

## Neutrino cosmology assignment

We shall need the expressions for the number density, energy density, and pressure of relativistic species in the early Universe.

$$\rho = \begin{cases} \frac{\pi^2}{30} g T^4 & \text{bosons} \\ \frac{7}{8} \frac{\pi^2}{30} g T^4 & \text{fermions} \end{cases} \quad (1)$$

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2} g T^3 & \text{bosons} \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} g T^3 & \text{fermions} \end{cases} \quad (2)$$

$$P = \rho/3 \quad (3)$$

Here,  $\zeta(3) = 1.20206\dots$  is the Riemann zeta-function.  $g$  is the number of internal degrees of freedom (spin states) for the species.  $g_\gamma = 2$ ,  $g_\nu = g_{\bar{\nu}} = 1$ ,  $g_{e^-} = g_{e^+} = 2$ . **NOTE:** We are using natural units where  $\hbar = c = k_B = 1$ . **EXAMPLE:** The energy density and number density of a boson species in SI or cgs units (so that  $[\rho] = \text{g}/\text{cm}^3$ , and  $[n] = \text{cm}^{-3}$ ) is

$$\rho = \frac{\pi^2}{30c^5\hbar^3} g (k_B T)^4 \quad (4)$$

$$n = \frac{\zeta(3)}{\pi^2 c^3 \hbar^3} g (k_B T)^3 \quad (5)$$

### 1. The expansion rate in the early Universe

The present density of non-relativistic matter ( $P = 0$ ) in the Universe is  $\Omega_M h^2 = \frac{\rho_M}{\rho_c} h^2 = 0.12$ . The critical density is  $\rho_c = 1.88 \times 10^{-29} h^2 \text{ g}/\text{cm}^3$ . The present photon temperature is 2.726 K.

Use the equation of energy conservation,

$$\frac{\partial \rho}{\partial t} + 3 \frac{\dot{a}}{a} (\rho + P) = 0 \quad (6)$$

which applies separately for matter and radiation to show that  $\rho_M \propto a^{-3}$  and  $\rho_R \propto a^{-4}$ .

Calculate the present value of  $\rho_\gamma/\rho_M$ .

At which value of  $a$  did the Universe become matter dominated ( $\rho_M > \rho_\gamma$ ), assuming that  $a_0 = 1$ ?

Assume that in the early Universe at  $T > 1$  MeV, the energy density is completely dominated by neutrinos, photons, electrons, and positrons. There are 3 different neutrino species (electron, muon and tau neutrinos).

Calculate  $H(T)$  in the early Universe at  $T = 1$  MeV (assuming that it is flat).

Show that in a radiation dominated Universe  $a \propto t^{1/2}$  and  $H = \frac{1}{2}t^{-1}$

Assuming that  $a(t = 0) = 0$ , calculate the age of the Universe at  $T = 1$  MeV.

## 2. Neutrino decoupling

In the early Universe, neutrinos can be created and destroyed by the process

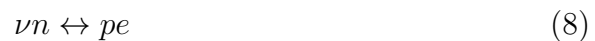


The thermally averaged cross section for this process is given by  $\langle\sigma|v|\rangle = KG_F^2T^2$ , where  $K$  is a constant of order unity. Assume that  $K = 1$ .

Use the condition  $\Gamma \equiv n_\nu\langle\sigma|v|\rangle = H$  to calculate the decoupling temperature of neutrinos.

## 3. Big Bang nucleosynthesis

If the process



proceeds in equilibrium, the neutron to proton ratio is

$$z = \frac{n_n}{n_p} = e^{-(m_n - m_p)/T}. \quad (9)$$

The process (8) has a thermally averaged cross section of  $\langle\sigma|v|\rangle \simeq 60G_F^2 T^2$ . The interaction rate for neutrons is then  $\Gamma_n = n_\nu \langle\sigma|v|\rangle$ .

Calculate the decoupling temperature for the process (8).

Calculate  $z$  at this temperature.

Now assume that there are 4 neutrinos instead of 3. What is the decoupling temperature of neutrons in this case?

Calculate  $z$  at this temperature.

After the reaction (8) decouples, neutrons will decay with a lifetime of  $\tau = 887$  s. The temperature at which helium is produced is  $T = 0.07$  MeV.

Calculate the neutron to proton ratio at this temperature for  $N_\nu = 3$  and  $N_\nu = 4$ .

Will the final helium abundance  $Y$  be higher or lower for  $N_\nu = 4$  than for the case of  $N_\nu = 3$ ?