The correspondence between rotating black holes and fundamental strings

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Work to appear, with

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Black holes are massive, large, highly degenerate objects

Black holes are massive, large, highly degenerate objects

Strings have very massive, large, highly degenerate states

Black hole entropy (Bekenstein)

$$S_{BH} \propto M^2$$

String degeneracy (Hagedorn)

$$S_{st} \propto M$$

Different?

Black hole entropy (Bekenstein)

$$S_{BH} \propto M^2$$

String degeneracy (Hagedorn)

$$S_{st} \propto M$$

In what units?

Gravitational units

$$S_{BH} = \frac{M^2}{M_P^2}$$

$$M_P^2 = G^{-1}$$

String units

$$S_{st} = \frac{M}{M_s}$$

$$M_s^2 = \alpha'^{-1}$$

$$M_s = g_s M_P$$

g_s: string coupling

Perturbatively $g_s \ll 1$

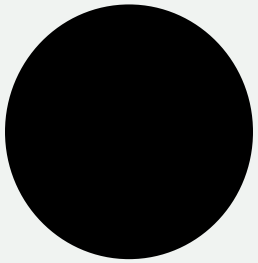
$$M_s \ll M_P$$

Black holes are strongly gravitating

Strings are weakly gravitating

$$\text{curvature} \sim \frac{1}{(GM)^2}$$

$$\sim \left(\frac{M_s}{g_s^2 M}\right)^2 \frac{1}{\ell_s^2}$$



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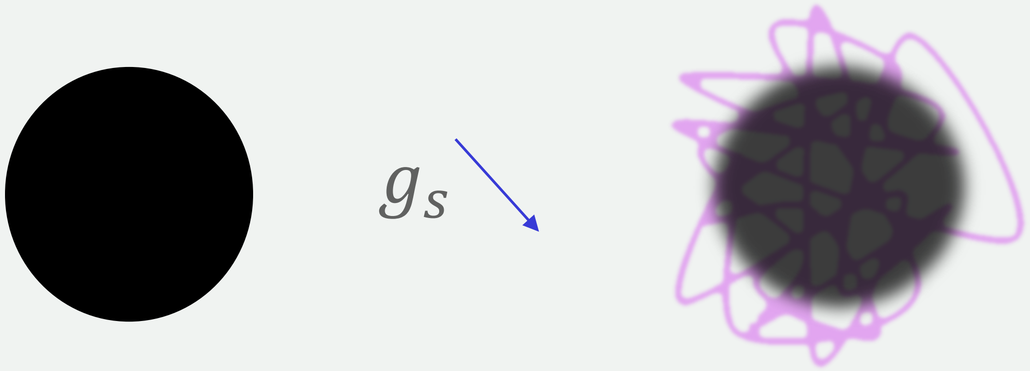
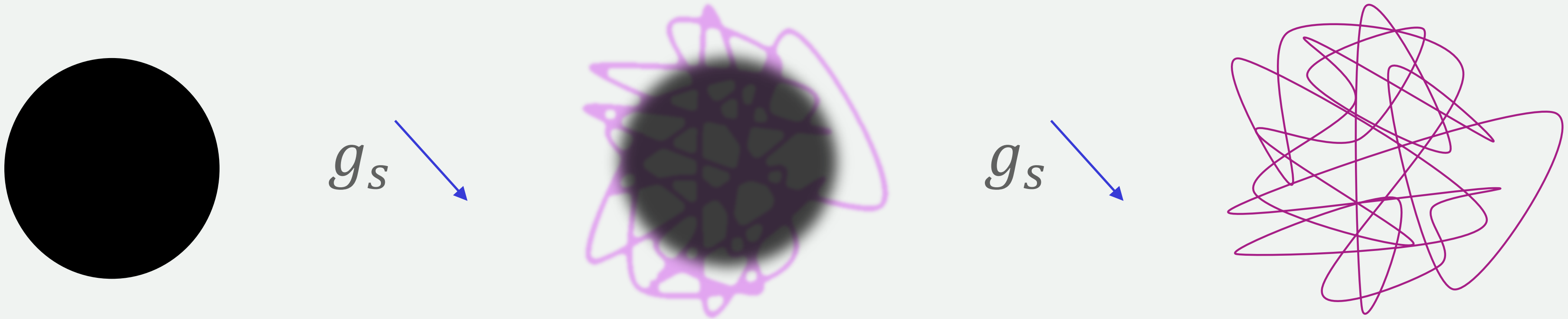


$$g_s^2 \sim \frac{M_s}{M} \ll 1$$

$$\text{curvature} \sim \frac{1}{(GM)^2}$$

$$\sim \left(\frac{M_s}{g_s^2 M}\right)^2 \frac{1}{\ell_s^2}$$

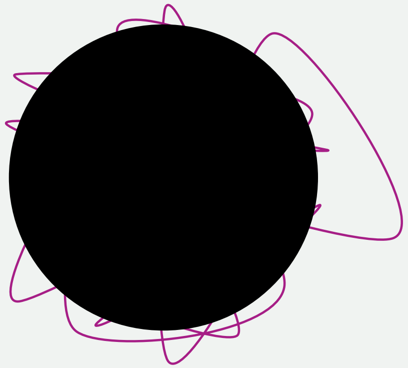
$$\text{curvature} \sim \frac{1}{\ell_s^2}$$



$$g_s^2 \sim \frac{M_s}{M} \ll 1$$

Tune the coupling g_s up and down

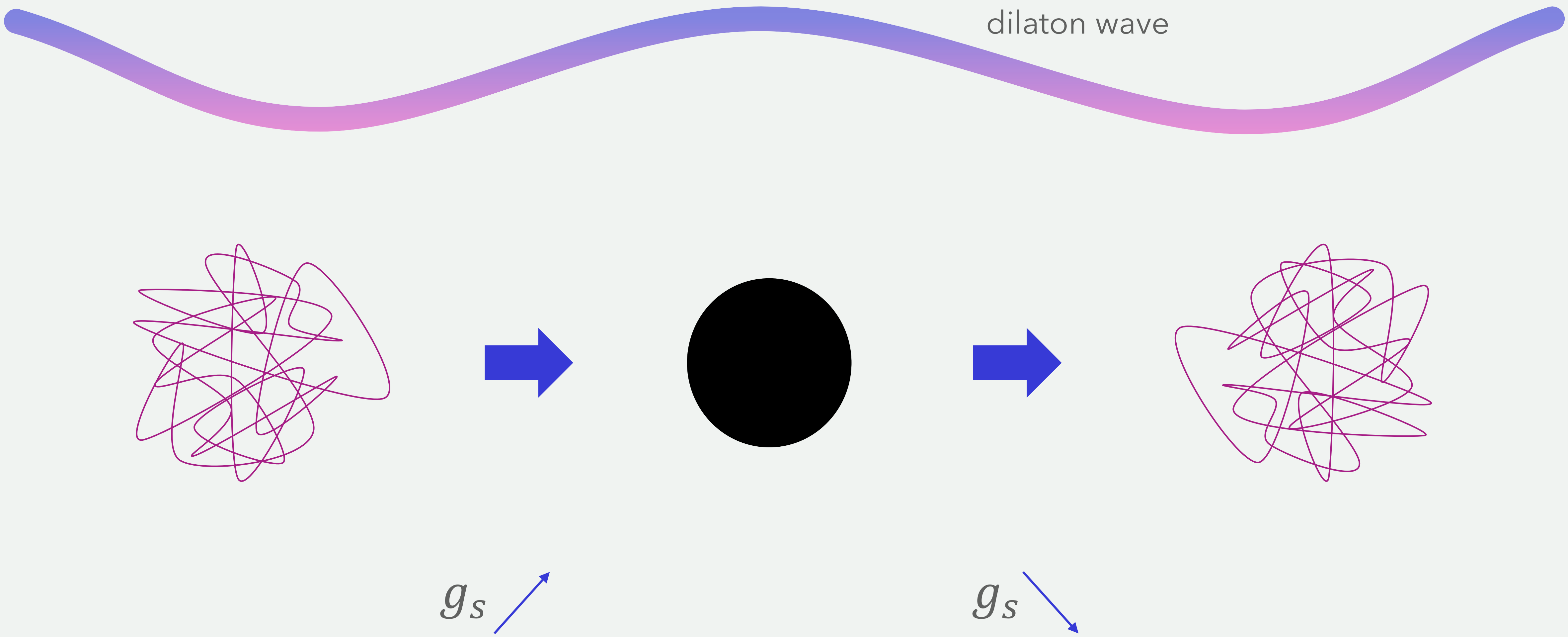
Adiabatically switch between massive string and black hole



Susskind 1993

Horowitz+Polchinski 1996

Veneziano+Damour 1998


dilaton wave
g_s
g_s

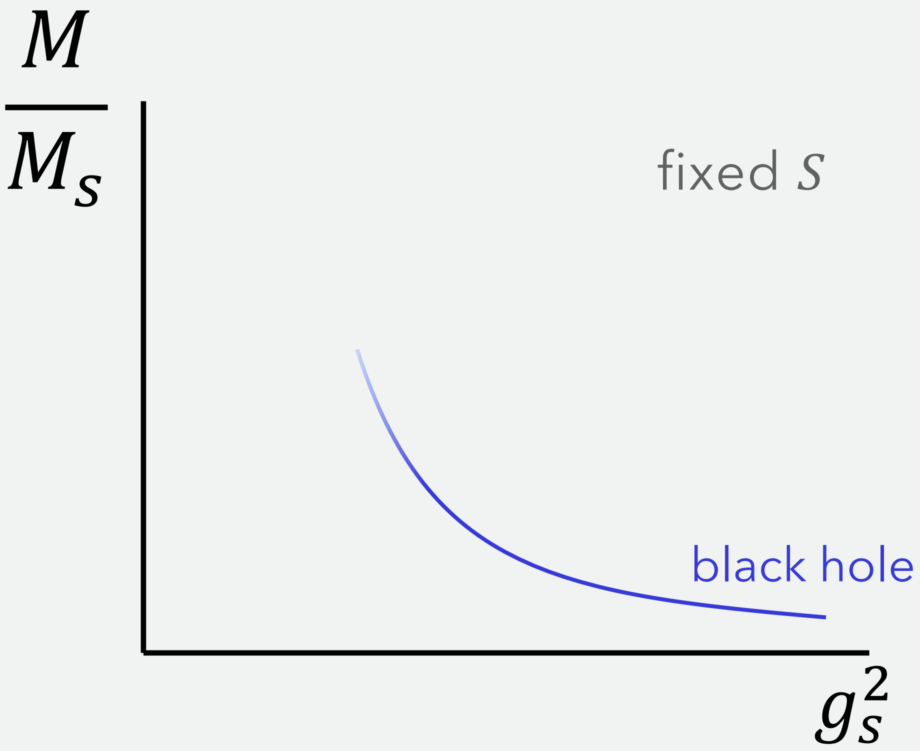
Susskind 1993

Fix entropy ≡ fix state

As g_s changes, mass gets renormalized by interactions

Start with black hole

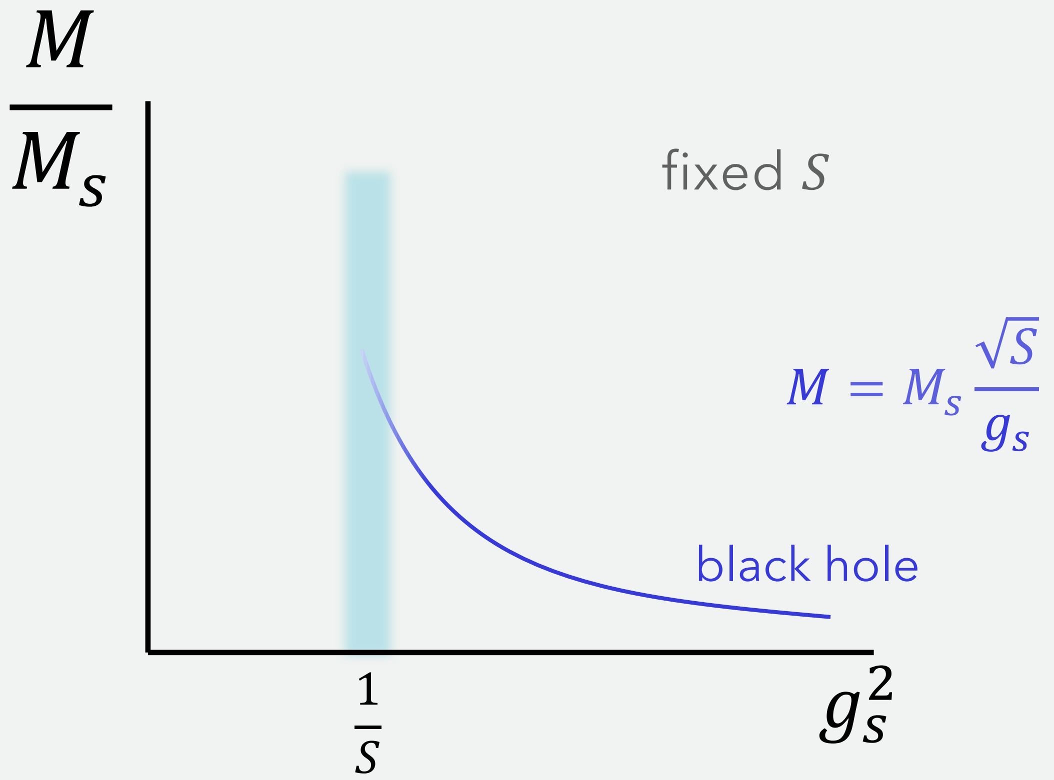
$$S_{BH} = g_s^2 \frac{M^2}{M_s^2} \Rightarrow M = M_s \frac{\sqrt{S}}{g_s}$$



Susskind 1993

Black hole becomes stringy

when $g_s^2 \sim \frac{M_s}{M} \sim \frac{1}{S}$

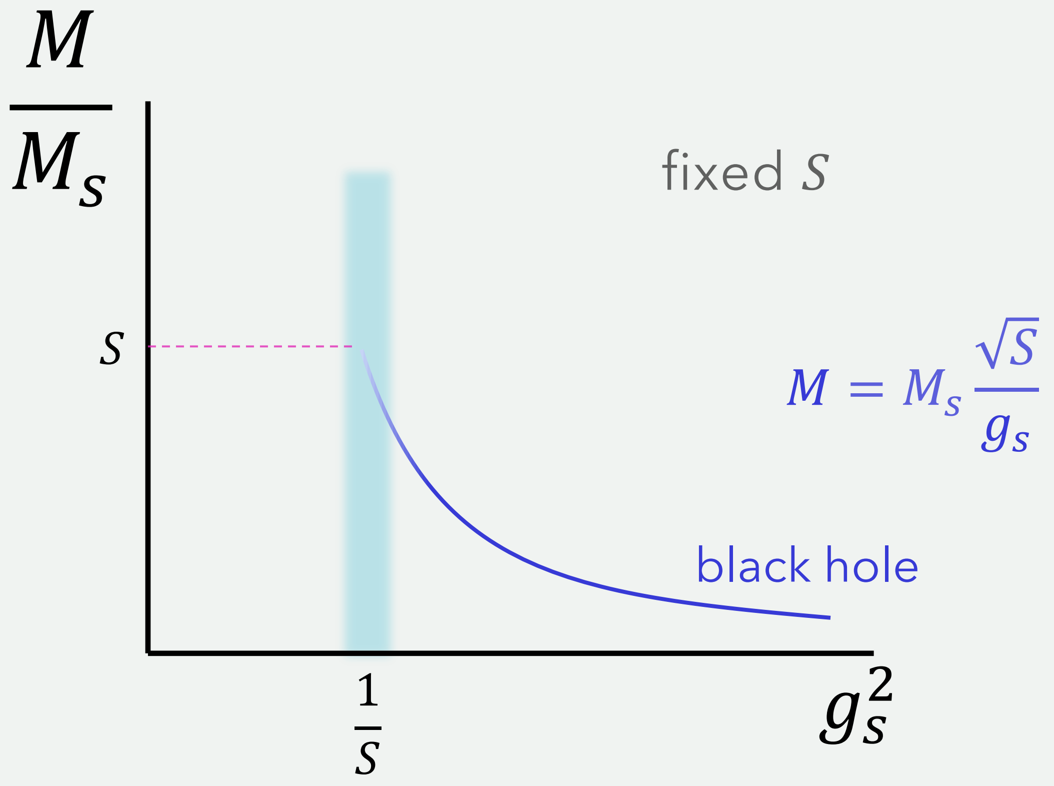


Susskind 1993

Black hole becomes stringy

when $g_s^2 \sim \frac{M_s}{M} \sim \frac{1}{S}$

At that point $S = \frac{M}{M_s}$

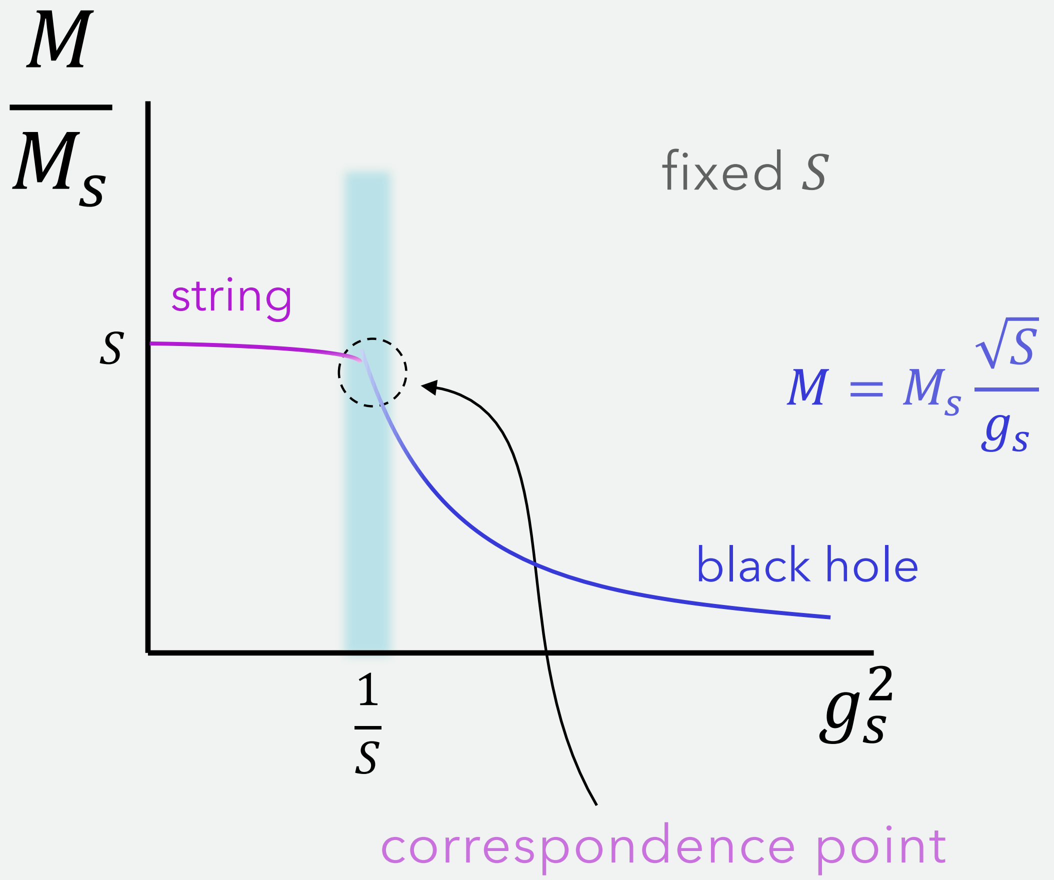


Susskind 1993

Black hole becomes stringy

when $g_s^2 \sim \frac{M_s}{M} \sim \frac{1}{S}$

At that point $S = \frac{M}{M_s} = S_{string}$



String/Black hole Correspondence

Parametric match of string states and black holes

Strings give microscopic account of black hole entropy

50 years back

Clues and puzzles

Kerr bound on black holes

$$J \leq M^2$$

Regge bound on strings

$$J \leq M^2$$

black holes = strings?

$$J \leq M^2$$

Misleading!

$$J \leq M^2$$

What units?

$$J \leq M^2$$

gravitational units

$$J \leq \frac{M^2}{M_P^2}$$

string units

$$J \leq \frac{M^2}{M_s^2}$$

$$M_S = g_S M_P \ll M_P$$

$$J_{Kerr} = \frac{M^2}{M_P^2} = g_s^2 \frac{M^2}{M_s^2} \ll J_{Regge} = \frac{M^2}{M_s^2}$$

Puzzles

$$J_{Kerr} \ll J_{Regge}$$

Massive string states with high enough spin

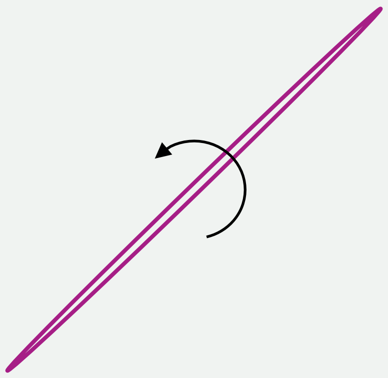
$$J_{Kerr} < J \leq J_{Regge}$$

don't have black hole counterparts

Strings with $J = J_{Regge}$

do not look like black holes at all

Non-degenerate, rigidly rotating rods



$$J = M^2 L_s^2$$

Black holes: $J \leq J_{Kerr} = \frac{M^2}{M_P^2}$

No problem

There exist string states with the same degeneracy, spin, and (parametric) mass

But $J \leq GM^2$ is a bound for 4D black holes

In $D \geq 5$ $\exists$ black holes with arbitrarily large spins

But Regge bound $J \leq \frac{M^2}{M_S^2}$ is for strings in any D

Ultraspinning black holes with

$$J \gg J_{Regge}$$

don't have string counterparts

Start with fast spinning string

As g_s grows, what does it turn into?

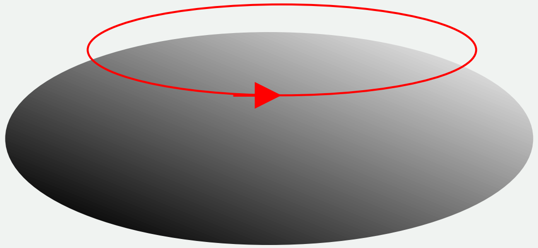
Start with fast spinning black hole

As g_s decreases, what does it turn into?

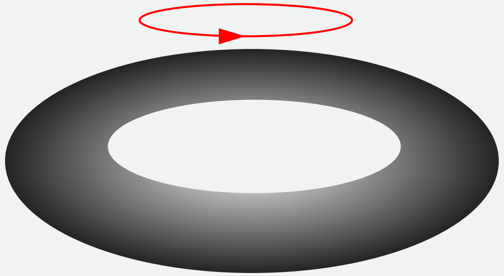
Rotating black holes in higher D

$D \geq 5$ with rotation in a single plane

$\exists$ black holes/rings with arbitrarily large J



Myers+Perry 1986



RE+Reall 2001

Two length scales

$$\ell_M = (GM)^{\frac{1}{D-3}}$$

$$\ell_J = \frac{J}{M}$$

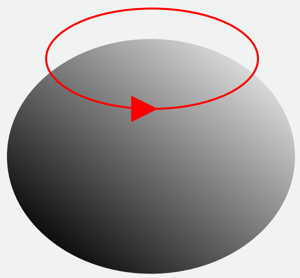
Ultraspinnning: $\ell_J \gg \ell_M$

RE+Harmark+Niarchos+Obers 2009

$\ell_J \lesssim \ell_M$

unique

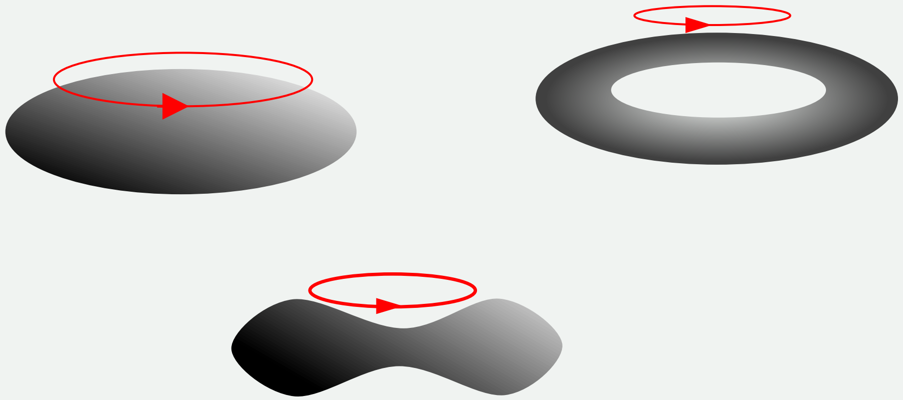
stable



$\ell_J \gtrsim \ell_M$

many

unstable to fragmentation

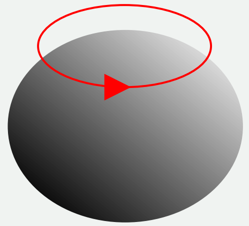


Dynamical "Kerr bound"

Only stable solution is M-P plump black hole

with $\ell_J \lesssim \ell_M \;\Rightarrow\; J \lesssim M(GM)^{\frac{1}{D-3}} = S$

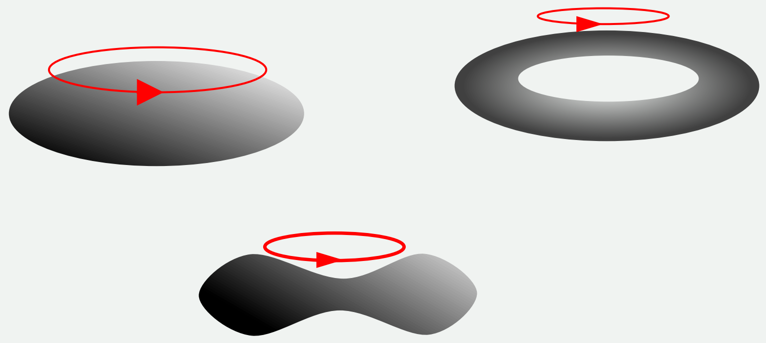
$$J < S$$



Ultraspinning

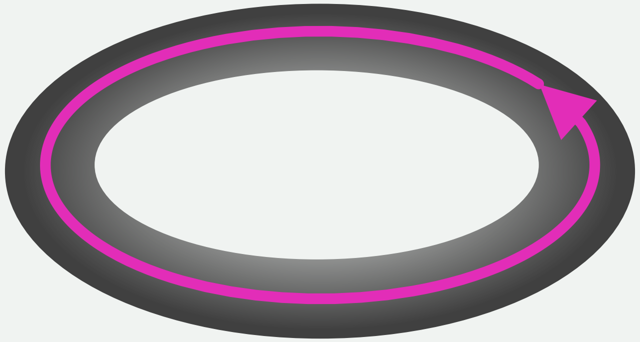
$$J > S$$

unstable



Dipole black rings

- No charge, but a dipole
- $J > S$, but can be stable



Ring of non-extremal fundamental string

String configurations

String balls

Classical Nambu-Goto (conformal gauge w/ $X^0 = p^0\tau$)

$$X^i = \frac{1}{2}(A^i(\tau - \sigma) + B^i(\tau + \sigma))$$

String balls

$$X^i = \frac{1}{2}\left(A^i(\tau - \sigma) + B^i(\tau + \sigma)\right)$$

A^i, B^i arbitrary

(up to $|\partial_\sigma A^i|^2 = |\partial_\sigma B^i|^2 = 1$)

A^i, B^i: random walk

Degeneracy $S \sim \text{Length} = M$

Mañes 2004

String balls

Quantum string construction

reproduces random walk results

Mitchell+Turok 1987

Add rotation

Russo+Susskind 1994

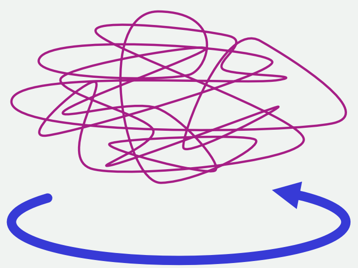
$$S \sim \sqrt{M^2 - J}$$

Large away from Regge bound

Shape

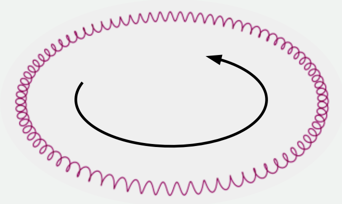
Rotating strings pancake along rotation plane

$$\ell_{par}^2 \sim M > \ell_{perp}^2 \sim \sqrt{M^2 - J}$$



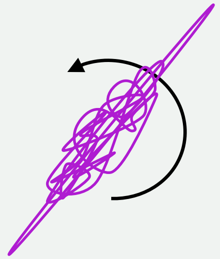
Other rotating strings

Plasmid strings



Blanco-Pillado+RE+Iglesias 2007

Wiggly rods

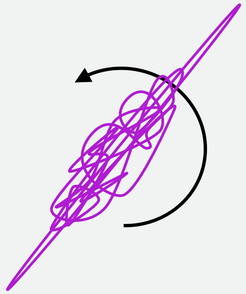


Can construct both classical and quantum

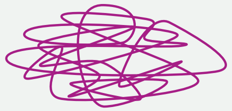
When interactions are turned on, $g_s > 0$, they all become unstable

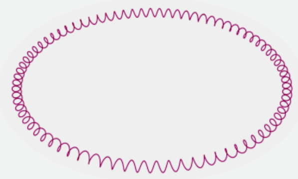
But some are shorter lived, some longer-lived

Shorter-lived: gravitational antennas



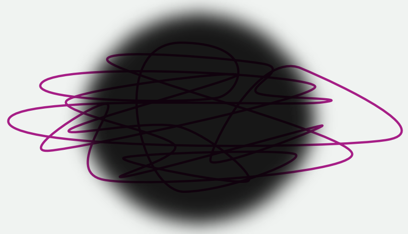
Longer-lived: ~ thermal radiation

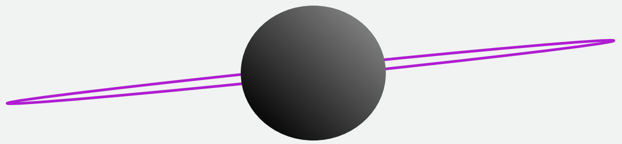




Ultraspinning black holes → black hole/string hybrids

Curvature on horizon first becomes string-size near the equator

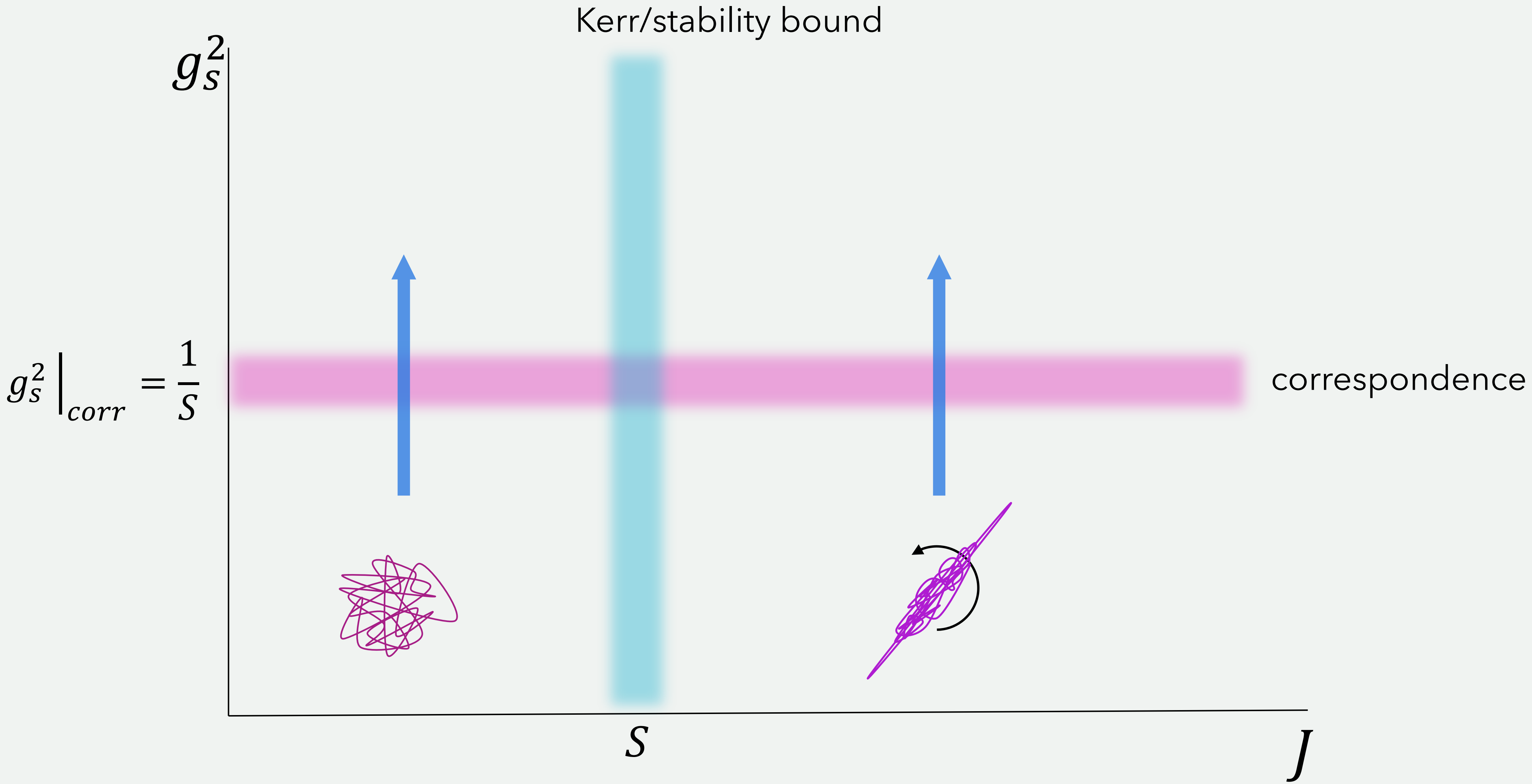




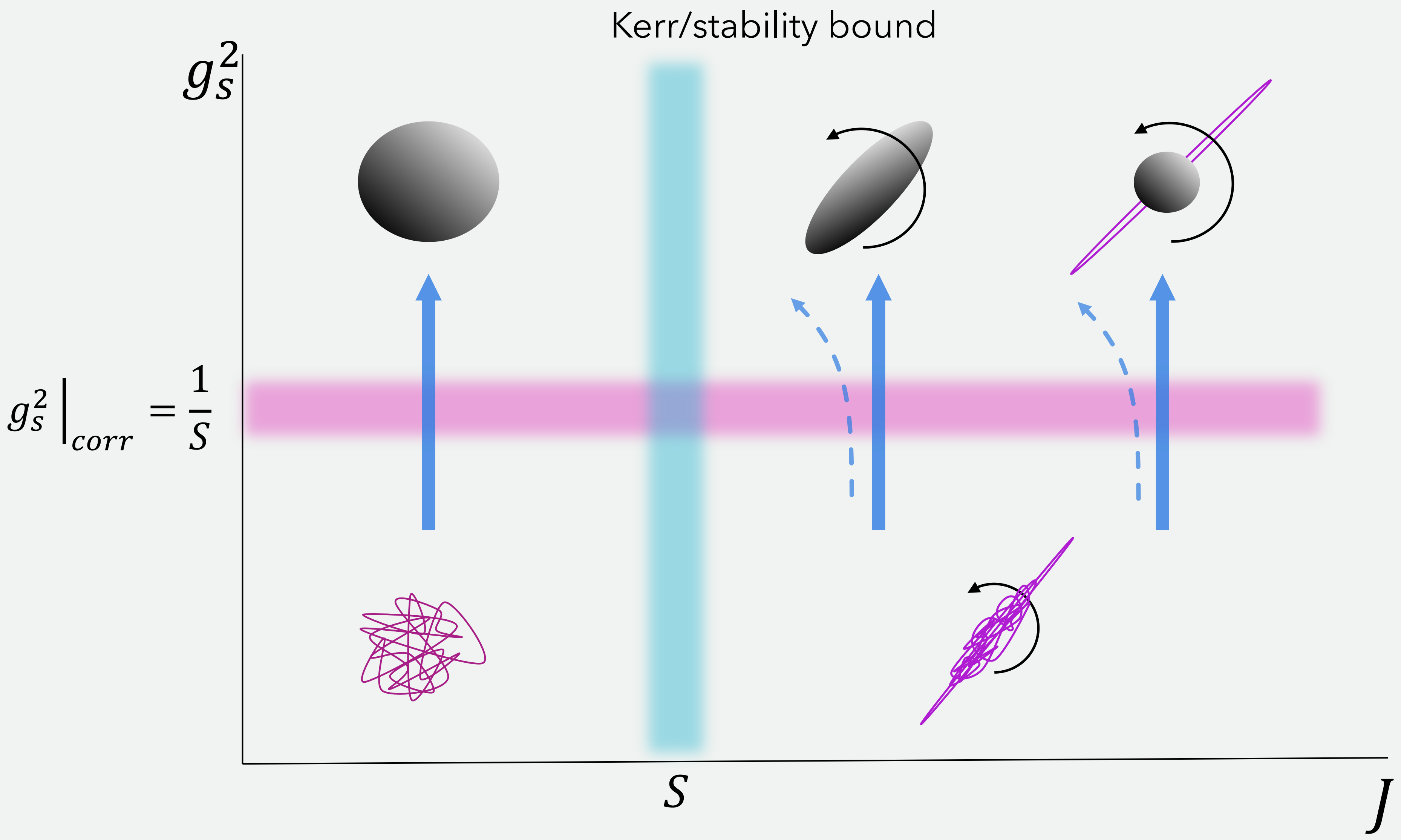
Deng+Gruzinov+Levin +Vilenkin 2023

This is enough to establish the correspondence across J as we vary g_s

Strings to black holes

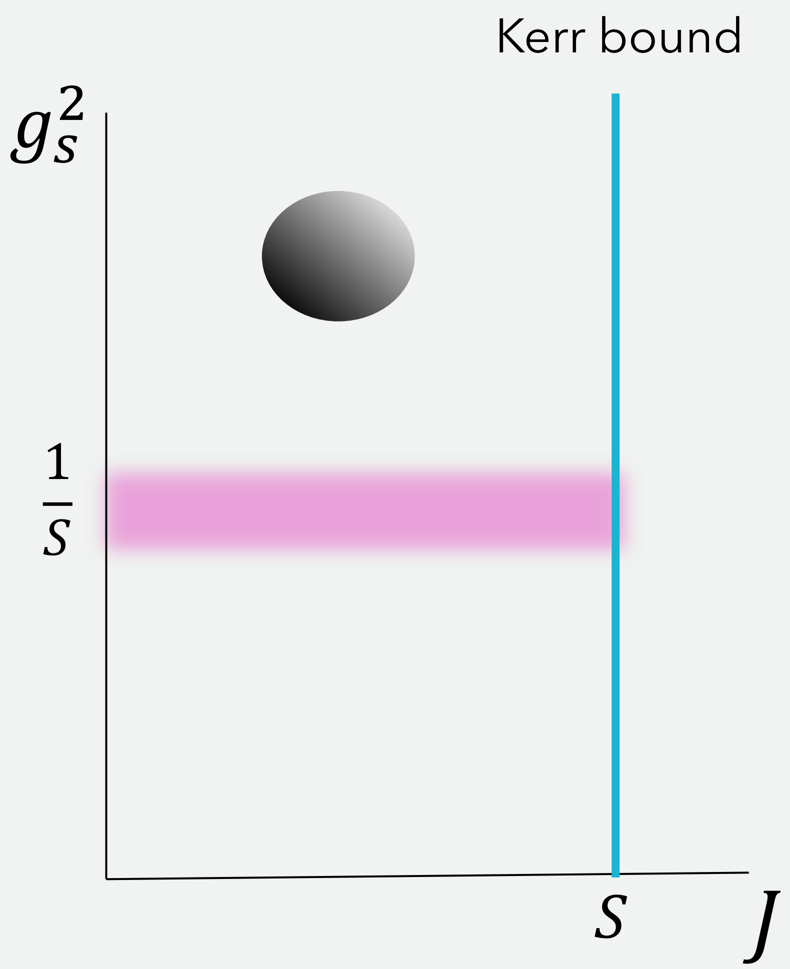


Strings to black holes

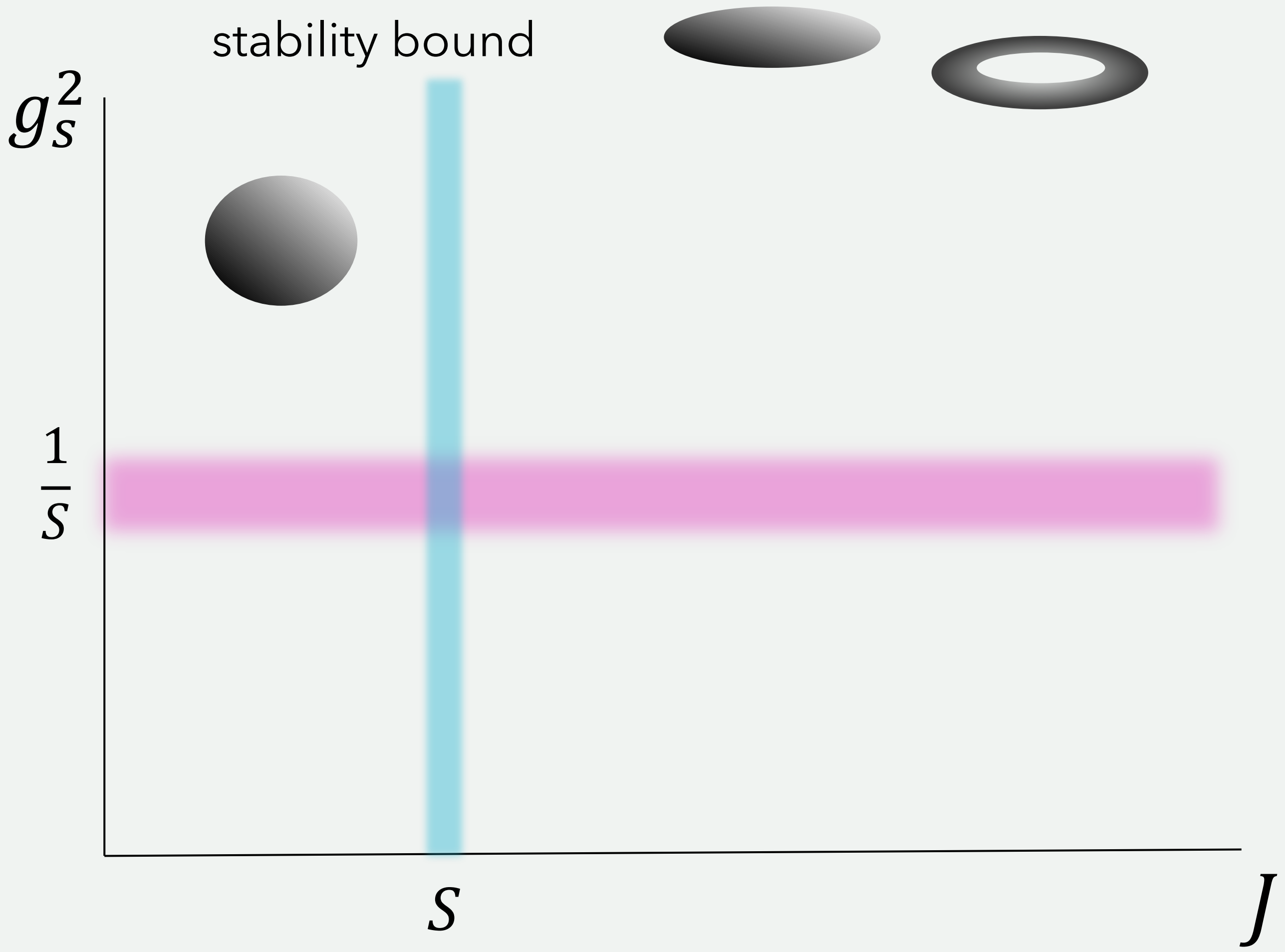


Black holes to strings

D=4

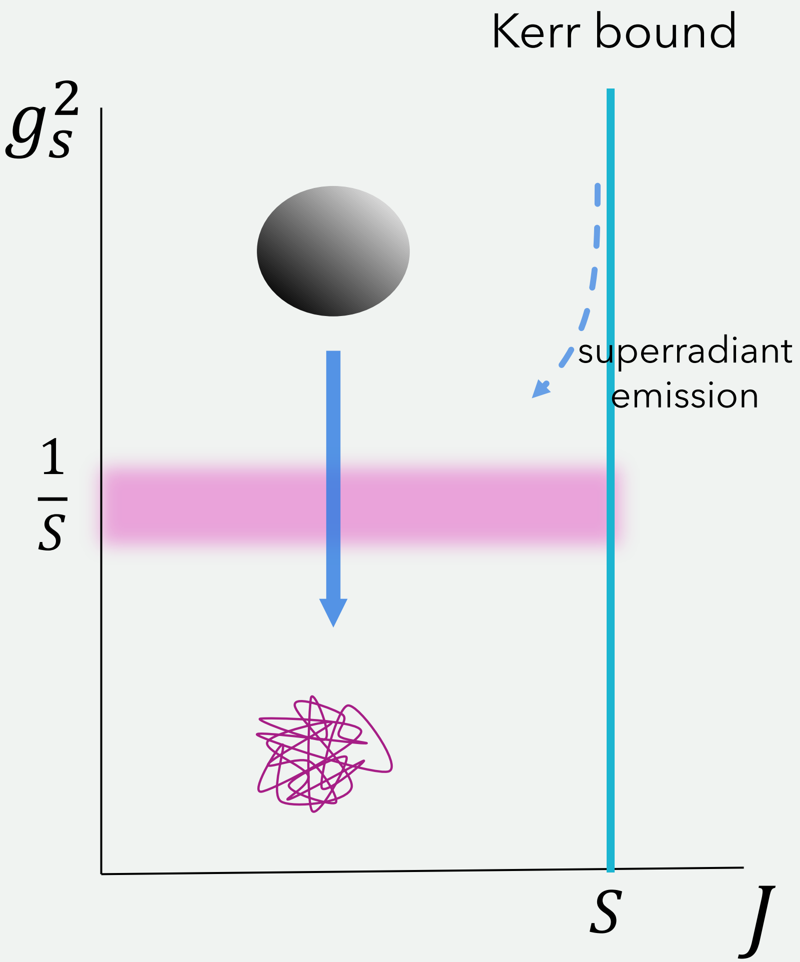


D≥6



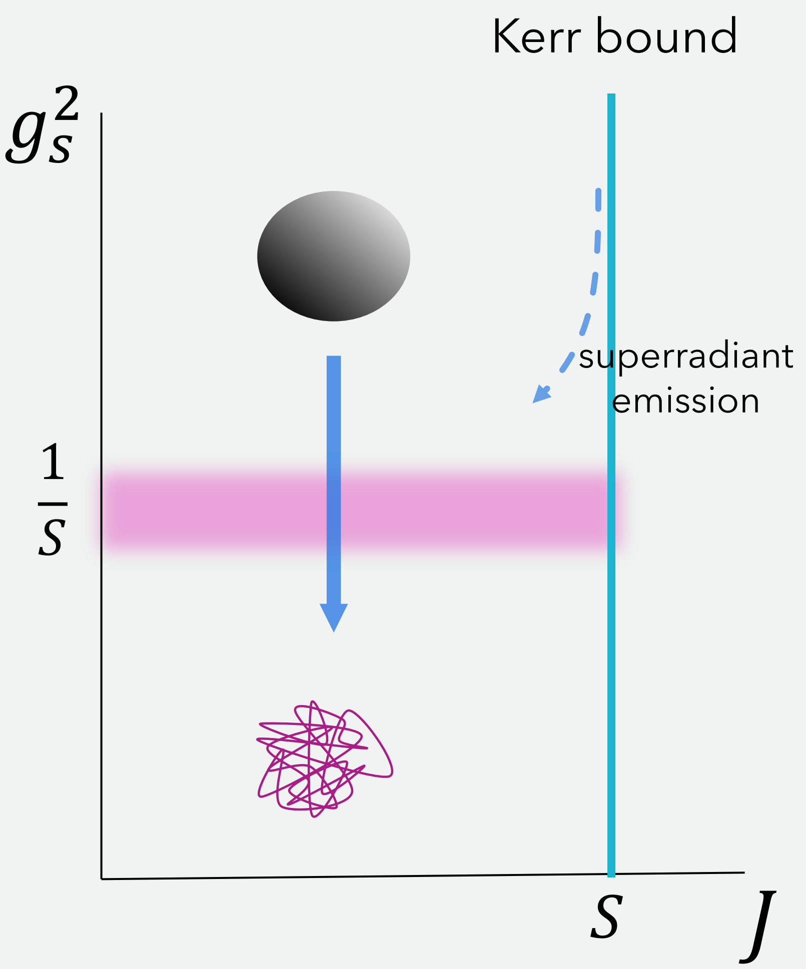
Black holes to strings

D=4

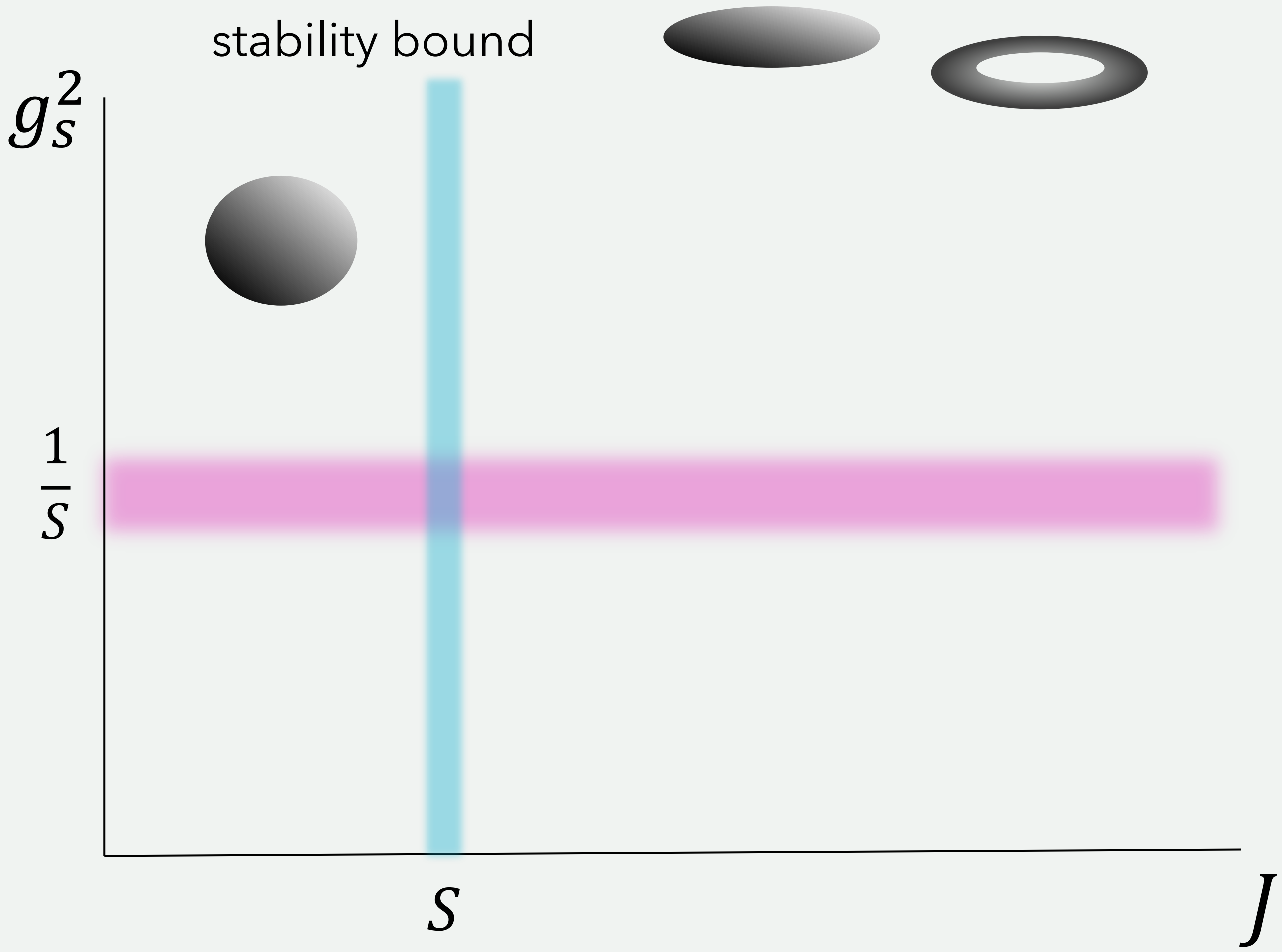


Black holes to strings

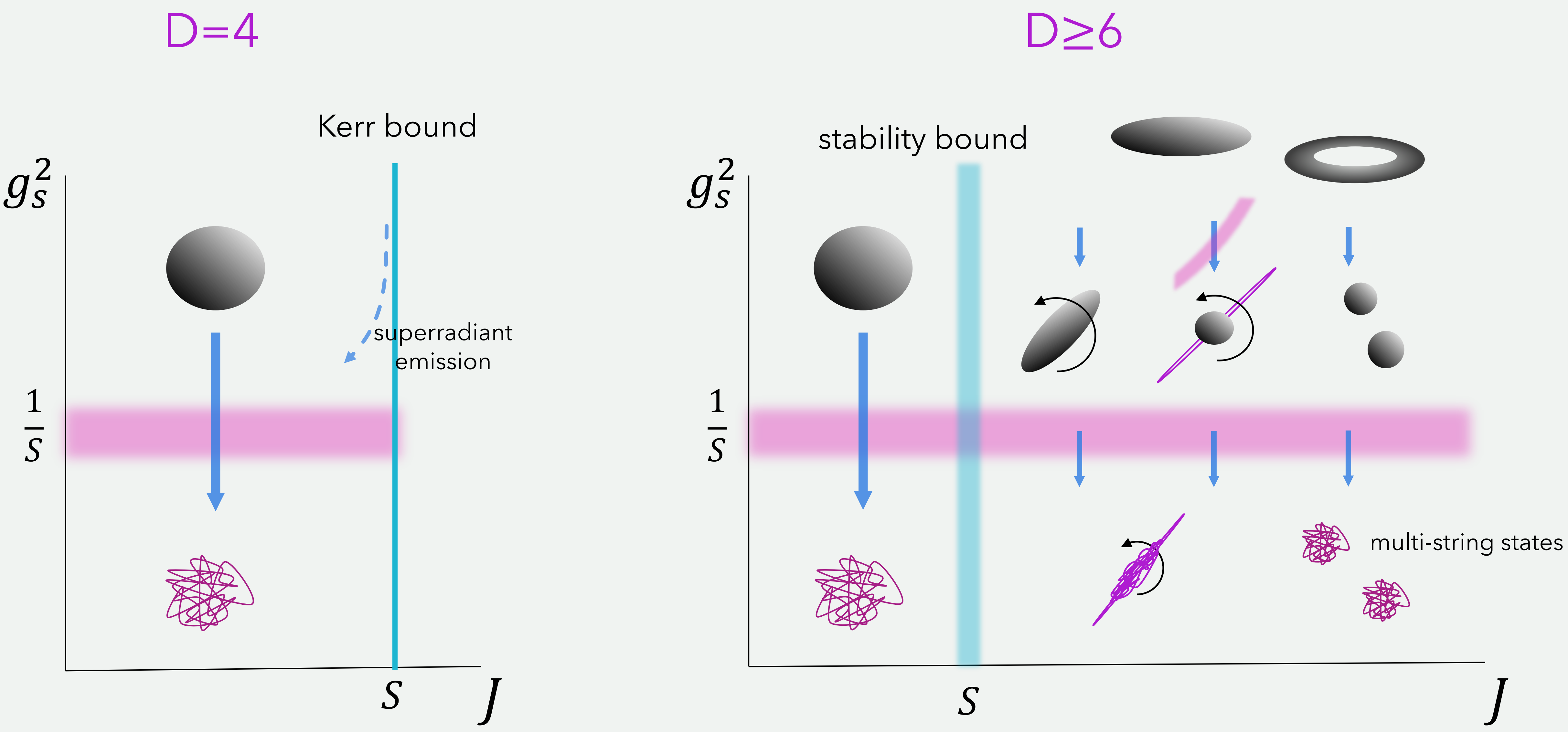
D=4



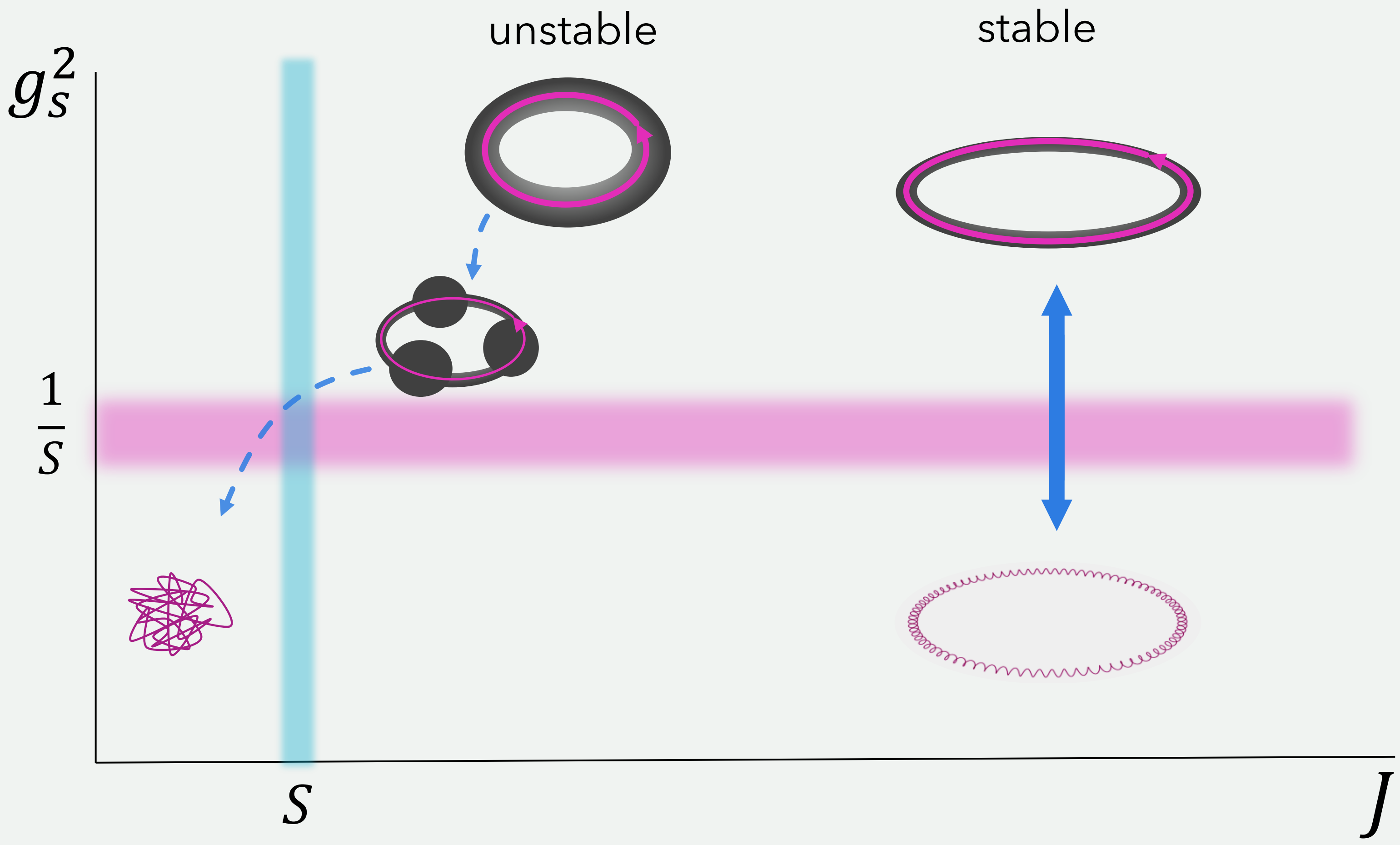
D≥6



Black holes to strings



Dipole black rings and plasmid strings



Puzzles arose because of implicit assumption:

“Stationary black holes and string states must be in 1-to-1 correspondence”

But when coupling is finite, time evolution for spinning states is not negligible

Stationary states can evolve into fast time dependence

Final remarks

Fundamental strings allow to go as close as possible to black holes in a Quantum theory of Gravity, while using a spacetime description

Correspondence describes the transition between them

Must understand how it works

Rotation:

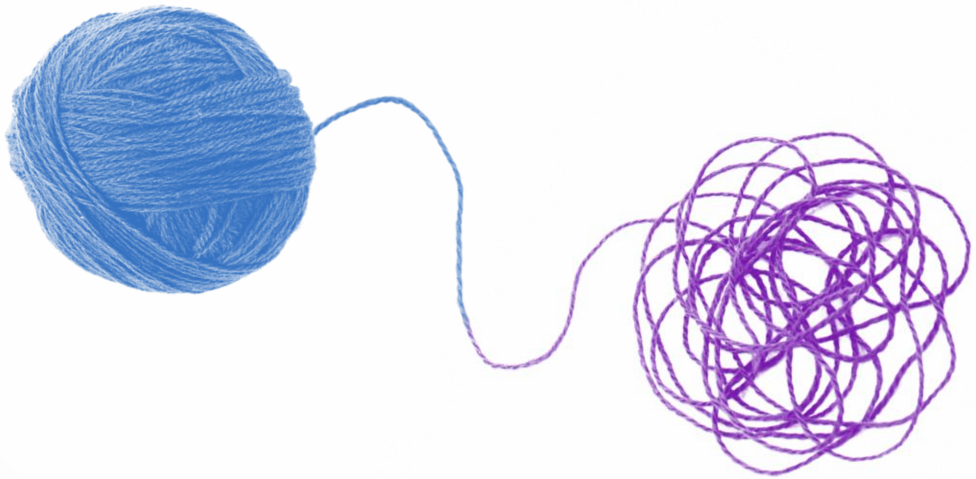
First step ✓

More work ahead

Nejc Čeplak

Andrea Puhm

Marija Tomašević



Thank you