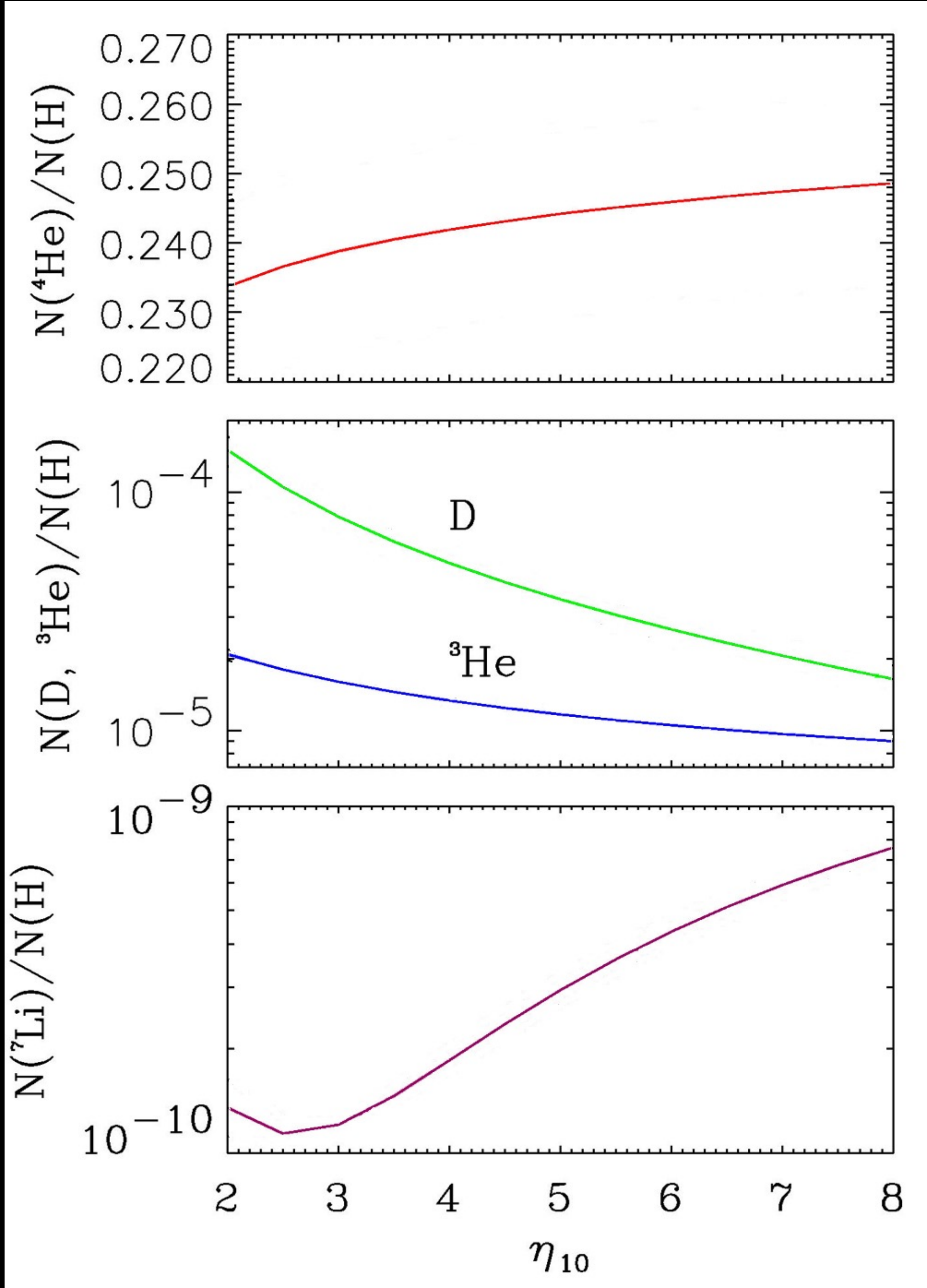




^{4}He
D
^{3}He
^{7}Li
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
N_i/N_H
1.00
0.10
0.01
T (MeV)

ABUNDANCES HAVE BEEN CALCULATED USING THE WELL-DOCUMENTED AND PUBLICLY AVAILABLE FORTRAN CODE NUC.F, WRITTEN BY LAWRENCE KAWANO

The amounts of various elements produced depend on the physical conditions during nucleosynthesis, primarily the values of N(T) and η



Helium-4: Essentially all available neutrons are processed into He-4, giving a mass fraction of

$$Y_P = \frac{4n_{He}}{n_N} = \left.\frac{2n_n}{n_n + n_p}\right|_{T_{BBN}} \approx 0.25 \quad \text{for} \quad n_n / n_p \sim 1/7$$

$$\left.\frac{n_n}{n_p}\right|_{T_{BBN}} \approx \exp(-\Delta m / T_{weak}) \frac{\exp(-t_{BBN} / \tau_n)}{2 - \exp(-t_{BBN} / \tau_n)} \sim 1/7$$

Y_p depends on η because T_{BBN} changes with η

$$T_{BBN} \approx -\frac{E_{B,D}}{\ln(\eta)}$$

D, He-3: These elements are processed to produce He-4. For higher η, T_{BBN} is higher and they are processed more efficiently

Li-7: Non-monotonic dependence because of two different production processes
Much lower abundance because of mass gap

Confronting theory with observations

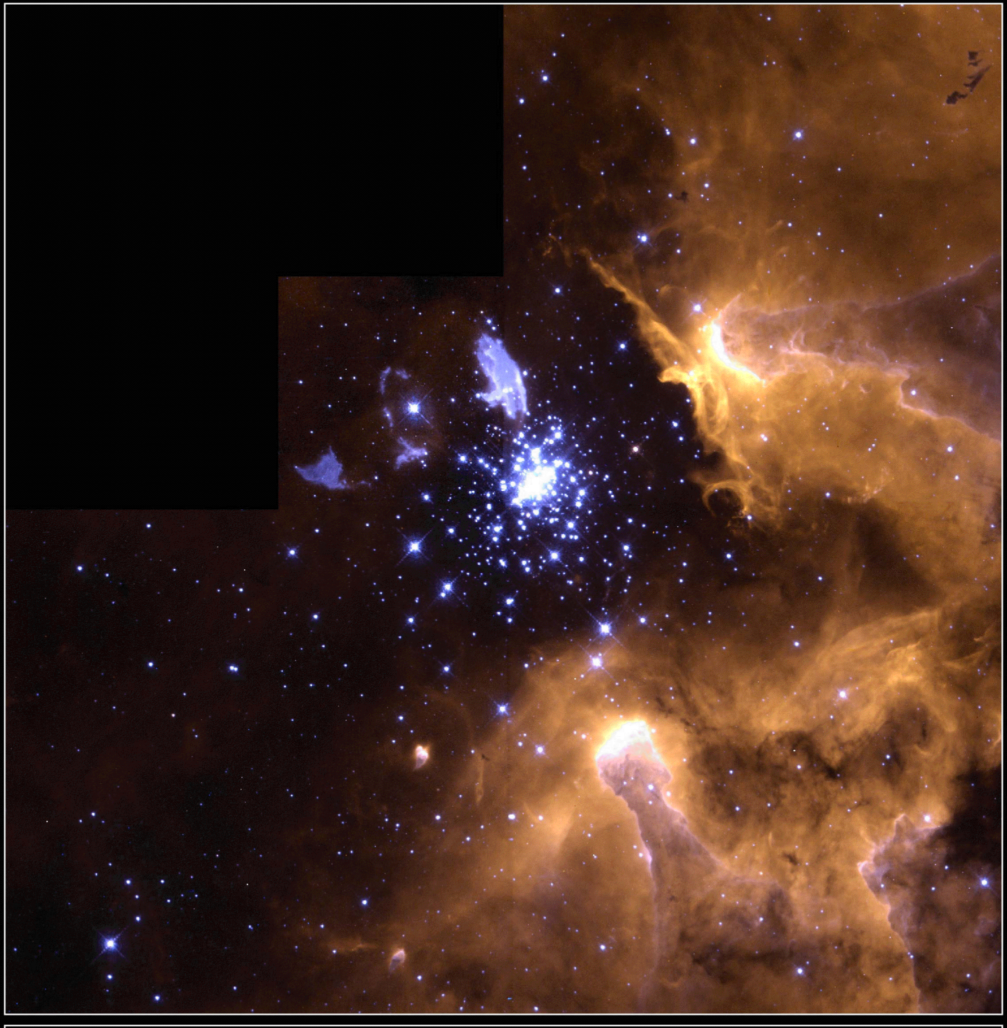
He-4:

He-4 is extremely stable and is in general always produced, not destroyed, in astrophysical environments

The Solar abundance is $Y = 0.28$, but this is processed material

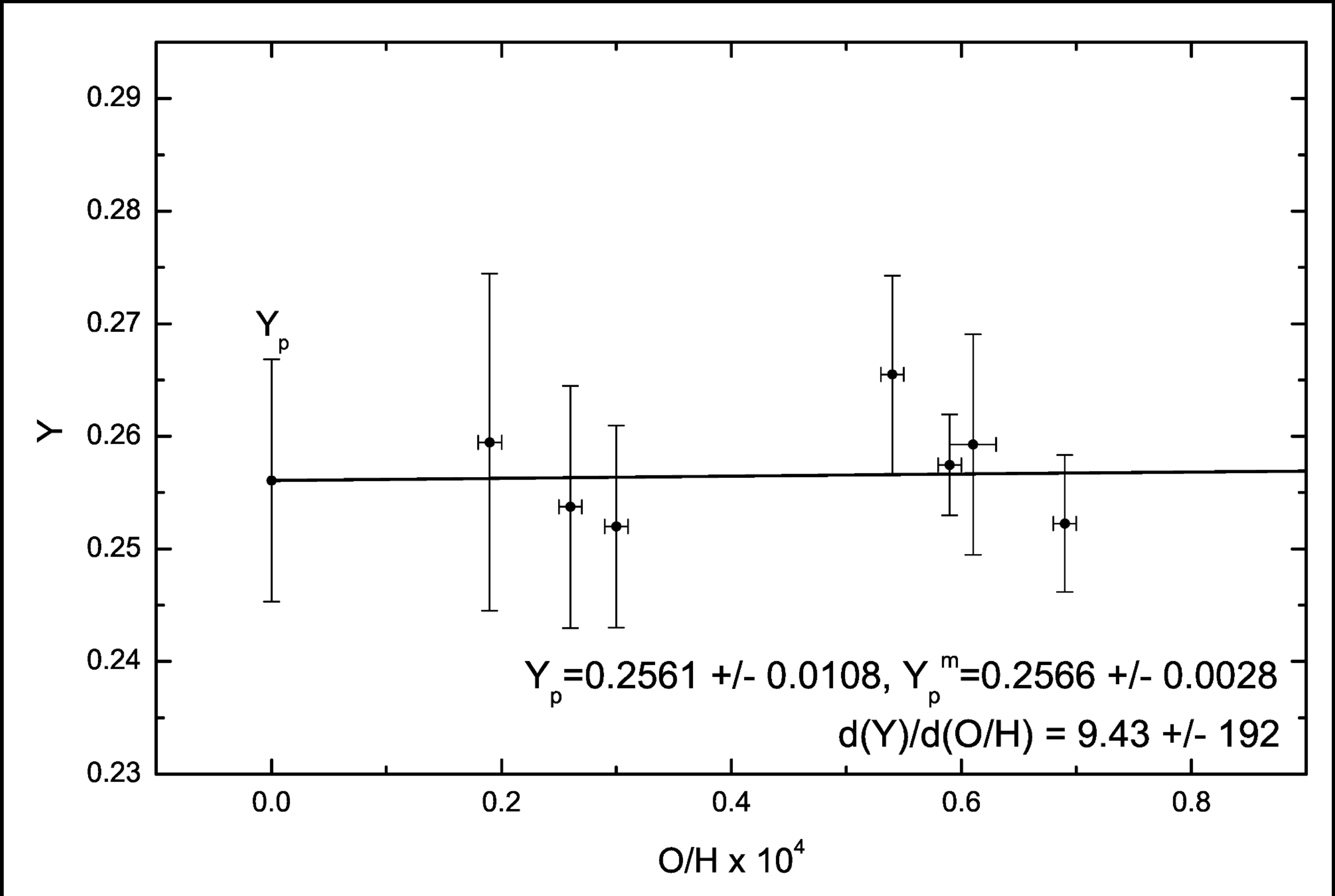
The primordial value can in principle be found by measuring He abundance in unprocessed (low metallicity) material.

Extragalactic H-II regions

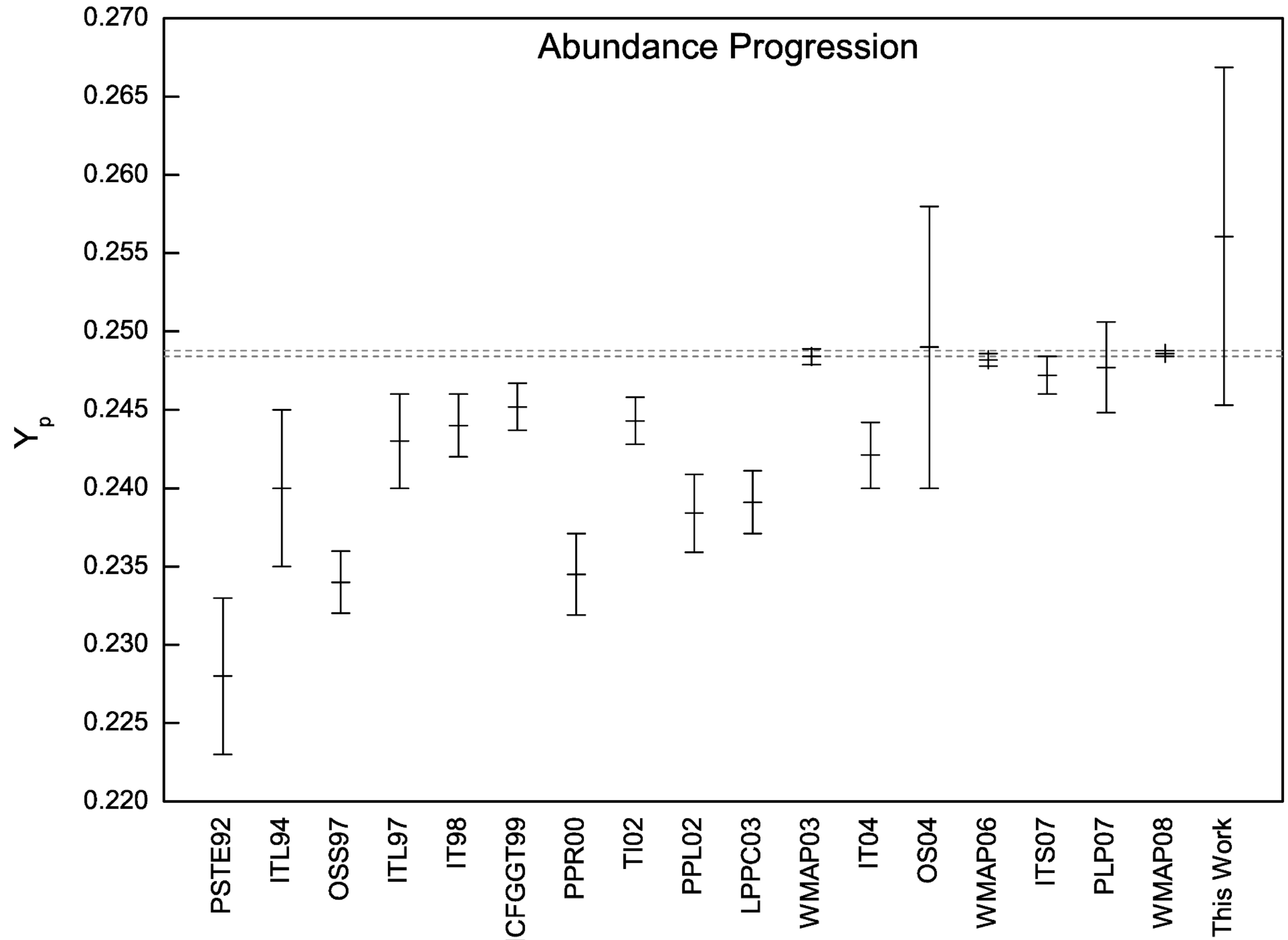


NGC 3603
Hubble Space Telescope • WFPC2

PRC99-20 • STScI OPO
Wolfgang Brandner (JPL/IPAC), Eva K. Grebel (University of Washington),
You-Hua Chu (University of Illinois, Urbana-Champaign) and NASA



Aver et al. 2010


Abundance Progression
Y_p
0.270
0.265
0.260
0.255
0.250
0.245
0.240
0.235
0.230
0.225
0.220
PSTE92
ITL94
OSS97
ITL97
IT98
ICFGGT99
PPR00
TI02
PPL02
LPPC03
WMAP03
IT04
OS04
WMAP06
ITS07
PLP07
WMAP08
This Work

G. Steigman

Deuterium: Deuterium is weakly bound and therefore can be assumed to be only destroyed in astrophysical environments

Primordial deuterium can be found either by measuring solar system or ISM value and doing complex chemical evolution calculations

OR

Measuring D at high redshift

The ISM value of

$$(D/H)_{ISM} = 1.60 \pm 0.09^{+0.05}_{-0.10} \times 10^{-5}$$

can be regarded as a firm lower bound on primordial D

1994: First measurements of D in high-redshift absorption systems

A very high D/H value was found

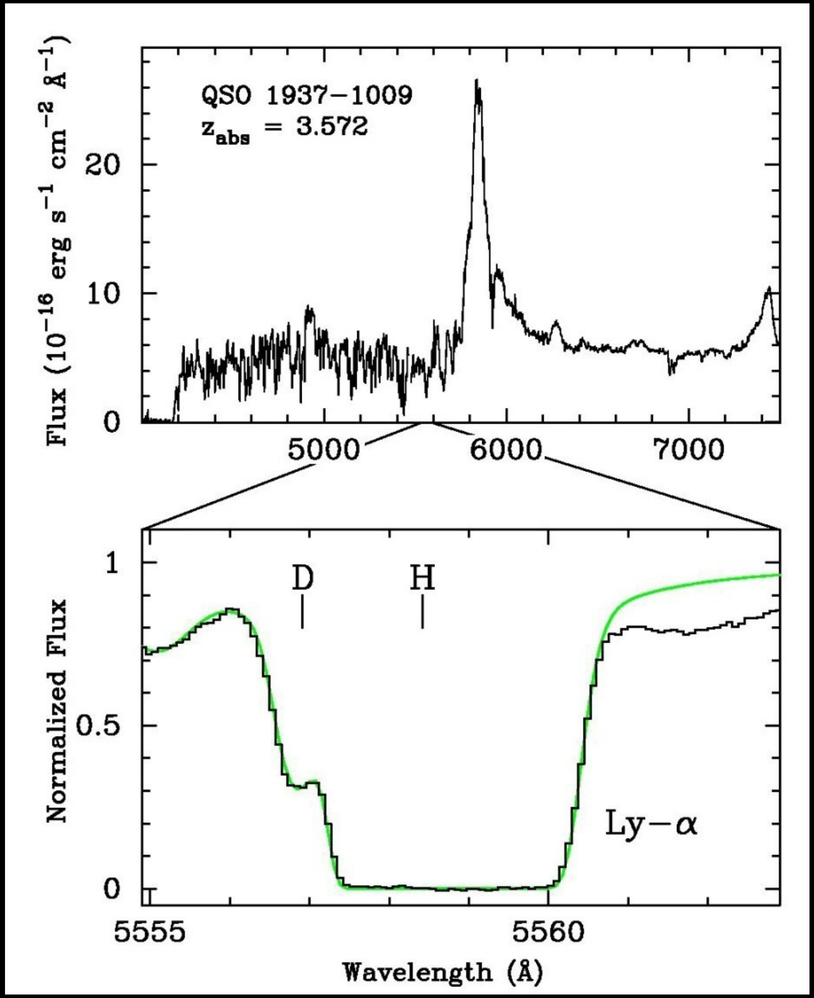
$$(D/H)_{High-z} \approx 1.9 - 2.5\times10^{-4}$$

Carswell et al. 1994
Songaila et al. 1994

However, other measurements found much lower values

$$(D/H)_{High-z} \approx 2.5\times10^{-5}$$

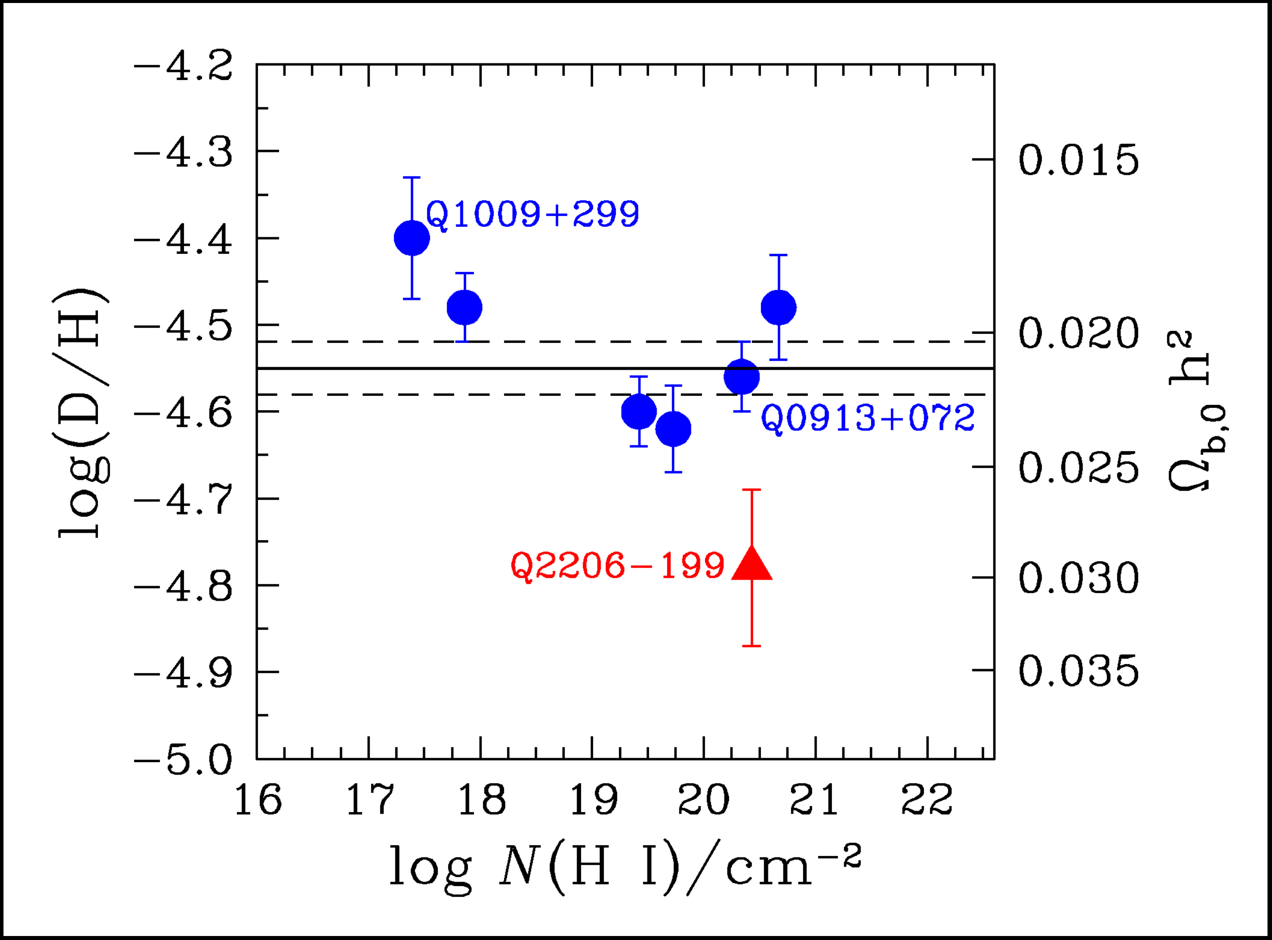
Burles & Tytler 1996



Burles & Tytler

The discrepancy has been resolved in favour of a low deuterium value of roughly

$$(D/H)_{High-z} \approx 3.4 \pm 0.5 \times 10^{-5}$$



Pettini et al 2010

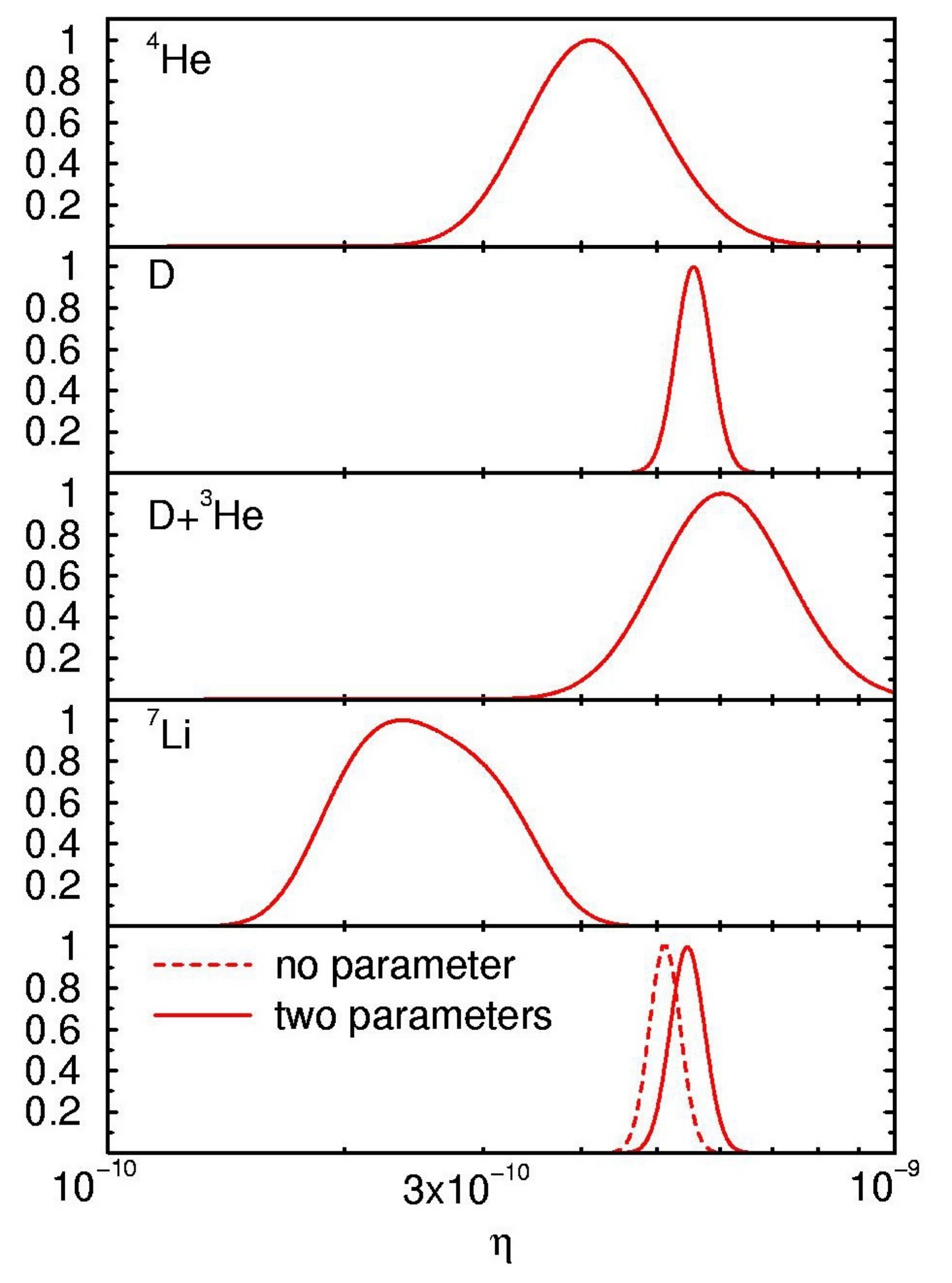
There is consistency between theory and observations

All observed abundances fit well with a single value of eta

This value is mainly determined by the High-z deuterium measurements

The overall best fit is

$$\eta \sim 6\times10^{-10}$$



BOUND ON THE RELATIVISTIC ENERGY DENSITY (NUMBER OF NEUTRINO SPECIES) FROM BBN

The weak decoupling temperature depends on the expansion rate

$$H = \sqrt{\frac{8\pi G\rho}{3}} = \sqrt{\frac{2\pi^3 G N(T) T^4}{15}}$$

And decoupling occurs when

$$\Gamma_{\text{int}} \approx G_F^2 T^5 \approx H \Rightarrow T_D \propto N(T)^{1/6}$$

N(*T*) is can be written as

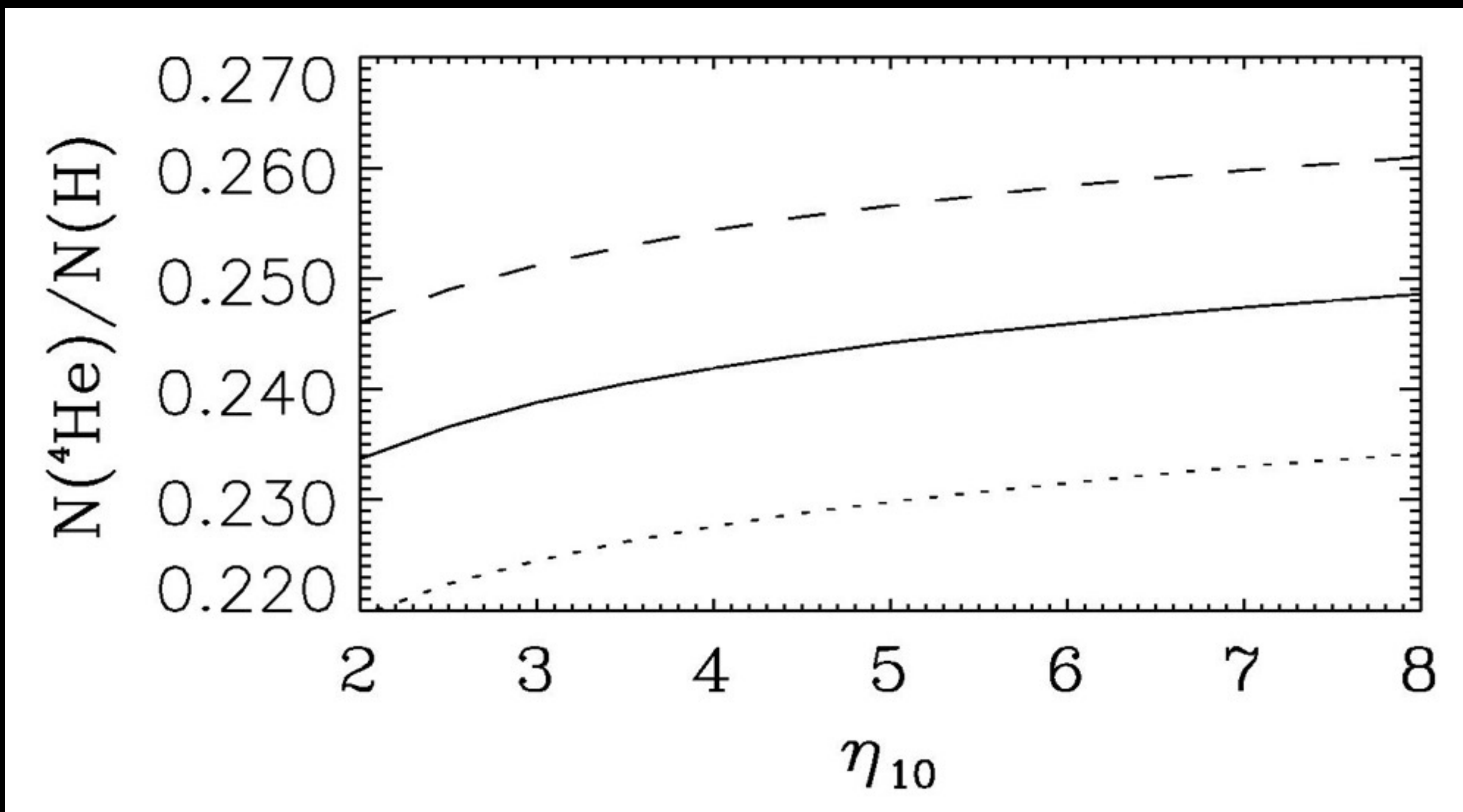
$$N(T) = N(T)_{e^+,e^-,\gamma} + N(T)_{\nu,SM} \frac{\rho_{\nu+\text{extra}}}{\rho_{\nu,SM}}$$

$$= N(T)_{e^+,e^-,\gamma} + N(T)_{\nu,SM}(3 + \Delta N_\nu)$$

Since

$$\left.\frac{n_n}{n_p}\right|_{BBN} \approx \exp(-\Delta m / T_D)$$

The helium production is very sensitive to N_ν

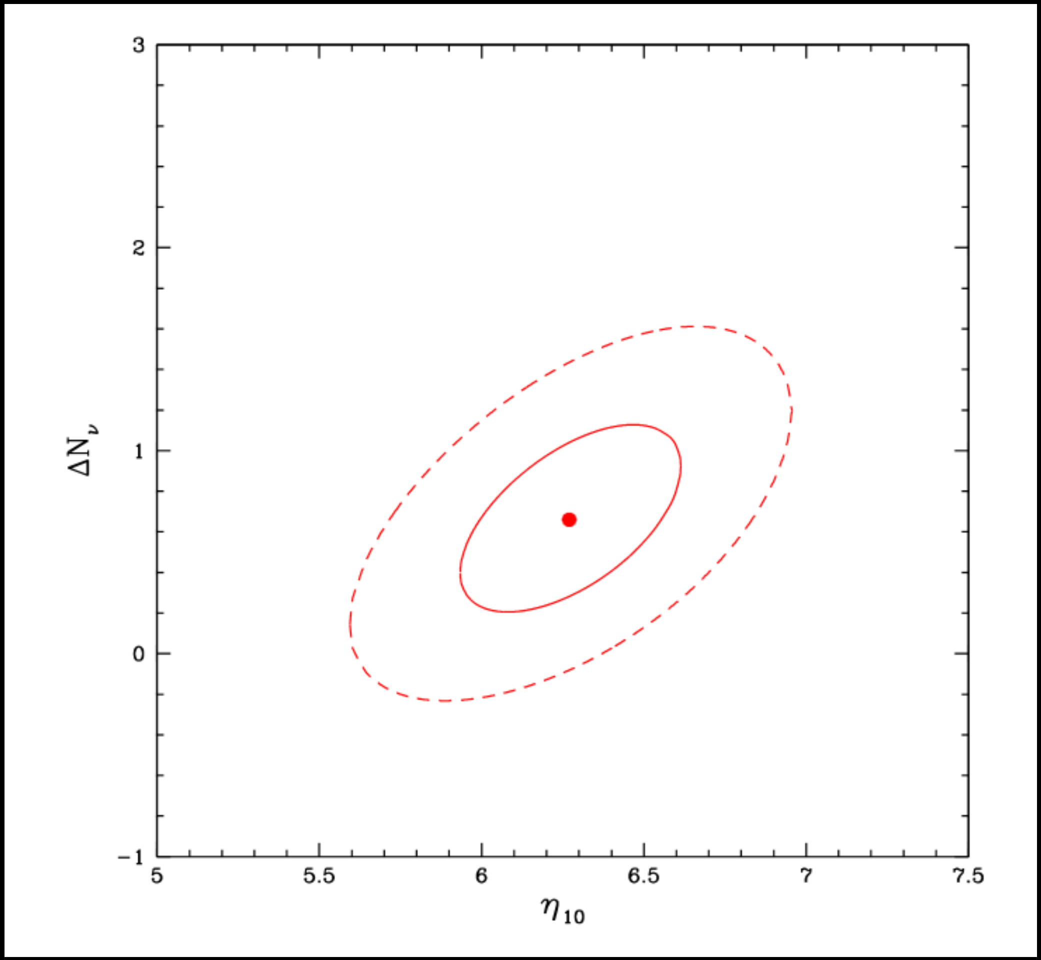


$N_\nu = 4$

$N_\nu = 3$

$N_\nu = 2$

Using BBN to probe physics beyond the standard model



Steigman 2012

Using BBN to probe physics beyond the standard model

Non-standard physics can in general affect either

- Expansion rate during BBN
  - extra relativistic species
  - massive decaying particles
  - quintessence
  - ….
- The interaction rates themselves
  - neutrino degeneracy
  - changing fine structure constant
  - ….

PERSPECTIVES

- Neutrino thermodynamics and decoupling is well understood in the standard model.

- Big bang nucleosynthesis provides a powerful probe of neutrino physics beyond the standard model. However, many of the abundance measurements are dominated by systematics which need to be better understood