Probing nonlagrangian theories with surface operators

based on

[arxiv:2204.12627] "Ironing out the crease" with N. Drukker

[arxiv:2212.05020] "Bootstrapping string dynamics in the 6d (2,0) theories" with C. Meneghelli

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Surface operators are 2d objects that arise in a variety of contexts, e.g.

- In 3d they appear as boundaries and interfaces
- In 4d they appear as Dirac strings, coupled 2d-4d systems, vortices, entanglement entropy, and more

The focus of my talk will be their role in the 6d $\mathcal{N} = (2, 0)$ SCFTs, which are famous examples of nonlagrangian theories.

I will discuss how

- Surface operators in these theories are fundamental observables analogous to Wilson lines
- We have powerful nonperturbative tools to study them, which also apply to other theories

The free theory

The simplest 6d $\mathcal{N} = (2, 0)$ theory is the theory of a free tensor multiplet in 6d. It consists of

- a two-form potential $B_{\mu\nu}$ with self-dual field strength $dB = *dB$.
- a scalar field Φ^i transforming under $SO(5)$
- some fermions

Writing a lagrangian is difficult because of the self-duality constraint. Nevertheless the EOMs are known.

We can construct a surface operator

$$V_{\Sigma,n} = \exp \int_\Sigma B$$

V is invariant under gauge transformations $B \to B + d\Lambda$
$\Rightarrow$ analogous to a Wilson line

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$$V_{\Sigma,n} = \exp \int_{\Sigma} B + n^i \Phi^i \, \mathrm{vol}_{\Sigma}$$

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Both B and Φ have dimension 2.

The interacting theories

They are predicted by string/M-theory considerations, e.g. they describe the low-energy dynamics of N coincident M5-branes

- No known lagrangian description
- Maximal supersymmetry and maximal dimension
- No exactly marginal deformation ("strongly coupled")
- Obey an ADE classification
- Underpin the construction of many theories in lower dimensions (e.g. class S) and shed light on dualities
- M2-branes ending on M5-branes give rise to surface operators $V_{\Sigma,n}$

These theories challenge the standard perturbative approach to QFT. In the absence of a lagrangian, we may ask

- Do they really exist?
- How do we even define them?
- How do we perform calculations?

To answer these questions we need new tools.

My approach to this problem: exploit the analogy between surface operators and Wilson lines in $\mathcal{N} = 4$ SYM.

Since Wilson lines have been so useful in understanding $\mathcal{N} = 4$ SYM, it suggests that surface operators are an excellent starting point to study the (2,0) theories.

Wilson lines:

- capture the quark-antiquark potential
- labelled by reps Λ of the gauge group
- dual to fundamental strings
- Feynman diagrams
- supersymmetric localization
- bootstrap
- ...

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~~Wilson lines:~~ Surface operators:

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- labelled by reps Λ of the ~~gauge group~~ ADE group
- dual to ~~fundamental strings~~ M2-branes
- ~~Feynman diagrams~~
- ~~supersymmetric localization~~ VOA
- bootstrap
- ...

Plan for the talk

The (generalised) string potential

Vertex operator algebra

Analytical conformal bootstrap at large N

Conclusion

The (generalised) string potential

Recall that in any gauge theory, a fundamental observable is the quark-antiquark potential calculated by a pair of Wilson lines

$$\langle W(0)W(R)\rangle = \exp\left(-U(R)T\right), \qquad U(R) = \frac{U_0}{R}$$

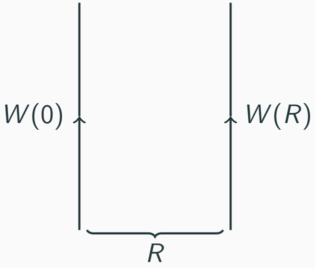


Figure 1: Parallel Wilson lines

For surface operators we can take 2 parallel planes to get the string potential

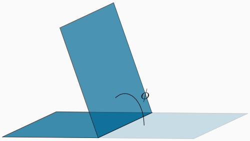
$$\langle V(0)V(R)\rangle = \exp\left(-\frac{U_0 \text{Area}}{R^2}\right)$$

$U_0(N, \Lambda)$ can be calculated in the free theory and at large N using holography [Maldacena, 1998, Drukker and Trépanier, 2022]

The crease

We can define a generalised potential $U(\phi, \theta, N, \Lambda)$ which gives us more parameters to play with.

Consider the crease



e.g. for the free theory

$$V_{\phi,\theta} = \exp \left[\int_{r<0} \mathrm{d}r\mathrm{d}s (B_{12} + \Phi^1) \right. \\ \left. + \int_{r>0} \mathrm{d}r\mathrm{d}s \cos\phi B_{12} + \sin\phi B_{13} + \cos\theta \Phi^1 + \sin\theta \Phi^2 \right]$$

The expectation value must be extensive in the length L, so by dimensional analysis is proportional to L/ϵ, ϵ a UV cutoff. This is removed by renormalisation.

To define a finite quantity, take the conformal transformation to a compact surface (with the topology of a sphere).

The change in topology gives rise to a conformal anomaly, which appears as a $\log \epsilon$ divergence.

Example: the sphere in the free theory

$$\langle V_{S^2} \rangle = \left\langle \exp \int_{S^2} B + \Phi^1 \,\mathrm{vol}_{S^2} \right\rangle$$
$$= 1 + \frac{1}{16\pi^2} \int_{S^2} \frac{\cos u \mathrm{d}u \mathrm{d}v \cos u' \mathrm{d}u' \mathrm{d}v'}{1 - \cos u \cos u' \cos(v - v') - \sin u \sin u' + \epsilon^2/2R^2}$$
$$+ \dots$$

Using spherical symmetry this reduces to

$$\frac{4\pi \cdot 2\pi}{16\pi^2} \int_{-\pi/2}^{\pi/2} \frac{\cos u \mathrm{d}u}{1 - \sin u + \epsilon^2/2R^2} = -\log\left(\frac{\epsilon}{2R}\right) + \mathcal{O}(\epsilon^2)\,.$$

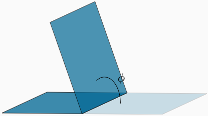
The appearance of the $\log \epsilon$ divergence indicates that the regularisation does not preserve conformal symmetry: it's a conformal anomaly.

The anomaly does not depend on ϕ, θ, so we can obtain a finite quantity by taking a difference

$$\log \langle V_{\phi,\theta} \rangle - \log \langle V_{S^2} \rangle = 2\pi U(\phi, \theta, N, \Lambda)$$

U can be calculated explicitly at $N = 1$ and $N \gg 1$, e.g.

$$U(\phi, \theta, N = 1) = \frac{1}{2\pi} \log \cos \frac{\phi}{2} + \frac{\cos \phi - \cos \theta}{8\pi \cos^2(\phi/2)}$$



In [Drukker and Trépanier, 2022], we find that

- U is interpreted as the potential density on $AdS_2 \times S^4$
- In the limit $\phi \to \pi, \theta \to 0$, we recover U_0
- When $\phi, \theta \ll 1$, U can be calculated using defect CFT techniques for any N, Λ
- When $\phi = \theta$, the crease is BPS and we conjecture that

$$U(\phi, \phi, N, \Lambda) = \frac{d(N, \Lambda)}{\pi} \log \cos(\phi/2)$$

with $d(N, \Lambda)$ an anomaly coefficient [Chalabi et al., 2020].

The quantity U can be defined for any surface operator. In upcoming work I will discuss the appearance of U in the critical $O(N)$ model.

Vertex operator algebra

In lagrangian theories, one can sometimes calculate observables exactly using supersymmetric localization. In the (2,0) theories, we can the VOA construction of [Beem et al., 2015] instead.

Consider the surface operator V defined over a plane (1/2 BPS). I will show how to calculate exact OPE coefficients $a_{\Delta,\ell,R}$ appearing in the 1-point function of certain BPS bulk operators $\mathcal{O}$ with V

$$\langle \mathcal{O}_{\Delta,\ell,R}(x,u,v)V\rangle = a_{\Delta,\ell,R}\frac{(u\cdot n)^R(x^\perp\cdot v)^\ell}{|x_\perp|^\Delta}\,.$$

u, v encode polarisations.

Review of the VOA

The (2,0) theory contains a set of 1/2 BPS operators Φ_k with $k = 2, \ldots, N$. Φ_2 is the superprimary of the stress tensor multiplet.

The 3-point functions are

$$\langle \Phi_{k_1}(x_1, u_1) \Phi_{k_2}(x_2, u_2) \Phi_{k_3}(x_3, u_3) \rangle = \lambda_{k_1 k_2 k_3} \left(\frac{-2 u_1 \cdot u_2}{x_{12}^4} \right)^{\frac{k_{123}}{2}} \left(\frac{-2 u_1 \cdot u_3}{x_{13}^4} \right)^{\frac{k_{132}}{2}} \left(\frac{-2 u_2 \cdot u_3}{x_{23}^4} \right)^{\frac{k_{231}}{2}}$$

Taking $x_i = (z_i, \bar{z}_i, 0, \ldots)$ and choosing $u_i = \bar{z}_i$ we get

$$\langle \Phi_{k_1}(z_1, \bar{z}_1; \bar{z}_1) \Phi_{k_2}(z_2, \bar{z}_2; \bar{z}_2) \Phi_{k_3}(z_3, \bar{z}_3; \bar{z}_3) \rangle = \frac{\lambda_{k_1 k_2 k_3}}{(z_{12})^{k_{123}} (z_{13})^{k_{132}} (z_{23})^{k_{231}}} \, .$$

This is a 3-point function of primary operators in a 2d CFT.

The deeper reason for the appearance of a 2d CFT structure is that the choice $x = (z, \bar{z}, \dots)$ and $u = \bar{z}$ defines a holomorphic twist of the (2,0) theory.

Concretely, $\Phi_k(z, \bar{z}, \bar{z})$ sit in the cohomology of a supercharge $\mathcal{Q}$, i.e. $\mathcal{Q}\Phi = 0$. $\bar{z}$ translations are $\mathcal{Q}$-exact, so $\Phi(z, \bar{z}, \bar{z}) = \Phi(z, 0, 0) + \mathcal{Q}(\dots)$ and the dependence on $\bar{z}$ drops out of the correlators.

Beem, Rastelli, van Rees conjecture that correlators of holomorphically-twisted operators obey the structure of the $\mathcal{W}_N$ algebra, with

$$\Phi_k(z, \bar{z}, \bar{z}) \mapsto W_k(z)$$

$W_k(z)$ are the generators of the $\mathcal{W}_N$ algebra.

This means that we can use tools from 2d CFTs to calculate the CFT data of a class of BPS operators in the 6d theory!

Example: 4-point functions of stress tensors

We consider the 4-point function

$$\langle T(0)\,T(z)\,T(1)\,T(\infty)\rangle$$

This is a meromorphic function, so is uniquely determined by its poles in z and their residues. Using the OPE

$$T(z)\,T(w) = \frac{c/2}{(z-w)^4} + \frac{2\,T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)} + \dots$$

we can obtain all the poles at $z = 0$

$$\frac{c/2}{z^4}\,\langle T(1)\,T(\infty)\rangle + \frac{2}{z^2}\,\langle T(0)\,T(1)\,T(\infty)\rangle + \frac{1}{z}\,\langle \partial T(0)\,T(1)\,T(\infty)\rangle$$

and similarly at $z = 1, z = \infty$. We find

$$\frac{\langle T(0)\,T(z)\,T(1)\,T(\infty)\rangle}{\langle T(0)\,T(z)\rangle\langle T(z)\,T(1)\rangle} = 1+z^4+\left(\frac{z}{z-1}\right)^4+\frac{8}{c}\left(\frac{z}{z-1}\right)^2(z^2-z+1)$$

Including surface operators

Surface operators fit naturally in the VOA, since $\mathcal{Q}V = 0$ for V a $1/2$ BPS plane orthogonal to the $z, \bar{z}$ plane.

They are identified with modules $|\mathsf{V}_\Lambda\rangle$ of the W_N algebras

$$V \mapsto \mathsf{V}_\Lambda(0)$$

These modules were studied in [Fateev and Lukyanov, 1988], they showed that

$$T(z)\mathsf{V}_\Lambda(0) \sim \frac{\Delta(N,\Lambda)}{z^2}\,\mathsf{V}_\Lambda(0) + \frac{2}{z}\,\partial\mathsf{V}_\Lambda(0) + \dots$$

where

$$\Delta(N,\Lambda) = -\frac{1}{2}(\Lambda,\Lambda) - 2(\rho,\Lambda)$$

Let's obtain some OPE coefficients. The 1-point function is

$$\langle \Phi_2 V \rangle \mapsto \left\langle \bar{\mathsf{V}}_\Lambda(\infty) T(z) \mathsf{V}_\Lambda(0) \right\rangle = \frac{\Delta(N,\Lambda)}{z^2} \,.$$

So

$$a_{\Phi_2} = \Delta(N,\Lambda)$$

The predicted OPE coefficient is $-d(N,\Lambda)$ [Chalabi et al., 2020], and indeed $-d(N,\Lambda) = \Delta(N,\Lambda)$. This is a direct confirmation of their result!

We can do more. Consider the 2-point function

$$\langle \Phi_2 \Phi_2 V_\Lambda \rangle \mapsto \left\langle \bar{\mathsf{V}}_\Lambda(\infty) T(z) T(1) \mathsf{V}_\Lambda(0) \right\rangle$$
$$= 1 + \frac{\Delta(N,\Lambda)^2}{c} 2Z^2 + \frac{\Delta(N,\Lambda)}{c} 4Z \,, \qquad Z \equiv \frac{(z-1)^2}{z} \,.$$

We can read the OPE coefficients from 2 conformal blocks decompositions, the "bulk" and "defect" channels.

In the bulk channel we find e.g.

$$(a\lambda)_{\mathcal{D}[2,0]} = \frac{4\Delta(N,\Lambda)}{c} \,, \qquad (a\lambda)_{\mathcal{D}[4,0]} = \alpha_0 \,, \qquad (a\lambda)_{\mathcal{B}[2,0]_{\ell=2n-2}} = -\alpha_n \,.$$

$$\alpha_n = \frac{(2n+2)!(2n+3)!}{(2n+3)!(4n+5)} \left((2n+5) \frac{\Delta(N,\Lambda)^2}{c} + \frac{1}{n+1} \frac{\Delta(N,\Lambda)}{c} \right)$$

from which we can extract the coefficients *a*.

- Exact results in a nonlagrangian theory!
- Strong indication that these theories are meaningful for any N
- The chiral algebra also contains much more information than what we analysed
- Confirmation of the classification of surface operators by Λ
- What about more complicated BPS observables? (e.g. the BPS potential)

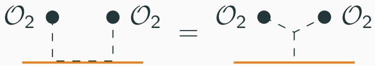
Analytical conformal bootstrap at large N

The bootstrap approach to defects is well-known. We consider the 2-point function

$$\langle \mathcal{O}_2(x_1, u_1)\mathcal{O}_2(x_2, u_2)V\rangle = \frac{|u_1^\perp|^2|u_2^\perp|^2}{|x_1^\perp|^4|x_2^\perp|^4}\mathcal{F}(z, \bar{z}, \omega)$$

$z, \bar{z}, \omega$ are conformal cross-ratios.

$\mathcal{F}$ admits two conformal blocks decomposition, which is a crossing symmetry constraint

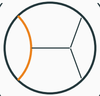


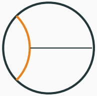
$$\mathcal{F} = \sum_{\hat{\mathcal{O}}_k}(b^2)_k\hat{\mathcal{G}}_k(z, \bar{z}, \omega) = \sum_{\mathcal{O}_l}(a\lambda)_l\mathcal{G}_l(z, \bar{z}, \omega)$$

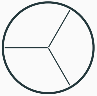
Solving these constraints is hard.

At large N we can use the supersymmetric inversion formula [Barrat, Gimenez-Grau, Liendo 2022]

$$\mathcal{F} \sim \int \text{Disc} \left(\quad \right) \sim \quad \times$$







Adapting this strategy we find

$$\mathcal{F}(z,\bar{z},\omega) = \frac{|z-\omega|^2|z-\omega^{-1}|^2}{|z-1|^4} F(z,\bar{z}) + \frac{\bar{z}(z-\omega)(z-\omega^{-1})(\omega-1)^2}{\omega(z-\bar{z})(z-\bar{z}^{-1})(\bar{z}-1)^2}\zeta(z) + \text{c.c.}$$

where F, ζ are given by

$$\zeta(z) = \frac{z^2}{(1-z)^4} + a_2^2 + a_2\lambda_{222}\frac{z}{(1-z)^2}\,,$$

$$\begin{aligned} F(z,\bar{z}) = 0 + a_2^2 + a_2\lambda_{222}\Bigg(&\frac{z\bar{z}}{(1-z\bar{z})^6}\left[2\left(1+z\bar{z}+(z\bar{z})^2\right)\left(1+18z\bar{z}+(z\bar{z})^2\right)-(z+\bar{z})\left(1+z\bar{z}\right)\left(1+28z\bar{z}+(z\bar{z})^2\right)\right] \\ &+6\frac{(z\bar{z})^2\log z\bar{z}}{(1-z\bar{z})^7}\left[(1+z\bar{z})\left(3+4z\bar{z}+3(z\bar{z})^2\right)-2\left(z+\bar{z}\right)\left(1+3z\bar{z}+(z\bar{z})^2\right)\right]\Bigg) + \ldots \end{aligned}$$

along with the dCFT data b^2, $a\lambda$ and anomalous dimensions.

- $\zeta(z)$ agrees with the result from the VOA
- We obtained the superconformal blocks and solved the superconformal Ward identities
- The setup is surprinsingly nice: the conformal blocks are (relatively) simple, and we can do all the computations explicitly
- Is there a systematic approach to calculating CFT data at large N?

Conclusion

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- This work was motivated by the analogy betwen surface operators and Wilson lines.

  Does this extend also to surface operators in other theories?

  What is the meaning of the string potential in 4d gauge theories?
- We have (at least) two powerful nonperturbative methods to study surface operators in the (2,0) theories:
  1. VOA
  2. Analytical conformal bootstrap

  In principle both techniques apply also to 4d defects in (2,0) theories, and surface operators in 4d $\mathcal{N} = 2$ theories.
- Can we use our better understanding of 6d (2,0) theories to learn something about theories in lower dimensions?

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