Balance Laws in Gravitational Two-Body Scattering

Massimiliano Maria Riva

Based on work with F. Vernizzi and L. K. Wong
[2302.09065]

Deutsches Elektronen-Synchrotron DESY,
Notkestr. 85, 22607 Hamburg, Germany

at Niels Bohr Institute, Copenhagen (DK),
15 June 2023

Outline

One Problem, Many Approaches

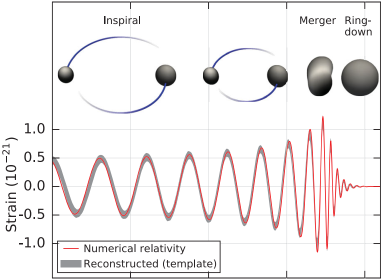


Figure: LIGO and VIRGO scientific collaboration, [1602.03837].

Template → EOB, IMR

A. Buonanno, T. Damour **[gr-qc/0001013]**

P. Ajith et al. **[0710.2335]**

Inspiral phase, PN Expansion

$$\text{Expansion in } \frac{Gm}{c^2 r} \sim \frac{v^2}{c^2}$$

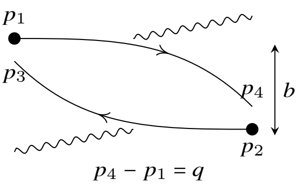
$$H = H_{0\text{PN}} + \frac{H_{1\text{PN}}}{c^2} + \frac{H_{2\text{PN}}}{c^4} + \frac{H_{2,5\text{PN}}}{c^5} + \frac{H_{3\text{PN}}}{c^6} \ldots$$

- Traditional GR approach *Bernard, Bini, Blanchet, Buonanno, Damour, Faye, Geralico, Jaranowski, Schäfer...*

  L. Blanchet **[1310.1528 - 1812.07490]**
- EFT methods *Foffa, Goldberger, Levi, Porto, Ross, Rothstein, Steinhoff, Sturani...*

  W. Goldberger **[hep-ph/0701129]**, *S. Foffa, R. Sturani* **[1309.3474]**, *R. A. Porto* **[1601.04914]**

The Post-Minkowskian Expansion



Extract Classical contribution

$c = \hbar = 1$

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G}\, h_{\mu\nu}$$

Quantum vs Classical PM

$$\underbrace{\frac{\ell_c}{b}} \ll \underbrace{\frac{Gm}{b}} \ll 1 \rightarrow \underbrace{\frac{q}{m}} \ll \underbrace{Gmq} \ll 1$$

Two main methods:

- Classical physics from full scattering amplitude

$$\langle p_4, p_3 | S | p_1, p_2 \rangle = 1 + i\mathcal{M}(q^2)$$

D. Neill, I. Z. Rothstein **[1304.7263]**, *N. E. J. Bjerrum-Bohr et al.* **[1806.04920]**, *Z. Bern et al.* **[1908.01493]**, *E. Herrmann et al.* **[2104.03957]**, *N. E. Bjerrum-Bohr et al.* **[2104.04510]**, *P. Di Vecchia et al.* **[2104.03256 - 2210.12118]**.

- Classical Effective Field Theory approach

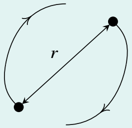
$$e^{iS_{\text{eff}}[x_a]} = \int \mathcal{D}[h] e^{i(S_{\text{grav}} + S_{\text{GF}} + S_{\text{sources}})}$$

W. D. Goldberger, I. Z. Rothstein **[hep-th/0409156]**, *G. Kälin, R. A. Porto* **[2006.01184]** *G. Mogull, J. Plefka, J. Steinhoff* **[2010.02865]**

The importance of Balance Laws

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\mathcal{F}_E \qquad\qquad \frac{\mathrm{d}J}{\mathrm{d}t} = -\mathcal{F}_J$$

Bound case



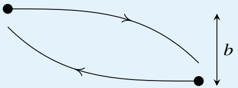
Wave phase from the instantaneous flux

$$\Delta\phi(t) \propto \int \mathrm{d}v\, v^3 \frac{\mathrm{d}E/\mathrm{d}v}{\mathcal{F}_E}$$

W. D. Goldberger, I. Z. Rothstein **[hep-th/0409156]**, *T. Damour et al.* **[1110.2938]**

L. Blanchet **[1310.1528 - 1812.07490]**

Unbound case



Linear Response formula

$$2\chi_{\mathrm{rad}}(E,J) = \frac{\partial \chi_{\mathrm{cons}}}{\partial E}\int \mathrm{d}t\, \mathcal{F}_E + \frac{\partial \chi_{\mathrm{cons}}}{\partial J}\int \mathrm{d}t\, \mathcal{F}_J$$

D. Bini, T. Damour **[1210.2834]**, *T. Damour* **[2010.01641]**

Supertranslation puzzle

DeWitt-Thorne Formula

$u := t - r$

$$\Delta J^{ij}_{\mathrm{DWT}} = \int \frac{\mathrm{d}u\mathrm{d}\Omega}{32\pi G}\, r^2 \left(2h^{a[i}_{\mathrm{TT}}\dot{h}^{j]a}_{\mathrm{TT}} - \dot{h}^{ab}_{\mathrm{TT}} x^{[i}\partial^{j]} h^{ab}_{\mathrm{TT}}\right) = O(G^2)$$

- Linear response gives the correct radiation-reaction contribution

$$2\chi^{\mathrm{LO}}_{\mathrm{rad}} = \overbrace{\frac{\partial \chi_{\mathrm{cons}}}{\partial J}}^{O(G)} \overbrace{\Delta J_{\mathrm{DWT}}}^{O(G^2)} = O(G^3)$$

T. Damour **[2010.01641]**

Supertranslation ambiguity

- In the BMS formalism, J is not supertranslation invariant

  E. T. Newman R. Penrose **J. Math. Phys. 7 (1966)**,

  A. Ashtekar, T. De Lorenzo, N. Khera **[1910.02907]**

- $\Delta J^{ij} = O(G^2) \underset{\mathrm{ST}}{\longrightarrow} \Delta J^{ij} = O(G^3)$

  G. Veneziano, G. A. Vilkovisky **[2201.11607]**

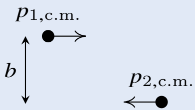
- Invariant definition of $J_{(\mathrm{inv})}$ leads to $\Delta J^{ij}_{(\mathrm{inv})} = O(G^3)$

  G. Compère, R. Oliveri, A. Seraj **[1912.03164]**,

  R. Javadinezhad, U. Kol, M. Porrati **[2202.03442]**

Non-covariance puzzle

Initial c.m. frame

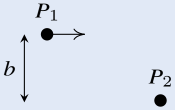


$$p^{\mu}_{1,\text{c.m.}} = (E_1, \boldsymbol{p})$$

$$p^{\mu}_{2,\text{c.m.}} = (E_2, -\boldsymbol{p})$$

T. Damour **[2010.01641]**

Initial rest frame of one body



$$P^{\mu}_1 = (\gamma m_1, \boldsymbol{P})$$

$$P^{\mu}_2 = (m_2, \boldsymbol{0})$$

G. U. Jakobsen et al. **[2101.12688]**,

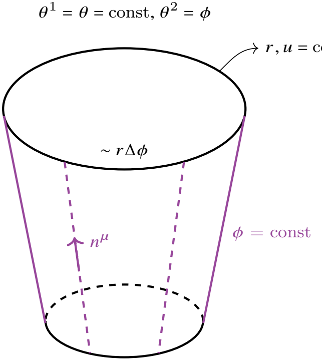
S. Mougiakakos, ***MMR****, F. Vernizzi* **[2102.08339]**

$$\Delta J^{ij}_{\text{DWT}} = 2\frac{G^2 m_1 m_2}{b^2}\frac{2\gamma^2 - 1}{\sqrt{\gamma^2 - 1}}\mathcal{I}(\gamma)\begin{cases} 2b^{[i}p^{j]} & \text{Initial c.m. frame} \\ 2b^{[i}P^{j]} & \text{Initial rest frame of } m_2 \end{cases}$$

$$\Delta J^{\mu\nu}_{\text{QFT}} = 2\frac{G^2 m_1 m_2}{b^2}\frac{2\gamma^2 - 1}{\sqrt{\gamma^2 - 1}}\mathcal{I}(\gamma)b^{[\mu}(p_1^{\nu]} - p_2^{\nu]})$$

- Agreement only in the initial c.m. frame
  A. V. Manohar, A. K. Ridgway, C.-H. Shen **[2203.04283]**
  P. Di Vecchia et al. **[2203.11915-2210.12118]**
- New Noetherian computation matches this result *D. Bini, T. Damour* **[2211.06340]**

The BMS formalism



Adapted coordinates (u, r, θ^A) with $A = 1, 2$

- Hypersurfaces of constant u are null

  $$g^{uu} = 0$$

- θ^A = const along the null-rays

  $$g^{uA} = 0$$

- Surface element r, u = const is $r^2 d^2\Omega$

  $$\det(g_{AB}) = r^2 \det(\Omega_{AB})$$

  Ω_{AB} metric of the 2-sphere.

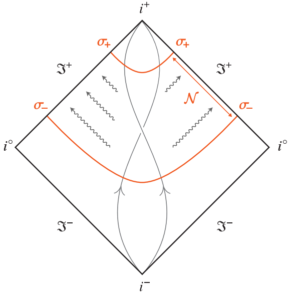
Most generic metric

$$ds^2 = -\mu e^{2\beta} \mathrm{d}u^2 - 2e^{2\beta} \mathrm{d}u \mathrm{d}r + \gamma_{AB}(r \mathrm{d}\theta^A + W^A \mathrm{d}u)(r \mathrm{d}\theta^B + W^B \mathrm{d}u)$$

H. Bondi **Nature 186,(1960)**, *R. K. Sachs* **Proc. R. Soc. Lond. A 270, (1962)**

H. Bondi, M. G. J. van der Burg, A. W. K. Metzner **Proc. R. Soc. Lond. A 269, (1962)**

The BMS formalism



$$ds^2 = -\mu e^{2\beta} \mathrm{d}u^2 - 2e^{2\beta} \mathrm{d}u\mathrm{d}r + \gamma_{AB}(r\,\mathrm{d}\theta^A + W^A \mathrm{d}u)(r\,\mathrm{d}\theta^B + W^B \mathrm{d}u)$$

- $\mathfrak{I}^+(\mathfrak{I}^-) \to$ future (past) time-like infinities, where massless particles end up (come from)
- Asymptotic expansion near $\mathfrak{I}^+$ to study emitted radiation

$D_A \Omega_{CD} = 0,\ \dot{X} \equiv \partial_u X$

Asymptotic Metric near $\mathfrak{I}^+$

$$\mu = 1 - \frac{2G M}{r} + \ldots, \quad \gamma_{AB} = \Omega_{AB} + \frac{1}{r} C_{AB} + \ldots, \quad \beta = -\frac{1}{32r^2} C_{AB} C^{AB} + \ldots$$

$$W^A = \frac{1}{2r} D_B C^{AB} + \frac{1}{r^2}\left(\frac{2}{3} G N^A - \frac{1}{16} D^A (C^{BC} C_{BC}) + \frac{1}{2} C^{AB} D^C C_{BC}\right) + \ldots$$

$M(u,\theta) \to$ Mass Aspect, $\quad N_A(u,\theta) \to$ Angular Momentum Aspect

$C_{AB}(u,\theta) \to$ Shear Tensor $\quad N_{AB}(u,\theta) \equiv \dot{C}_{AB}(u,\theta) \to$ News Tensor

The BMS formalism

- Symmetries that preserve asymptotic flatness

$$\delta g_{\mu\nu}\big|_{\mathfrak{I}^+} = (\mathcal{L}_\xi g)_{\mu\nu}\big|_{\mathfrak{I}^+} = 0$$

$$n^\mu = (1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \qquad \partial_A = (\partial_A n^\mu)\partial_\mu$$

- Asymptotic Killing vector $\xi = \left(\alpha(\theta) + \frac{u}{2} D_A Y^A(\theta)\right)\partial_u + Y^A(\theta)\partial_A$
- Spherical harmonics decomposition

$$\alpha(\theta) = a \cdot n + \alpha_{\ell \geq 2}(\theta)$$

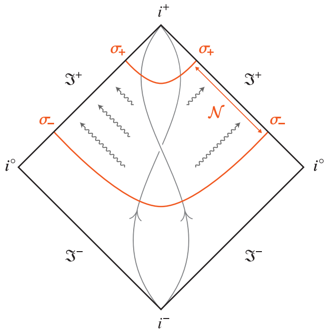
Spacetime translations ($a \cdot n$) — Supertranslations ($\alpha_{\ell \geq 2}(\theta)$)

- $Y^A(\theta)$ generate Lorentz transformations

- Metric near $\mathfrak{I}^+$ determined by M, N_A and C_{AB}
- Flat spacetime not unique but changes under supertranslation

$$C_{AB} = 0 \xrightarrow[\text{ST}]{} C_{AB} = -(2D_A D_B - \Omega_{AB} D^2)\alpha \neq 0$$

BMS Linear Momentum and Flux



BMS Supermomentum and Flux

$$P(\sigma_+) - P(\sigma_-) = -F_P(\mathcal{N}),$$

$$P(\sigma) = \int_\sigma \frac{d^2\Omega}{4\pi}\,\alpha M\,,$$

$$F_P(\mathcal{N}) = \int_\mathcal{N} \frac{du d^2\Omega}{32\pi G}\,\alpha(N^{AB}N_{AB} - 2D_AD_BN^{AB})$$

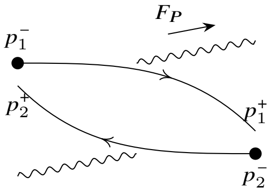
R. M. Wald, A. Zoupas **[gr-qc/9911095]**

É. É. Flanagan and D. A. Nichols **[1510.03386]**

$$P(\mathfrak{I}^+_+) - P(\mathfrak{I}^+_-) = -F_P(\mathfrak{I}^+)\,,$$

- The $\ell \leq 1$ harmonics give the linear momentum balance law
- Confirmed explicitly up to $O(G^3)$
- Flux coincides with DeWitt-Thorne formula

$$C_{AB} \rightarrow \lim_{r\rightarrow\infty} r h^{\mathrm{TT}}_{\mu\nu}(x)$$



BMS Angular Momentum and Flux

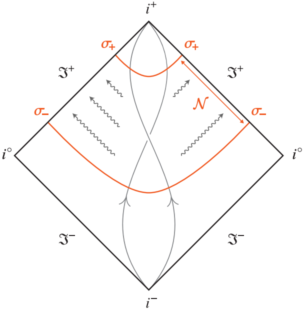
BMS Angular Momentum and Flux

$$\hat{N}_A \coloneqq N_A - u D_A M$$

$$J(\sigma) = \int_\sigma \frac{\mathrm{d}^2\Omega}{8\pi G} Y^A\left(G\hat{N}_A - \frac{1}{16}D_A(C_{BC}C^{BC}) - \frac{1}{4}C_{AB}D_C C^{BC}\right),$$

$$F_J(\mathcal{N}) = \int_\mathcal{N} \frac{\mathrm{d}u\mathrm{d}^2\Omega}{32\pi G} Y^A\Big(N^{BC}D_A C_{BC} - 2D_B(N^{BC}C_{AC}) + \frac{1}{2}D_A(N^{BC}C_{BC}) - \frac{1}{2}uD_A(N^{BC}N_{BC})\Big)$$

R. M. Wald, A. Zoupas **[gr-qc/9911095]**, *É. É. Flanagan and D. A. Nichols* **[1510.03386]**



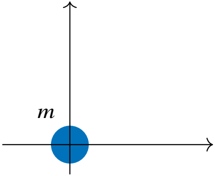
$$J(\sigma_+) - J(\sigma_-) = -F_J(\mathcal{N})$$

- Spatial components of the flux gives the DeWitt-Thorne formula

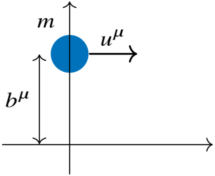
$$C_{AB} \to \lim_{r\to\infty} r h^{\mathrm{TT}}_{\mu\nu}(x)$$

- $J(\sigma)$ generically different from mechanical angular momentum

Pedagogical example: Boosted Schwarzschild



$$p^\mu = (m, \mathbf{0})$$
$$\mathcal{J}^{\mu\nu} = 0$$



$$p^\mu = m u^\mu$$
$$\mathcal{J}^{\mu\nu} = 2 b^{[\mu} p^{\nu]}$$

$$M = m^4/(-n \cdot p)^3 \qquad \hat{N}_A = 3 M D_A (B+S) + (B+S) D_A M$$

$$C_{AB} = -(2 D_A D_B - \Omega_{AB} D^2) S$$

- $B = (n \cdot b)$ translation of amount b^μ
- $S = 2G(n \cdot p) \log\left(\frac{-n \cdot p}{m}\right) + \beta$

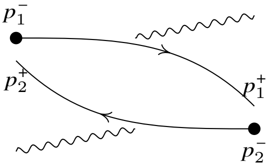
- Bondi momentum P coincides with momentum of the system

$$J = \mathcal{J} + j(M, S)$$

- $j(M, S)$ is the supertranslation ambiguity
- Choose β such that $S = 0$.

New definition of Angular Momentum

one body $\to J = \mathcal{J} + j(M, S)$



$$M^\pm = \sum_{a=1}^{2} \frac{m_a}{(-n \cdot p_a^\pm)^3}$$

$$S^- = \sum_{a=1}^{2} 2G(n \cdot p_a^-) \log\left(\frac{-n \cdot p_a^-}{m_a}\right) + \beta$$

$$S^+ = \sum_{a=1}^{2} 2G(n \cdot p_a^+) \log\left(\frac{-n \cdot p_a^+}{m_a}\right) + \beta + O(G\Delta\mathcal{E})$$

- $N_A^\pm$ ***not*** a superposition

  D. Bini, T. Damour **[2211.06340]**

Asymptotic Bondi J

Different supertranslation at $\mathfrak{I}_\pm^+$

$$\mathcal{J}^\pm = J^\pm - j(M^\pm, S^\pm)$$

$$S^\pm = Z^\pm + C^\pm + \beta$$

Translation $Z^\pm \equiv \mathbb{P}_{\ell \le 1} S^\pm$

Supertranslation $C^\pm \equiv \mathbb{P}_{\ell \ge 2} S^\pm$

The Angular Momentum Flux

- $\Delta M = M^+ - M^-, \qquad \Delta C = C^+ - C^-, \qquad \Delta Z = Z^+ - Z^-, \qquad \Delta S = \Delta Z + \Delta C.$

$$\mathcal{J}^\pm = J^\pm - j(M^\pm, S^\pm)$$

Asymptotic Bondi J

Different supertranslations at $\mathfrak{I}^+_\pm$

$$S^\pm = Z^\pm + C^\pm + \beta$$

- Freedom to fix β to remove either $S^\pm$ but ***not both at the same time***
- Set $Z^- = 0$ to "restore" the origin at $\mathfrak{I}^+_-$, then $Z^+ = \Delta Z$.

$$\mathcal{J}^- = J^- - j(M^-, C^-), \qquad \mathcal{J}^+ = J^+ - j(M^+, C^+) - j(M^+, \Delta Z)$$

$$\mathcal{J}^+ - \mathcal{J}^- = -\Delta_{\mathcal{J}}$$

New Angular Momentum Flux

$$\Delta_{\mathcal{J}} := F_J + j(\Delta M, C^-) + j(M^+, \Delta S)$$

- Bondi angular momentum balance law $J^+ - J^- = -F_J$
- Rewrite $j(M^+, C^+) - j(M^-, C^-) = j(\Delta M, C^-) + j(M^+, \Delta C)$

The Angular Momentum Flux

- $\Delta M = \frac{1}{4G} D^A D^B \int_{-\infty}^{+\infty} \mathrm{d}u \, N_{AB} - \frac{1}{8G} \int_{-\infty}^{+\infty} \mathrm{d}u \, N_{AB} N^{AB}$ $\qquad N_{AB} \equiv \dot{C}_{AB}$

Radiative Flux

$$\Delta_{\mathcal{J}}^{(\mathrm{rad})} = \int \frac{\mathrm{d}u \mathrm{d}^2\Omega}{32\pi G} Y^A \Big(\mathbb{N}^{BC} D_A \mathbb{C}_{BC} - 2 D_B (\mathbb{N}^{BC} \mathbb{C}_{AC}) + \frac{1}{2} D_A (\mathbb{N}^{BC} \mathbb{C}_{BC}) - \frac{1}{2} u D_A (\mathbb{N}^{BC} \mathbb{N}_{BC}) \Big)$$

- $\mathbb{C}_{AB}(u,\theta) := C_{AB}(u - C^-(\theta), \theta) - C^-_{AB}(\theta), \qquad \mathbb{N}_{AB}(u,\theta) := N_{AB}(u - C^-(\theta), \theta)$

$$\Delta_{\mathcal{J}} := \overbrace{F_J + j(\Delta M, C^-)} + \underbrace{j(M^+, \Delta S)}$$

Soft/Static Flux

$$\Delta_{\mathcal{J}}^{(\mathrm{stat})} = \int \frac{\mathrm{d}^2\Omega}{8\pi} M^+ \Big(2 Y^A D_A \Delta S - \Delta S D_A Y^A \Big)$$

Supertranslation invariance

$$\mathcal{J}^+ - \mathcal{J}^- = -\Delta_{\mathcal{J}} \qquad \Delta_{\mathcal{J}} = \Delta_{\mathcal{J}}^{(\mathrm{rad})} + \Delta_{\mathcal{J}}^{(\mathrm{stat})}$$

Supertranslation $\tilde{u} = u - \alpha_{\ell\geq 2}(\theta)$

$$\tilde{C}^{\pm} = C^{\pm} + \alpha(\theta), \quad \tilde{C}'_{AB}(\tilde{u}) = C_{AB}(\tilde{u}+\alpha), \quad \tilde{N}_{AB}(\tilde{u}) = N_{AB}(\tilde{u}+\alpha)$$

- When $\dot{M} = 0$, M is invariant $\longrightarrow$ $M^{\pm}$ and ΔM are invariant
- $\Delta S = \Delta Z + \Delta C$ is invariant.
- $\mathbb{C}_{AB}$ and $\mathbb{N}_{AB}$ are invariant under supertranslation

$$\mathbb{C}_{AB}(u,\theta) \coloneqq C_{AB}(u - C^-(\theta), \theta) - C^-_{AB}(\theta), \qquad \mathbb{N}_{AB}(u,\theta) \coloneqq N_{AB}(u - C^-(\theta), \theta)$$

$$\Delta_{\mathcal{J}}^{(\mathrm{rad})} = \int \frac{\mathrm{d}u\mathrm{d}^2\Omega}{32\pi G} Y^A \Big(\mathbb{N}^{BC} D_A \mathbb{C}_{BC} - 2 D_B (\mathbb{N}^{BC}\mathbb{C}_{AC}) + \frac{1}{2} D_A (\mathbb{N}^{BC}\mathbb{C}_{BC}) - \frac{1}{2} u D_A (\mathbb{N}^{BC}\mathbb{N}_{BC}) \Big)$$

- Manifestly invariant under supertranslation

$$\Delta_{\mathcal{J}}^{(\mathrm{stat})} = \int \frac{\mathrm{d}^2\Omega}{8\pi} M^+ \Big(2 Y^A D_A \Delta S - \Delta S D_A Y^A \Big)$$

- Manifestly invariant under supertranslation

Solution to the first puzzle

$$\mathcal{J}^+ - \mathcal{J}^- = -\Delta_{\mathcal{J}} \qquad \Delta_{\mathcal{J}} = \Delta_{\mathcal{J}}^{(\mathrm{rad})} + \Delta_{\mathcal{J}}^{(\mathrm{stat})}$$

$$\Delta_{\mathcal{J}}^{(\mathrm{rad})} = \int \frac{\mathrm{d}u\mathrm{d}^2\Omega}{32\pi G} Y^A \Big(\mathbb{N}^{BC} D_A \mathbb{C}_{BC} - 2D_B(\mathbb{N}^{BC}\mathbb{C}_{AC}) + \frac{1}{2} D_A(\mathbb{N}^{BC}\mathbb{C}_{BC}) - \frac{1}{2} u D_A(\mathbb{N}^{BC}\mathbb{N}_{BC}) \Big)$$

- Manifestly invariant under supertranslation
- $\{\mathbb{C}_{AB}, \mathbb{N}_{AB}\} \sim O(G^2)$, hence $\Delta_{\mathcal{J}}^{(\mathrm{rad})} \sim O(G^3)$
- Consistent with definition of invariant angular momentum

G. Compère, R. Oliveri, A. Seraj **[1912.03164]**, *G. Veneziano, G. A. Vilkovisky* **[2201.11607]**, *R. Javadinezhad, U. Kol, M. Porrati* **[2202.03442]**

$$\Delta_{\mathcal{J}}^{(\mathrm{stat})} = \int \frac{\mathrm{d}^2\Omega}{8\pi} M^+ \Big(2Y^A D_A \Delta S - \Delta S D_A Y^A \Big)$$

- Manifestly invariant under supertranslation
- $M^+ \sim O(1)$, $\Delta S \sim O(G^2)$, hence $\Delta_{\mathcal{J}}^{(\mathrm{stat})} \sim O(G^2)$
- Depends on the memory $\Delta C_{AB} \equiv -(2D_A D_B - \Omega_{AB} D^2)\Delta C$

T. Damour **[2010.01641]**

Result up to G^2

$$n^\mu = (1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \quad \bar{n}^\mu = (1, -\sin\theta\cos\phi, -\sin\theta\sin\phi, -\cos\theta), \quad \eth_A = (\partial_A n^\mu)\eth_\mu$$

$$\Delta\mathcal{J}^{\mu\nu} = -\Delta^{\mu\nu}_{\mathcal{J}\,(\text{stat})} + O(G^3)$$

$$\Delta^{\mu\nu}_{\mathcal{J}\,(\text{stat})} = \int \frac{d^2\Omega}{4\pi} M^+ \Big[2\eth^{[\mu}\Delta S - \Delta S \bar{n}^{[\mu}\Big] n^{\nu]}$$

- $\Delta S = \sum_{a=1}^{2} \left[2G(n\cdot p_a)\log\left(\frac{-n\cdot p_a}{m_a}\right)\right]_{-\infty}^{+\infty} + O(G\Delta\mathcal{E}),\qquad M^+ = \sum_{a=1}^{2} \frac{m_a}{(-n\cdot p_a^+)^3}$

$$\Delta^{\mu\nu}_{\mathcal{J}} = \Delta^{\mu\nu}_{\mathcal{J}\,(\text{stat})} + O(G^3) = \frac{2G^2 m_1 m_2}{b^2}\frac{2\gamma^2-1}{\sqrt{\gamma^2-1}}\mathcal{I}(\gamma) b^{[\mu}(p^{\nu]}_{1-} - p^{\nu]}_{2-}) + O(G^3)$$

- Agreement in all Bondi frame at G^2 with $\Delta J^{\mu\nu}_{\text{QFT}}$
  A. V. Manohar, A. K. Ridgway, C.-H. Shen **[2203.04283]**,
  P. Di Vecchia et al. **[2203.11915-2210.12118]**
- Agreement up to $O(G^3)$ with soft flux
  P. Di Vecchia et al. **[2203.11915-2210.12118]**

Special case of the binary's center of mass

$f^{\pm}_{AB} \to \lim_{r\to\infty} r h^{\pm}_{\mu\nu}$

- In Lorentz indices $C_{AB} \to \lim_{r\to\infty} r h_{\mu\nu}$
- $\hat{r}^{\mu} = n^{\mu} - \hat{t}^{\mu} = (0, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$
- Spherical harmonics decomposition $M^{\pm} = (3\hat{r} - \hat{t}) \cdot P^{\pm} + \frac{1}{4G} D^A D^B \overbrace{f^{\pm}_{AB}}$

$$\Delta^{\mu\nu}_{\mathcal{J}\,(\text{stat})} = \int \frac{\mathrm{d}u\mathrm{d}\Omega}{32\pi G} r^2 \Big[4h^{\rho[\mu}_{-}\dot{h}^{\nu]}{}_{\rho} - 2\dot{h}_{\rho\sigma} n^{[\mu}\eth^{\nu]} h^{\rho\sigma}_{-} - n^{[\mu}\bar{n}^{\nu]}\dot{h}_{\rho\sigma}h^{\rho\sigma}_{-} \Big]$$
$$+ \int \frac{\mathrm{d}\Omega}{4\pi} \underbrace{(3\hat{r}\cdot P_{-})} \Big[2\eth^{[\mu}\Delta S - \Delta S \bar{n}^{[\mu} \Big] n^{\nu]} - (3\hat{t}\cdot P_{-}) \int \frac{\mathrm{d}\Omega}{4\pi} \Delta S \underbrace{\bar{n}^{[\mu}n^{\nu]}} + O(G^3)$$

- $\hat{r}\cdot(p^-_1 + p^-_2) = 0$ in the initial c.m.
- $\bar{n}^{[\mu}n^{\nu]}$ contributes only to $0i$ components
- Spatial component of first line gives Thorne-DeWitt formula at $O(G^2)$
- Without ΔC **and** ΔZ, the final result would not be covariant

Comparison with other proposal

$$\mathcal{J}^- = J^- - j(M^-, C^-), \qquad \mathcal{J}^+ = J^+ - j(M^+, C^+) - j(M^+, \Delta Z)$$

$$J_{(\mathrm{inv})}(\sigma) = J(\sigma) - j(M(\sigma), C(\sigma))$$

G. Compère, R. Oliveri, A. Seraj **[1912.03164]**

P.-N. Chen et al. **[2102.03235 - 2107.05316]**, *P. Mao, J.-B. Wu, X. Wu* **[arXiv:2301.08032]**

$$\Delta\mathcal{J} - \Delta J_{(\mathrm{inv})} = -j(M^+, \Delta Z)$$

$$J_{(\mathrm{JP})}(\sigma) = J(\sigma) - j(M(\sigma), C^-) + j\big(M(\sigma), \mathcal{C}(\boldsymbol{\theta})\big)$$

R. Javadinezhad, U. Kol, M. Porrati **[2202.03442]**, *R. Javadinezhad, M. Porrati* **[2211.06538]**

$$\Delta\mathcal{J} - \Delta J_{(\mathrm{JP})} = -j(\Delta M, \mathcal{C}(\boldsymbol{\theta})) - j(M^+, \Delta C) - j(M^+, \Delta Z)$$

- $-j\big(M(\sigma), \mathcal{C}(\boldsymbol{\theta})\big) - j(M^+, \Delta C) = O(G^3)$

- Agreement on the radiative flux
- Disagreement on the soft part outside of the c.m. frame because of $j(M^+, \Delta Z)$
- $\Delta J_{(\mathrm{inv})}$ and $\Delta J_{(\mathrm{JP})}$ are not Lorentz-covariant

Conclusions and Future directions

$$\mathcal{J}^- = J^- - j(M^-, C^-), \qquad \mathcal{J}^+ = J^+ - j(M^+, C^+) - j(M^+, \Delta Z)$$

$$\mathcal{J}^+ - \mathcal{J}^- = -\Delta_{\mathcal{J}}^{(\mathrm{rad})} - \Delta_{\mathcal{J}}^{(\mathrm{stat})}$$

- New definition of supertranslation invariant definition of the angular momentum $\mathcal{J}^{\mu\nu}$ for the two-body scattering
- Resolution of the two puzzles
- Key role plays by $j(M^+, \Delta Z)$

- Properly define $j(M^+, \Delta Z)$ for generic gravitational system
- Compare the $O(G^3)$ contribution → compute NLO $C_{AB}(u, \theta)$
- Understand the effect of non-linear memory → compute $O(G^4)$

  A. Strominger, A. Zhiboedov **[1411.5745]**

  S. Pasterski, A. Strominger, A. Zhiboedov **[1502.06120]**

  M. Campiglia, A. Laddha **[1509.01406]**
- New way of computing $\Delta\mathcal{J}$

  A. V. Manohar, A. K. Ridgway, C.-H. Shen **[2203.04283]**

  P. Di Vecchia et al. **[2203.11915-2210.12118]**, *D. Bini, T. Damour* **[2211.06340]**

Thank you for your attention!

State of the Art of the PM Approximation (point-particles)

	0PN	1PN	2PN	3PN	4PN	5PN	6PN		
1PM	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	$\dots$	G^1
2PM		1	v^2	v^4	v^6	v^8	v^{10}	$\dots$	G^2
3PM			1	v^2	v^4	v^6	v^8	$\dots$	G^3
4PM				1	v^2	v^4	v^6	$\dots$	G^4
5PM					1	v^2	v^4	$\dots$	G^5

- 3PM conservative dynamics

  Z. Bern et al. **[1908.01493]** *G. Kälin, Z. Liu, R. A. Porto* **[2007.04977]**

- 3PM radiative dynamics

  E. Herrmann et al. **[2101.07255 - 2104.03957]**, *G. U. Jakobsen et al.* **[2101.12688]**, *S. Mougiakakos,* ***MMR****, F. Vernizzi* **[2102.08339]**, ***MMR****, F. Vernizzi* **[2110.10140]**

- 3PM conservative + radiative alternative methods

  P. Di Vecchia et al. **[2008.12743 - 2101.05772 - 2104.03256 - 2210.12118]**

  N. E. Bjerrum-Bohr et al. **[2104.04510 - 2105.05218]**

  G. Kälin, J. Neef, R. A. Porto **[2207.00580]**, *G. U. Jakobsen et al.* **[2207.00569]**

- 4PM Conservative dynamics

  Z. Bern et al. **[2101.07254 - 2112.10750]** *C. Dlapa et al.* **[2106.08276 - 2112.11296]**

- 4PM Conservative + Radiative dynamics

  C. Dlapa et al. **[2210.05541 - 2304.01275]**

Two-parameter family for $J(\sigma)$

$\hat{N}_A \coloneqq N_A - uD_A M$

two-parameter family of angular momentum

$$J^{(\alpha,\beta)}(\sigma) = \int_\sigma \frac{\mathrm{d}^2\Omega}{8\pi G} Y^A\left(G\hat{N}_A - \frac{\beta}{16} D_A(C_{BC}C^{BC}) - \frac{\alpha}{4} C_{AB}D_C C^{BC}\right),$$

G. Compère, R. Oliveri, A. Seraj **[1912.03164]**, *A. Elhashash, D. A. Nichols* **[2101.12228]**

- Flat spacetime Bondi functions

$$M = 0, \qquad N_A = 0, \qquad C_{AB} = (2D_A D_B - \Omega_{AB} D^2)\Phi$$

Flat spacetime angular momentum

$$J^{(\alpha,\beta)}(\sigma) = (\beta - \alpha) \int_\sigma \frac{\mathrm{d}^2\Omega}{256\pi G} \left(D_A Y^A\right)\left[(D^2\Phi)^2 - 4D_C\Phi D^C\Phi\right],$$

- Vanishes for $\alpha = \beta$
- For $\alpha = \beta = 1$ it is balanced by the flux of *R. M. Wald, A. Zoupas* **[gr-qc/9911095]**
- For non-radiative to non-radiative transition, one can relax the previous condition and find the same results

A. Elhashash, D. A. Nichols **[2101.12228]**