

$$\frac{|m(K_0) - m(\bar{K}_0)|}{m_{K-av}} < 10^{-18}$$

$$m_{K-av} \approx \frac{1}{2} \cdot 10^9 \text{ eV}$$

$$(m(K_0) - m(\bar{K}_0)) (m(K_0) + m(\bar{K}_0)) < 2 \cdot 10^{-18} m_{K-av}^2$$

$$|m^2(K_0) - m^2(\bar{K}_0)| \approx \frac{1}{2} \text{ eV}^2$$

CPT tests

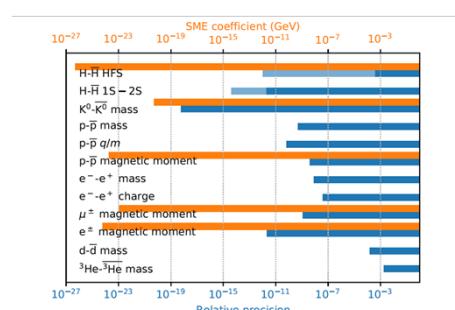
CPT invariance tested in several matter-antimatter systems:

neutral kaons

electron/positron

proton/antiproton

H/anti-H



Several experiments at the Antiproton Decelerator and ELENA(Extra Low Energy Antiproton) @CERN
E. Widmann, arXiv:2111.04056 [hep-ex]

Current bounds

- We can use data of various experiments to calculate the neutrino and antineutrino oscillation parameters:

- Solar neutrino data: $\theta_{12}, \Delta m_{21}^2, \theta_{13}$
- Neutrino mode in LBL: $\theta_{23}, \Delta m_{31}^2, \theta_{13}$
- KamLAND data: $\bar{\theta}_{12}, \Delta \bar{m}_{21}^2, \bar{\theta}_{13}$
- SBL reactors: $\bar{\theta}_{13}, \Delta \bar{m}_{31}^2$
- Antineutrino mode in LBL: $\bar{\theta}_{23}, \Delta \bar{m}_{31}^2, \bar{\theta}_{13}$

- No bounds on CP-phases since all values are allowed

Parameter	Main contribution	Other contributions
θ_{12}	SOL	KamLAND
θ_{13}	REAC	ATM, LBL and SOL+KamLAND
θ_{23}	ATM, LBL	-
δ_{CP}	LBL	ATM
Δm_{21}^2	KamLAND	SOL
$ \Delta m_{31}^2 $	LBL, ATM, REAC	-
MO	LBL, REAC and ATM	-

SOL: Solar
LBL: Long baseline accelerator experiments

ATM:

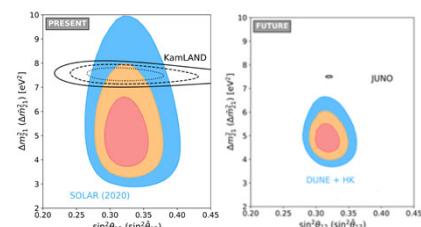
Aerospheric neutrinos

REAC: Short-baseline reactor experiments

Current bounds

- We use the same data (except atmospheric neutrinos) as for the global fit to obtain

$$\begin{aligned} |\Delta m_{21}^2 - \Delta \bar{m}_{21}^2| &< 4.7 \times 10^{-5} \text{ eV}^2, \\ |\Delta m_{31}^2 - \Delta \bar{m}_{31}^2| &< 2.5 \times 10^{-4} \text{ eV}^2, \\ |\sin^2 \theta_{12} - \sin^2 \bar{\theta}_{12}| &< 0.14, \\ |\sin^2 \theta_{13} - \sin^2 \bar{\theta}_{13}| &< 0.029, \\ |\sin^2 \theta_{23} - \sin^2 \bar{\theta}_{23}| &< 0.19. \end{aligned}$$



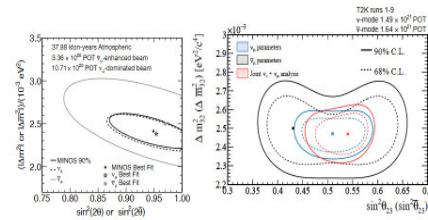
T2K results, a hint ?

- T2K studied neutrino and anti-neutrino oscillations separated

$$\sin^2 \theta_{23} = 0.51, \quad \Delta m_{32}^2 = 2.53 \times 10^{-3} \text{ eV}^2$$

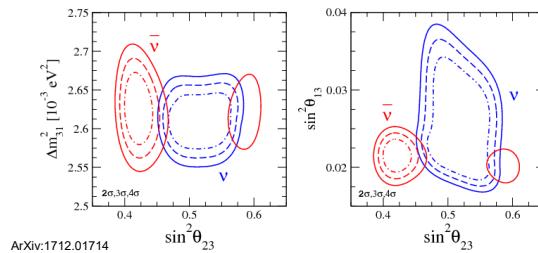
$$\sin^2 \bar{\theta}_{23} = 0.42, \quad \Delta \bar{m}_{32}^2 = 2.55 \times 10^{-3} \text{ eV}^2$$

- Results are consistent with
- CPT-conservation



DUNE about T2K

If these values turn out to be the true values, DUNE would measure CPT-violation at more than 3σ confidence level



- In experiments and in fits normally you assume CPT-conservation
- If CPT is not conserved this leads to impostor (fake) solutions in the fits
- To perform the standard fit you would calculate

$$\chi_{\text{total}}^2 = \chi^2(\nu) + \chi^2(\bar{\nu})$$
and then minimize this function

$$h(x, y) = f(x) + g(y)$$

$$\partial_x f(x) = 0 \quad \partial_y g(y) = 0$$

$$x = y$$

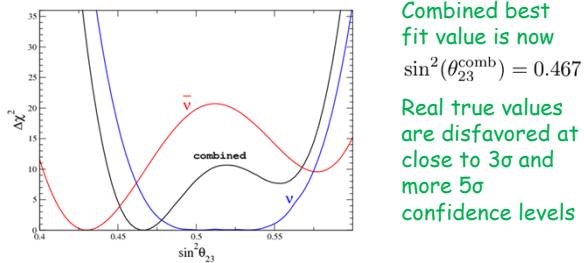
$$h(x) = f(x) + g(x)$$

$$\partial_x f(x) = \partial_x g(x) = 0$$

$$\partial_x f(x) = -\partial_x g(x)$$

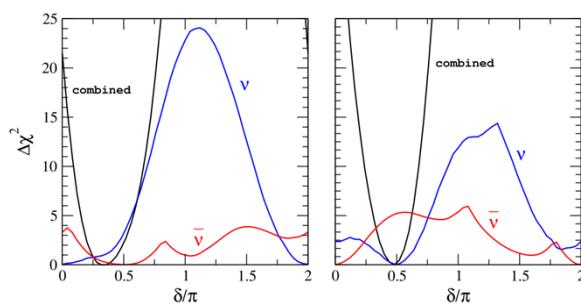
Obtaining impostor solutions

- This was done for $\sin^2(\theta_{23}) = 0.5, \sin^2(\bar{\theta}_{23}) = 0.43$



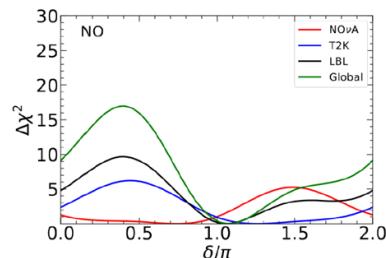
This can also happen

$$\delta = \begin{cases} \pi/2 \\ 0 \end{cases} \quad \text{and} \quad \bar{\delta} = \begin{cases} 0 \\ \pi/2 \end{cases}$$



G.B., C. Ternes and M. Tortola, JHEP 07 (2020) 155

$\theta_{13} \neq \bar{\theta}_{13}$ can account for different behavior in neutrino and antineutrino channels



all values of δ and $\bar{\delta}$ remain allowed at $\sim 1\sigma$

Tension between NOvA, T2K and SK atm. and $\delta_{bf} = 1.08\pi$

- Disfavours:
 - $\delta = \pi/2$ at 4.0σ
 - $\delta = 0$ at 3.0σ
 - $\delta = 3\pi/2$ with $\Delta\chi^2 = 4.9$

The increasing precision in neutrino oscillation measurements requires a thorough analysis of the assumptions considered.



Distinguishing CPT violation from NSNI

The muon neutrino survival probability in matter can be written as

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_\nu \sin^2 \left(\frac{\Delta m_\nu^2 L}{4E} \right).$$

in
matter

$$\begin{aligned} \Delta m_\nu^2 \cos 2\theta_\nu &= \Delta m^2 \cos 2\theta + \epsilon_{\tau\tau} A, & \Delta m_\nu^2 \cos 2\theta_\nu &= \Delta m^2 \cos 2\theta - \epsilon_{\tau\tau} A, \\ \Delta m_\nu^2 \sin 2\theta_\nu &= \Delta m^2 \sin 2\theta + 2\epsilon_{\mu\tau} A. & \Delta m_\nu^2 \sin 2\theta_\nu &= \Delta m^2 \sin 2\theta - 2\epsilon_{\mu\tau} A. \end{aligned}$$

$$4\Delta m^4 = \Delta m_\nu^4 + \Delta m_\nu^4 + 2\Delta m_\nu^2 \Delta m_\nu^2 \cos(2\theta_\nu - 2\theta_\tau)$$

$$\sin^2(2\theta) = \frac{(\Delta m_\nu^2 \sin(2\theta_\nu) + \Delta m_\nu^2 \sin(2\theta_\tau))^2}{\Delta m_\nu^4 + \Delta m_\nu^4 + 2\Delta m_\nu^2 \Delta m_\nu^2 \cos(2\theta_\nu - 2\theta_\tau)}$$

$$\begin{aligned} 2\epsilon_{\tau\tau}^m A &= \Delta m_\nu^2 \cos(2\theta_\nu) - \Delta m_\nu^2 \cos(2\theta_\tau) \\ 4\epsilon_{\mu\tau}^m A &= \Delta m_\nu^2 \sin(2\theta_\nu) - \Delta m_\nu^2 \sin(2\theta_\tau) \end{aligned}$$



G.B., C. Ternes and M. Tortola, Eur.Phys.J.C 79 (2019) 5, 390

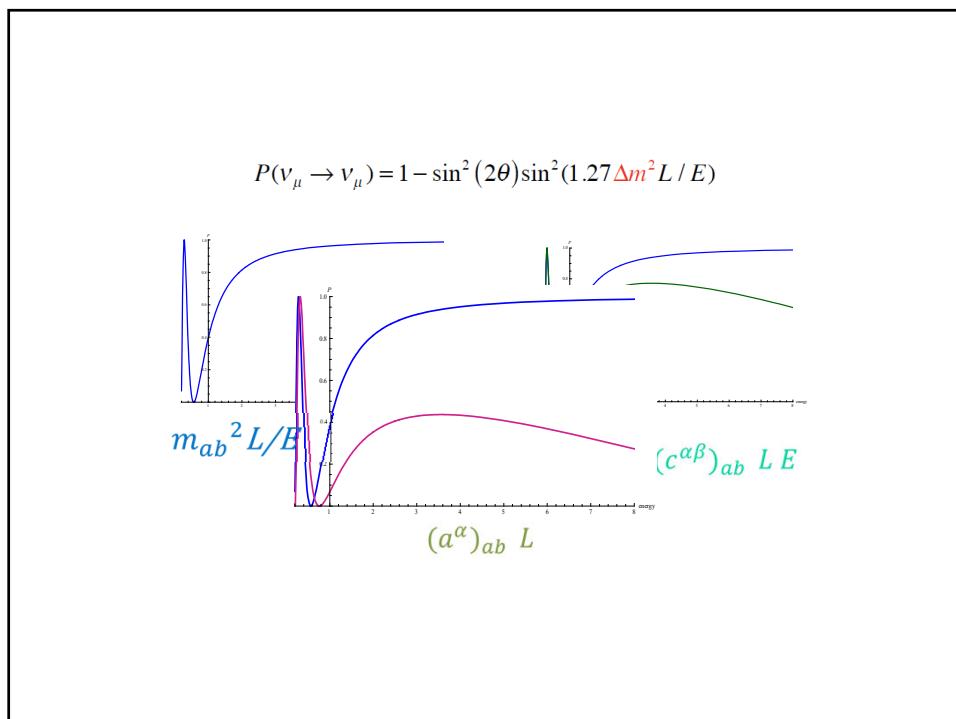
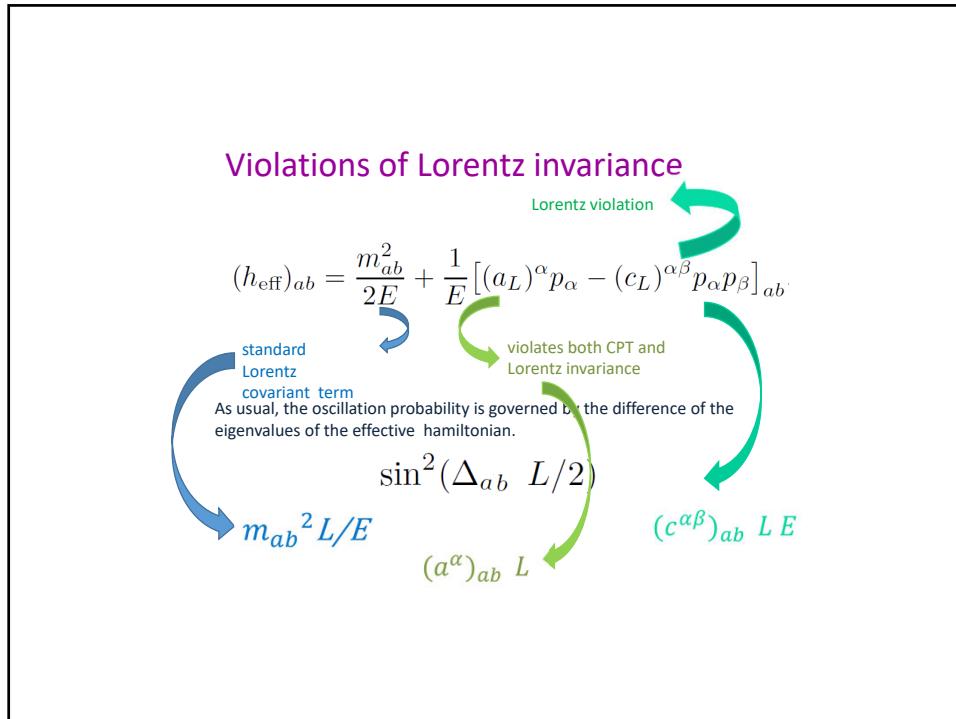
Violations of Lorentz invariance

$$(h_{\text{eff}})_{ab} = \frac{m_{ab}^2}{2E} + \frac{1}{E} [(a_L)^\alpha p_\alpha - (c_L)^{\alpha\beta} p_\alpha p_\beta]_{ab}.$$

standard
Lorentz
covariant term

Lorentz violation

violates both CPT and
Lorentz invariance



Neutrinos,
In and Beyond the Standard Model:

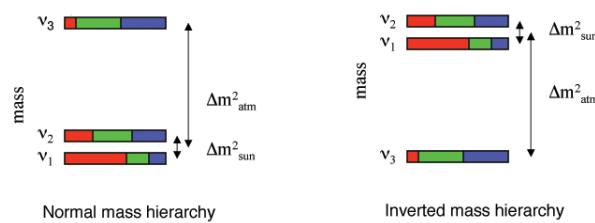
NEUTRINO MASS:

$$\delta m_{atm}^2 = 2.7^{+0.4}_{-0.3} \times 10^{-3} eV^2 \quad L/E = 500 \text{ km/GeV}$$

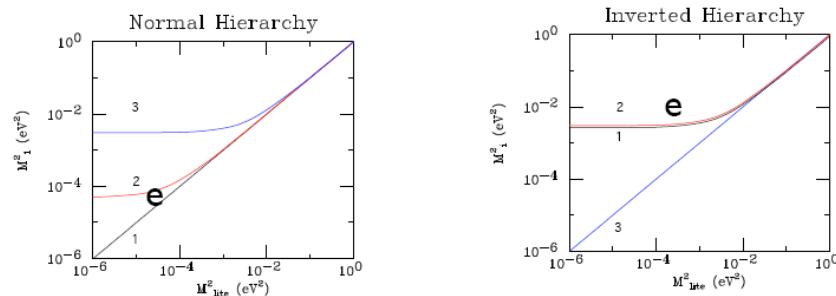
$$\delta m_{solar}^2 = 8.0 \pm 0.4 \times 10^{-5} eV^2 \quad L/E = 15 \text{ km/MeV}$$



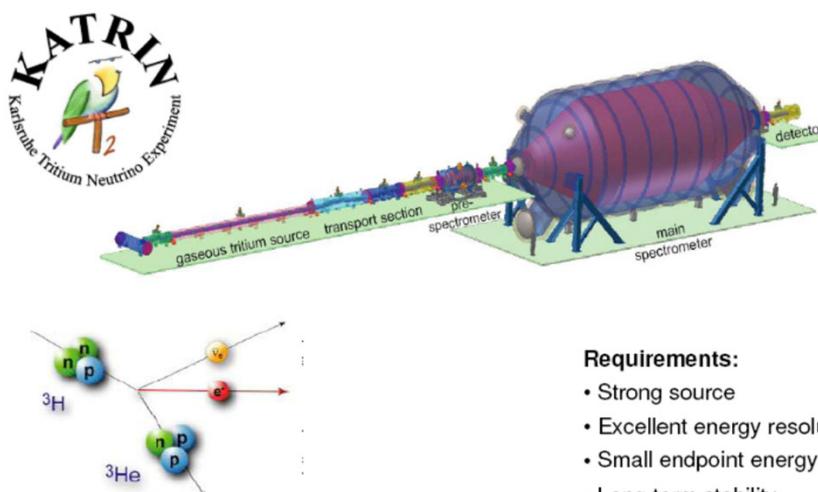
$$m_\nu^{Heavy} > \sqrt{\delta m_{atm}^2} = 50 \text{ meV}$$



Masses:



States 1 and 2 are ν_e rich.

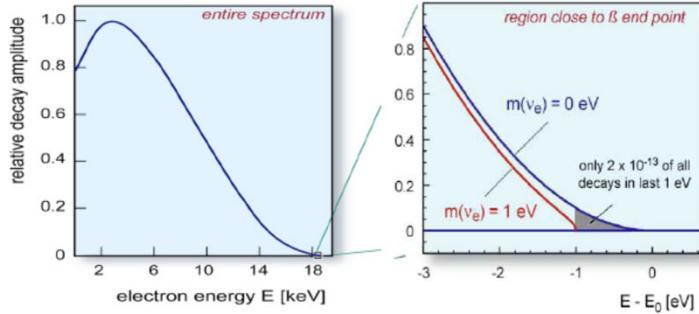


KATRIN Task:
Investigate Tritium endpoint with sub-eV precision

KATRIN Aim:
Improve m_ν sensitivity $10 \times (2\text{eV} \rightarrow 0.2\text{eV})$

Requirements:

- Strong source
- Excellent energy resolution
- Small endpoint energy E_0
- Long term stability
- Low background rate



Decay Rate:

$$|\langle {}^3He + e^- + \bar{\nu} | T | {}^3H \rangle|^2 \sim pE(E_0 - E) \sum_k |U_{ek}|^2 \sqrt{(E_0 - E)^2 - m_k^2}$$

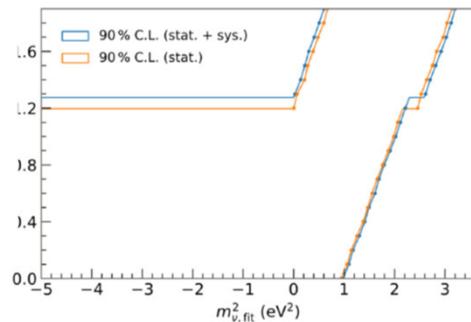
if ν 's quasi-degenerate: $m_1 \approx m_2 \approx m_3$

$$|\langle {}^3He + e^- + \bar{\nu} | T | {}^3H \rangle|^2 \sim pE(E_0 - E) \sqrt{(E_0 - E)^2 - m_\nu^2}$$

- KATRIN upper limit on neutrino mass:

LT $m(\nu) < 1.1$ eV (90% CL)

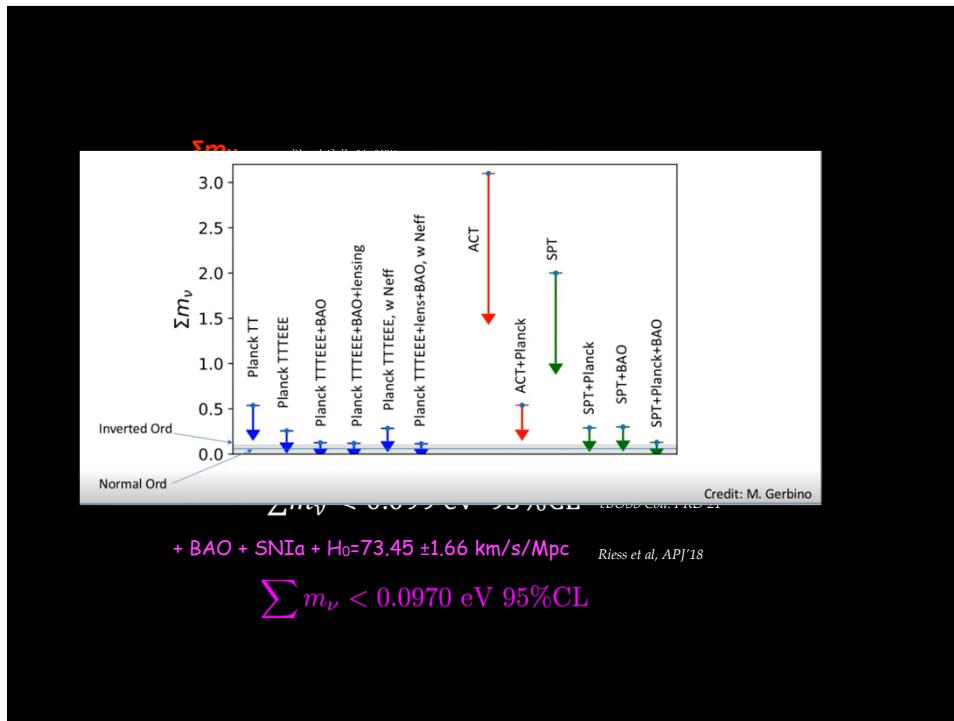
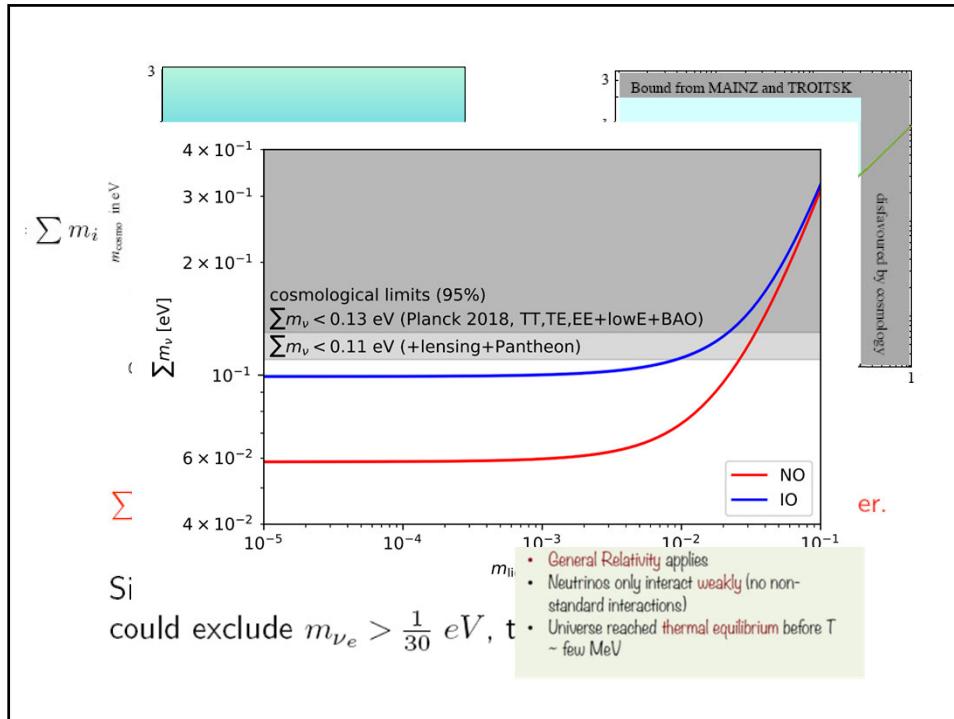
FC $m(\nu) < 0.8$ eV (90% CL)
 < 0.9 eV (95% CL)



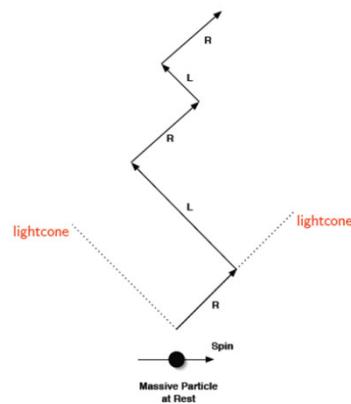
- 1000 days of measurements at nominal pd ($5 \cdot 10^{17}$ molecules cm⁻²)
3 tritium campaigns (65 days each)
per calendar year

sensitivity $m(v_e) = 0.2$ eV (90% CL)

0.35 eV (5 σ)



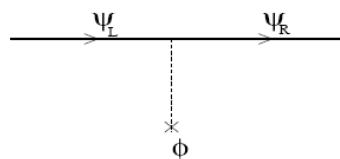
What is Fermion Mass ???



A mass can be thought of as a $L \leftrightarrow R$ transition:

$$m \bar{\psi}_L \psi_R + h.c.$$

In the SM fermion masses originate in the interaction with the Higgs field:



$$\lambda_f \bar{\psi}_L \Phi \psi_R + h.c. \rightarrow m_f = \lambda_f v$$

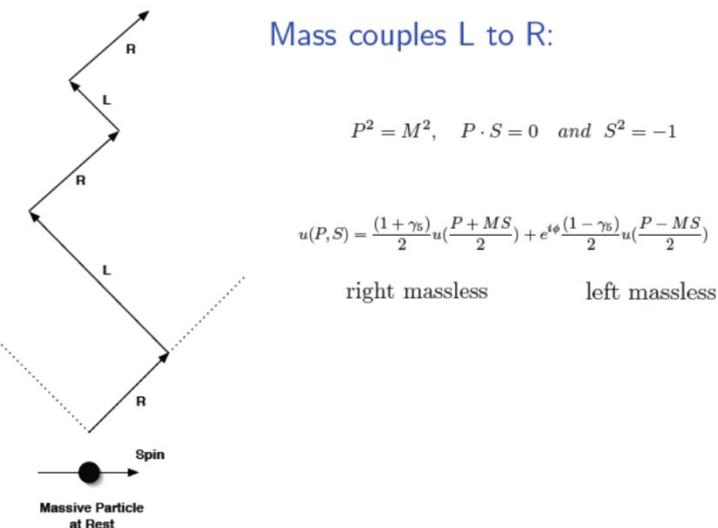
Fermion Masses:

	electron	positron	
Left Chiral	e_L	\bar{e}_R	$SU(2) \times U(1)$
Right Chiral	e_R	\bar{e}_L	$U(1)$

CPT: $e_L \leftrightarrow \bar{e}_R$ and $e_R \leftrightarrow \bar{e}_L$

Mass couples L to R:

e_L to e_R AND also \bar{e}_R to \bar{e}_L Dirac Mass terms.



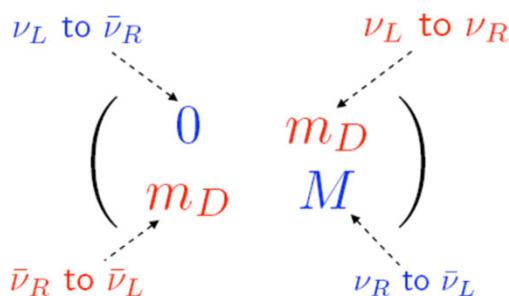
A coupling of e_L to \bar{e}_R OR e_R to \bar{e}_L would be (Majorana) mass term but this violates conservation of electric charge!

Seesaw / Dirac Neutrinos / Light Sterile Neutrinos

	Nu	CPT:	Anti-Nu	
Left Chiral	ν_L	\Leftrightarrow	$\bar{\nu}_R$	Dirac Masses
Right Chiral	ν_R	\Leftrightarrow	$\bar{\nu}_L$	Majorana Masses

Coupling of

- ν_L to ν_R AND $\bar{\nu}_R$ to $\bar{\nu}_L$ are the Dirac masses.
- ν_L to $\bar{\nu}_R$ forbidden by weak isospin.
- ν_R to $\bar{\nu}_L$ allowed and coefficient is unprotected. ($\rightarrow M$)



Two Majorana neutrinos
with masses m_D^2/M and M

Seesaw:
Yanagida, Gell-man-
Ramond-Slansky

- Coupling of ν_R to $\bar{\nu}_L$ allowed and coefficient is unprotected. ($\rightarrow M$)

Also applies to sterile neutrinos.

Light Sterile Neutrinos and/or Dirac Neutrinos Unexpected!!!

The consequences of this alternative are profound:

- Physics beyond the SM at a scale M !
- Majorana fermions carry no conserved charge: L is violated !

$$\nu_L \rightarrow e^{i\alpha} \nu_L$$

does not leave the Majorana mass term invariant.

- Most welcome for baryogenesis: a mechanism to understand the matter-antimatter asymmetry in the Universe emerges naturally
- Most welcome by string theory: it is difficult to get global $U(1)$ charges conserved

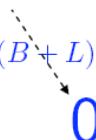
Leptogenesis

Baryon Asymmetry is created by a Lepton Asymmetry produced by the decays of super heavy Majorana Neutrinos.

$$\frac{\Gamma(N \rightarrow l^+ \phi^-) - \Gamma(N \rightarrow l^- \phi^+)}{\Gamma(N \rightarrow l^+ \phi^-) + \Gamma(N \rightarrow l^- \phi^+)}$$

$\Gamma(N \rightarrow l^\pm \phi^\mp)$ depends on the Majorana Phases in the MNS mixing matrix.

$$B_{now} = \frac{1}{2}(B - L) + \frac{1}{2}(B + L) = \frac{1}{2}(B - L)_{ini} = -\frac{1}{2}L_{ini}$$



$$0$$

Final asymmetry:

$$Y_B = 10^{-2} \underbrace{\epsilon_1}_{\text{CP-asym}} \underbrace{\kappa}_{\text{eff. factor}}$$

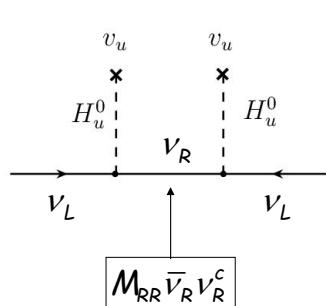
$$\epsilon_1 = \frac{\Gamma(N \rightarrow \Phi l) - \Gamma(N \rightarrow \Phi \bar{l})}{\Gamma(N \rightarrow \Phi l) + \Gamma(N \rightarrow \Phi \bar{l})}$$

κ efficiency factor which depends on the non-equilibrium dynamics.

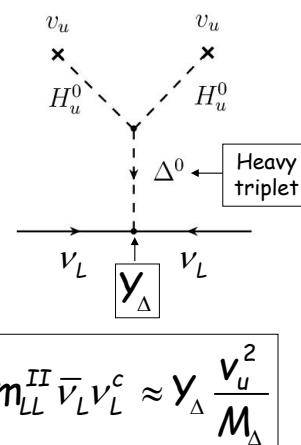
A relation between the baryon number of the Universe and the neutrino flavour parameters!

Types of see-saw mechanism

Type I see-saw mechanism



Type II see-saw mechanism



$$m_{LL}^I \approx -m_{LR} M_{RR}^{-1} m_{LR}^T$$

$$m_{LL}^{II} \bar{\nu}_L \nu_L^c \approx Y_\Delta \frac{v_u^2}{M_\Delta}$$

Naturalness may be over rated ...

Does this look natural ??

næbdyr



How Can We Demonstrate That $\bar{\nu}_i = \nu_i$?

We assume neutrino interactions are correctly described by the SM. Then the interactions conserve L ($\nu \rightarrow \ell^-$; $\bar{\nu} \rightarrow \ell^+$).

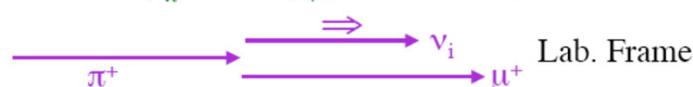
An Idea that Does Not Work
[and illustrates why most ideas do not work]

Produce a ν_i via—

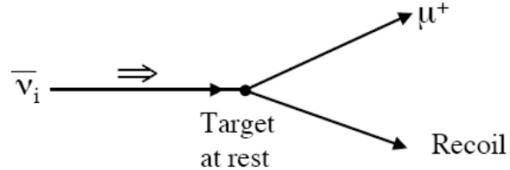


Give the neutrino a Boost:

$$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$$



The SM weak interaction causes—



$\nu_i = \bar{\nu}_i$ means that $\nu_i(h) = \bar{\nu}_i(h)$.

↑ ↑
 helicity

If $\nu_i \xrightarrow{\text{ }} = \bar{\nu}_i \xrightarrow{\text{ }} ,$

our $\nu_i \xrightarrow{\text{ }}$ will make μ^+ too.

Minor Technical Difficulties

$$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$$

$$\Rightarrow \frac{E_\pi(\text{Lab})}{m_\pi} > \frac{E_\nu(\pi \text{ Rest Frame})}{m_\nu}$$

$$\Rightarrow E_\pi \text{ (Lab)} > 10^4 \text{ TeV} \quad \text{if } m_\nu \sim 1 \text{ eV}$$

Fraction of all π -decay that get helicity flipped

$$\approx \left(\frac{m_\nu}{E_\nu (\pi \text{ Rest Frame})} \right)^2 \sim 10^{-16} \quad \text{if } m_\nu \sim 1 \text{ eV}$$

For Majorana Neutrinos

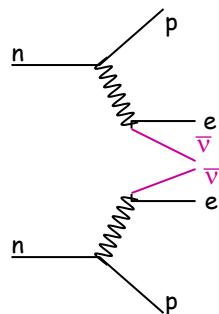


Not Observed

Allowed

BUT Suppressed by $\frac{m_\nu^2}{E^2} \sim 10^{-20}$!!!

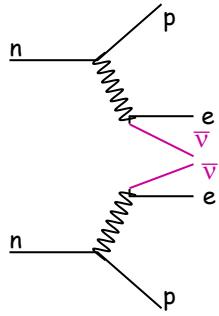
➤ How we can find out ?



SM double weak process

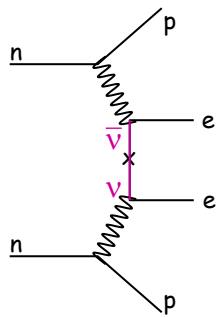
4 body decay: continuous spectrum for the energy sum

➤ How we can find out ?



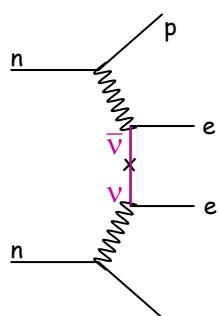
SM double weak process

4 body decay: continuous spectrum for the e energy sum

Only allowed for Majorana ν

2 body decay: e energy sum is a delta

$\bar{\nu}_i$ is emitted (RH + $O(m_i/E)$ LH)



Amp[ν_i contribution] $\sim m_i$

$$\text{Amp}[O\nu\beta\beta] \propto \left| \sum m_i U_{ei}^2 \right|$$

effective mass

Neutrinoless double beta decay

- Most sensitive (terrestrial) probe of the absolute neutrino mass
- Unique way of proving Majorana nature of ν
- If Majorana ν is the only mechanism, ==>



$$\begin{aligned} \langle m \rangle_{\beta\beta} &\equiv \left| \sum_{i=1}^3 m_i U_{ei}^2 \right| \\ &= \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\beta} + m_3 s_{13}^2 e^{2i(\gamma-\delta)} \right| \end{aligned}$$

