Neutrino cosmology assignment

We shall need the expressions for the number density, energy density, and pressure of relativistic species in the early Universe.

$$\rho = \begin{cases} \frac{\pi^2}{30} g T^4 & \text{bosons} \\ \frac{7}{8}\frac{\pi^2}{30} g T^4 & \text{fermions} \end{cases} \quad (1)$$

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2} g T^3 & \text{bosons} \\ \frac{3}{4}\frac{\zeta(3)}{\pi^2} g T^3 & \text{fermions} \end{cases} \quad (2)$$

$$P = \rho/3 \quad (3)$$

Here, $\zeta(3) = 1.20206...$ is the Riemann zeta-function. g is the number of internal degrees of freedom (spin states) for the species. $g_\gamma = 2$, $g_\nu = g_{\bar{\nu}} = 1$, $g_{e^-} = g_{e^+} = 2$. **NOTE:** We are using natural units where $\hbar = c = k_B = 1$. **EXAMPLE:** The energy density and number density of a boson species in SI or cgs units (so that $[\rho] = \text{g/cm}^3$, and $[n] = \text{cm}^{-3}$) is

$$\rho = \frac{\pi^2}{30 c^5 \hbar^3} g (k_B T)^4 \quad (4)$$

$$n = \frac{\zeta(3)}{\pi^2 c^3 \hbar^3} g (k_B T)^3 \quad (5)$$

1. The expansion rate in the early Universe

The present density of non-relativistic matter ($P = 0$) in the Universe is $\Omega_M h^2 = \frac{\rho_M}{\rho_c} h^2 = 0.12$. The critical density is $\rho_c = 1.88 \times 10^{-29} h^2\,\text{g/cm}^3$. The present photon temperature is 2.726 K.

Use the equation of energy conservation,

$$\frac{\partial \rho}{\partial t} + 3\frac{\dot{a}}{a}(\rho + P) = 0 \quad (6)$$

which applies separately for matter and radiation to show that $\rho_M \propto a^{-3}$ and $\rho_R \propto a^{-4}$.

Calculate the present value of ρ_γ/ρ_M.

At which value of a did the Universe become matter dominated ($\rho_M > \rho_\gamma$), assuming that $a_0 = 1$?

Assume that in the early Universe at $T > 1$ MeV, the energy density is completely dominated by neutrinos, photons, electrons, and positrons. There are 3 different neutrino species (electron, muon and tau neutrinos).

Calculate $H(T)$ in the early Universe at $T = 1$ MeV (assuming that it is flat).

Show that in a radiation dominated Universe $a \propto t^{1/2}$ and $H = \frac{1}{2}t^{-1}$

Assuming that $a(t = 0) = 0$, calculate the age of the Universe at $T = 1$ MeV.

2. Neutrino decoupling

In the early Universe, neutrinos can be created and destroyed by the process

$$\nu\bar{\nu} \leftrightarrow e^+e^- \tag{7}$$

The thermally averaged cross section for this process is given by $\langle\sigma|v|\rangle = KG_F^2T^2$, where K is a constant of order unity. Assume that $K = 1$.

Use the condition $\Gamma \equiv n_\nu\langle\sigma|v|\rangle = H$ to calculate the decoupling temperature of neutrinos.

3. Big Bang nucleosynthesis

If the process

$$\nu n \leftrightarrow pe \tag{8}$$

proceeds in equilibrium, the neutron to proton ratio is

$$z = \frac{n_n}{n_p} = e^{-(m_n - m_p)/T}. \tag{9}$$

The process (8) has a thermally averaged cross section of $\langle \sigma |v| \rangle \simeq 60 G_F^2 T^2$. The interaction rate for neutrons is then $\Gamma_n = n_\nu \langle \sigma |v| \rangle$.

Calculate the decoupling temperature for the process (8).

Calculate z at this temperature.

Now assume that there are 4 neutrinos instead of 3. What is the decoupling temperature of neutrons in this case?

Calculate z at this temperature.

After the reaction (8) decouples, neutrons will decay with a lifetime of $\tau =$ 887 s. The temperature at which helium is produced is $T = 0.07$ MeV.

Calculate the neutron to proton ratio at this temperature for $N_\nu = 3$ and $N_\nu = 4$.

Will the final helium abundance Y be higher or lower for $N_\nu = 4$ than for the case of $N_\nu = 3$?