NBIA-Neutrinos Everywhere Gabriela Barenboim

July 17-19,22

Neutrino theory & phenomenology

Academia is a thrilling, complex and wild ecosystem, like the ocean. It indeed offers many possibilities for a fulfilling career, but you need to find your way to navigate the system. As the point is not to survive but to thrive, we will help you become a shark, neutrino-wise. No matter whether you are a minnow or a goldfish now, your shark fin is about to start growing. Just self asses your starting point and keep paddling your neutrino-feet.

1. **Neutrino oscillations (minnow $\longrightarrow$ sardine)**

   Suppose that neutrinos have mass. Then, we expect that the weak charge eigenstates of leptons are not the same as the mass eigenstates. But we can always align the charged lepton weak eigenstates with their mass eigenstates. But then the neutrinos will be mixtures. Consider for simplicity only 2 lepton doublets (instead of all 3). Write the weak doublets as

   $$\begin{pmatrix} \nu_1 \cos\alpha + \nu_2 \sin\alpha \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_2 \cos\alpha - \nu_1 \sin\alpha \\ \mu \end{pmatrix}$$

   where α is the neutrino mixing angle and ν_1, ν_2 are the mass eigenstates with masses m_1, m_2.

   If I produce a ν_1 neutrino with four momentum p at a time $t = 0$ and let it propagate freely, its wavefunction at later times is just

   $$\nu_1(t) = e^{-ip.x}\nu_1(0) \tag{1}$$

   (a) Suppose I produce a muon neutrino ν_μ with energy E. Assume that muon neutrinos can oscillate to only one other species of neutrino (x). Assume that the neutrinos are highly relativistic, i.e. $E \sim p \gg m$.

   Show that the probability that the muon has oscillated to the other species, after travelling a distance L, is approximately

   $$P(\nu_\mu \longrightarrow \nu_x) \simeq \sin^2(2\alpha)\ \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \tag{2}$$

   where Δm^2 is the difference of the square of the masses of the two neutrino mass eigenstates.

Large number of muon and electron neutrinos are produced by cosmic rays hitting the Earth's atmosphere.

(b) Assuming that the primary source of these neutrinos is pion and muon decays, estimate the ratio

$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \tag{3}$$

(c) This ratio starts to increase for neutrinos with energies above ~ 1 GeV. Why? (Hint: assume that the Earth's atmosphere is 20 km deep, and use Special Relativity.) The SuperK experiment measured the ratio above for 1 GeV neutrinos and got an answer only about 60% of what you computed. They also found that the ratio varied with the zenith angle: it was larger for neutrinos coming from directly overhead ~ 20 km and smaller for neutrinos that had to pass through the Earth, ~ 12000 km.

(d) Assuming this result is due to muon neutrino oscillating into some other kind of neutrino, and assuming that the mixing angle is large, estimate Δm^2 for atmospheric neutrino oscillations.

(e) use this to make an argument why the Noνa (DUNE) detector, situated 800 (1200) km from Fermilab, ougth to see the same oscillation effects. (Use the fact that you can make a nice neutrino beam starting with the 120 GeV protons in the Fermilab Main Injector).

2. **More oscillations (sardine $\longrightarrow$ dolphin)** With neutrinos, mass eigenstates do not coincide with interaction eigenstates so they do oscillate. But wait... in the quark sector, mass eigenstates are not interaction eigenstates either, we all know the CKM matrix.

   (a) So why they do not oscillate, or they do ?

   (b) Can we see quark oscillations ?

   (c) so when can we see oscillations ?

3. **Majorana or Dirac (dolphin $\longrightarrow$ tuna)**

   We all know $\pi^+ \longrightarrow \nu_\mu \ \mu^+$ (by the way why not $\pi^+ \longrightarrow \nu_e \ e^+$).

   (a) Calculate the momentum of the daughter particles in the pion rest frame.

(b) Now make a boost to the pion (lab frame) so that every particle moves forward. What is the pion energy in this frame (the lab frame)

(c) Now, lets make the boosted pion decay in flight and its neutrinos hit a target. If (the now) right handed neutrino is also a left handed antineutrino, we will start producing wrong sign muons (easy right?). Suppose we do it, what is the fraction that get helicity flipped ?

4. **Subtleties on oscillations (tuna $\longrightarrow$ shark)**

   Do non-relativistic neutrinos oscillate ? The question of observability of oscillations is closely related to the question of coherence in production, propagation and detection. If **at any stage** any of them is violated, oscillations will not be observable. We have used that all neutrinos were produced with the same energy in the relativistic case, and claimed that they have all the same momentum. What happens now ? What would happen if we use the wave packet formalism ?

5. **Neutrino masses (test your fin)**

   We said in the lectures that Majorana masses are bad because they would violate charge conservation. However if right-handed neutrinos exist, they do not carry any electric charge or weak charge. So I can give them a Majorana mass and the only penalty is that I violate lepton number.

   (a) Assume that neutrinos get both a Dirac mass and that the right-handed neutrinos get a Majorana mass. For one neutrino species, diagonalize this 2 by 2 mass matrix. Assume that the three neutrino species that we see have Dirac masses equal to the masses of e, μ and τ leptons, compute the required Majorana masses to get three light mass eigenstates in the range .01 to 1 eV.

   (b) Obtain an upper bound on the masses on neutrinos by assuming that the number density of each neutrino species is about 1/10 the number density of photons, in our Universe. Use the fact that the number density of baryons is measured to be about 10^{-10} that of photons.