Research in theoretical high-energy physics at NBIA

Matthias Wilhelm

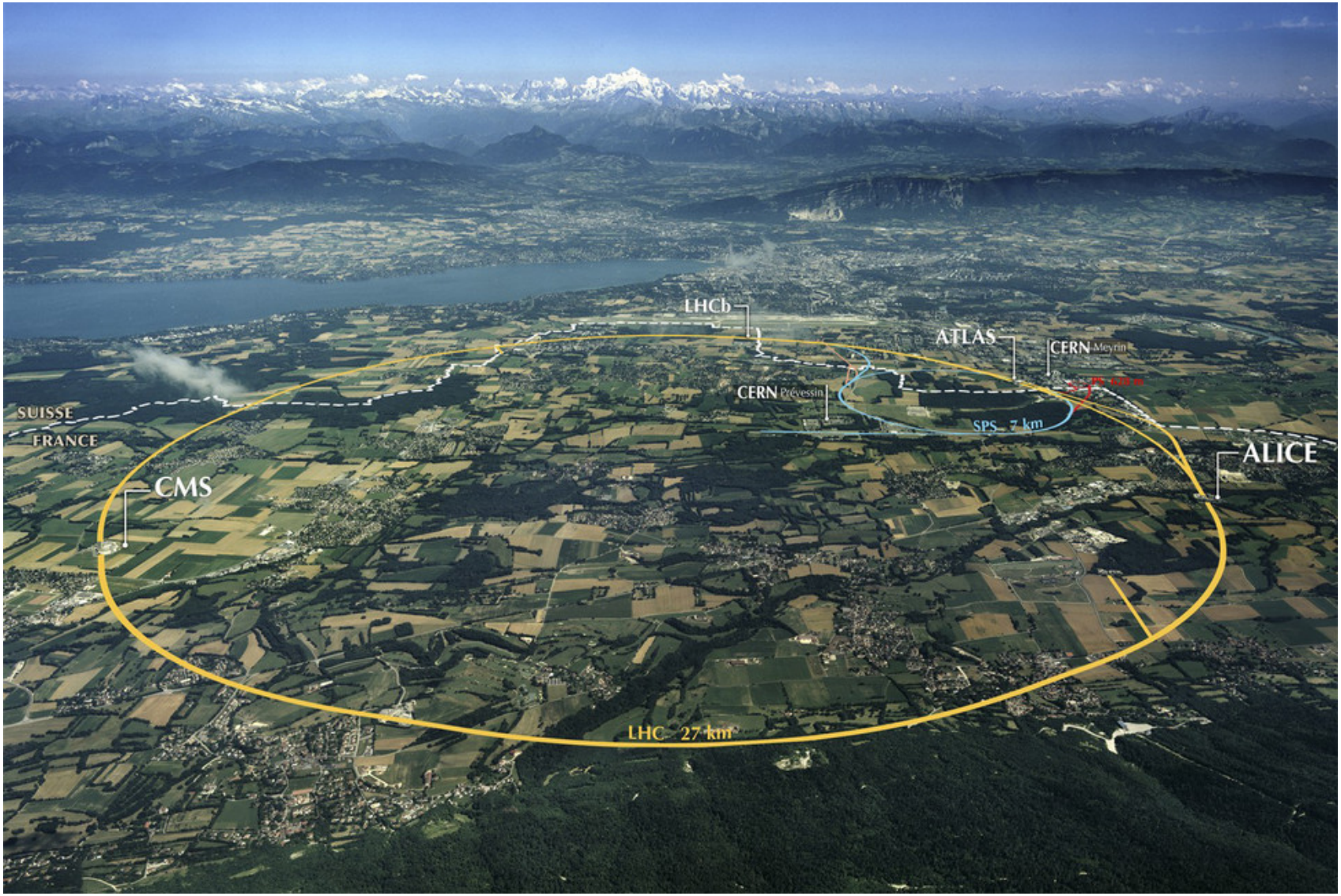
NBIA MSc Day 2023

October 11th, 2023

The Niels Bohr International Academy

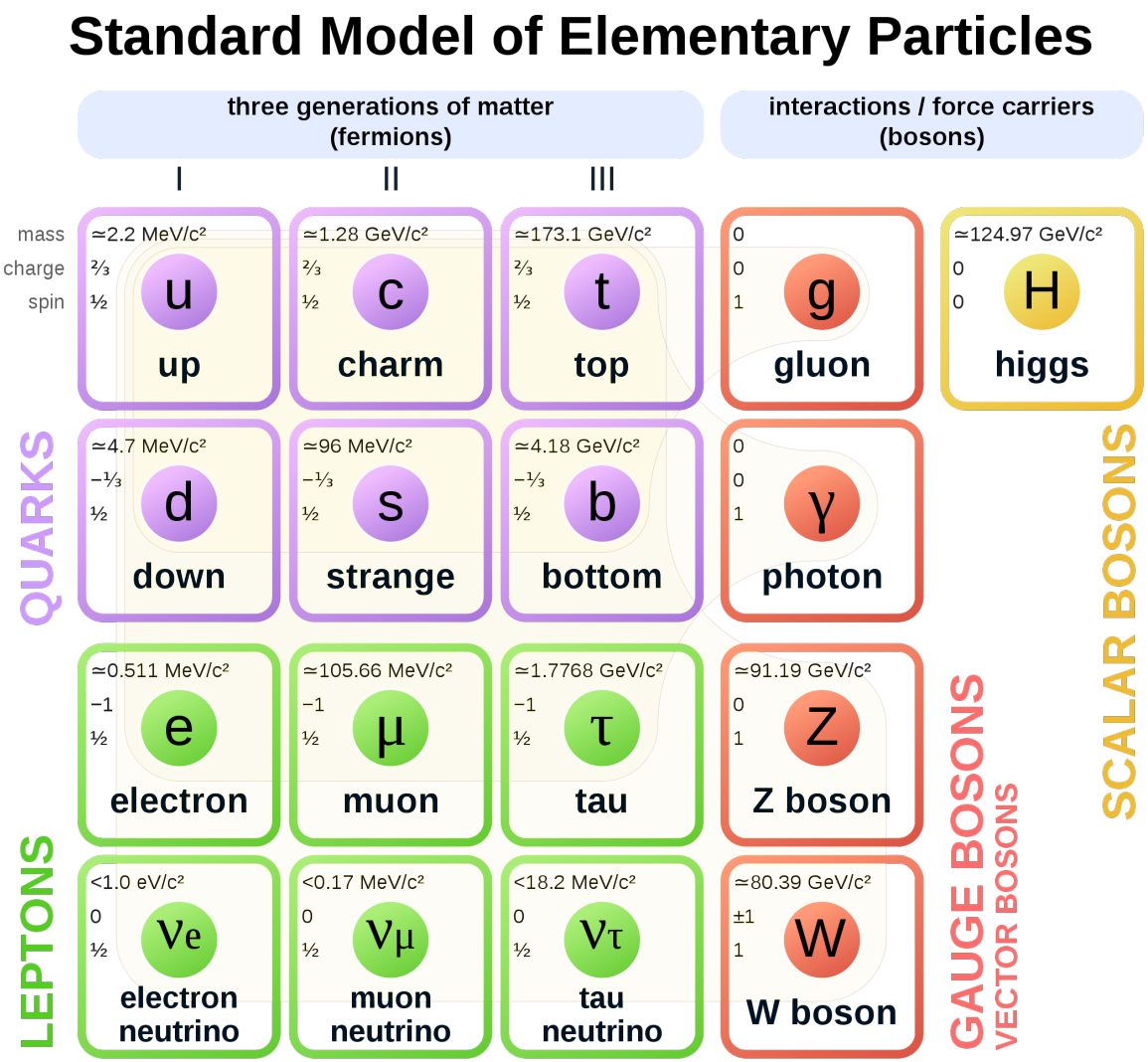
VILLUM FONDEN

Large Hadron Collider

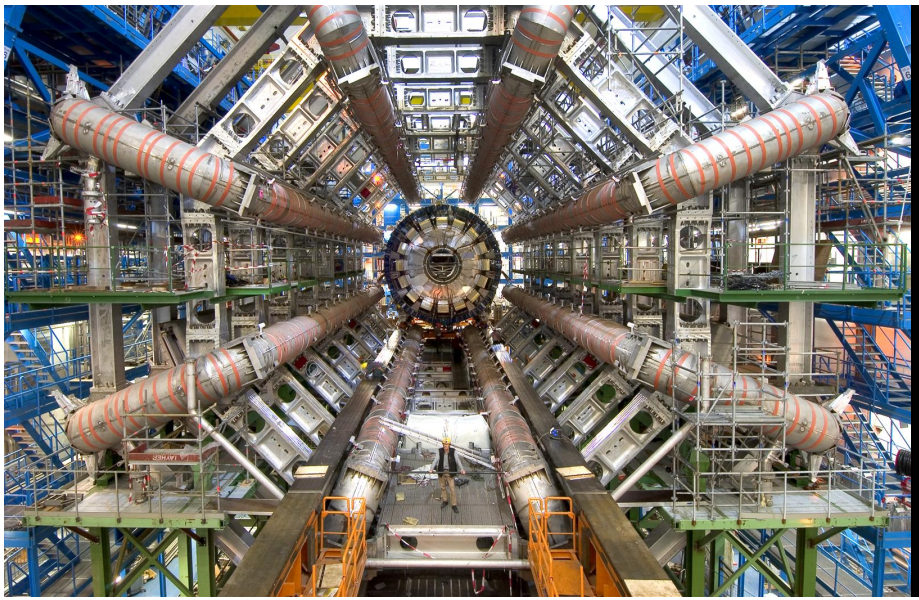


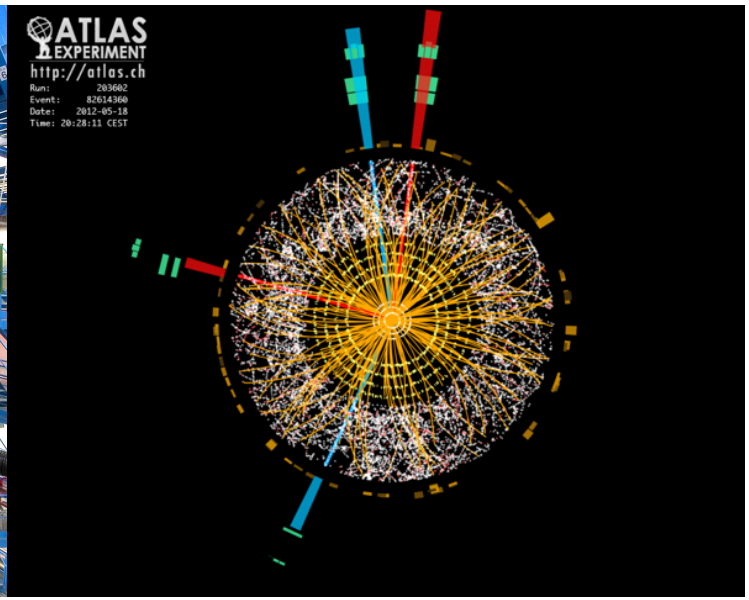
The world's largest machine = most powerful microscope

Standard model of particle physics



How do we see these particles?

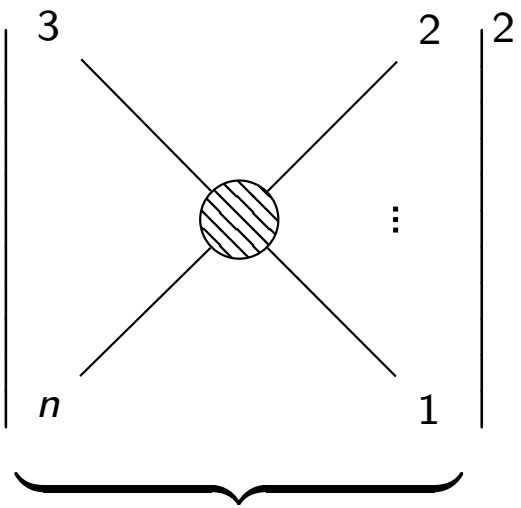




Short lived $\Rightarrow$ Only see decay products in detectors!

Theoretical description

Cross section $=$ probability of two incoming particles to scatter into $n-2$ outgoing particles:

$\sigma =$ 

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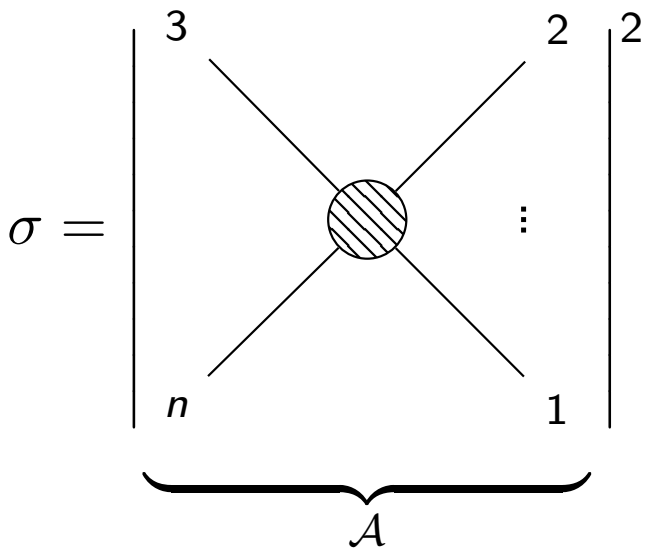


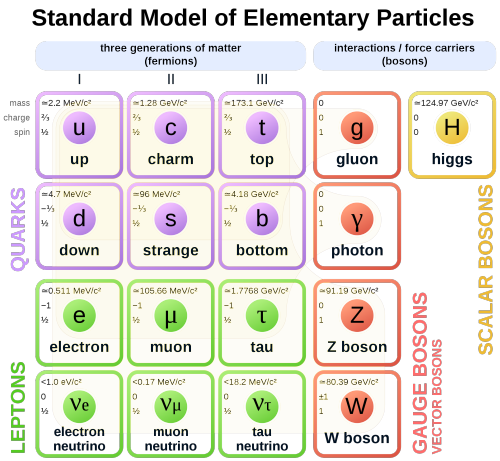
Amplitude $\mathcal{A}$ can be calculated using Quantum Field Theory

What is Quantum Field Theory?

Quantum Field Theory

$=$ Quantum mechanics + special relativity

- describes all known interactions among all known particles except gravity via so-called gauge theories



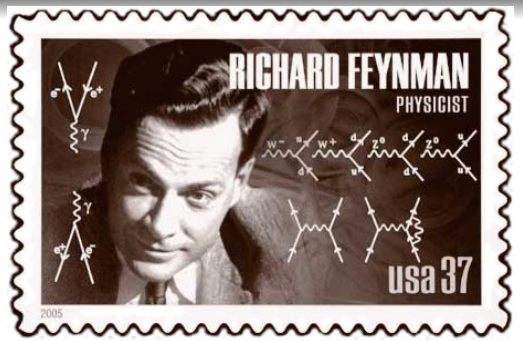
$\supset$ Quantum Electrodynamics (QED) and Quantum Chromodynamics (QCD)

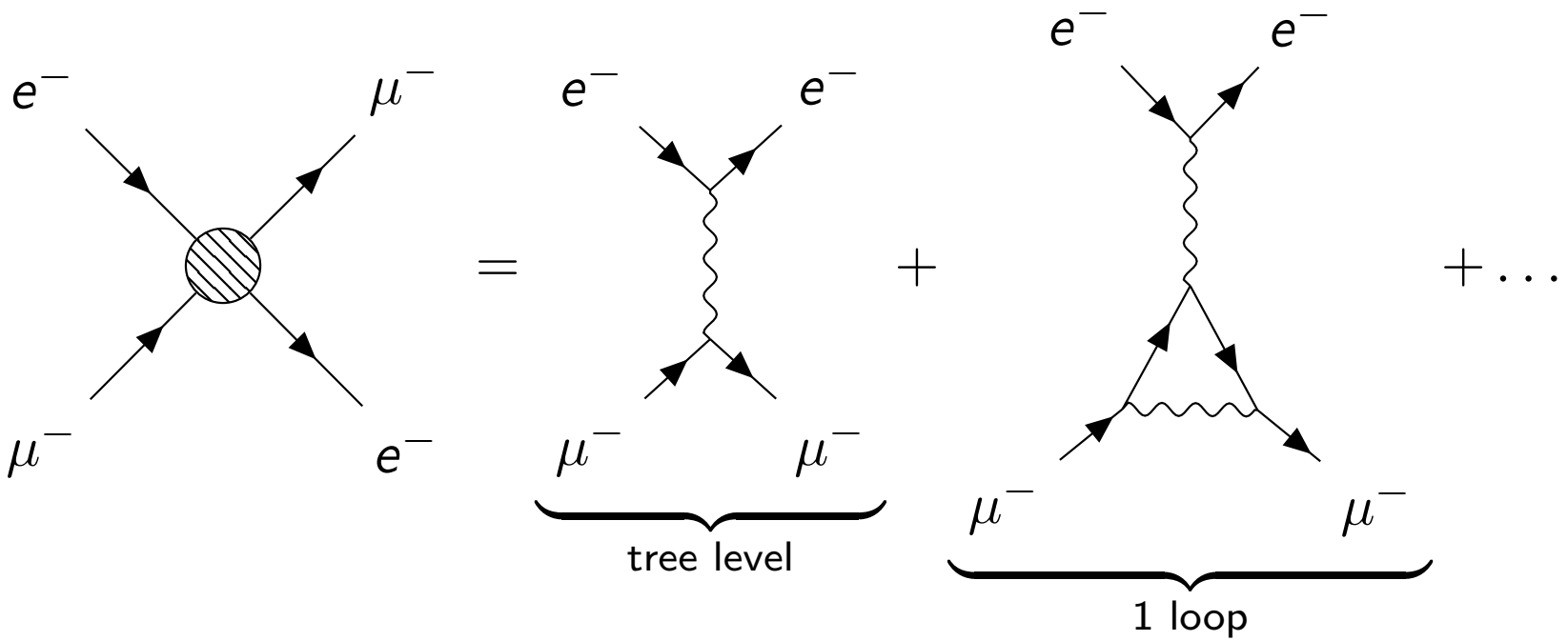
- describes classical gravity (general relativity) $\rightarrow$ Emil

$\rightarrow$ Course “Quantum Field Theory I”

Amplitudes from Quantum Field Theory

Feynman diagrams
= sums over possible particle histories





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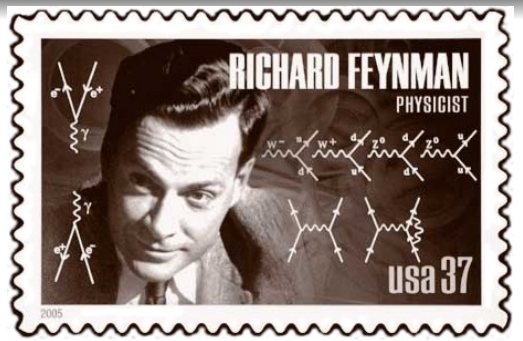

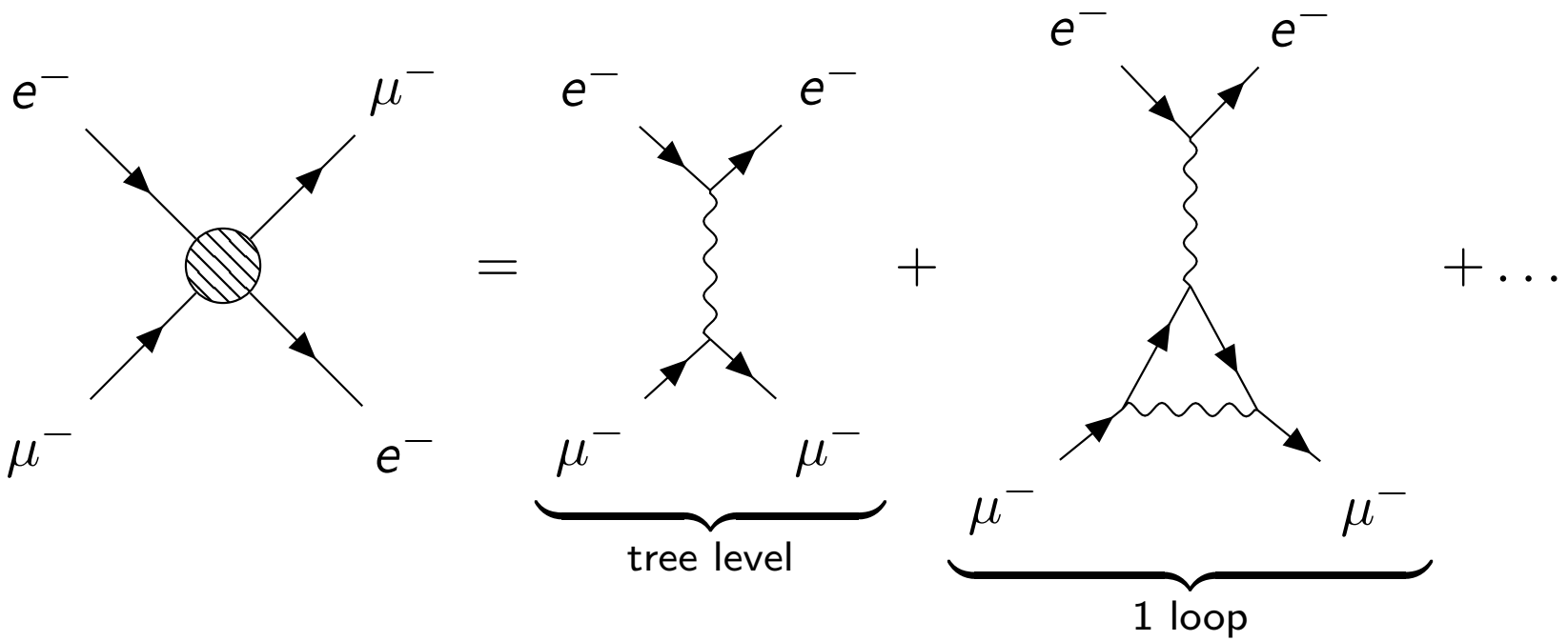

tree level leading order in perturbation theory

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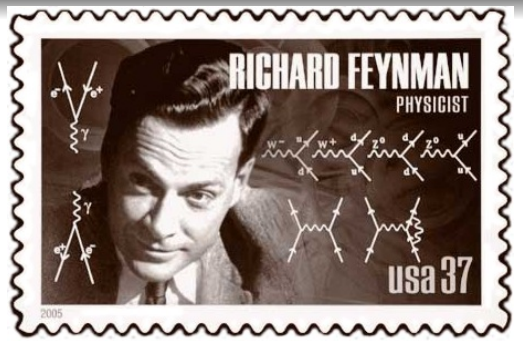

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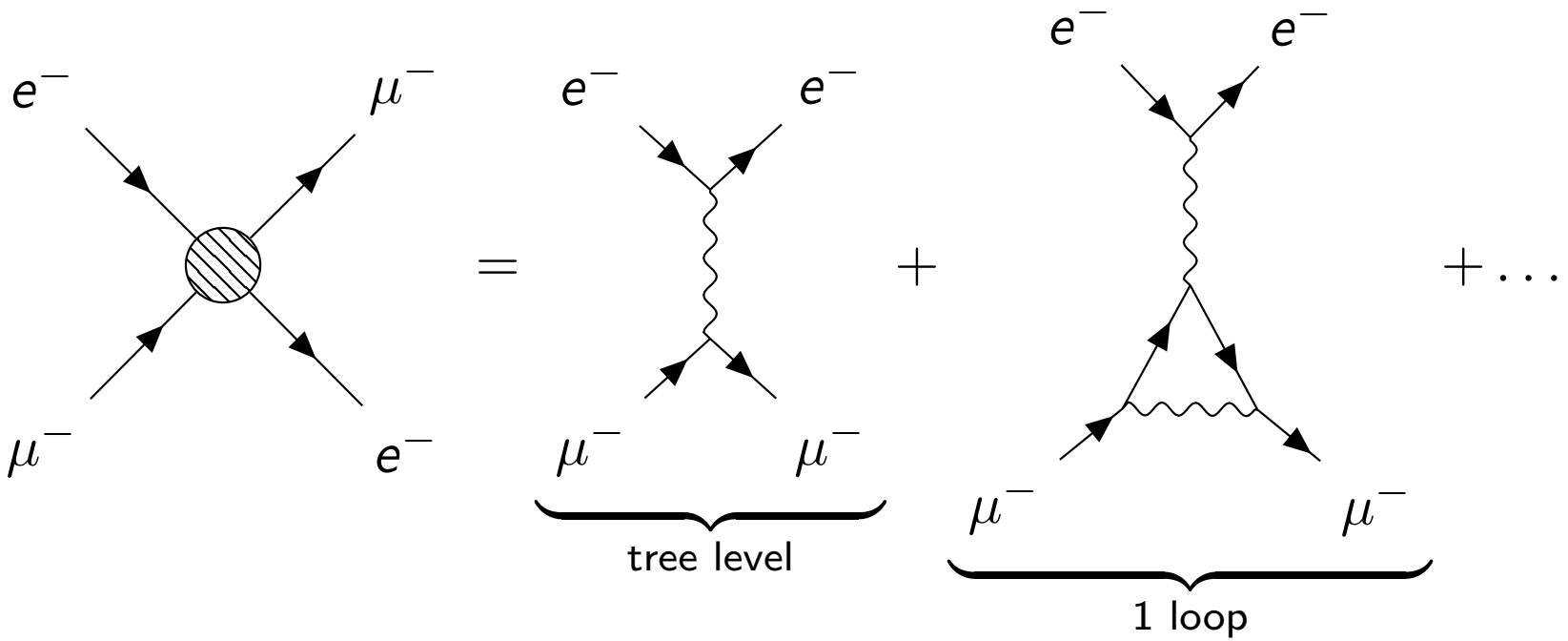

tree level leading order in perturbation theory

1 loop next-to-leading order in perturbation theory

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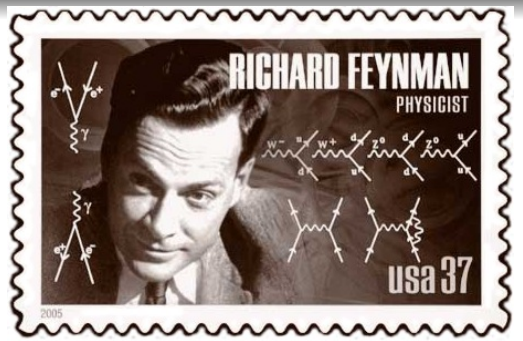

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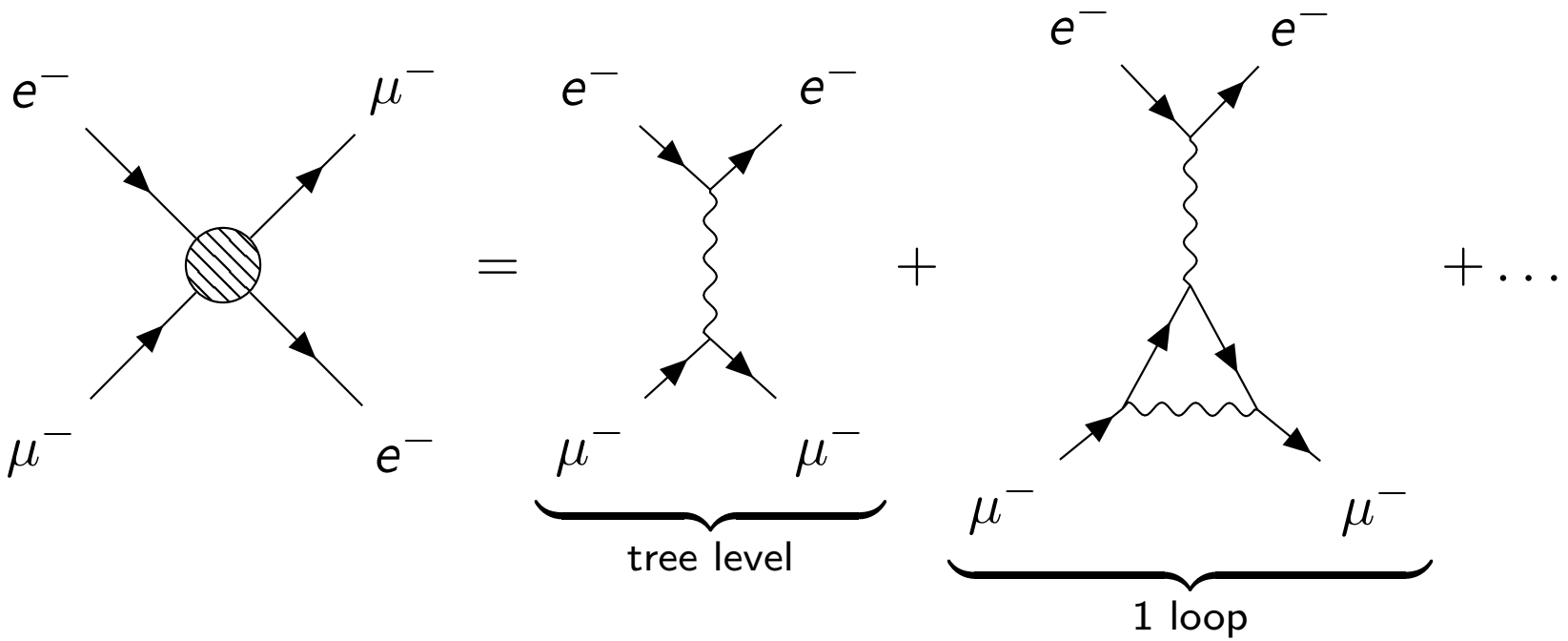

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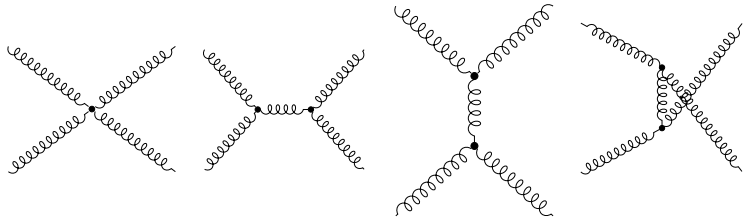

2 gluons $\to$ 2 gluons: 4 diagrams
2 gluons $\to$ 3 gluons: 25 diagrams
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2 gluons $\to$ 8 gluons: $>$ 1 million diagrams

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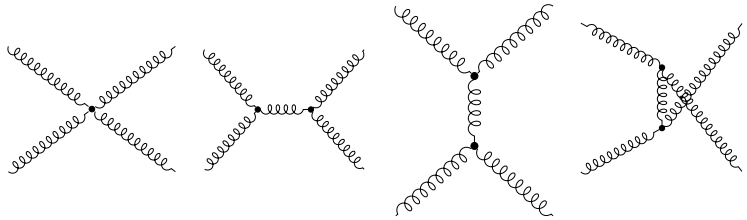

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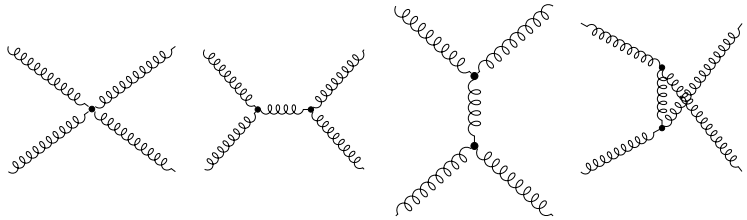

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n-gluon helicity amplitude [Parke-Taylor (1986)] [Mangano, Parke, Xu (1987)]

$$\mathcal{A}_6(1^-, 2^-, 3^+, \ldots, 6^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle \ldots \langle 61\rangle}$$

$\pm$: polarization of the gluon with four-momentum p_i
$\langle ij\rangle = \sqrt{|s_{ij}|}e^{i\phi_{ij}}$ with $s_{ij} = (p_i + p_j)^2$

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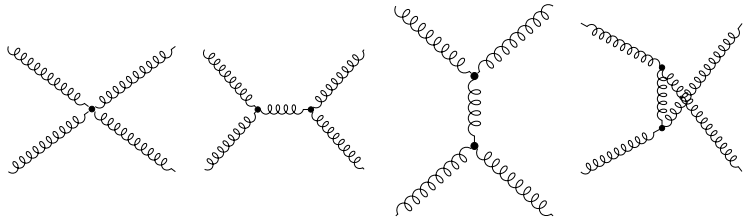

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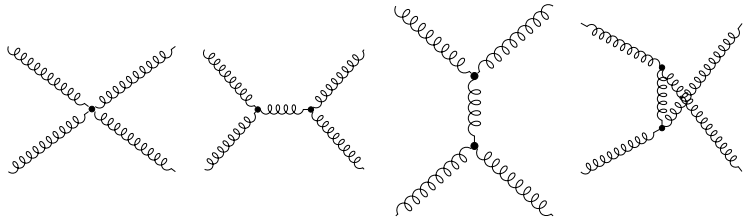

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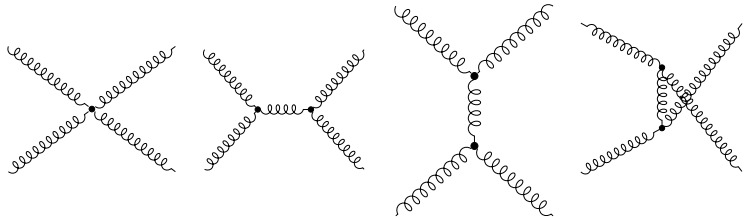

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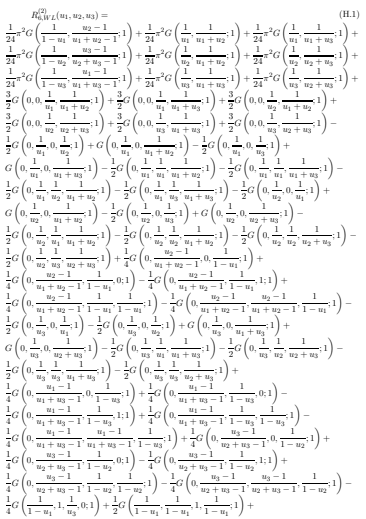
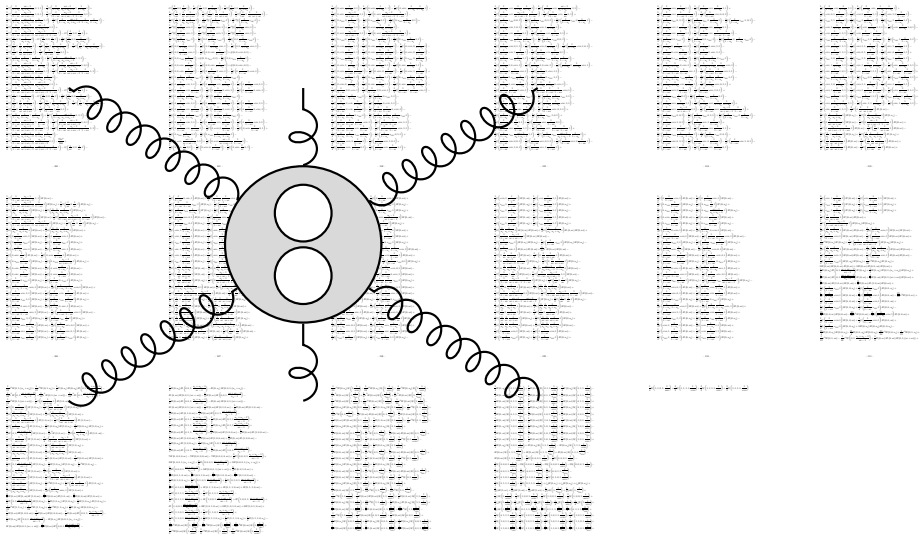
$\to$ Course "Modern methods in particle scattering"

Hidden simplicity II: Polylogarithms

Two-loop six-gluon reminder function (= non-trivial part of amplitude) in the maximally (super)symmetric gauge theory

[Del Duca, Duhr, Smirnov (2010)]





$$u_1 = \frac{s_{12}s_{45}}{s_{123}s_{345}} \qquad u_2 = \frac{s_{23}s_{56}}{s_{234}s_{123}} \qquad u_3 = \frac{s_{34}s_{61}}{s_{345}s_{234}}$$

Hidden simplicity II: Polylogarithms

$$\text{18 pages} = \sum_{i=1}^{3} \left(L_4(x_i^+, x_i^-) - \frac{1}{2} \operatorname{Li}_4(1 - 1/u_i) \right) - \frac{1}{8} \left(\sum_{i=1}^{3} \operatorname{Li}_2(1 - 1/u_i) \right)^2 + \frac{1}{24} J^4 + \frac{\pi^2}{12} J^2 + \frac{\pi^4}{72}$$

[Gancharov, Spradlin, Vergu, Volovich (2010)]

$$x_i^{\pm} = u_i x^{\pm}, \qquad x^{\pm} = \frac{u_1 + u_2 + u_3 - 1 \pm \sqrt{\Delta}}{2 u_1 u_2 u_3}, \qquad \Delta = (u_1+u_2+u_3-1)^2 - 4 u_1 u_2 u_3$$

$$L_4(x^+, x^-) = \frac{1}{8!!} \log(x^+ x^-)^4 + \sum_{m=0}^{3} \frac{(-1)^m}{(2m)!!} \log(x^+ x^-)^m (\ell_{4-m}(x^+) + \ell_{4-m}(x^-))$$

$$\ell_n(x) = \frac{1}{2} \left(\operatorname{Li}_n(x) - (-1)^n \operatorname{Li}_n(1/x) \right), \qquad J = \sum_{i=1}^{3} (\ell_1(x_i^+) - \ell_1(x_i^-))$$

Classical polylogarithms $\operatorname{Li}_n(x) = \int_0^x \frac{dt}{t} \operatorname{Li}_{n-1}$, $\qquad \operatorname{Li}_1(x) = -\log(1-x)$

Hidden simplicity II: Polylogarithms

Exploiting the simplicity:

Bootstrapping

$=$ ansatz for result from polylogarithms

$+$ fix coefficients via physical constraints

$\Rightarrow$ Avoid Feynman diagrams and Feynman integrals altogether!

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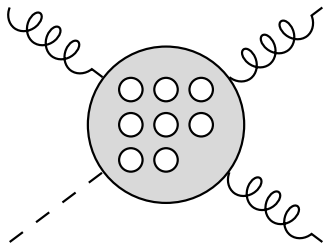
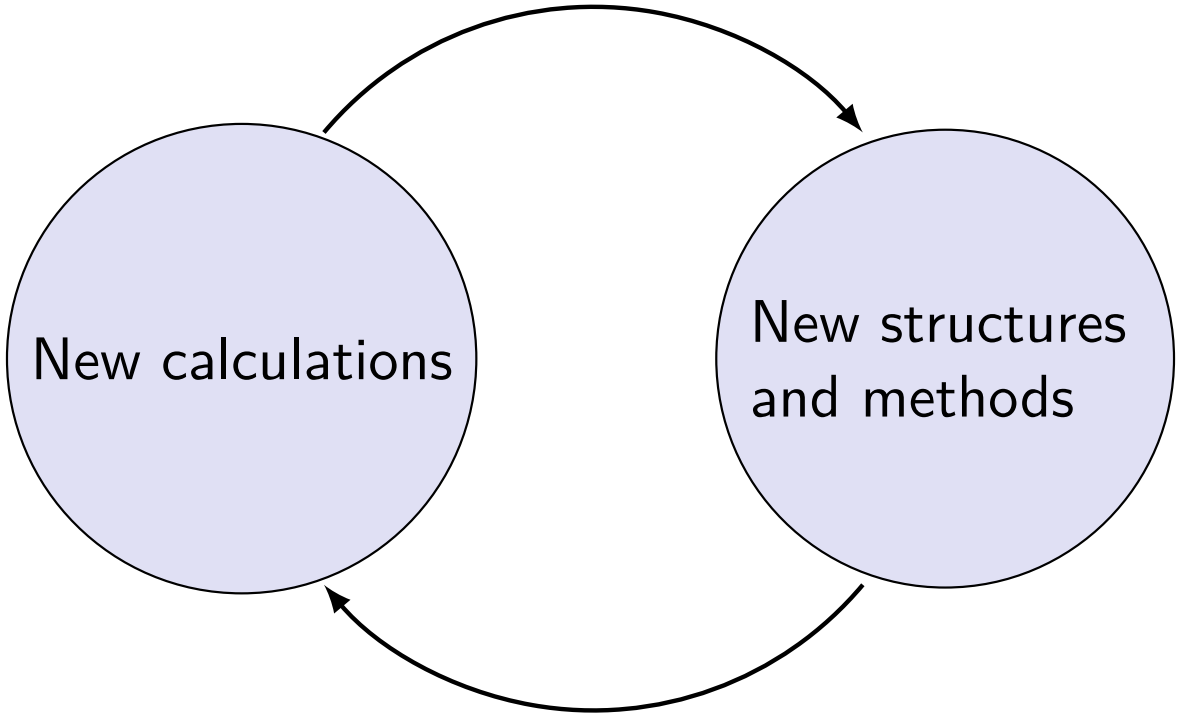
Higgs $\to$ 3 gluons (in some approximation) up to 8-loop order!



[Dixon, McLeod, MW (2020)], [Dixon, Gurdogan, McLeod, MW (2021)]

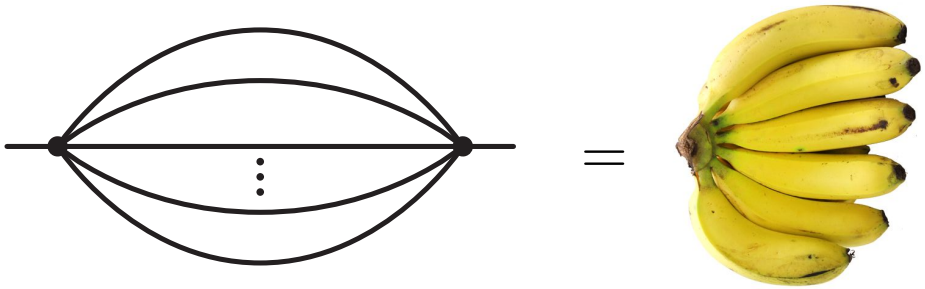
Virtuous circle

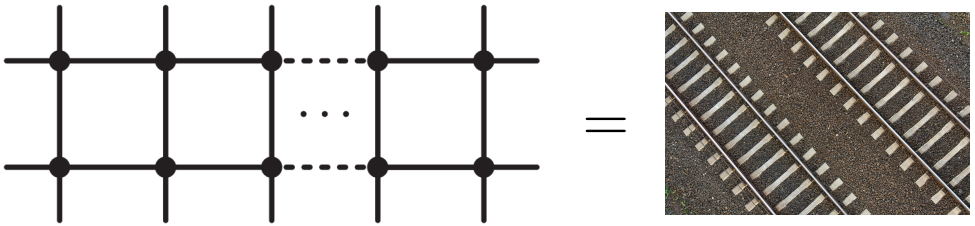


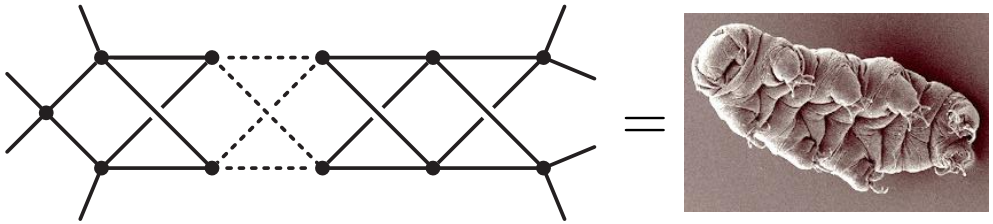
$\Rightarrow$ Precision predictions for the LHC to test our understanding of particle physics and to find new physics beyond the standard model of particle physics!

Beyond polylogarithms

New functions for collider physics and gravitational waves







Hidden structures and simplicity? How to exploit?

..., Frellesvig, Morales, MW, ...

Interested?

Study track: High-Energy Theory and Cosmology

	Block 1	Block 2	Block 3	Block 4
Year 1	Advanced Quantum Mechanics	General Relativity and Cosmology	Quantum Field Theory 1	Fundaments of High-Energy Astrophysics and Particle Astrophysics
	Elementary Particle Physics	Particle Physics and the Early Universe	Modern Methods for Particle Scattering	*Choose one of:* Introduction to String Theory* Introduction to Gauge/Gravity Duality** Advanced Topics in QFT & Gravity***

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Potential supervisor $\rightarrow$ Talk to me!

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