NBIA summer school Copenhagen 2023

Sheet 1: Neutrino astronomy
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The easiest way to calculate expected event rates in neutrino telescopes, such as IceCube, is the **effective area** A_{eff} describing the detector response. The number of expected neutrino events N for a flux from a point source can be obtained with

$$N = \int_0^\infty dE \int_0^{t_{\text{obs}}} dt\, A_{\text{eff}}(E,\delta)\,\phi(E,t)\,, \tag{1}$$

where $\phi(E,t)$ is the neutrino flux (use units $\text{GeV}^{-1}\,\text{cm}^{-2}\,\text{s}^{-1}$), δ is the declination of the source, and t_{obs} is the observation time.

1) Declination dependence of A_{eff}

Use the following effective areas:

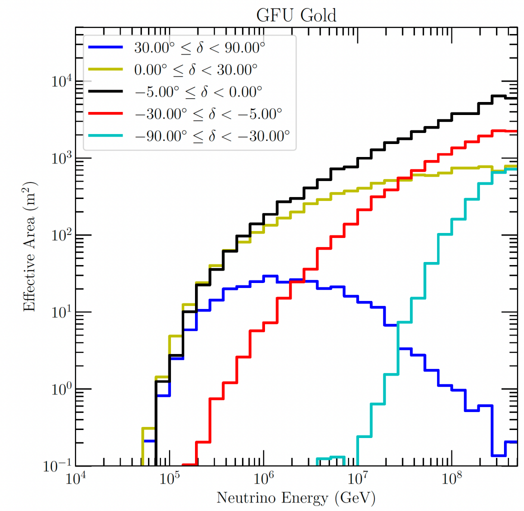

Figure 1: Effective area (for $\nu_\mu + \bar{\nu}_\mu$) as a function of neutrino energy for the event selection "GFU Gold". Taken from Ref. [1]

a) Why does it make sense to show A_{eff} for different declination bands? What kind of effect makes $A_{\text{eff}}(E)$ decline at high energies for large values of (positive) δ?

b) Assume now that the sources are isotropically distributed and identical. Which of the shown declination bands would dominate the number of neutrinos observed for IceCube?

Hint: Multiply the solid angle (proportional to the number of sources) with $A_{\text{eff}}(1\,\text{PeV})$ to compare the relative contributions.

2) Diffuse flux sensitivity of IceCube

Estimate the diffuse flux sensitivity limit of IceCube after ten years of operation. Use $A_{\rm eff}$ from the declination band $0° \leq \delta < 30°$ and assume a (constant in time) power law neutrino flux $\phi(E) = K_0 \cdot E^{-2}$ with a normalization factor K_0 in the background-free regime.[1]

Hint: Compute K_0 by setting $N \stackrel{!}{=} 2.44$ (90% CL Feldman-Cousins limit for no background, see Tab. 12 in Ref. [2]). Re-write Eq. (1) first into an integral over $x \equiv \log_{10} E$. You may then numerically integrate it, by using the approximated parameterization $A_{\rm eff} = 10^{\left(3.6\cdot(x-4.5)^{0.3}-2\right)}\,\mathrm{m}^2$ ($x \geq 4.5$) or by reading out the curve from the figure.

3) Differential limit

The quantity $E/(A_{\rm eff}(E)\, t_{\rm obs} \ln 10)$ is sometimes shown as a "differential limit".[2] How can this quantity be interpreted?

Hint: Identify this combination in the previous calculation after re-writing the integral. Show the differential limit together with the sensitivity limit in one figure.

4) Stacking analysis for GRBs

a) Compute a sensitivity limit, assuming that $N_{\rm GRB}$ identical GRBs at equal redshift have been stacked (looked at) and no neutrinos have been seen, in the background-free regime.

Hint: Here one uses a fluence (time-integrated flux) per GRB $\mathcal{F} = \tilde{K}_0 \cdot E^{-2}$ (use units $\mathrm{GeV}^{-1}\,\mathrm{cm}^{-2}$). The computation becomes similar to 2), solving for $\tilde{K}_0$, but the time integral in Eq. (1) is not necessary. Do not forget to sum the event rate over $N_{\rm GRB}$!

b) Convert this fluence limit per GRB into a quasi-diffuse flux limit from all GRBs, assuming that there are $\dot{N}_{\rm tot} \simeq 1000\,\mathrm{yr}^{-1}$ observable GRBs per year.[3] Compare to current GRB stacking limits, e.g. Fig. 7 in Ref. [3]. How does the result scale with $N_{\rm GRB}$? Why is the background-free assumption relatively well justified here?

References

[1] IceCube collaboration, *IceCat-1: the IceCube Event Catalog of Alert Tracks*, 2304.01174.

[2] G.J. Feldman and R.D. Cousins, *A Unified approach to the classical statistical analysis of small signals*, *Phys. Rev. D* **57** (1998) 3873 [physics/9711021].

[3] IceCube collaboration, *Extending the search for muon neutrinos coincident with gamma-ray bursts in IceCube data*, *Astrophys. J.* **843** (2017) 112 [1702.06868].

[1] Note that for realistic applications, the appriate effective area has to be taken!

[2] The pre-factor is sometimes a matter of definition/choice/purpose.

[3] To obtain a flux per steradian, divide the diffuse flux by the total solid angle 4π: $\phi_{\rm QD} = \frac{1}{4\pi}\,\dot{N}_{\rm tot}\,\mathcal{F}$.