Global symmetries, dualities and universal properties of quantum matter

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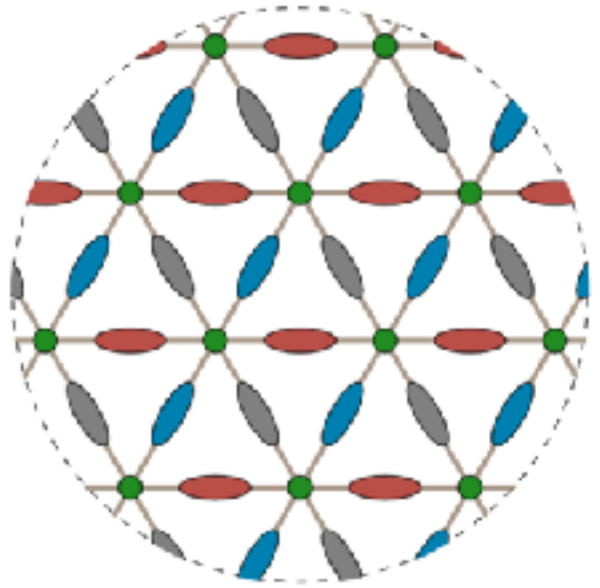
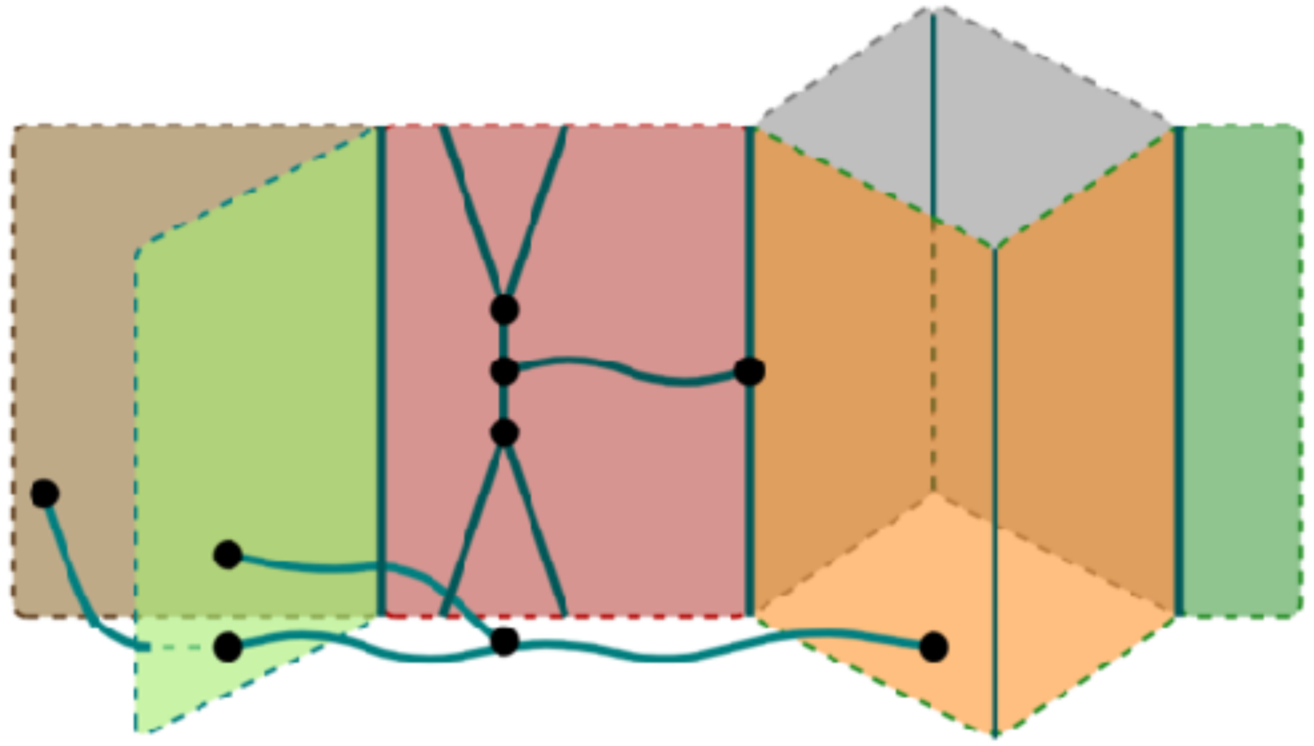
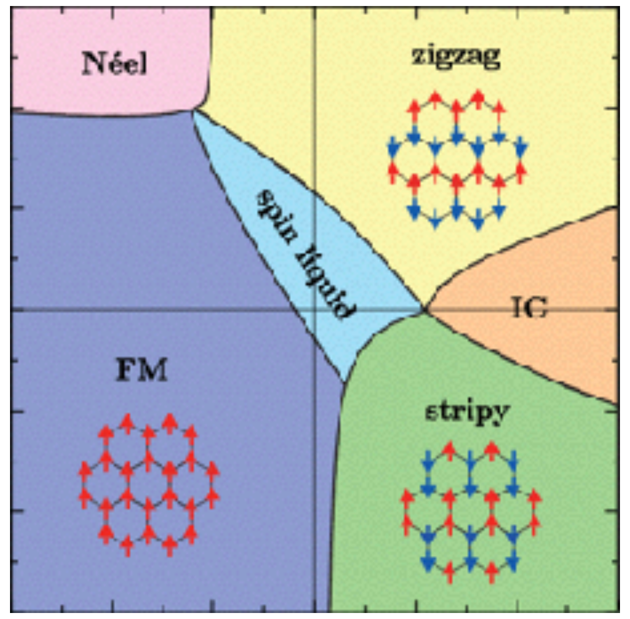






Goal:

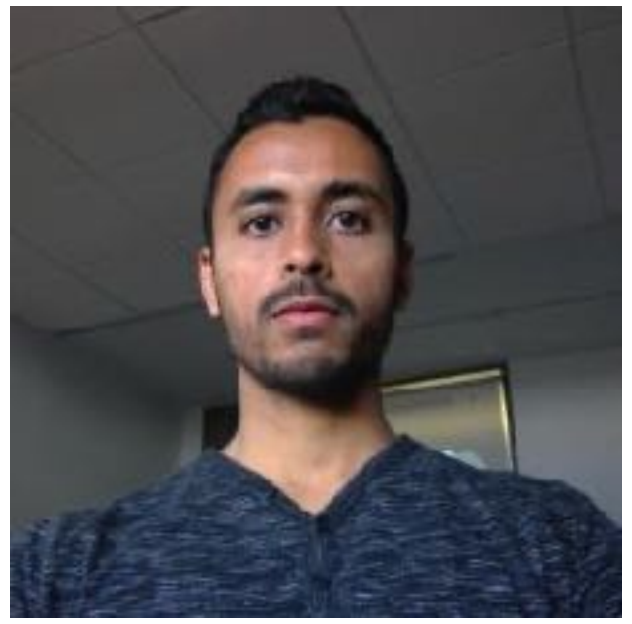
Non-perturbative methods to organize/predict universal properties of interacting quantum matter.

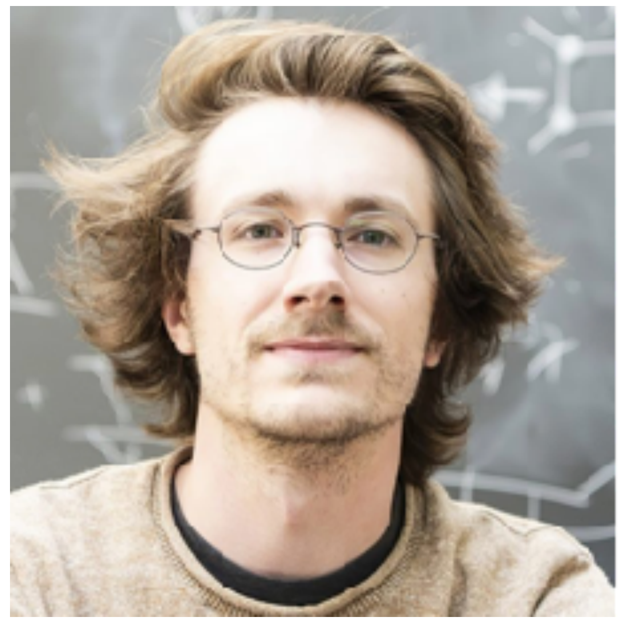
Toolbox:

~~**Symmetry & Topology**~~

Symmetry $\simeq$ Topology

In collaboration with



Lakshya Bhardwaj, Oxford

Lea Bottini, Oxford

Sakura Schäfer Nameki, Oxford

Clement Delcamp, IHES

arXiv:2301.01259

SciPost Phys. Core 6, 066 (2023)
arXiv:2307.01266



Faroogh Moosavian, Oxford

Heidar Moradi Kent

Ömer Aksoy MIT

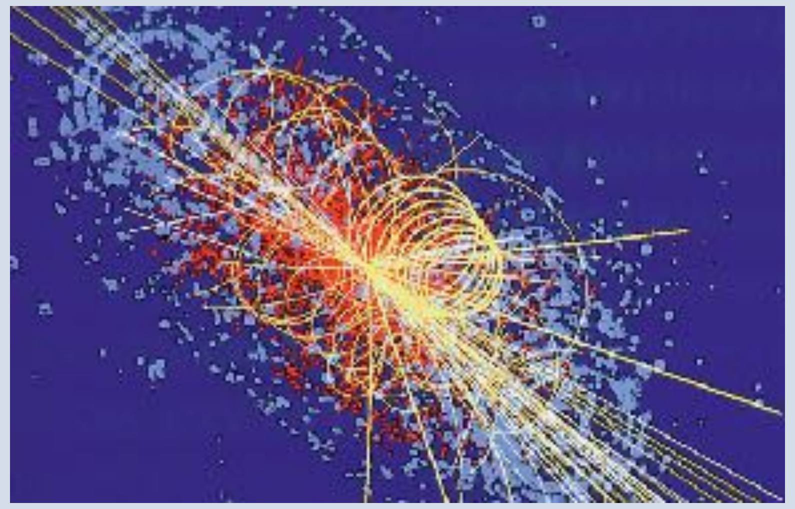
arXiv:2308.00743

Motivation and Background

Universality

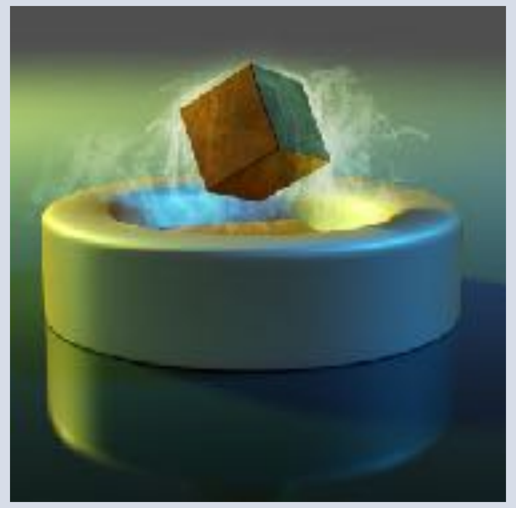
Physical Universe

Outer space

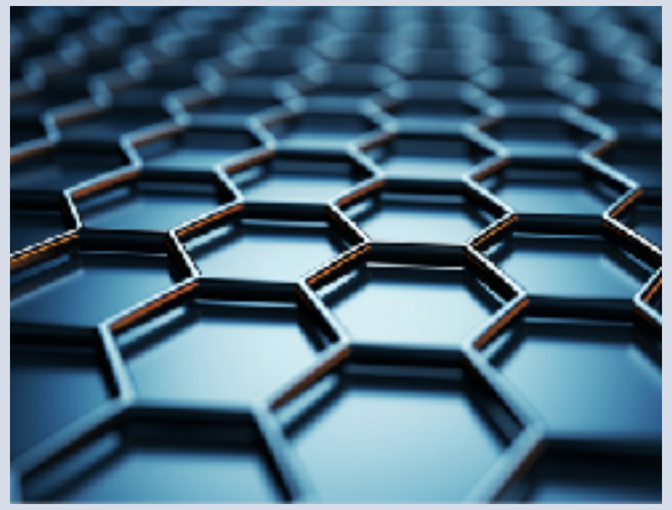


Fundamental particles

Material (tunable) universe

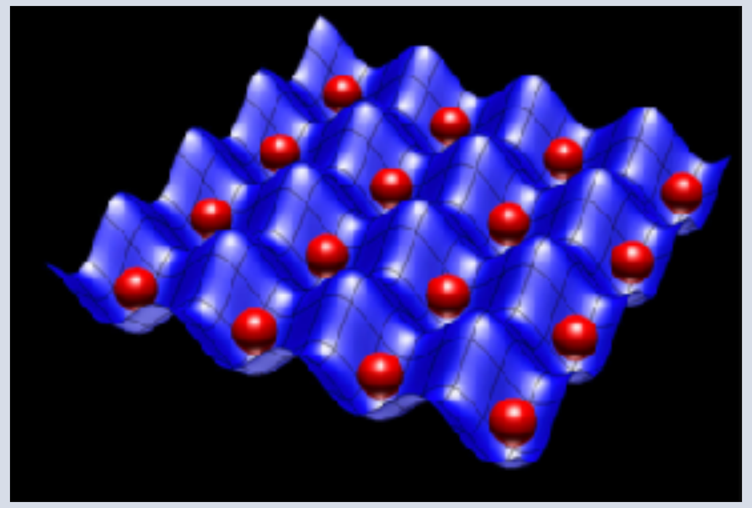


High-Temperature superconductor

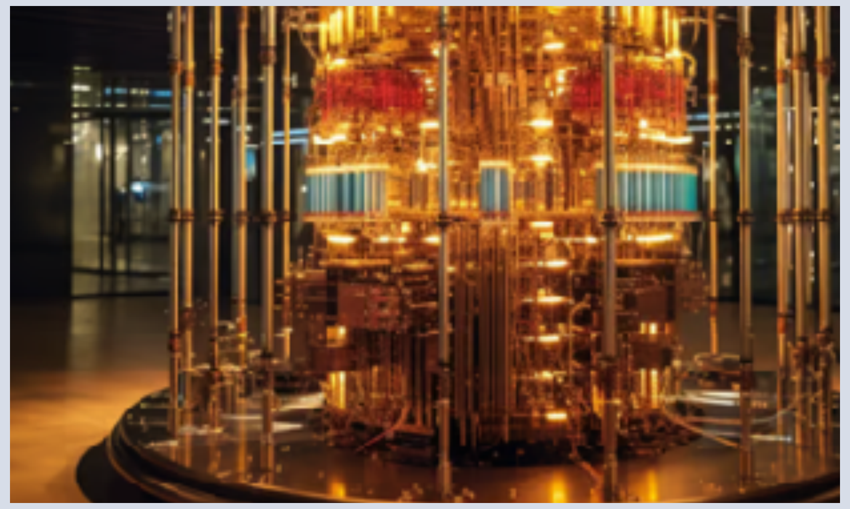


Graphene

Artificial (programmable) universe



Cold atom Experiments



Quantum Computers

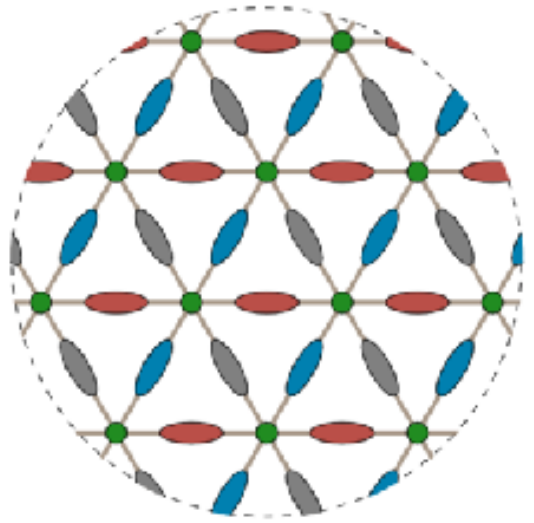
Universal properties

Emergence and Universality

Mona Lisa, Leonardo Da Vinci, ~ 1505

Zoom out →

Seated Woman, Pablo Picasso, 1927



Many-Body Quantum System

Zoom out →

?

Universal properties

Emergence and Universality

UV/short-distance

IR/long-distance

Microscopic lattice model

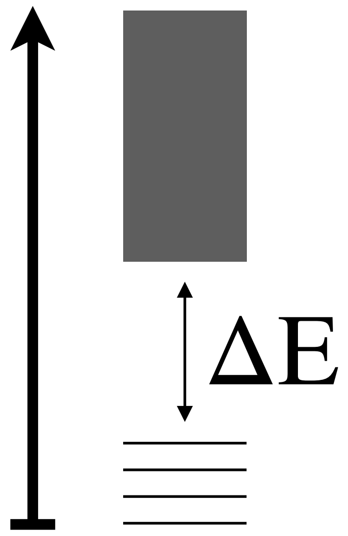
Effective descriptions

Renormalization group fixed-points

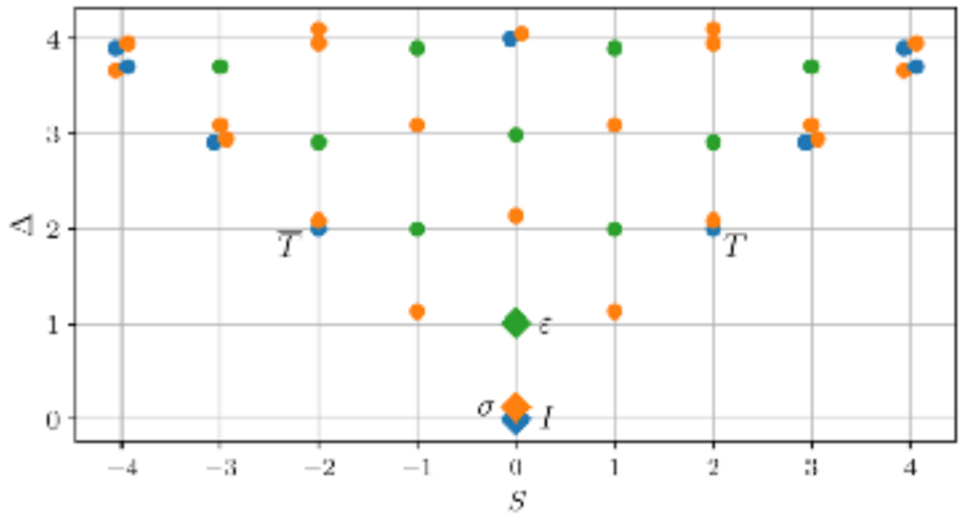
e.g. spin models,
Hubbard model, etc.

e.g. Effective field theory,
gauge systems, dimer models, etc.

Gapped



Gapless

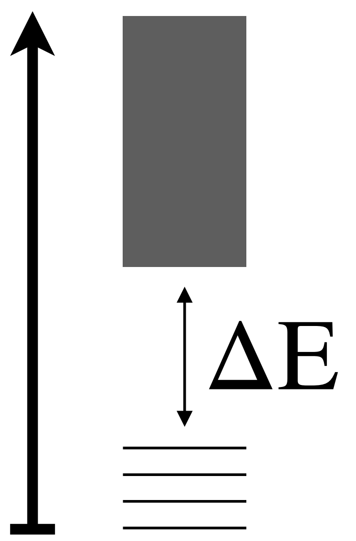


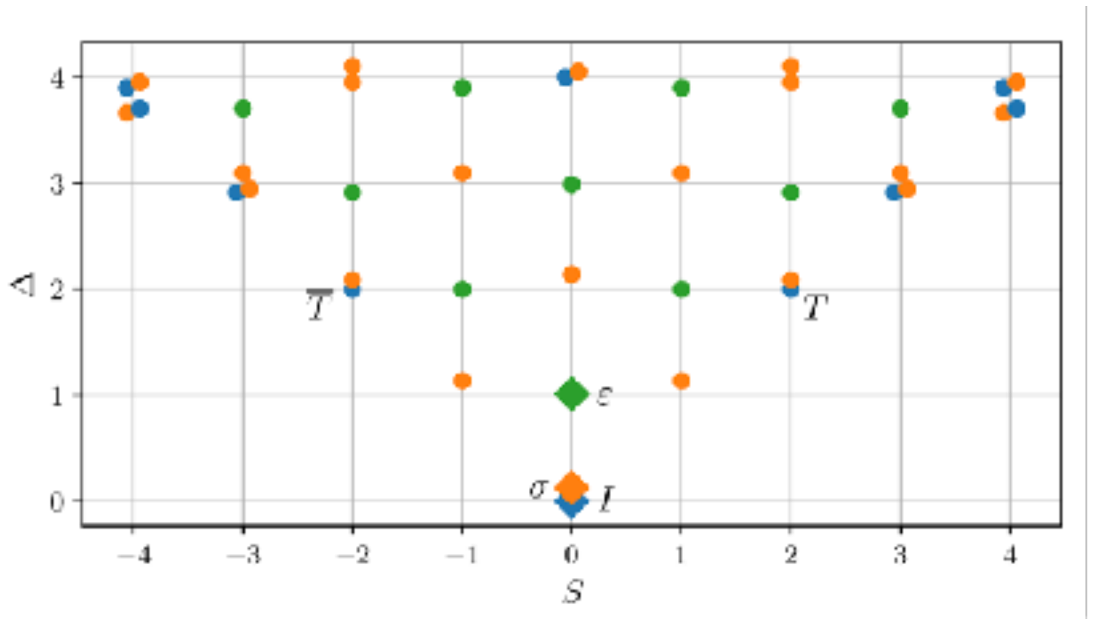
Tensor decomposable
Hilbert space

Possibly not
Tensor decomposable
Hilbert space

Renormalization group fixed points

Gapped





Gapless

Topological quantum field theory (TQFT)

Conformal field theory (CFT)

Describe universal properties of ground states/low-lying states.

Classification of (***)-gapped phases of matter	$\cong$	**Classification of (***)-TQFTs**

Hard problem:

Classify and characterise all phases & phase transitions

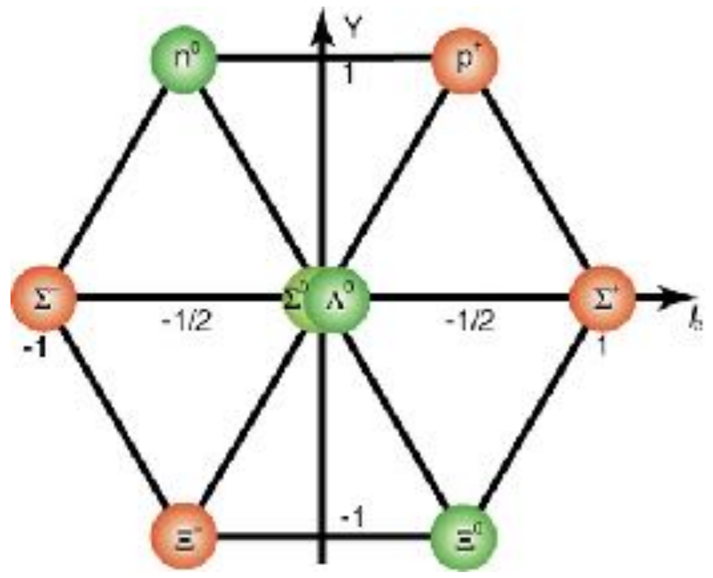
Easier problem:

Classify and characterise symmetric phases & phase transitions

Global symmetries are very useful

Organisation of matter

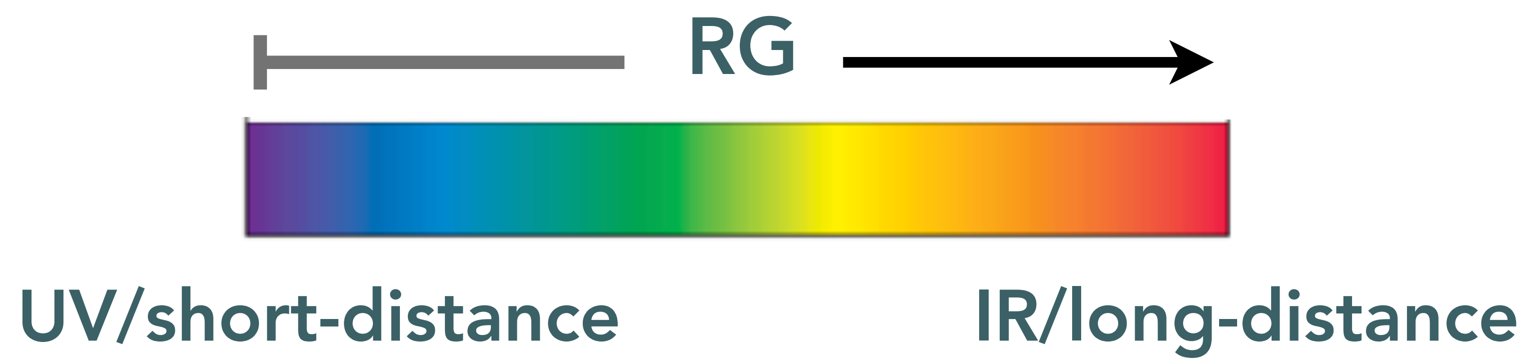
[Representation Theory]



Eight-fold way

Constraints on dynamics

['t Hooft anomalies]



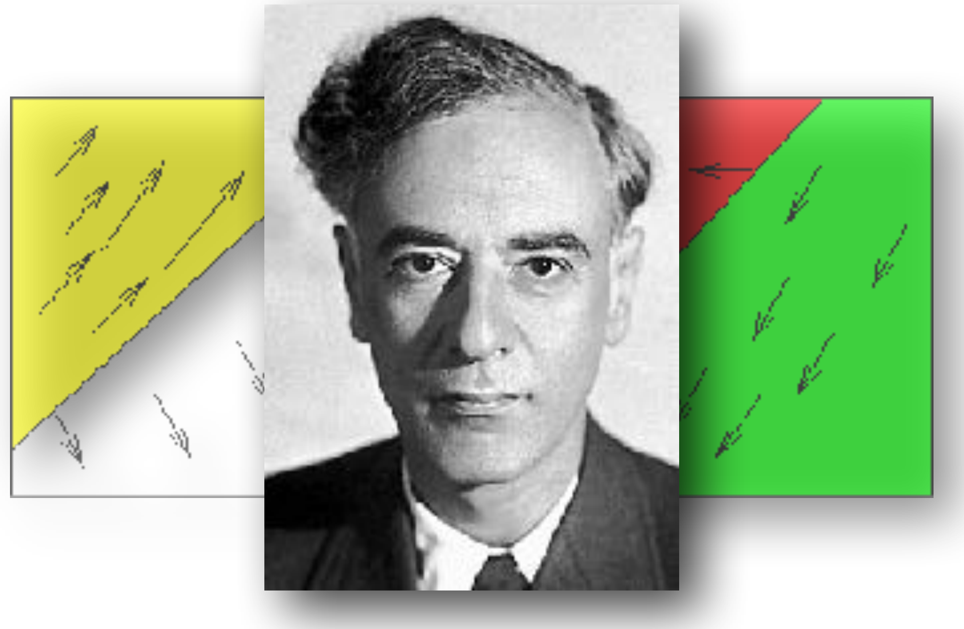
Classification of phases

[Spontaneous Symmetry breaking]

Landau paradigm

Classification of symmetric phases.

Landau Paradigm: **Phases distinguished by symmetry breaking patterns.**



G — disordered $\langle \Phi \rangle = 0$ — λ_c — $H \subset G$ — ordered $\langle \Phi \rangle \neq 0$ — λ

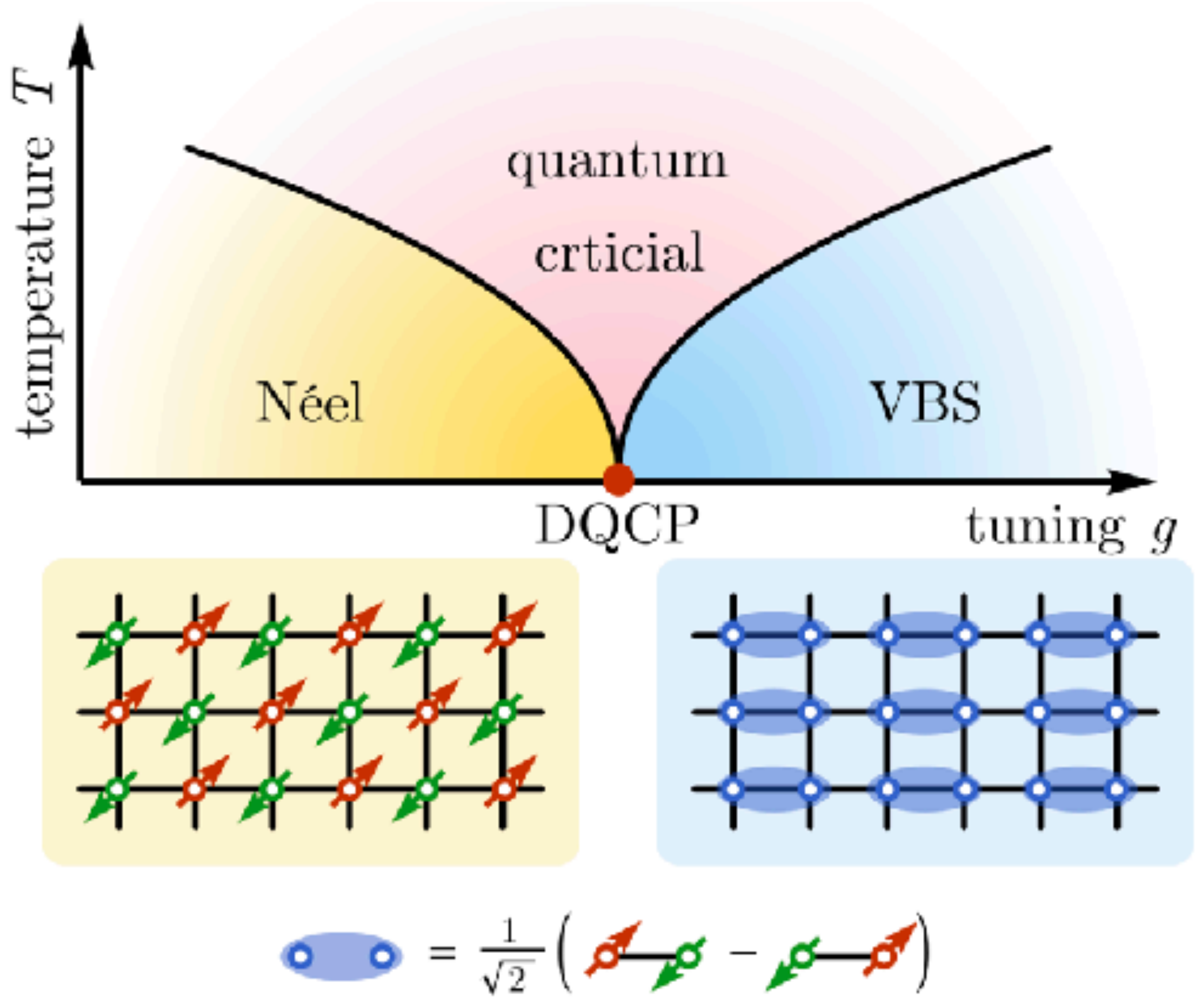
Examples:
Magnets, superconductors,…

Beyond-Landau Paradigm:

I. Deconfined quantum criticality:

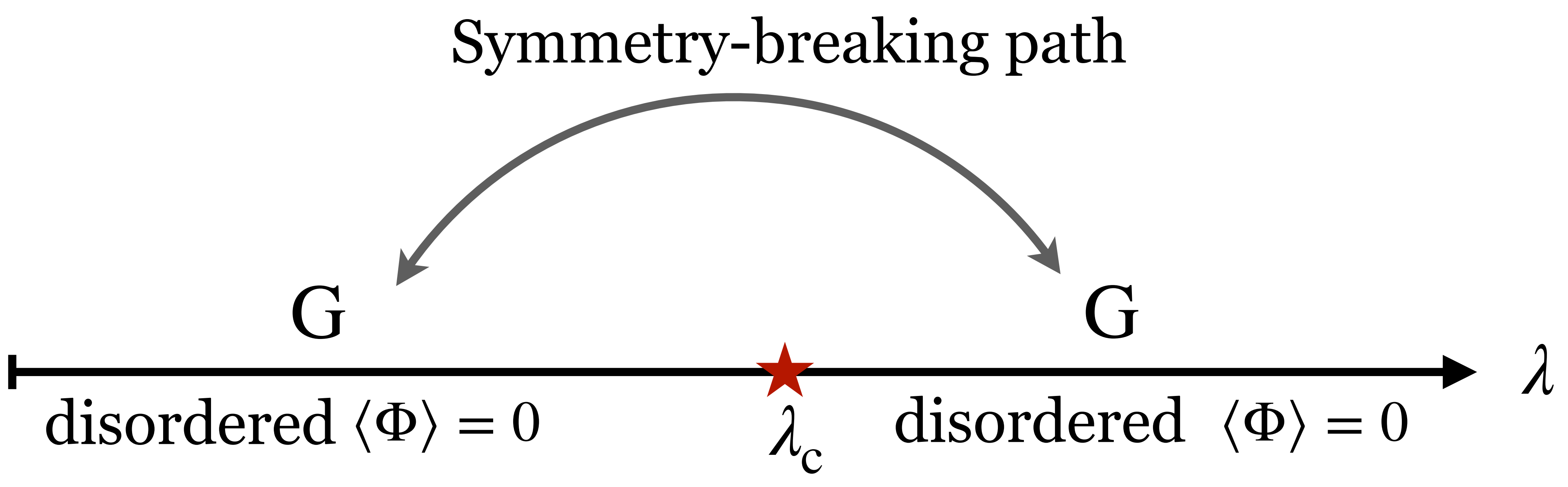
$H_1 \not\subset H_2 \,, H_2 \not\subset H_1$

$H_1 \subset G$ — ordered $\langle \Phi_1 \rangle \neq 0$ — λ_c — $H_2 \subset G$ — ordered $\langle \Phi_2 \rangle \neq 0$ — λ



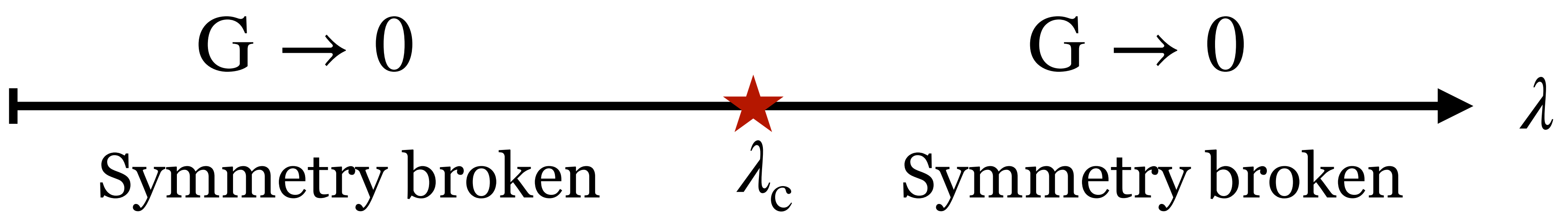
Classification of symmetric phases. Beyond-Landau Paradigm:

II. Symmetry protected topological phases



- Examples: Topological Insulators and superconductors.
- Distinguished by topological response.
- TQFT based classification.

III. Topologically ordered phases



- Examples: Toric code/deconfined finite gauge theory.
- Distinguished by topological correlation functions.
- TQFT based classification.

Generalized Symmetry :

A new perspective unifying

Landau and beyond physics

(Generalized Landau Paradigm)

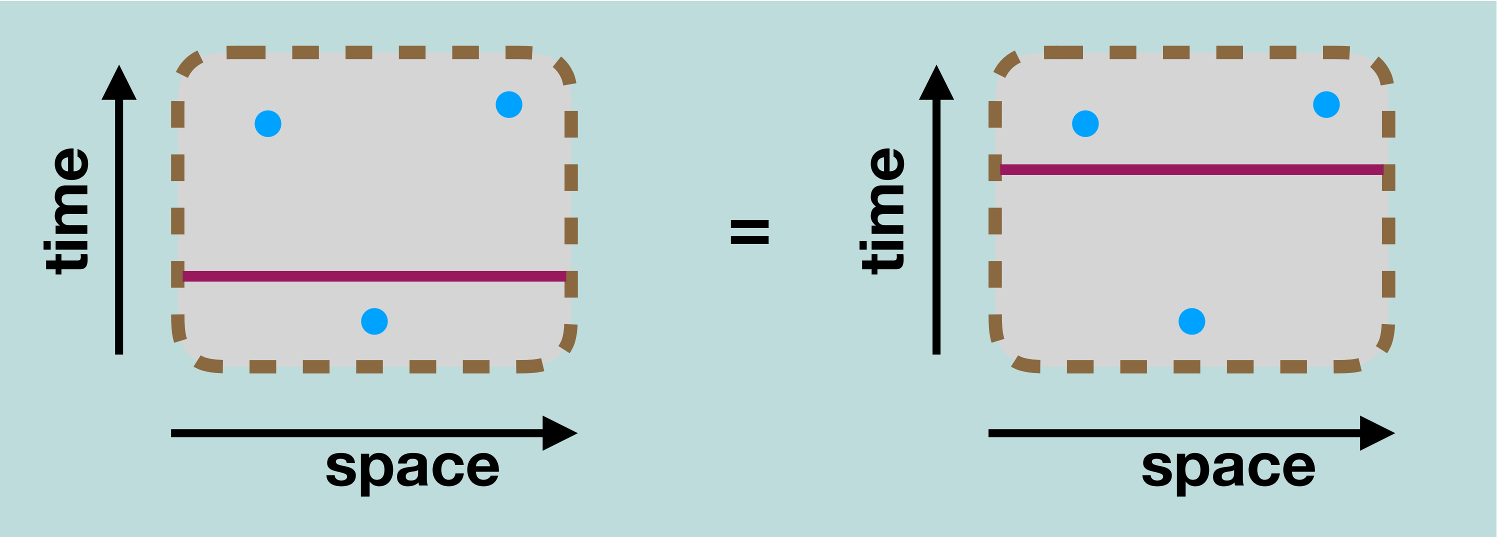
Global symmetry: a new perspective

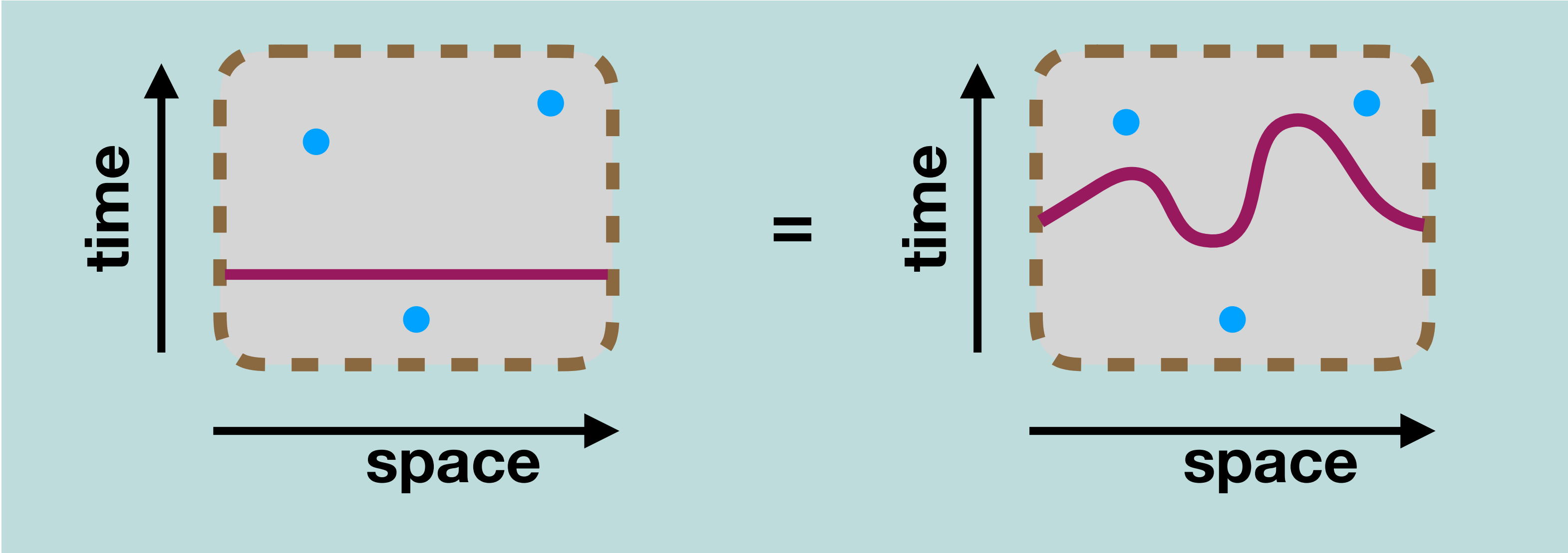
Central insight: Symmetry operators are topological (inside correlation functions)!

[Gaiotto-Kapustin-Seiberg-Willett '14]

Conventional global symmetry is defined via a collection of operators $\{U_g\}$.

Properties of symmetry operators	Re-statement
1. Act on all of space	1. Co-dimension-1 in spacetime
2. Commute with the Hamiltonian $[U_g, H] = 0$	2. Topological

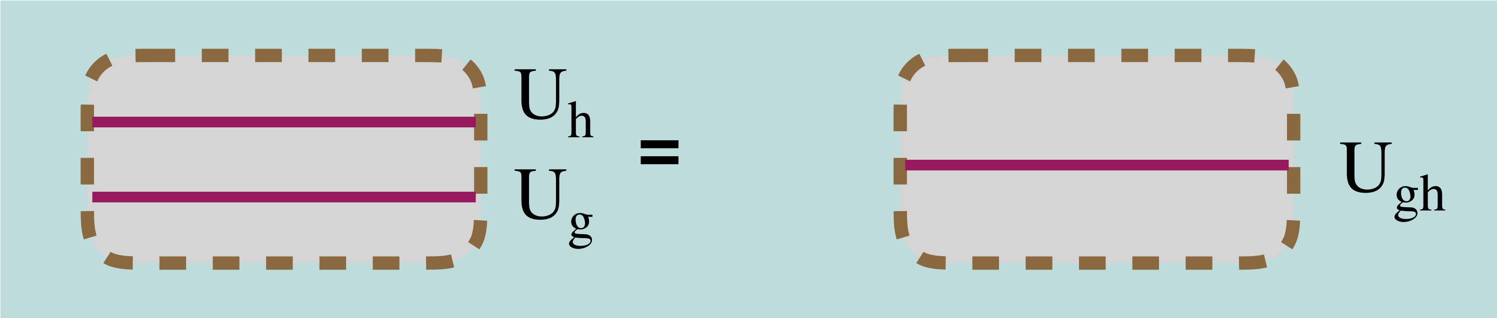




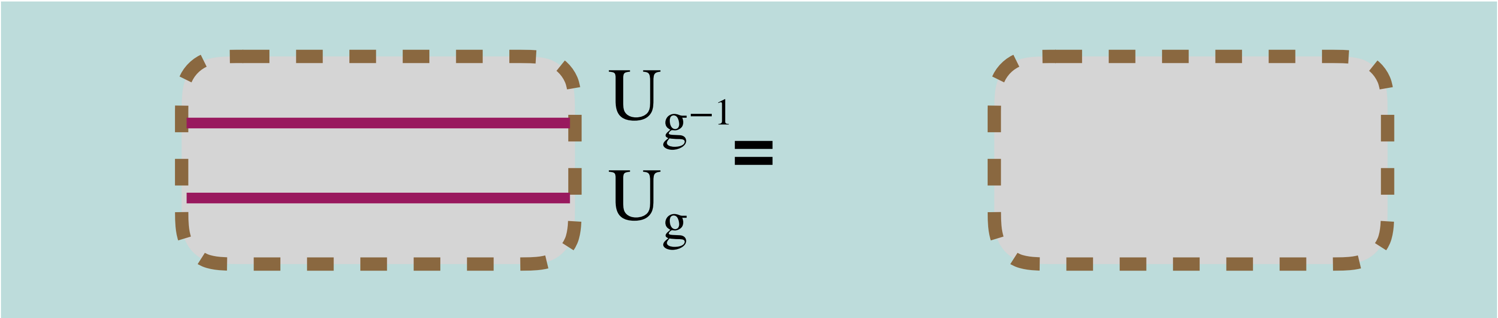
Generalized symmetries

Properties of symmetry operators	Re-statement

3. Compose as a group $U_g U_h = U_{g \cdot h}$



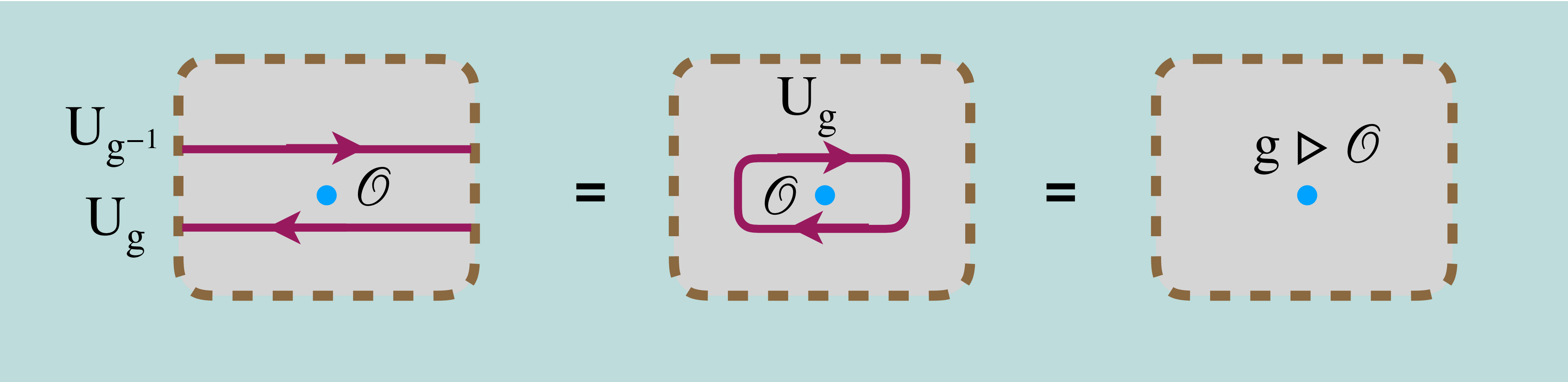
3. Invertible $U_g U_{g^{-1}} = 1$



4. Act on operators via conjugation

$$U_g : \mathcal{O} \longrightarrow U_g \mathcal{O} U_g^{-1}$$

4. Action via linking



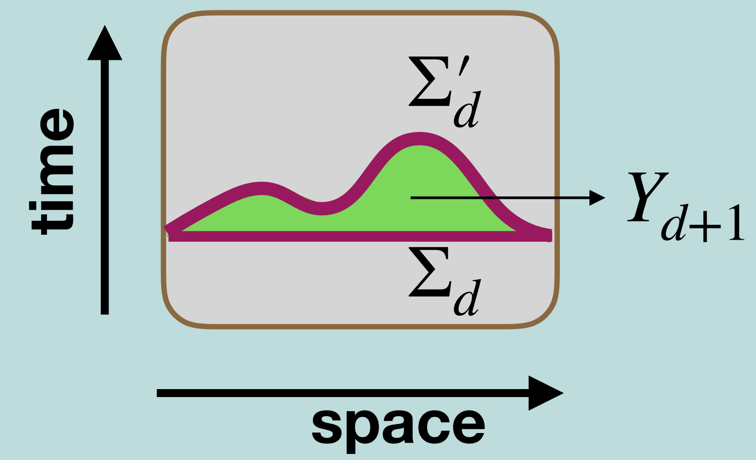
Examples: **Global symmetry via topological operators**

1. U(1) particle number conservation :

- **Symmetry operator :**

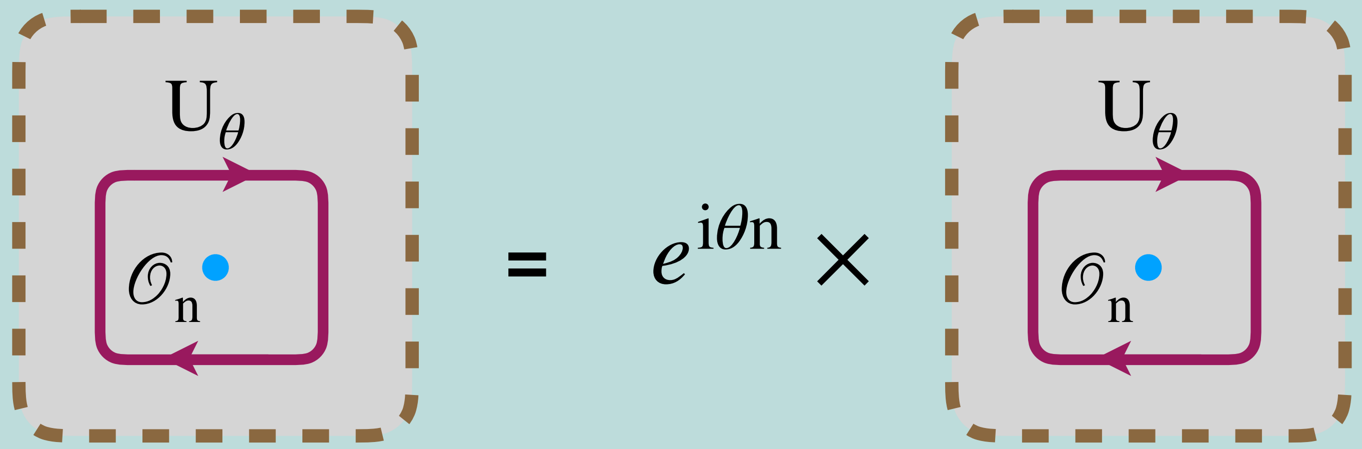
$$\mathrm{U_g}[\Sigma] = \exp\left\{\mathrm{i g}\mathcal{Q}_\Sigma\right\}\,, \qquad \mathcal{Q}_\Sigma = \int_\Sigma \star \mathrm{J}\,, \qquad \mathrm{d}\star\mathrm{J} = 0$$

- **Current conservation implies topological invariance.**



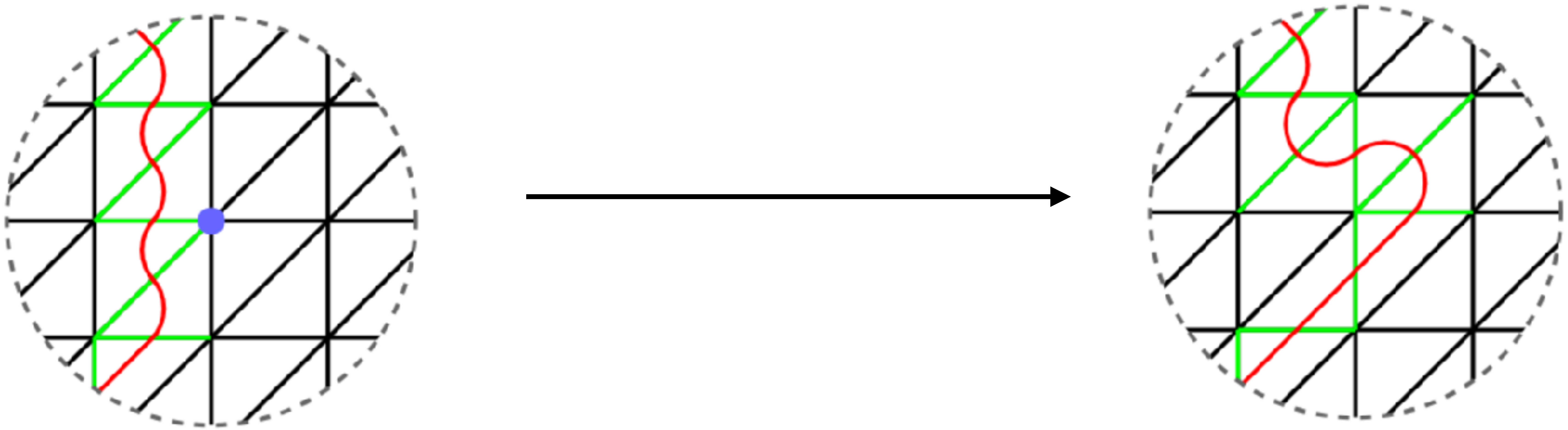
$$\mathcal{Q}_\Sigma = \mathcal{Q}_{\Sigma'} + \int_Y \cancel{\mathrm{d}\star\mathrm{J}}$$

- **Action via linking**



2. Z_2 global symmetry :

- **Invariance under background gauge transformations implies topological invariance of symmetry operator**

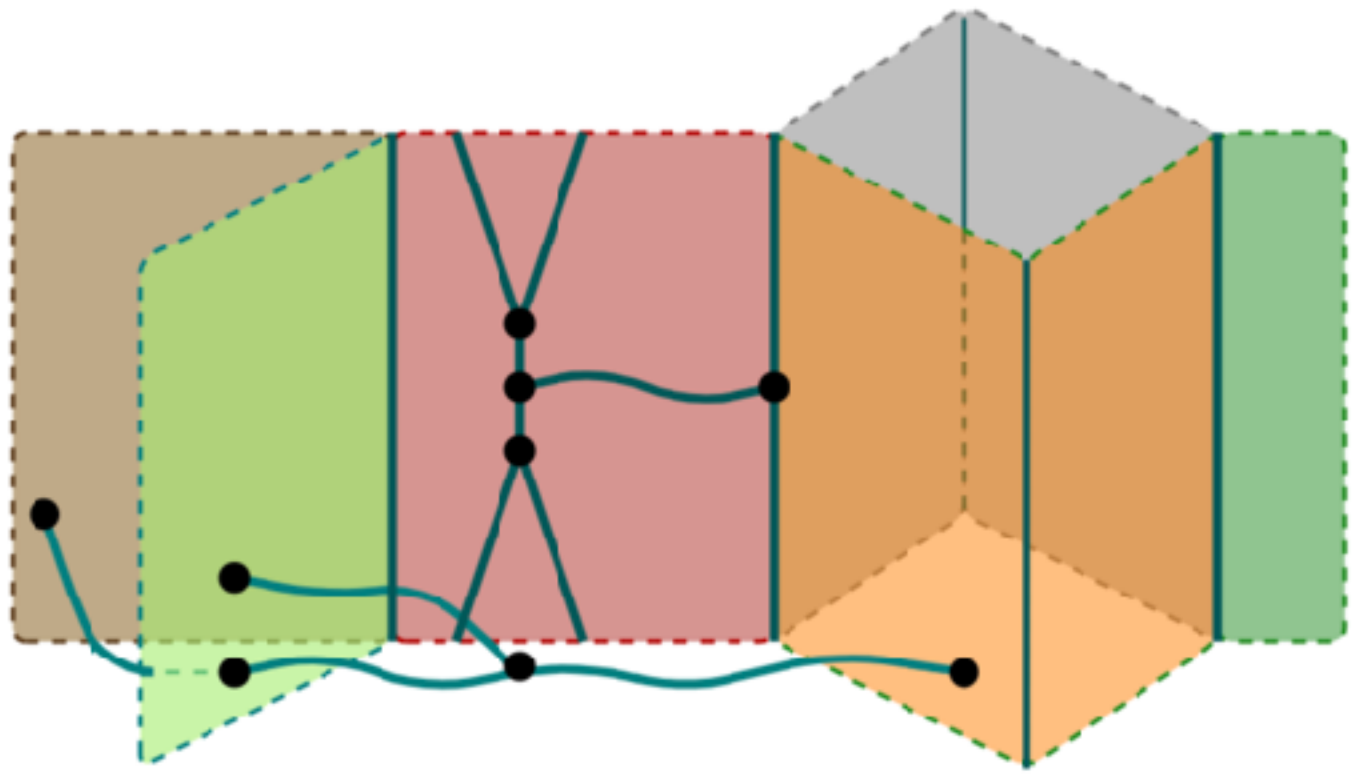


Symmetry operator   Background gauge field   gauge transformation

Generalized global symmetries

Properties of symmetry operators	Conventional symmetries	Higher group symmetries	Non-invertible
co-dimension	1	$\leq$ q	General
Fusion rules	Group like	~ Group like	Non-group like

Mathematical Language:



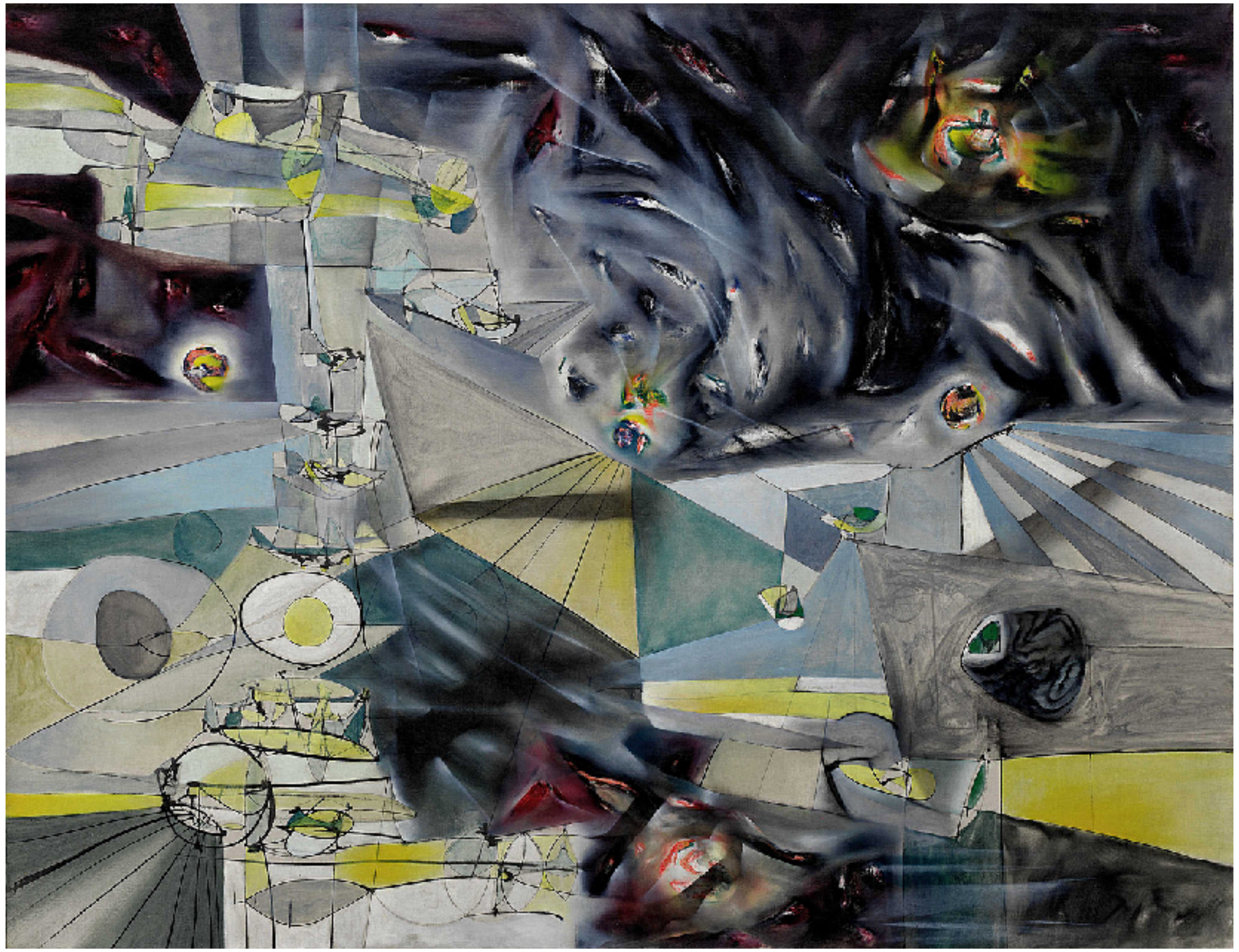
Higher- fusion categories:

A framework to organise the collection of topological defects/operators of all (co-)dimensions along with their fusions + ….

Symmetry structure of (d+1)-dimensional quantum system $\cong$ d-Fusion category

Generalized Landau Paradigm

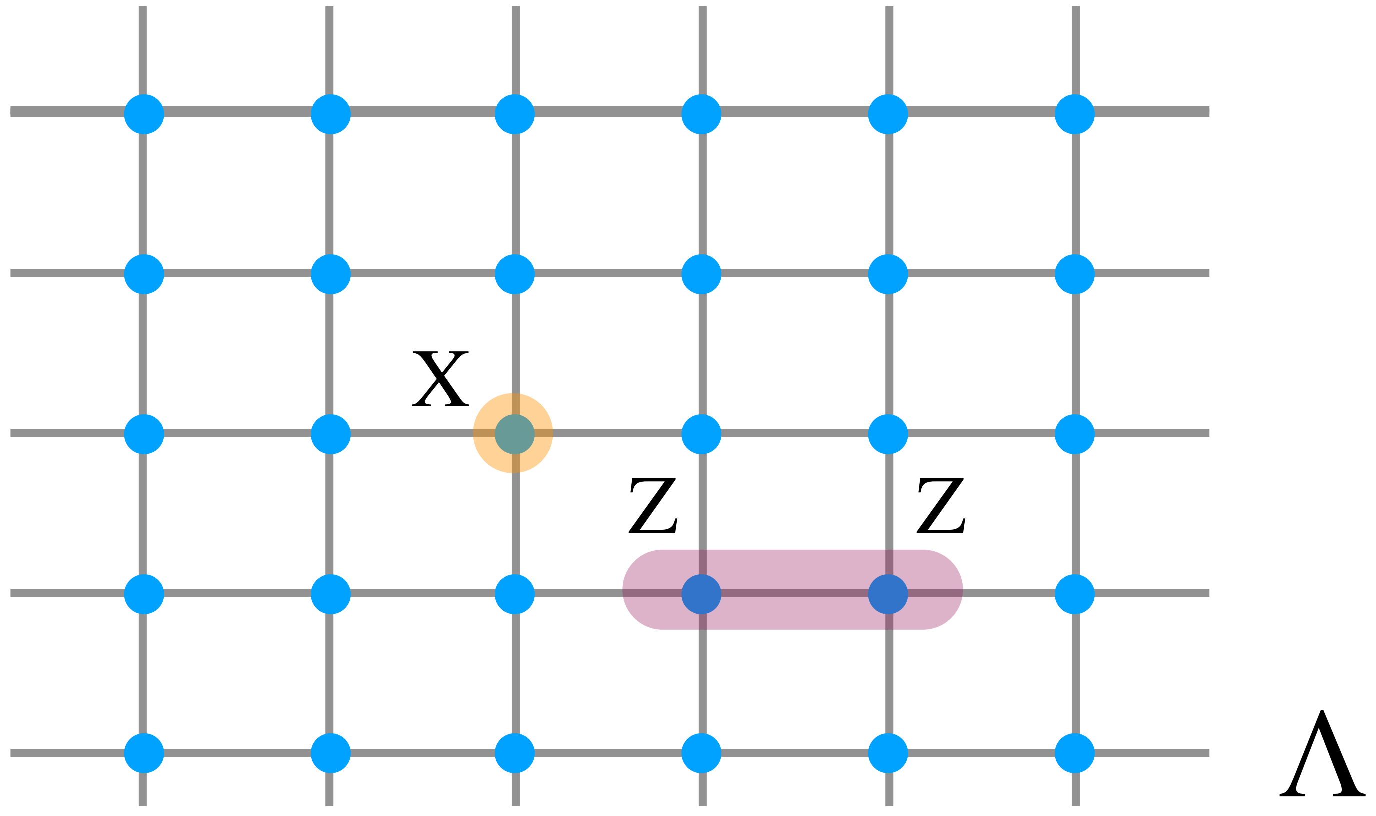
- **Incorporating generalized symmetries and their symmetry breaking.**
- **Generalized charges of generalized symmetries are Order Parameters.**
- **Upshot: Can be used to unify several beyond-Landau phases and phase transitions.**
- **Example: Topological order = Generalized symmetry breaking phase.**



Space of symmetric quantum systems and Gauging-related dualities

Space of Z_2 symmetric quantum systems

- **Consider a lattice Λ, with each vertex equipped with 2-dimensional local Hilbert space.**



- $V_j = \mathbb{C}^2_j \circlearrowleft X_j, Y_j, Z_j$

$$X_j = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Z_j = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- **Global Z2 symmetry generated by** $\prod_j X_j$ **.**

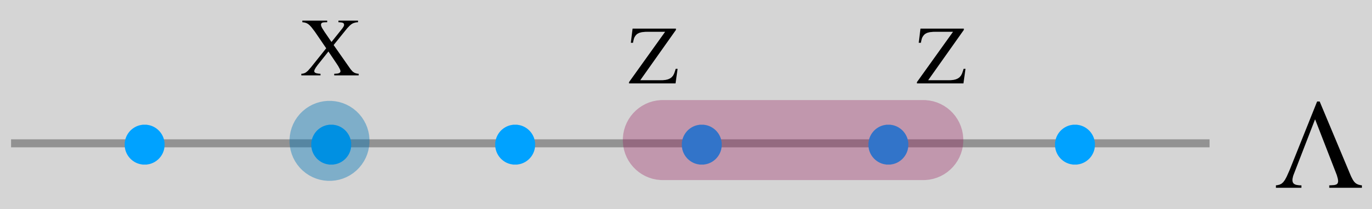
- **Space of symmetric operators is a bond algebra** $\mathscr{B}_{\mathbb{Z}_2}(\Lambda) = \langle X_j, Z_j Z_{j'} \rangle$.

Quantum dualities

- Non-trivial map between two quantum systems
- Imposes many constraints
- Relate strongly and weakly coupled systems.

Duality and gauging

- **Bond algebra isomorphisms are realised as dualities**

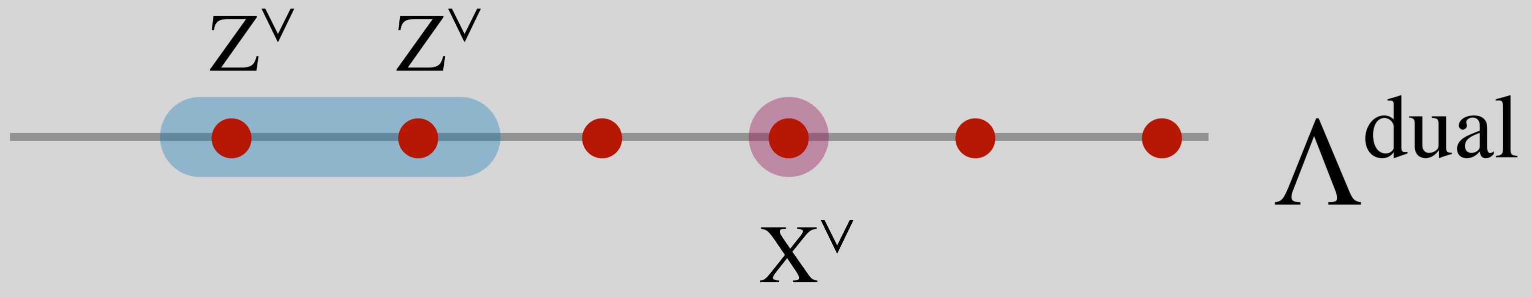


$$\mathscr{B}_{\mathbb{Z}_2}(\Lambda) = \langle\, X_j \,,\, Z_j Z_{j+1} \,\rangle$$

$$\mathscr{U} = \prod_j X_j$$

Kramers-Wannier duality

Gauging Z_2

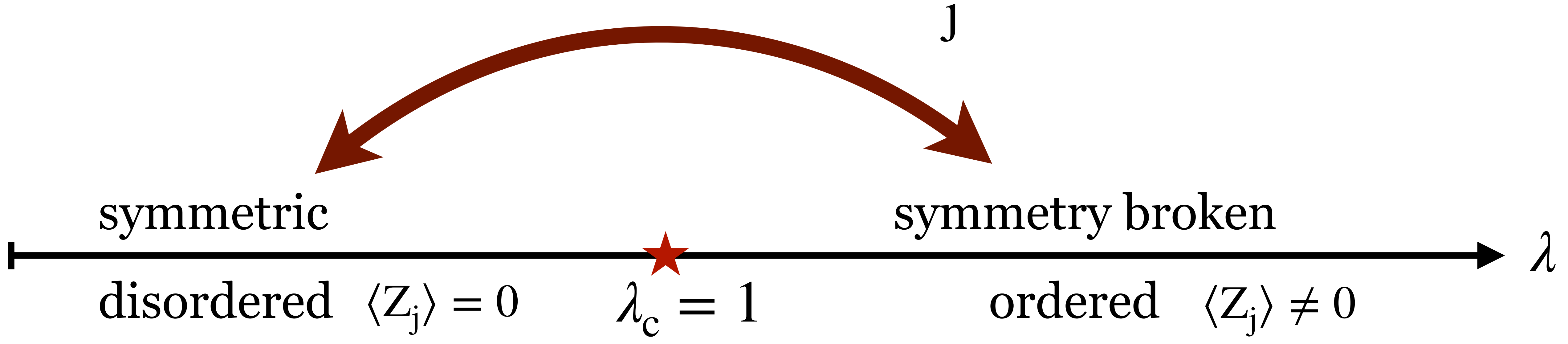


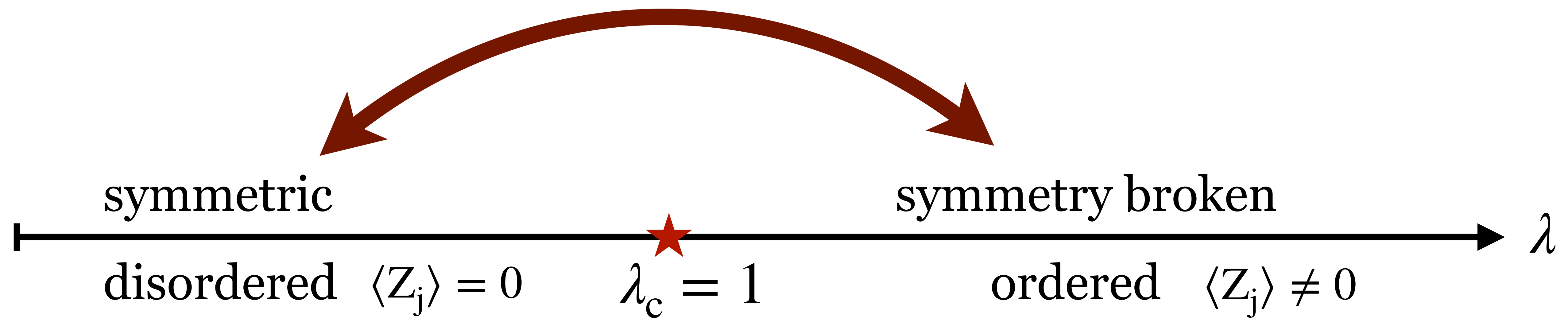
$$\mathscr{B}^{\vee}_{\mathbb{Z}_2^{\vee}}(\Lambda^{\mathrm{dual}}) = \langle\, Z_{j-1/2} Z_{j+1/2} \,,\, X_{j+1/2} \,\rangle$$

$$\mathscr{U}^{\vee} = \prod_j X_{j+1/2}$$

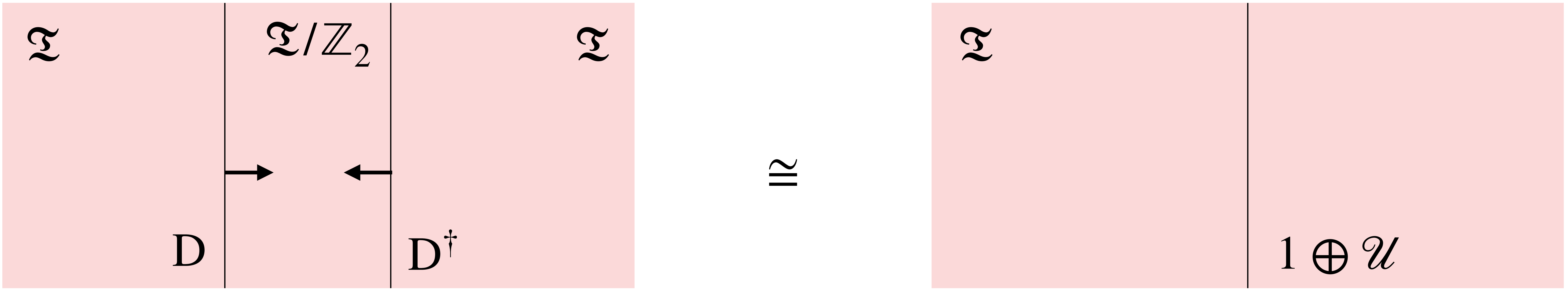
- **Transverse Field Ising model :**

$$\mathscr{H} = -\sum_j [X_j + \lambda Z_j Z_{j+1}]$$





- **Kramers Wannier self dual systems have emergent non-invertible symmetries.**



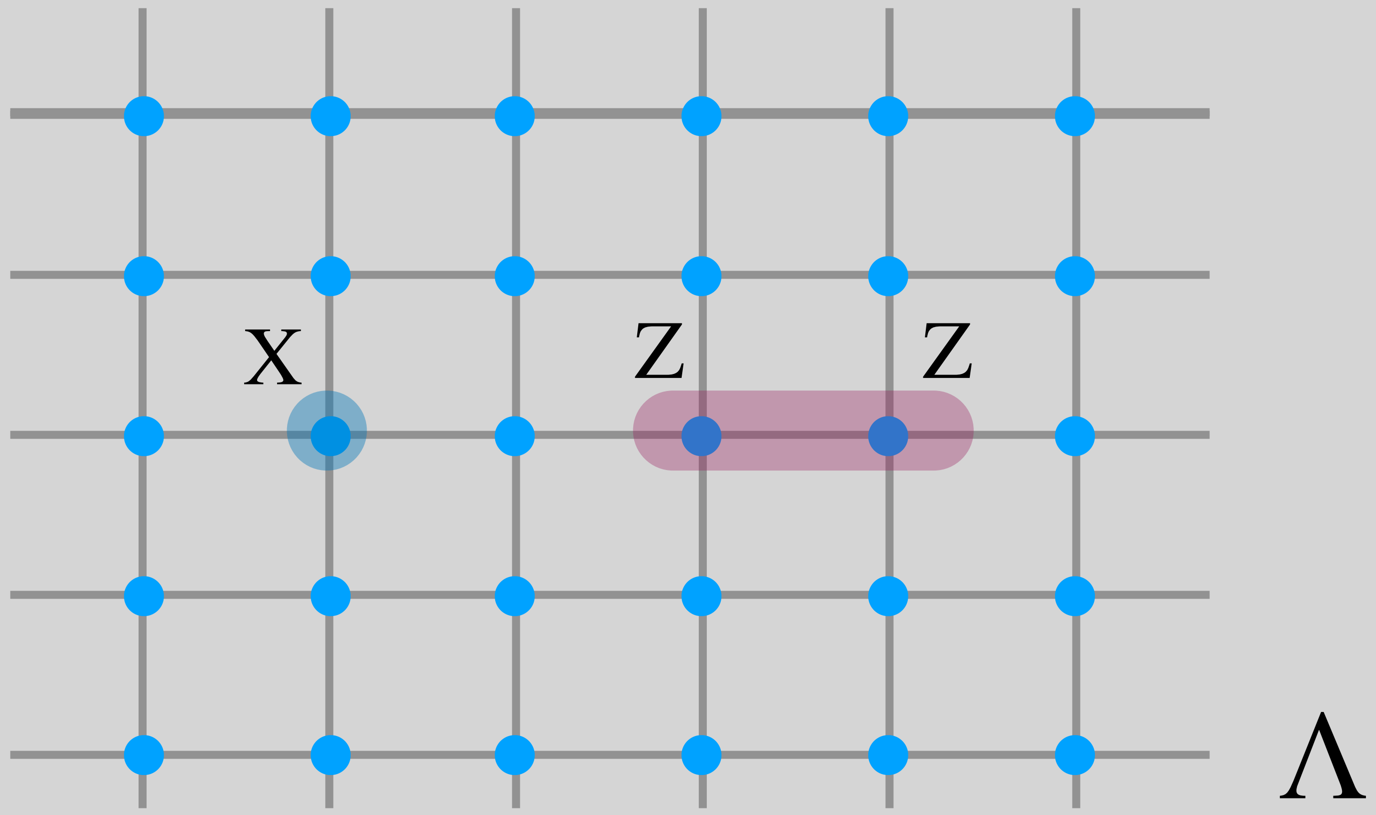
- **All Kramers Wannier self dual deformations have Ising Fusion Category symmetry**

$$\mathscr{H} = -\sum_j [X_j + \lambda Z_j Z_{j+1}] \quad + \quad \ldots$$

$$\mathscr{U} \times \mathscr{U} = 1\,, \quad \mathscr{U} \times D = D\,, \quad D \times D = 1 + \mathscr{U}$$

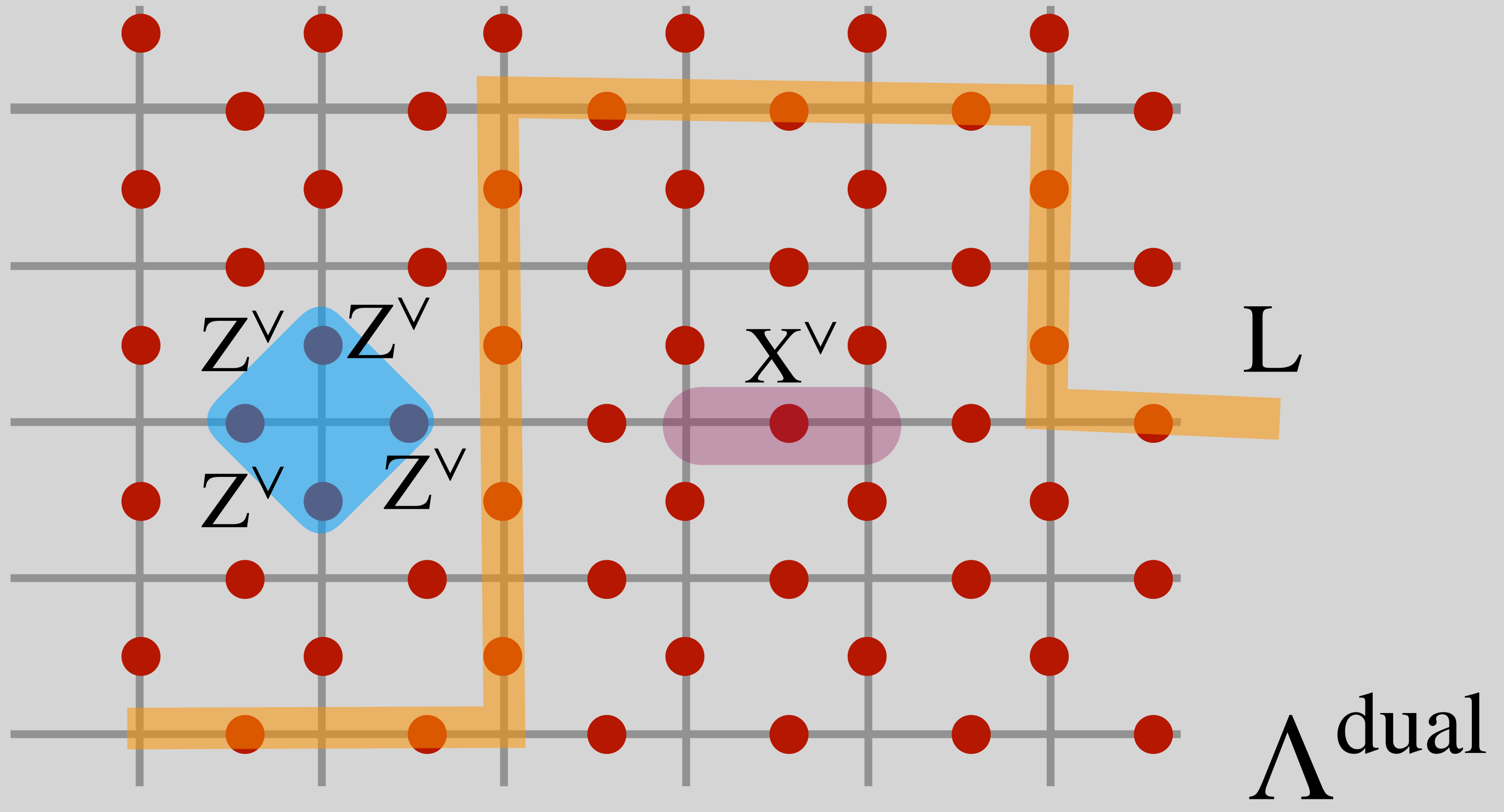
- **Ising Category symmetric phases can be classified.**

A 2+1 dimensional example.

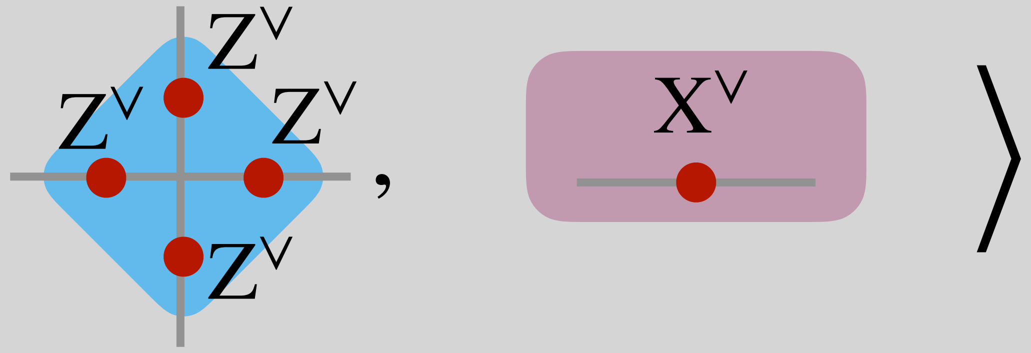


$\mathscr{B}_{\mathbb{Z}_2}(\Lambda) = \langle\, X_j \;,\; Z_j Z_{j+1} \,\rangle$

$\mathscr{U} = \prod_j X_j$ **(0-form symmetry)**



$\mathscr{B}^{\vee}_{\mathbb{Z}_2^{\vee}}(\Lambda^{\text{dual}}) = \big\langle$



$\big\rangle$

$\mathscr{U}^{\vee}(L) = \prod_{e\subset L} X^{\vee}_j$ **(1-form symmetry)**

- **Mapping of phases:**

• Symmetric (trivially disordered)	• Topological (deconfined) Z2 gauge theory
• Symmetry protected topological	• Topological (twisted) Z2 gauge theory
• Symmetry broken	• Symmetric/ confined Z2 gauge theory

Generalizations

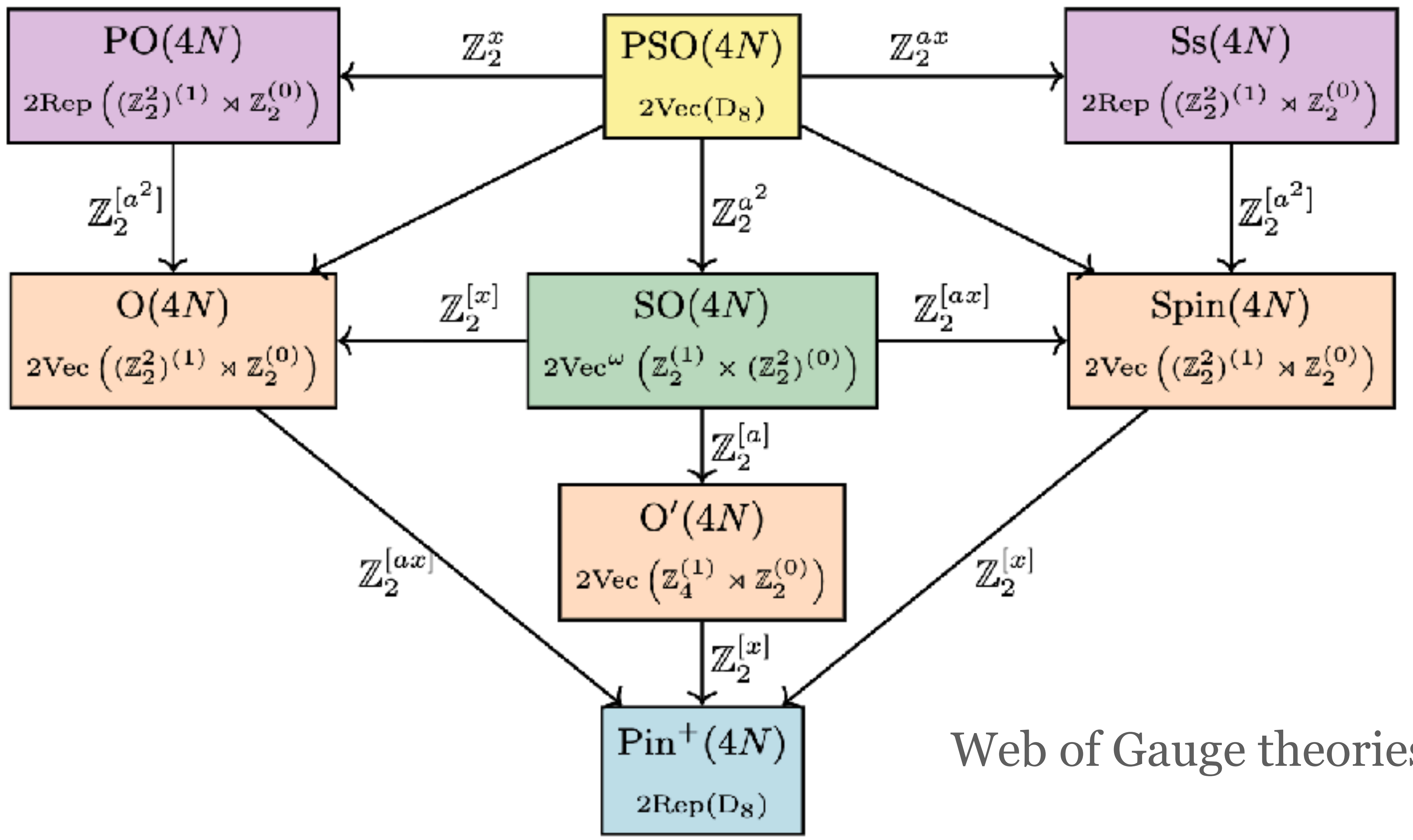
- **In 1+1 dimensions :**
  - **Gaging non-trivial central extensions $\Longrightarrow$ Mixed 't Hooft anomalies.**
  - **Gauging a non-Abelian group G $\Longrightarrow$ Non-invertible symmetry Rep(G).**
- **In 2+1 dimensions :**
  - **Gauging normal subgroup $\Longrightarrow$ Higher groups**
  - **Gauging central extensions $\Longrightarrow$ Mixed 't Hooft anomalies**
  - **Gauging non-normal subgroups $\Longrightarrow$ Non-invertible symmetries**
- **In 3+1 dimensions : .... Many more rich examples**

Generalizations

Webs of quantum dualities

Message:

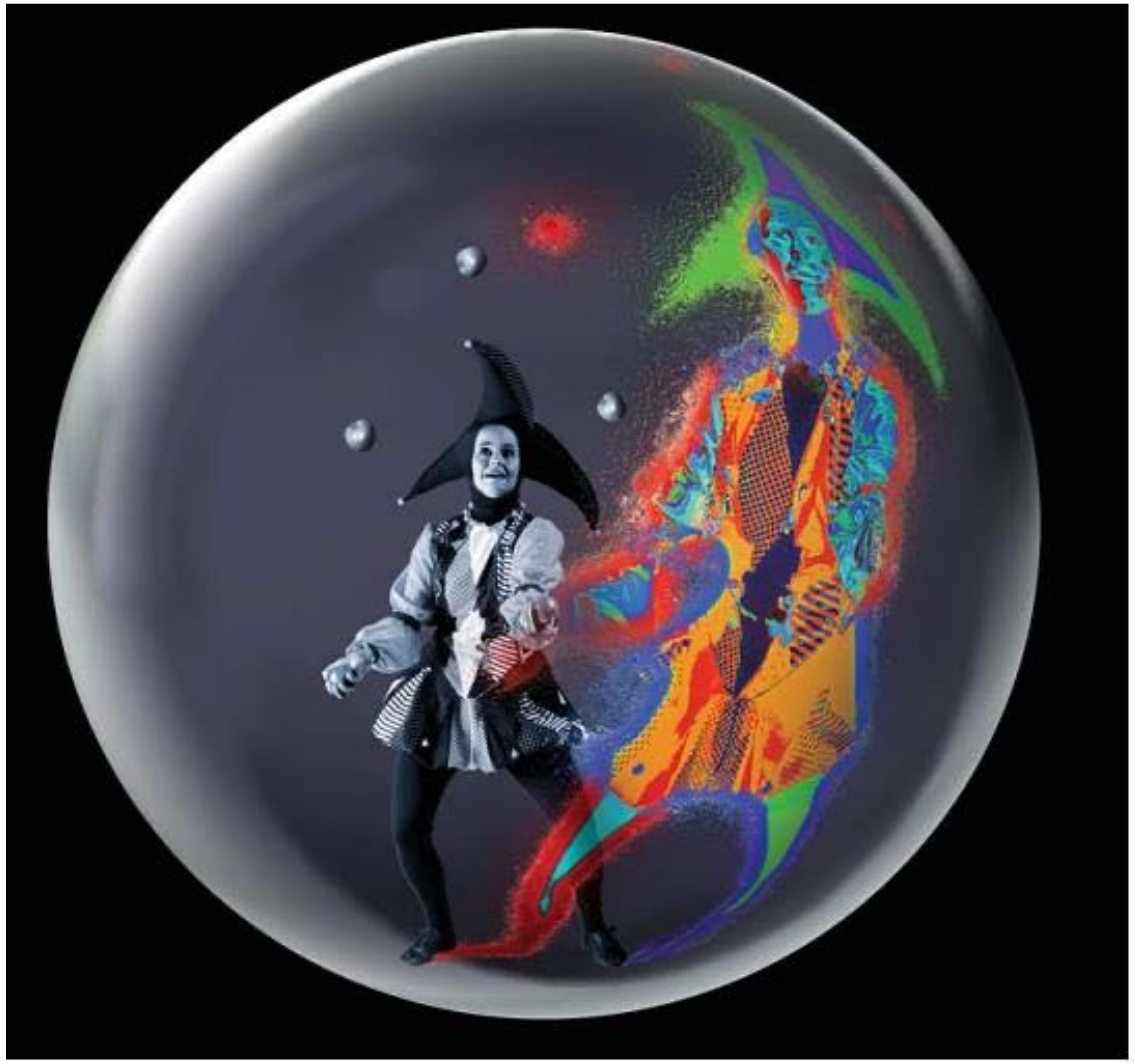
Gauging is a rich source of dualities and therefore a systematic exploration of phase diagrams.



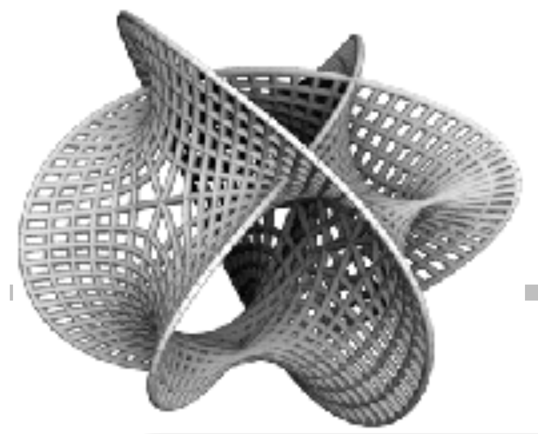
Web of Gauge theories

{ ○, ○, ○, ○ } = Different symmetries

Bhardwaj-Bottini-Nameki-**Tiwari** (Scipost ’22, Scipost ’23 I,II),



Topological Holography



Topological Holography

Main idea :

Finite symmetry in D dimensions 	=	Topological Order in D+1 dimensions 

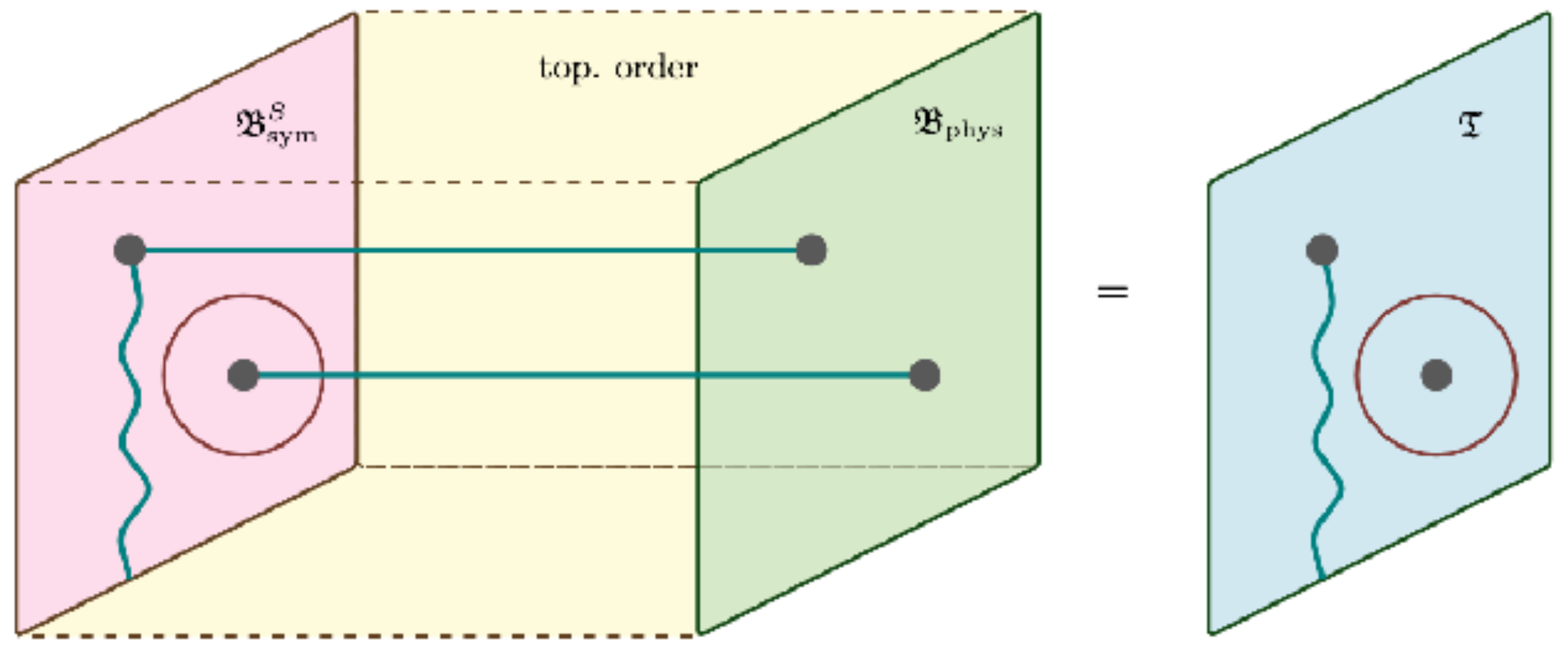
Convenient and powerful :

(i) Classification of phases,

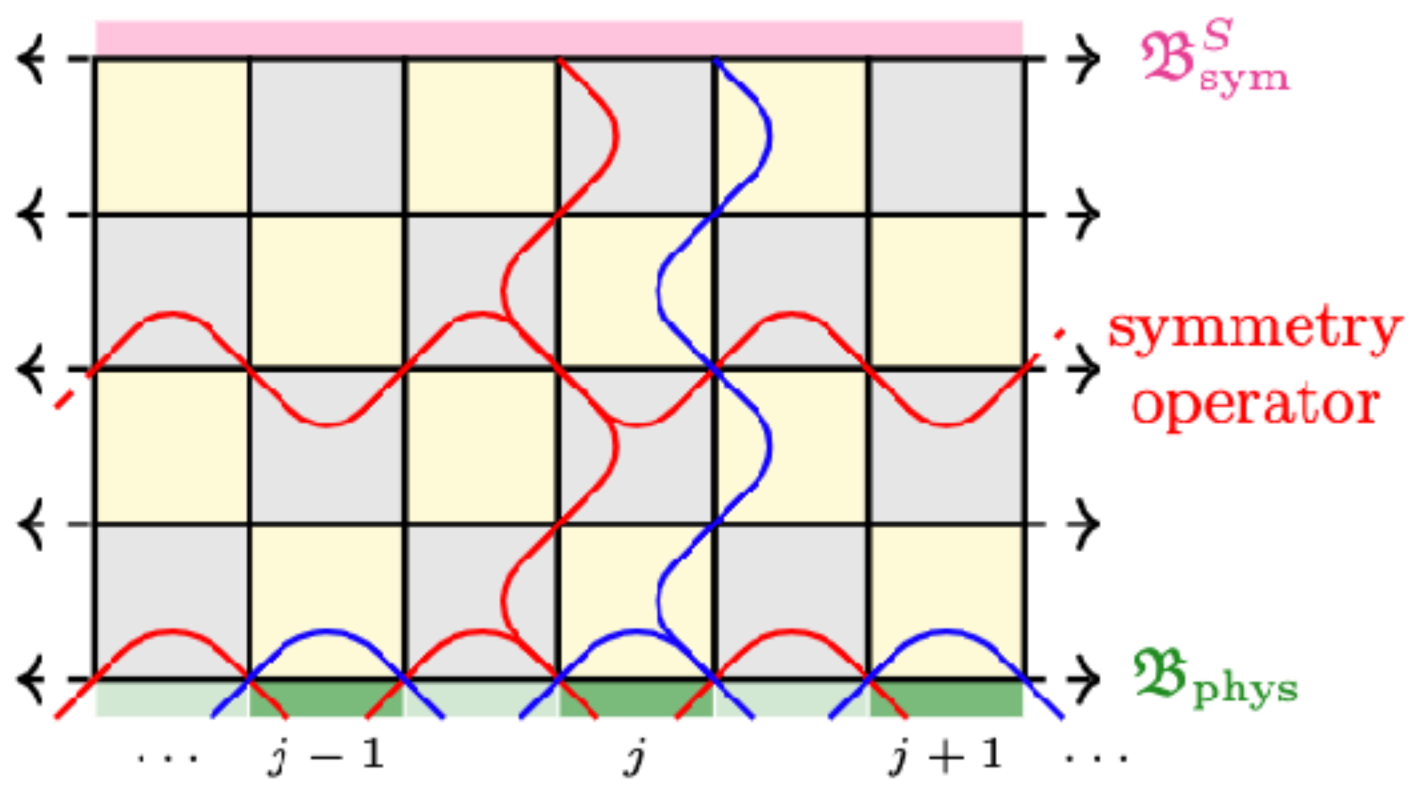
(ii) Properties of excitations,

(iii) Phase transitions,

(iv) Universal dualities.



Spacetime picture



Spatial lattice picture

Holographic "bulk" only knows about universal physics!

Moradi-Moosavian-Tiwari (Scipost 23),

Holographic dictionary

D-dimensional quantum system with symmetry category $\mathscr{C}$ ⟷ (D+1)-dimensional topological order $Z(\mathscr{C})$

Dictionary when G is a finite abelian group :

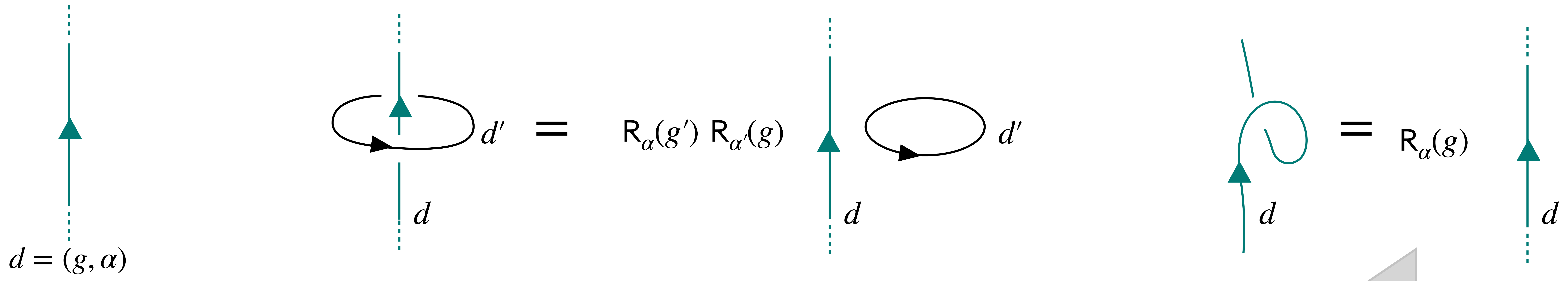
quantum system (D-dim)	Topological order (D+1)
G 0-form symmetry	G-topological gauge theory
Symmetry operators	Magnetic (D-1) surface operators
Charged operators	Electric line operators
Gapped phases	Gapped boundaries
Dualities	0-form symmetries
′t Hooft anomaly	Dijkgraaf-Witten Topological action

[Moradi-Moosavian-Tiwari, Scipost 2023]

An example: 1+1d, finite group symmetry

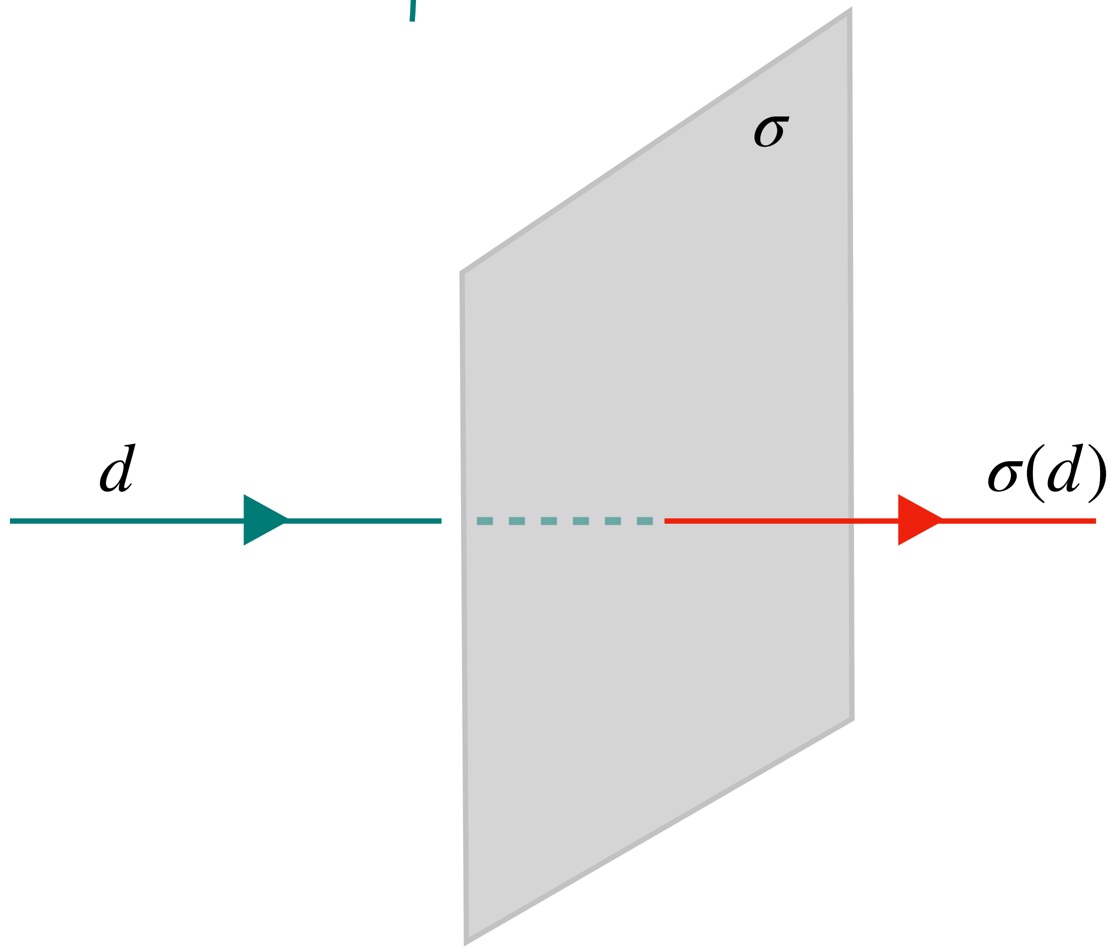
- **Quantum system:** 1+1 dimensional with G (finite Abelian) symmetry.
- **Holographic bulk:** G topological gauge theory.

- **Line operators (1-form symmetries):** $\mathscr{A}[G] = \underset{\text{flux}}{G} \times \underset{\text{charge}}{\mathrm{Rep}(G)} \simeq G \times G$



- **0-form symmetries: "braided auto-equivalences"**

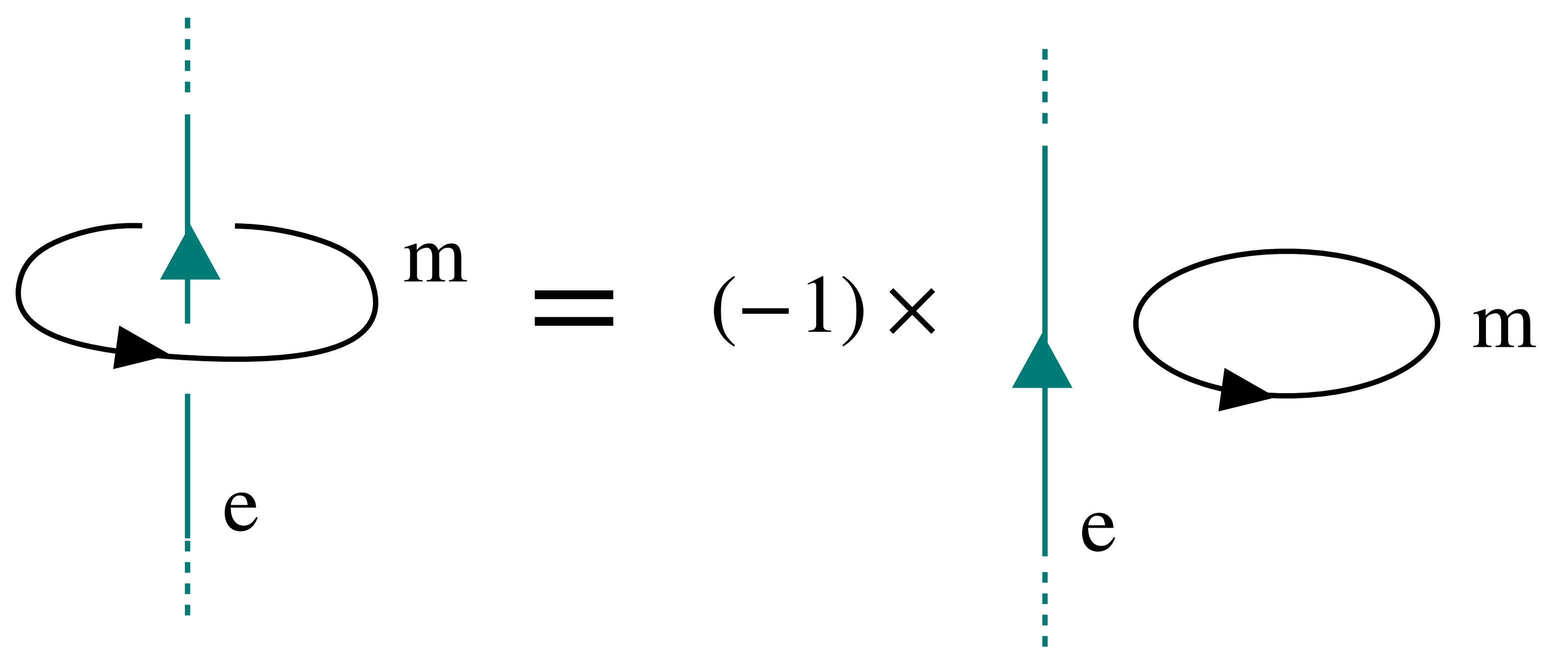
  [Fuchs-Priel-Schweigert-Valentino], [Barkeshli-Bonderson-Cheng-Wang], [Bombin], [Teo-Hughes-Fradkin]



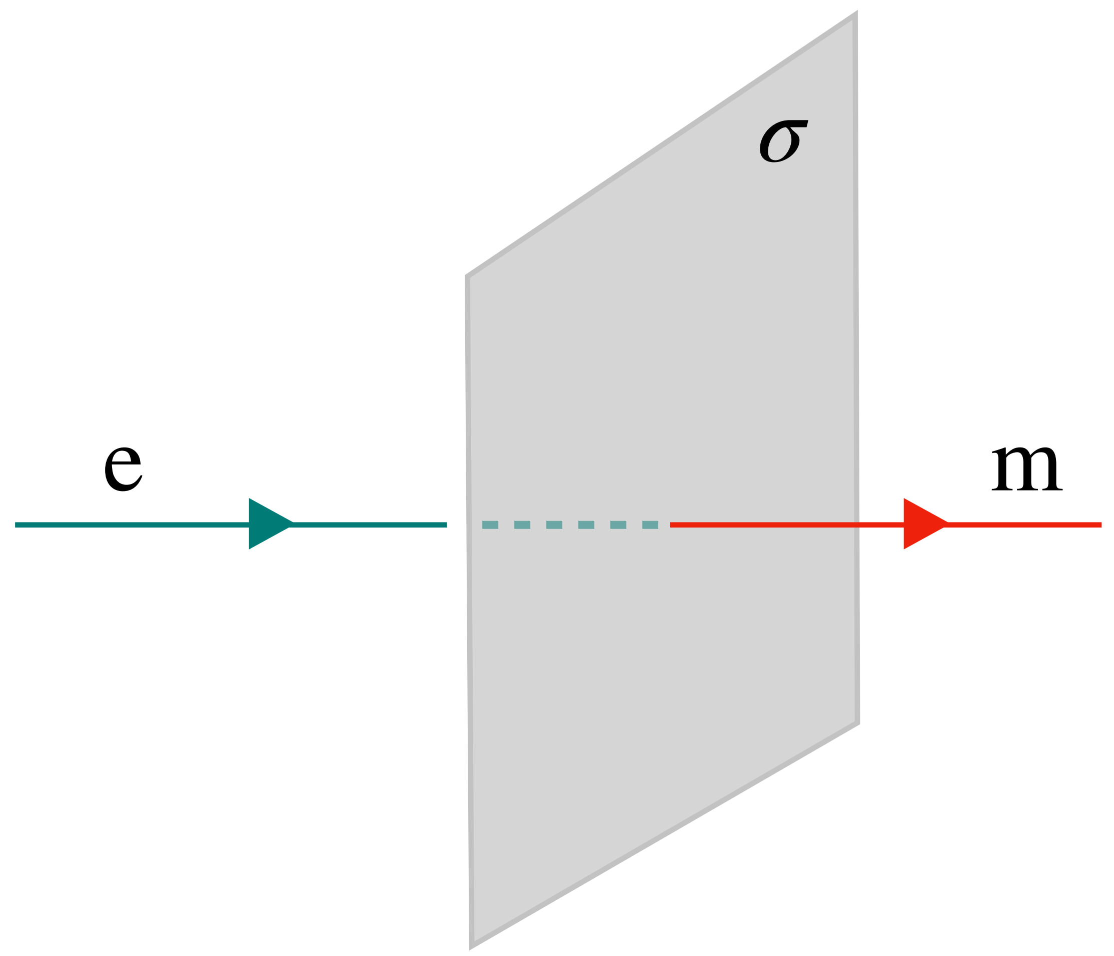
G=Z2

- **Quantum system:** 1+1 dimensional with Z2 symmetry.
- **Holographic bulk:** Z2 topological gauge theory/toric code.

- **Line operators** $\{1, e, m, f = e \times m\}$

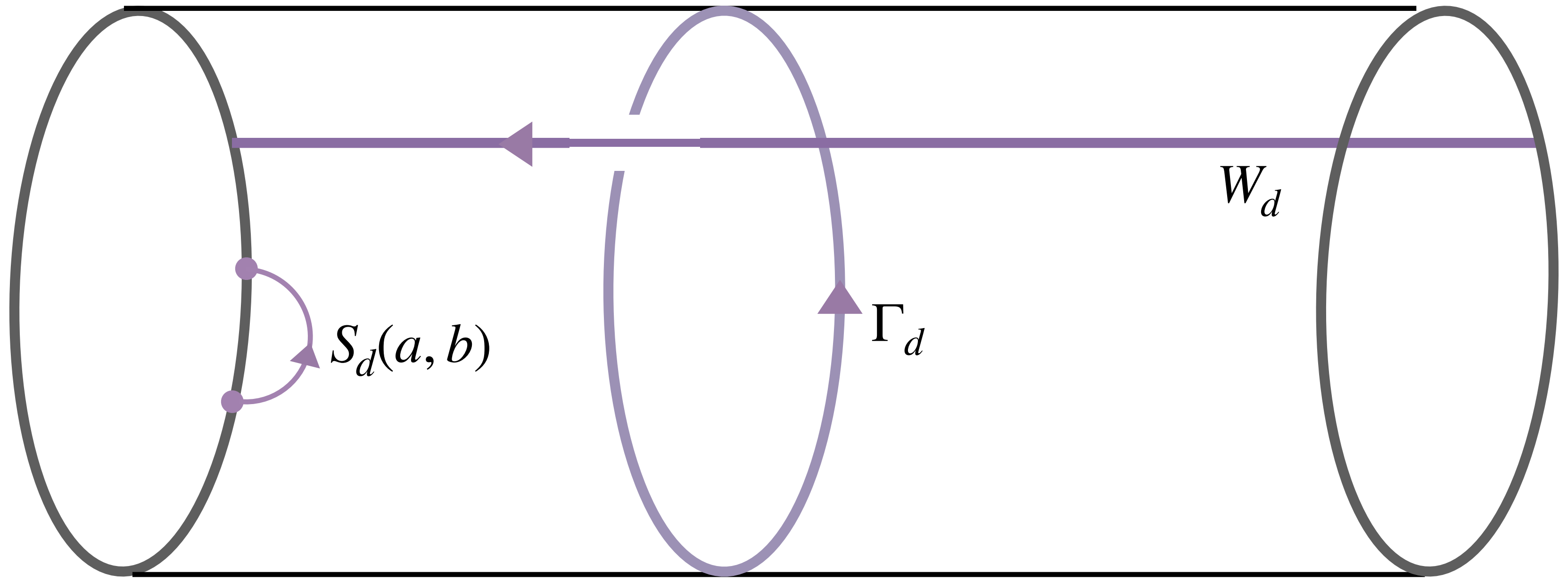


- **0-form symmetry: e-m duality**

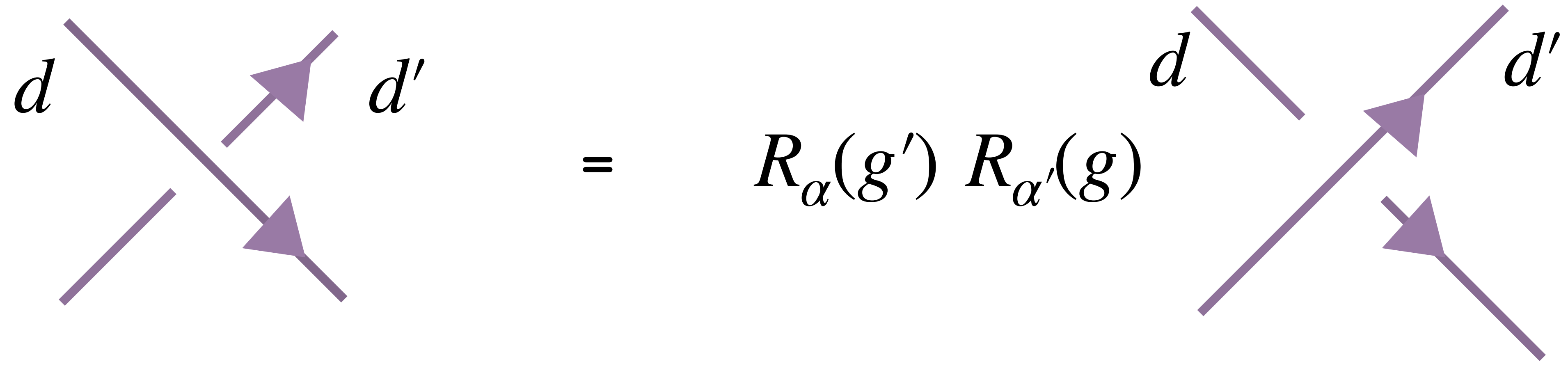


Effective spin chains from holographic bulk

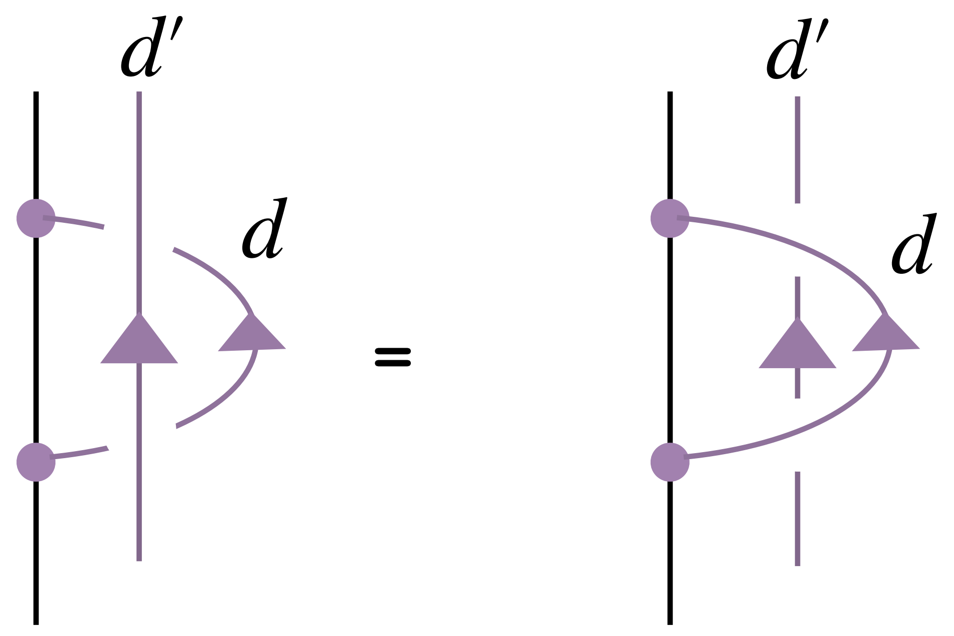
- Define bulk on a spatial cylinder.



- Algebra of line operators

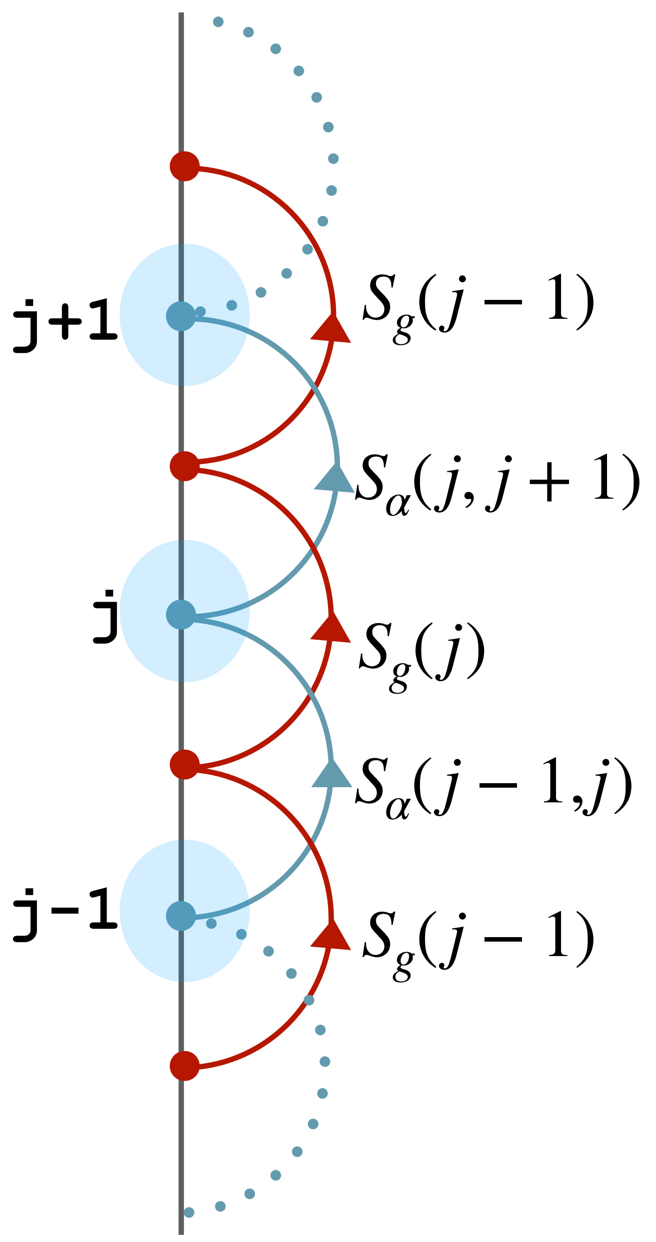


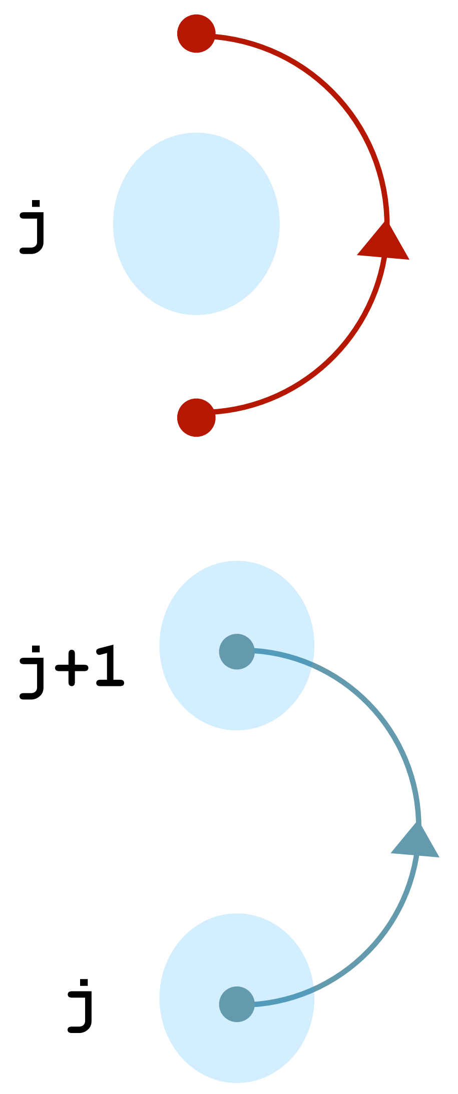
- Boundary Hilbert space: decomposition into super-selection sectors: $\mathscr{H}^\partial = \bigoplus_{d'=(g',\alpha')} \mathscr{H}^\partial_{d'}$ labelled by Γ_d eigenvalues.



symmetry twisted sectors
+
symmetry eigenspaces

Bulk line operators to Boundary operators

- 

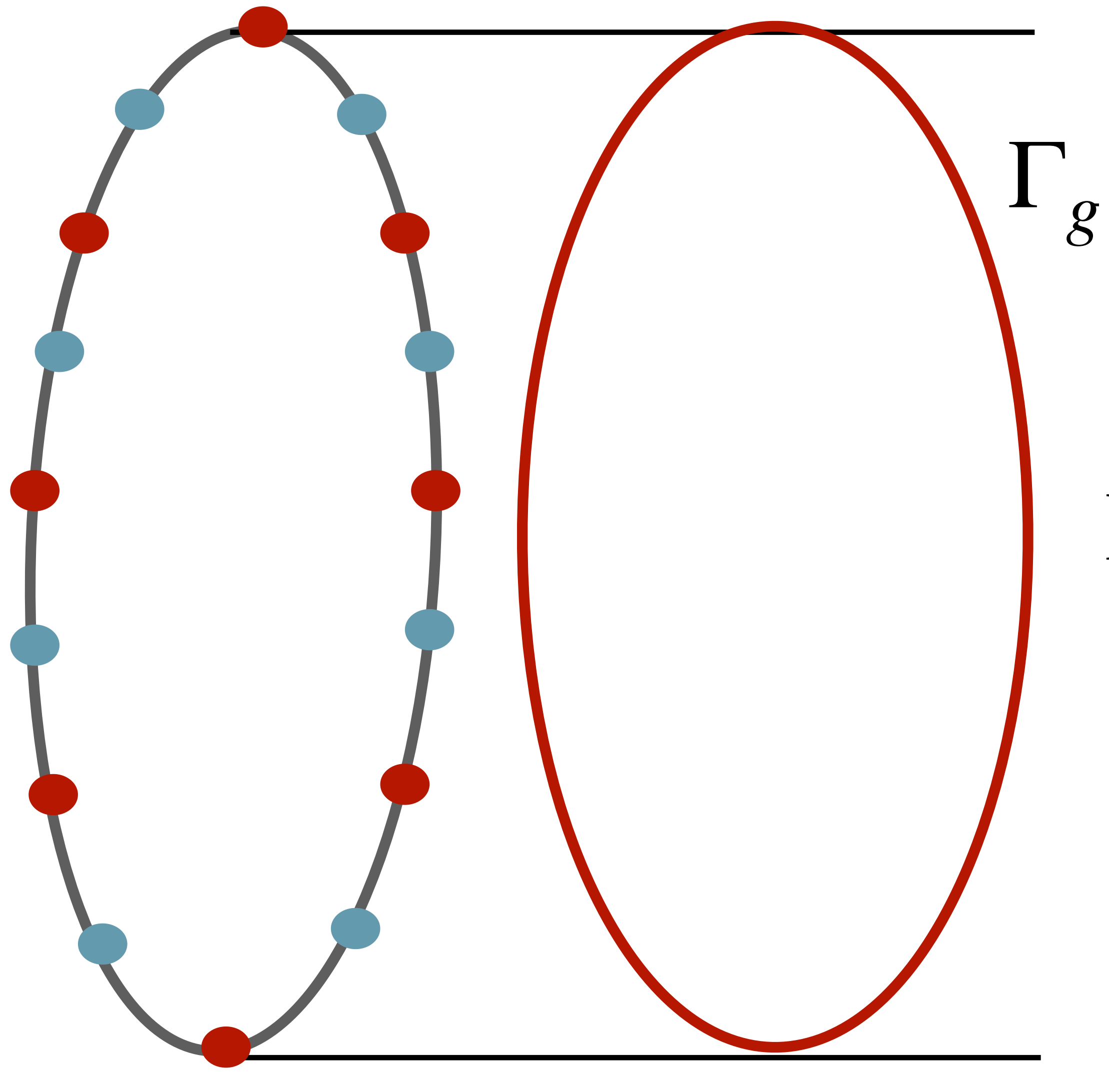


$S_g(j) \longrightarrow \mathcal{O}_{g,j}$ ⟶ X_j

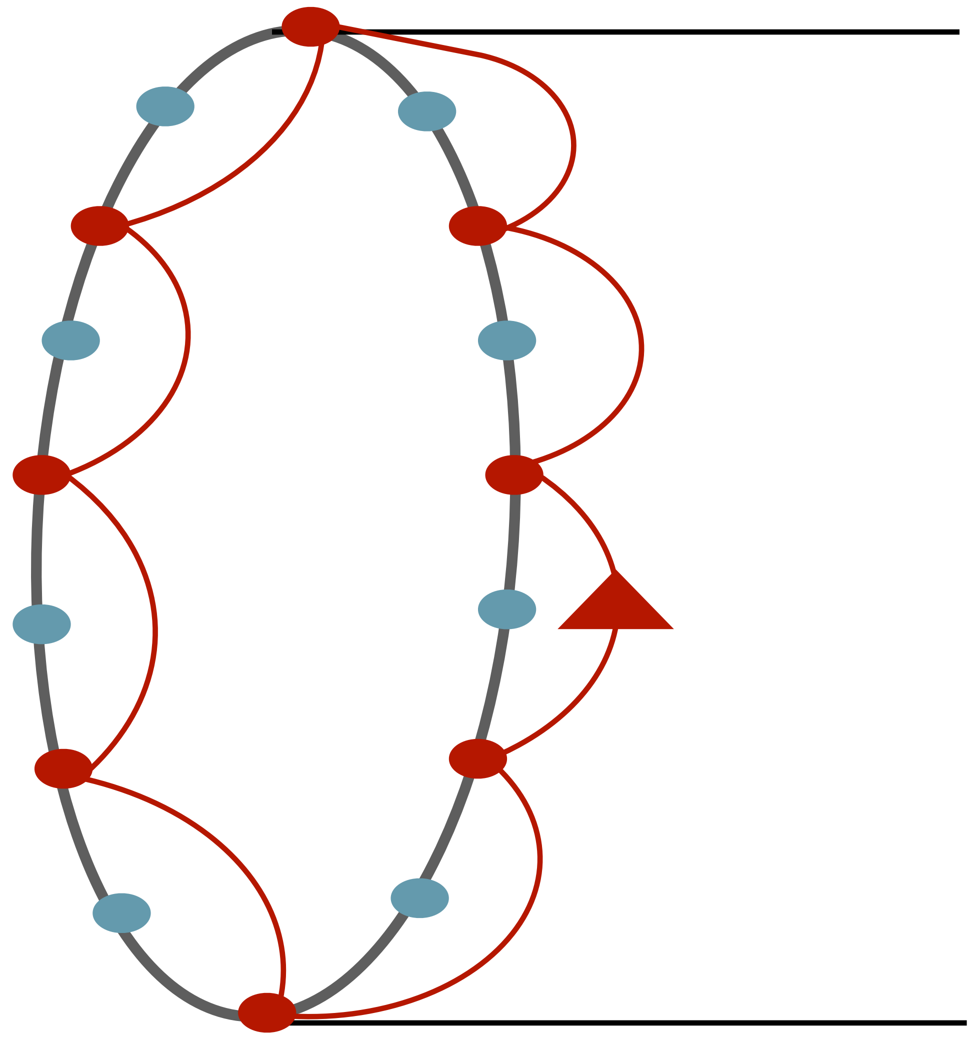
$S_\alpha(j,j+1) \longrightarrow \mathcal{O}_{\alpha,j}\mathcal{O}^\dagger_{\alpha,j+1}$ ⟶ Z_jZ_{j+1}

G=Z_2

- Symmetry operator:

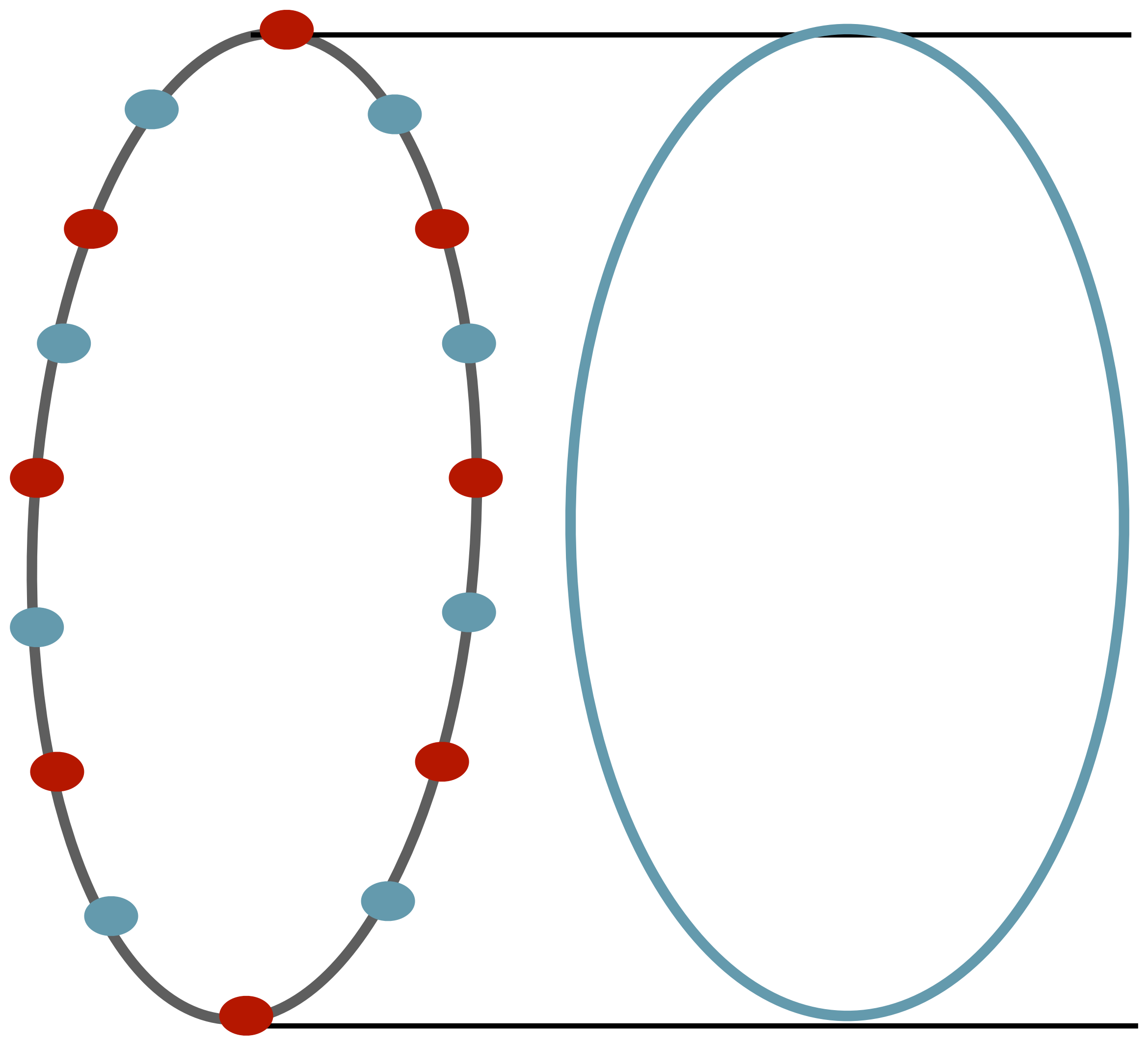


$$\Gamma_g = \prod_j S_g(j) \longrightarrow \mathcal{U}_g = \otimes_i \mathcal{O}_{g,j}$$



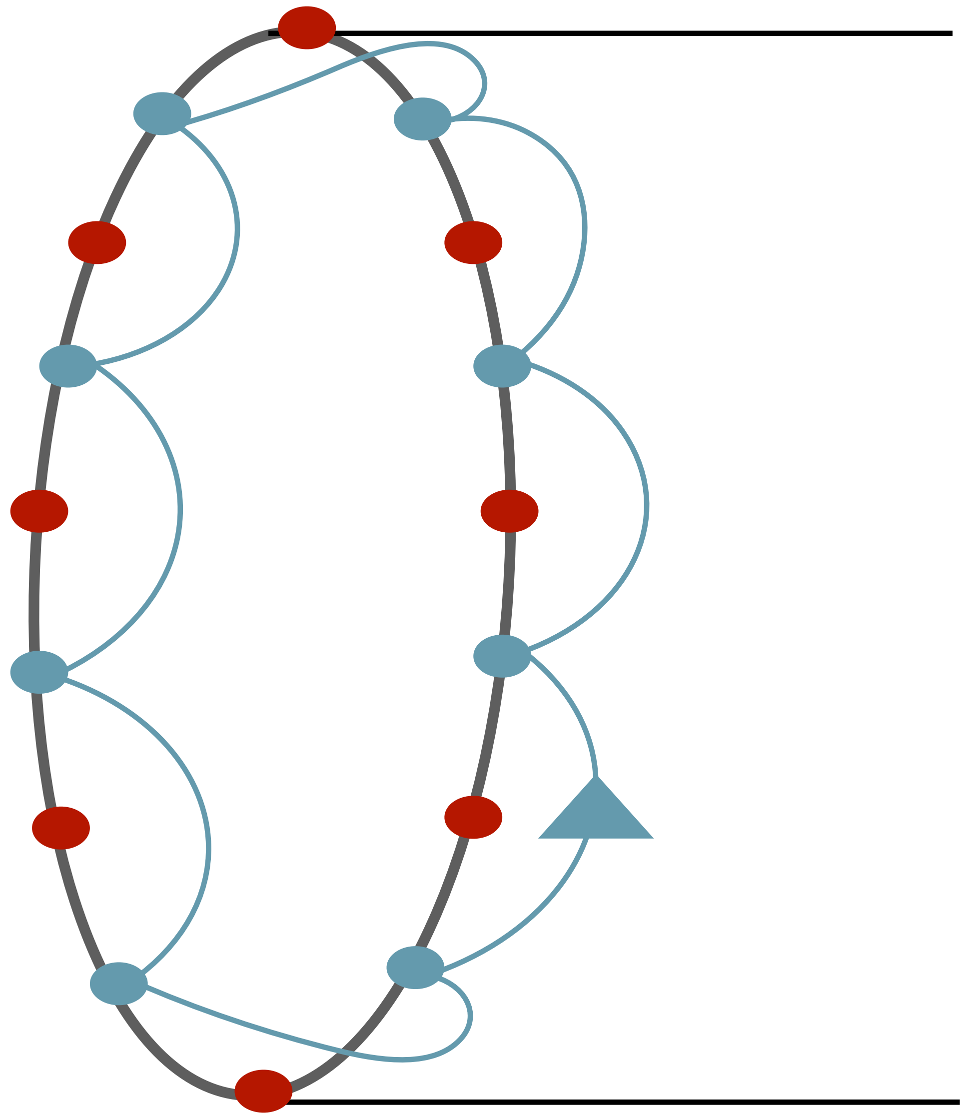
Bulk line operators to Boundary operators

- **Symmetry twist operator**



Γ_α

$$\Gamma_\alpha = \prod_j S_\alpha(j, j+1) \longrightarrow \mathsf{T}_\alpha = \otimes_j \mathcal{O}_{\alpha,j}\ \mathcal{O}^\dagger_{\alpha,j+1}$$



G=Z$_2$

$$\mathsf{T}_\alpha = \prod_j Z_j^\alpha Z_{j+1}^\alpha, \qquad \alpha = 0,1$$

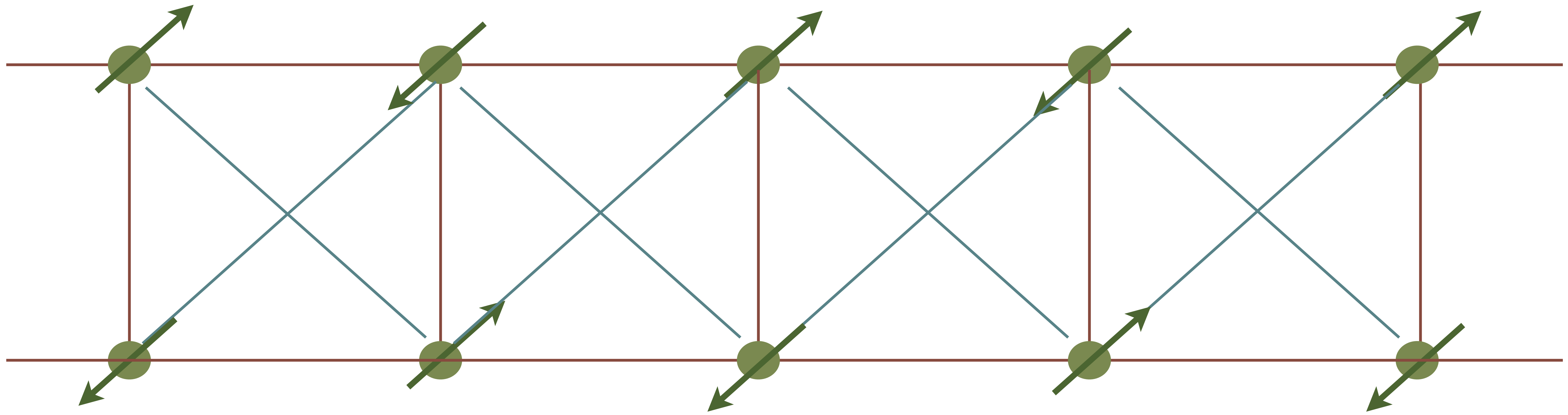
Z2 x Z2 symmetric quantum systems

- 2+1d Topological gauge theory:

  ➡ 1-form and 0-form symmetries

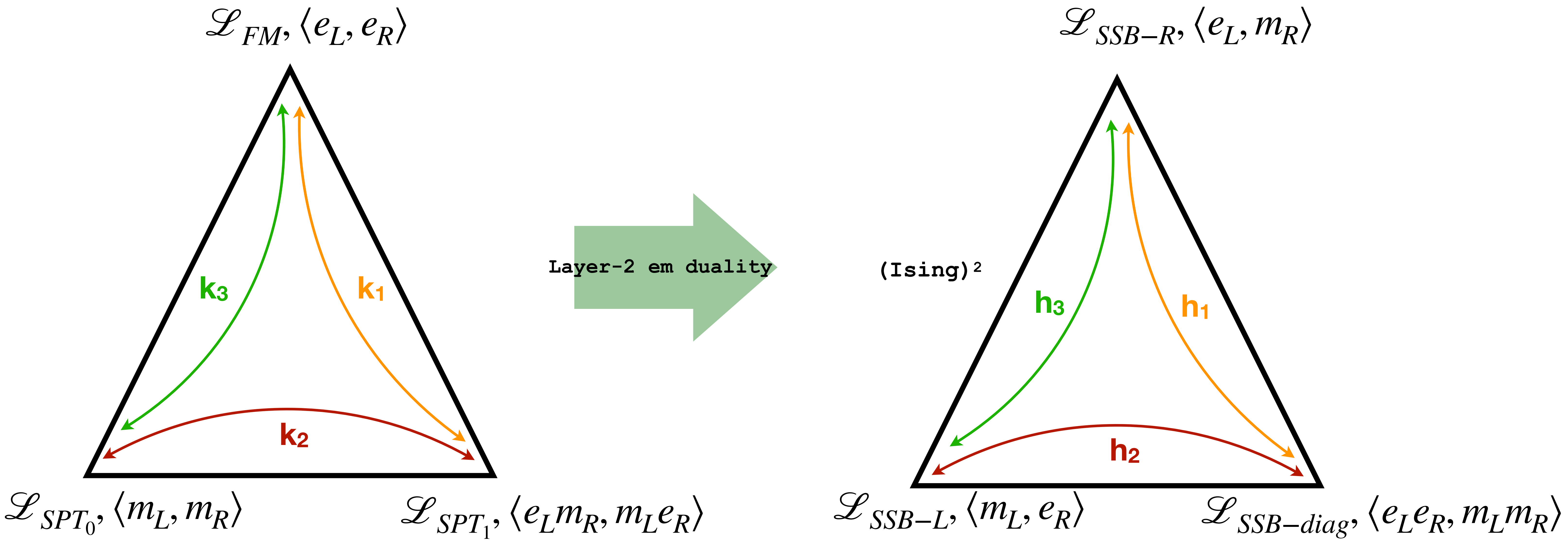
$$\mathscr{A}[G] = \mathbb{Z}_2 \times \mathbb{Z}_2 = \langle e_L, e_R, m_L, m_R \rangle$$
$$\mathscr{D}[G] = (S_3 \times S_3) \rtimes \mathbb{Z}_2$$

  ➡ 6 Gapped boundaries/6 gapped phases.

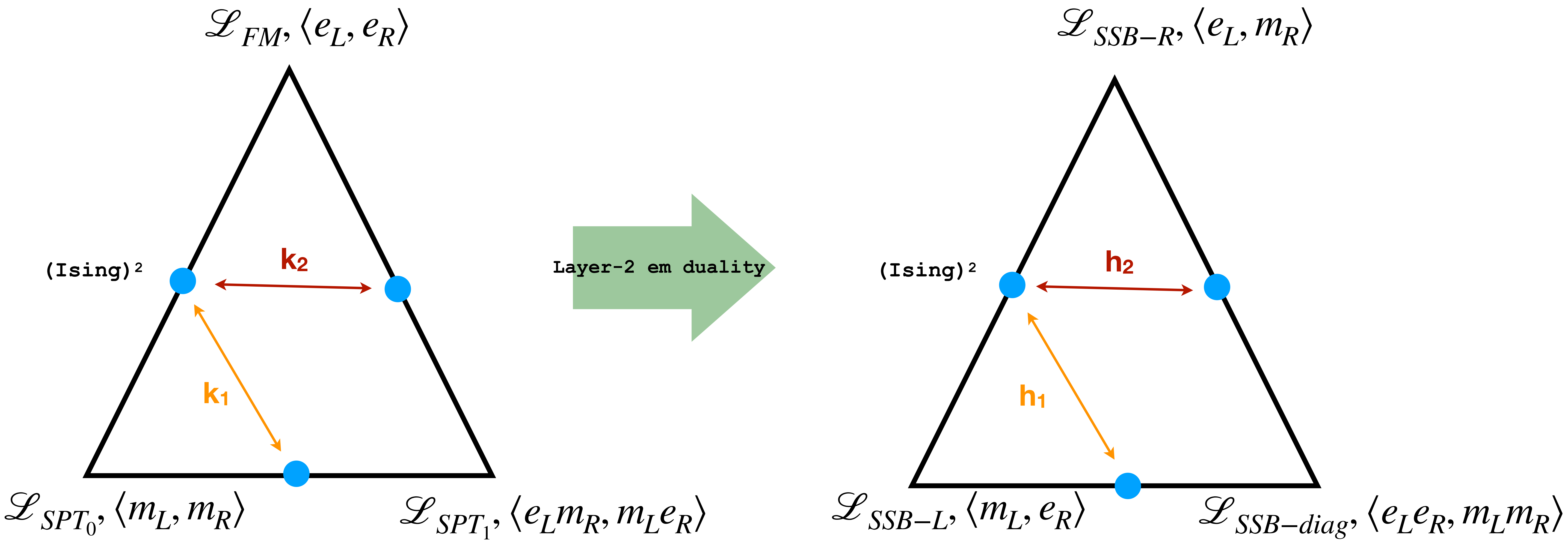


2-leg spin Ladder

➡ **Action on gapped phases:**

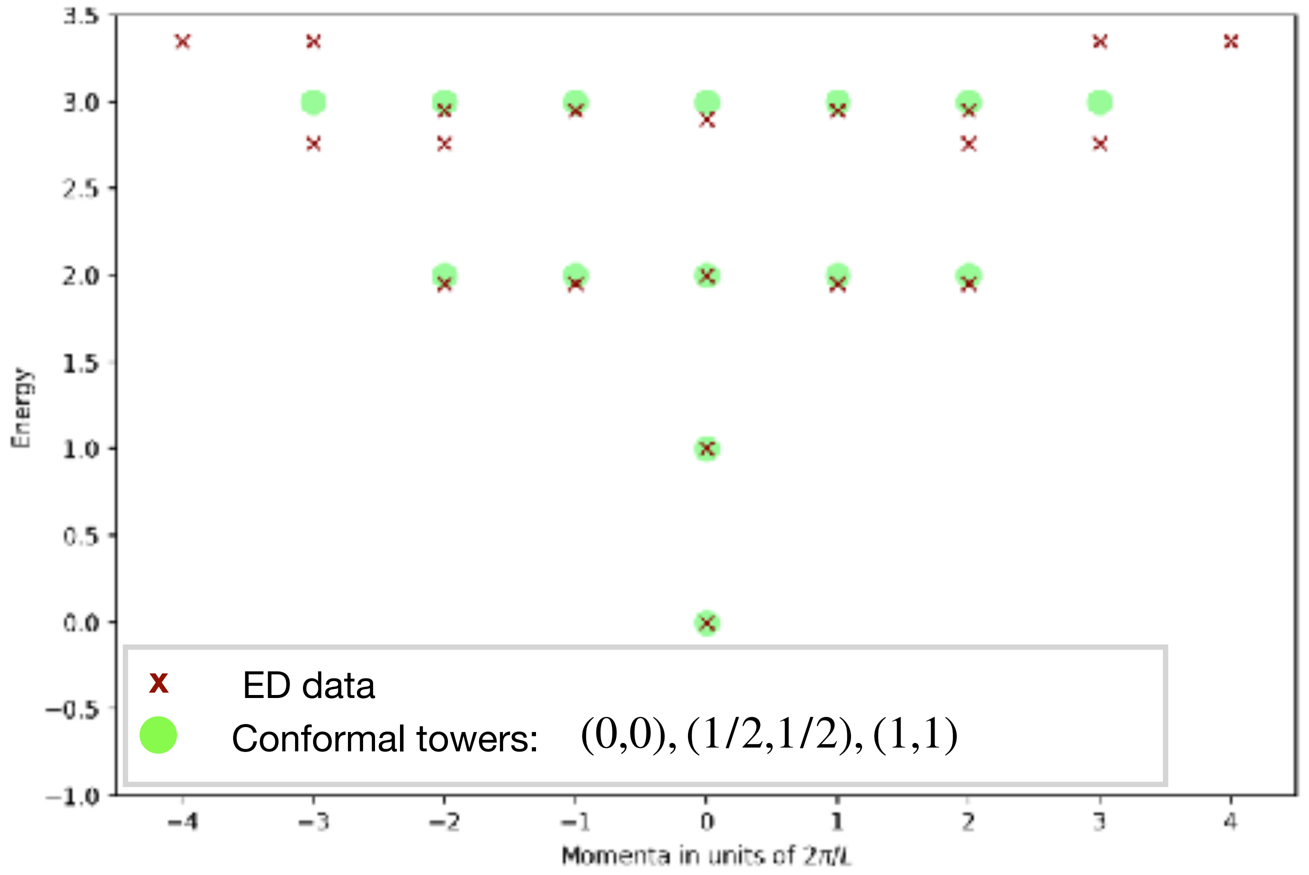


➡ **Action on critical points:**

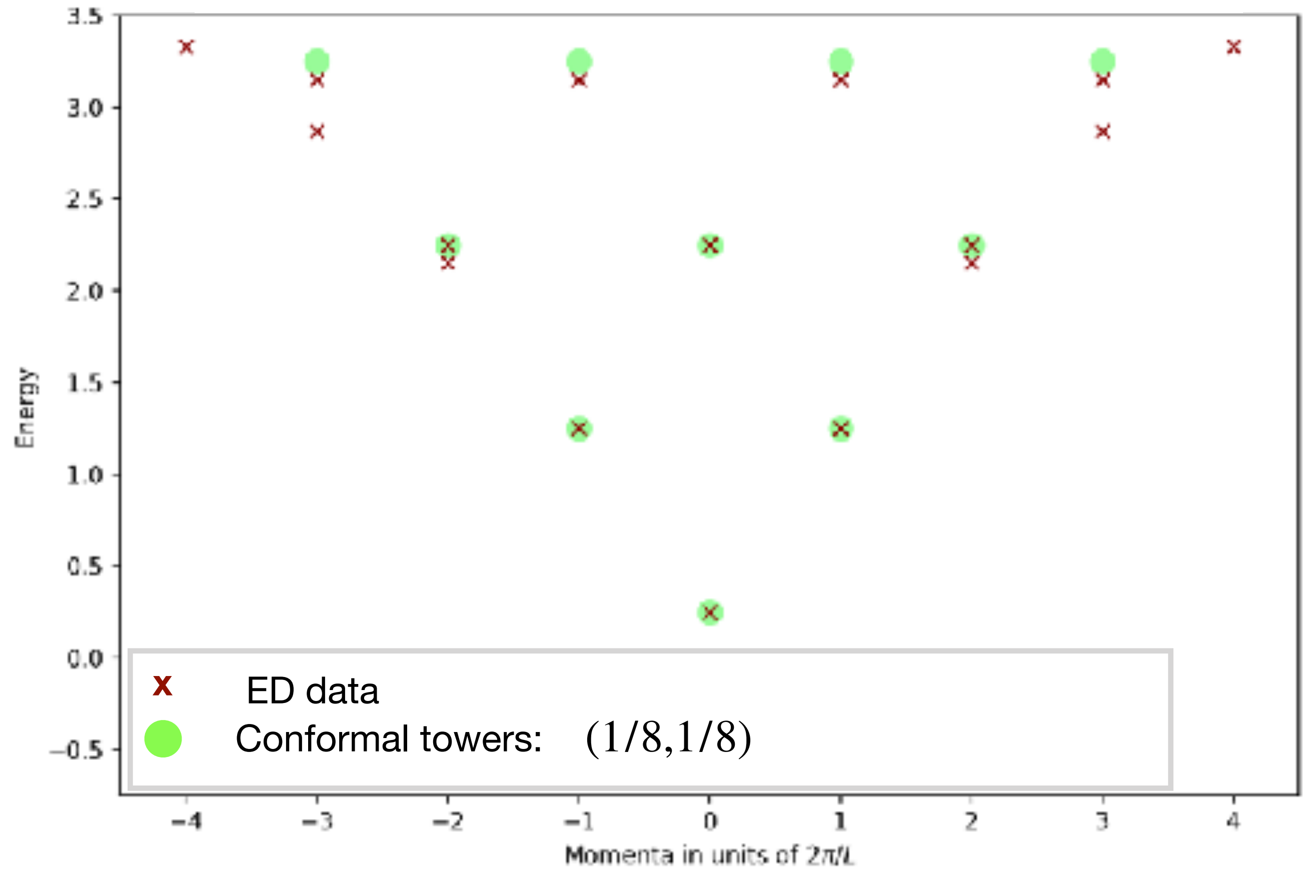


Orbifold CFT between SPT_0 and SPT_1 (using k_1 duality):

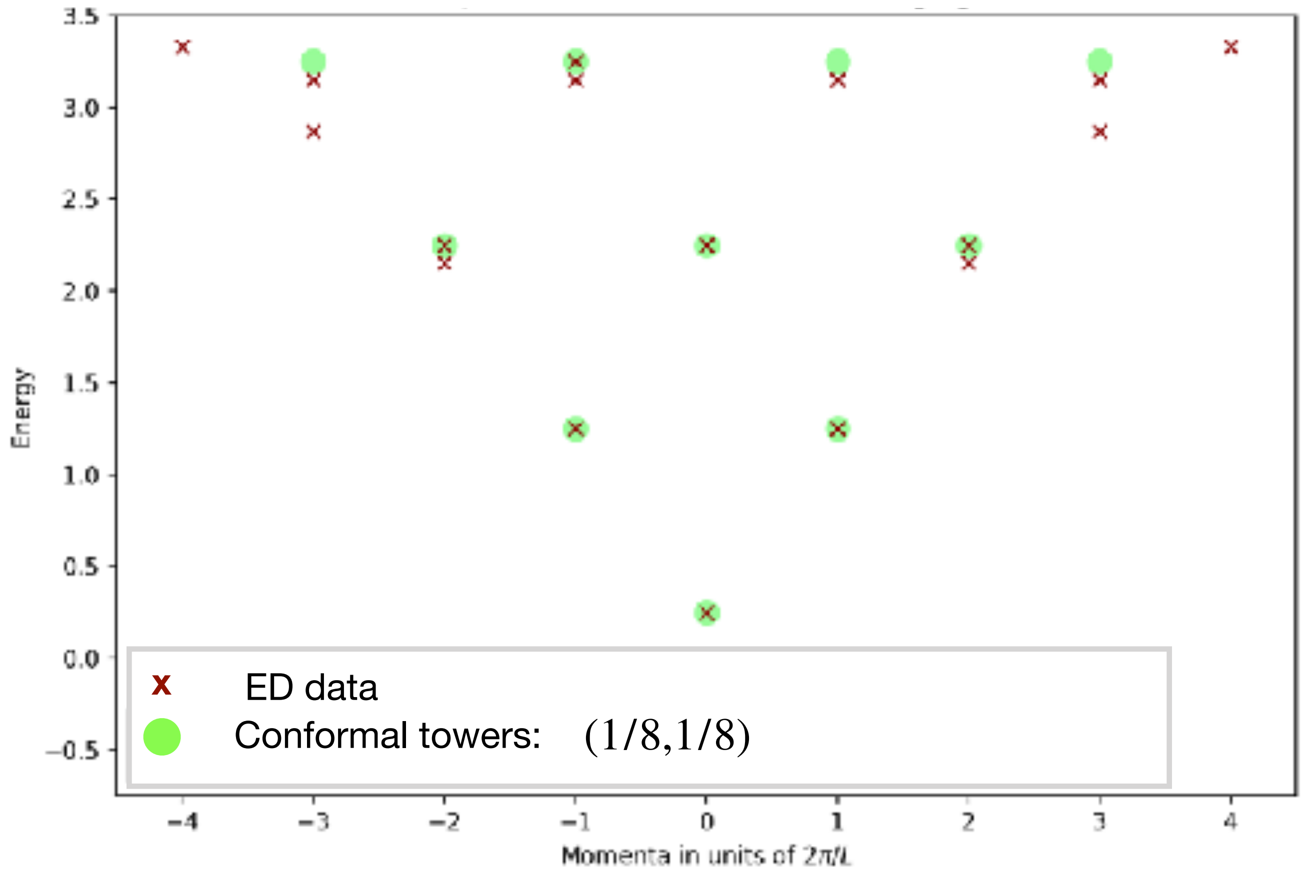
➡ **ED vs conformal spectrum**



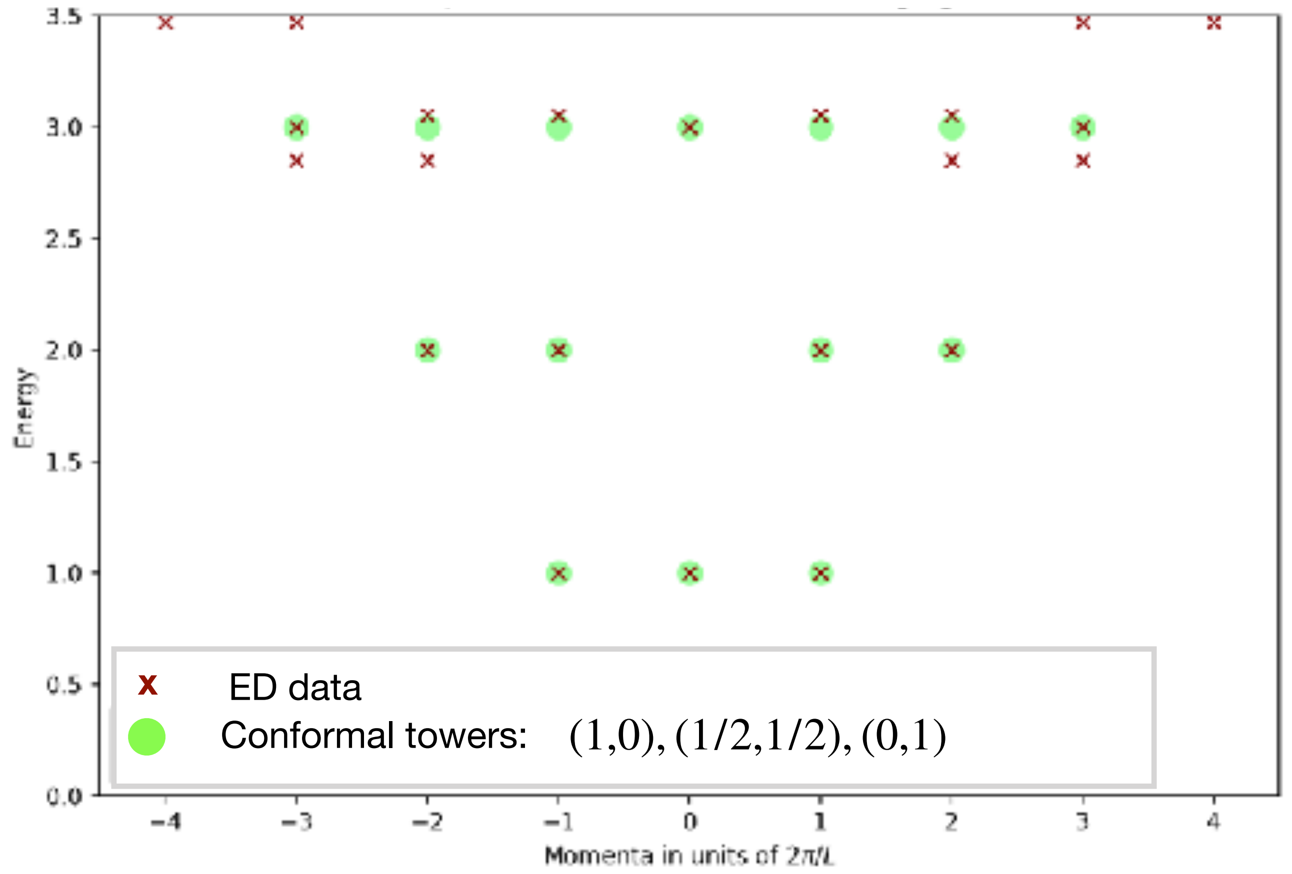
$1_L 1_R \longrightarrow 1_L 1_R$



$e_R \longrightarrow m_L e_R$

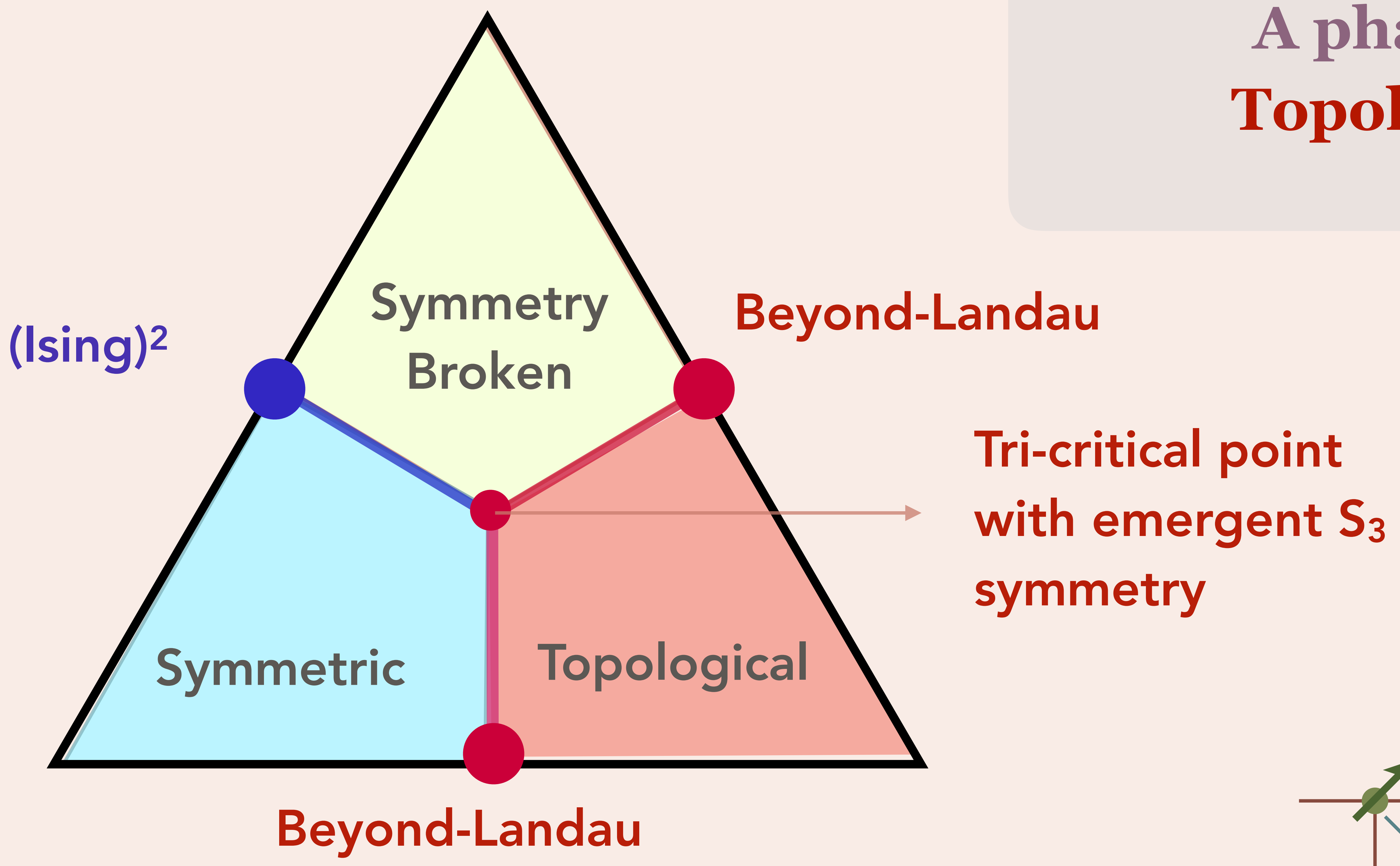


$e_L \longrightarrow e_L m_R$

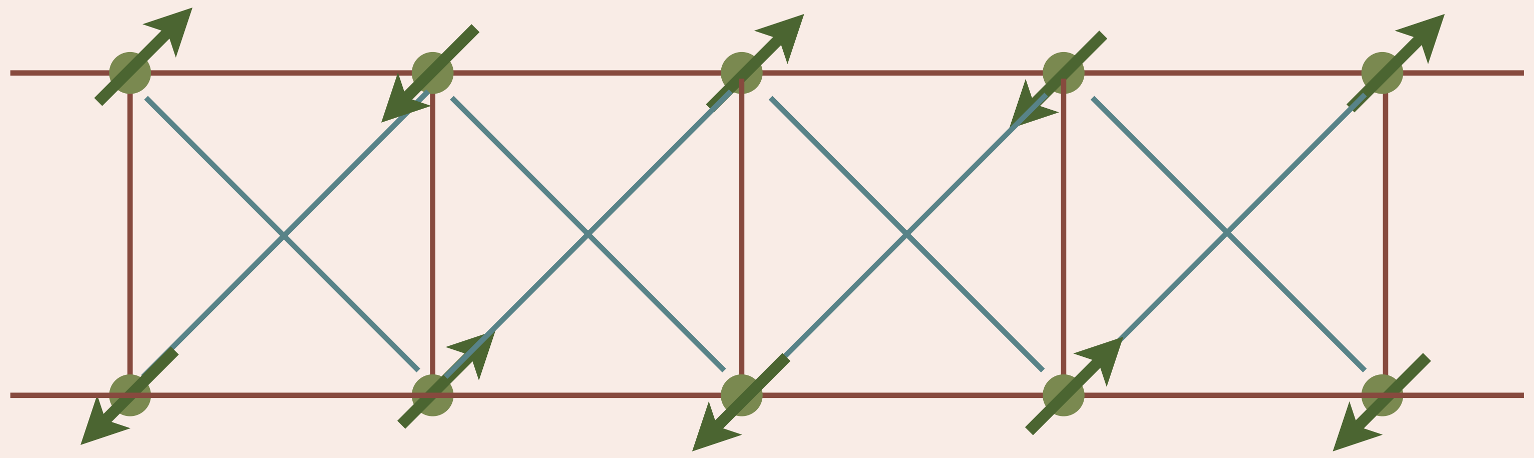


$e_L\, e_R \longrightarrow f_L f_R$

A phase diagram from **Topological holography**



Phase diagram:
$Z_2 \times Z_2$ symmetric systems



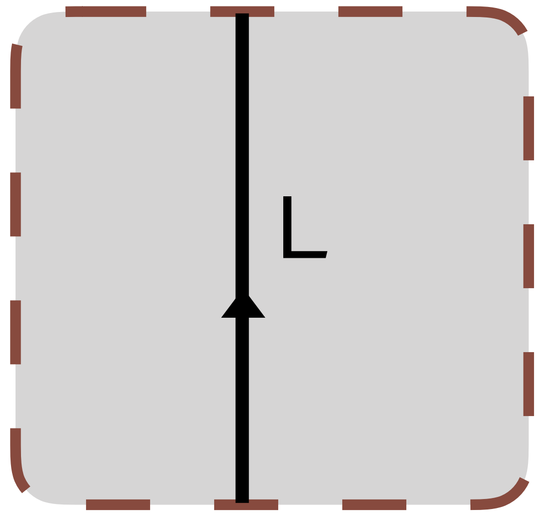
2-leg spin Ladder

Higher Category of Symmetric Gapped phases

Fusion 1-category symmetries

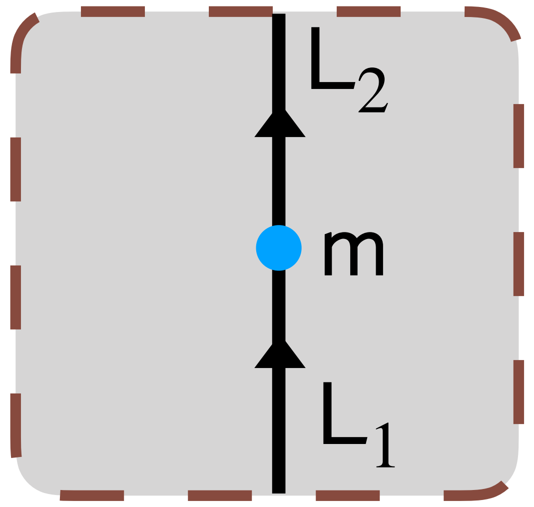
- **Describe finite symmetries of 1+1d systems.**

- **Objects :**



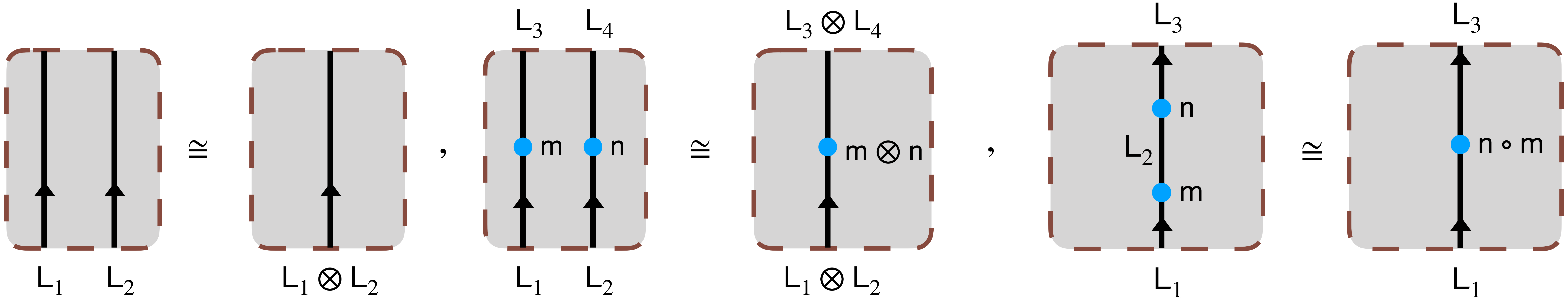
Topological lines

- **Morphisms :**

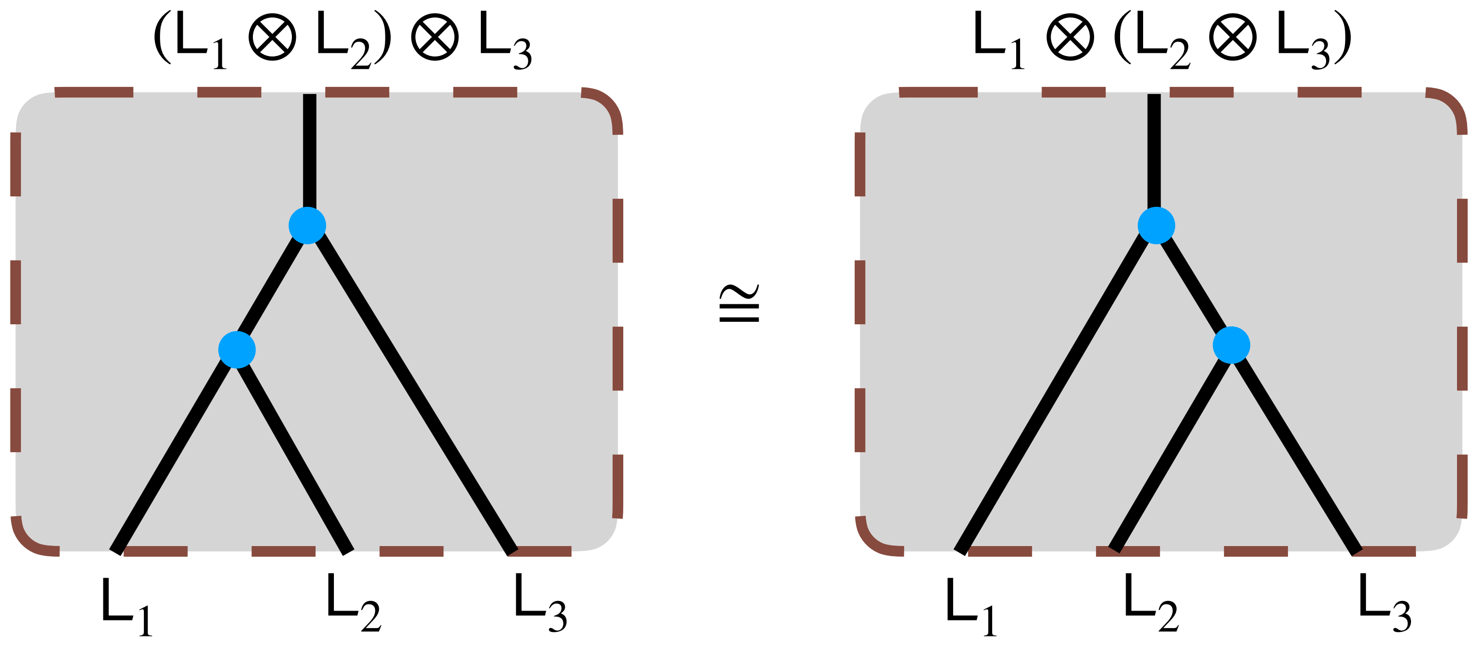


Topological local operators

- **Fusion :**



- **Associators :**

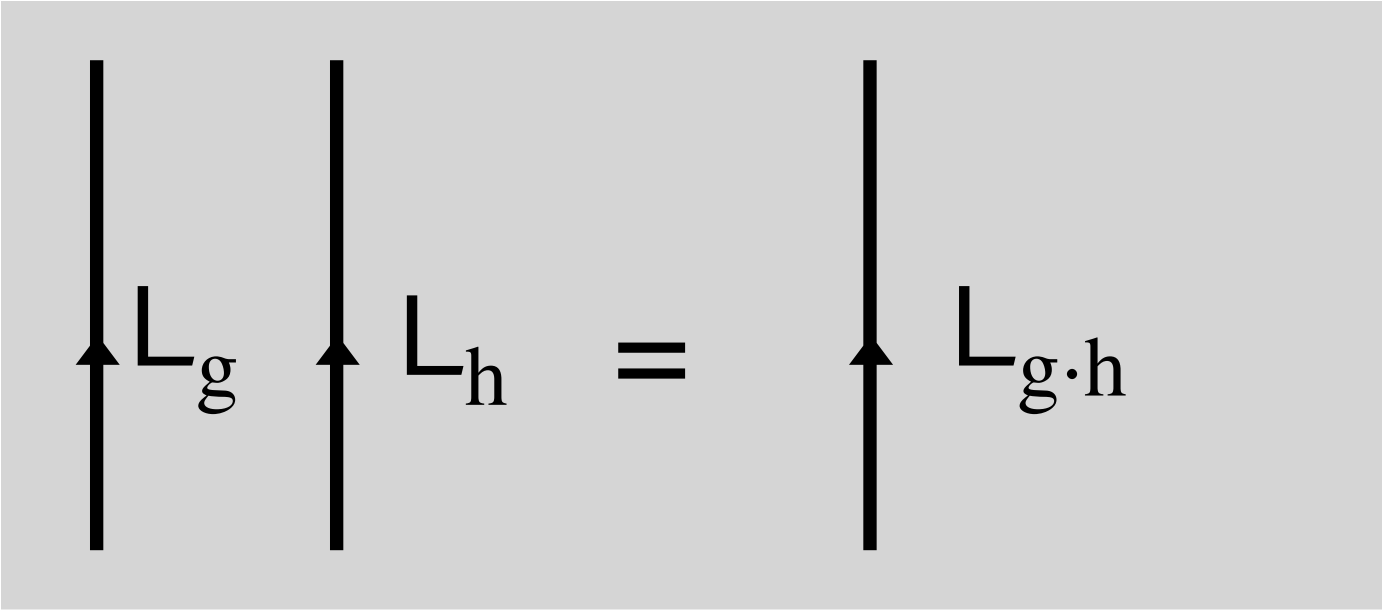


+ coherence relations

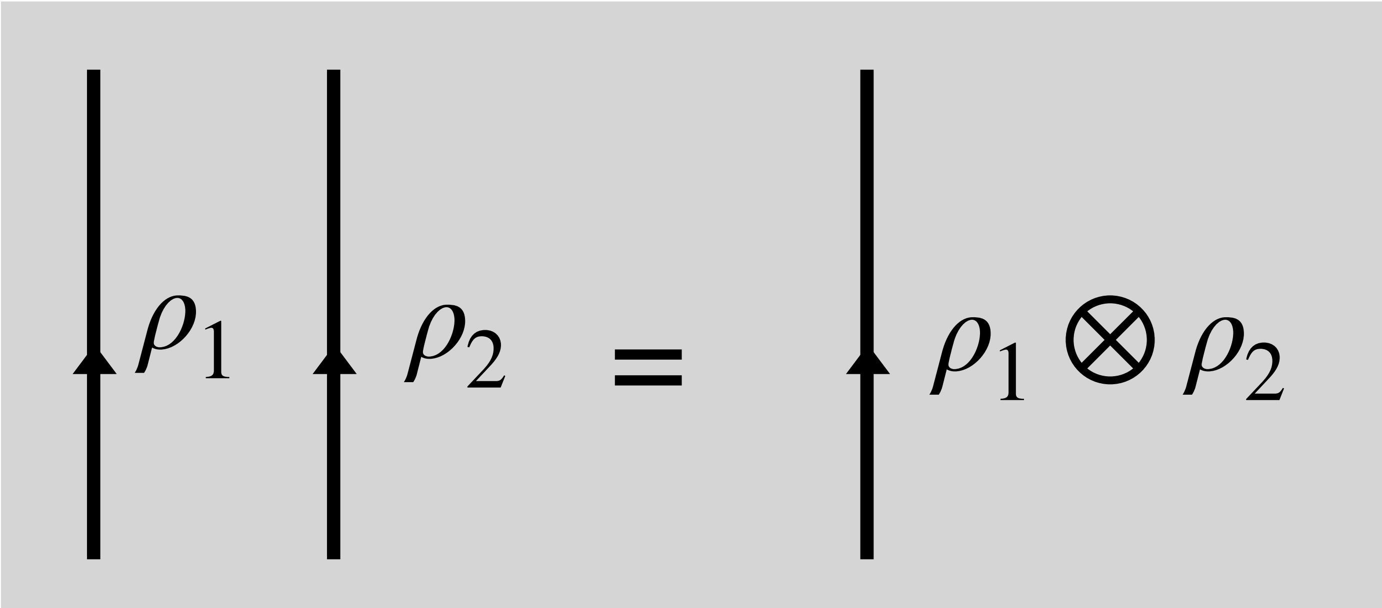
[Etingof-Gelaki-Niksych-Ostrik]

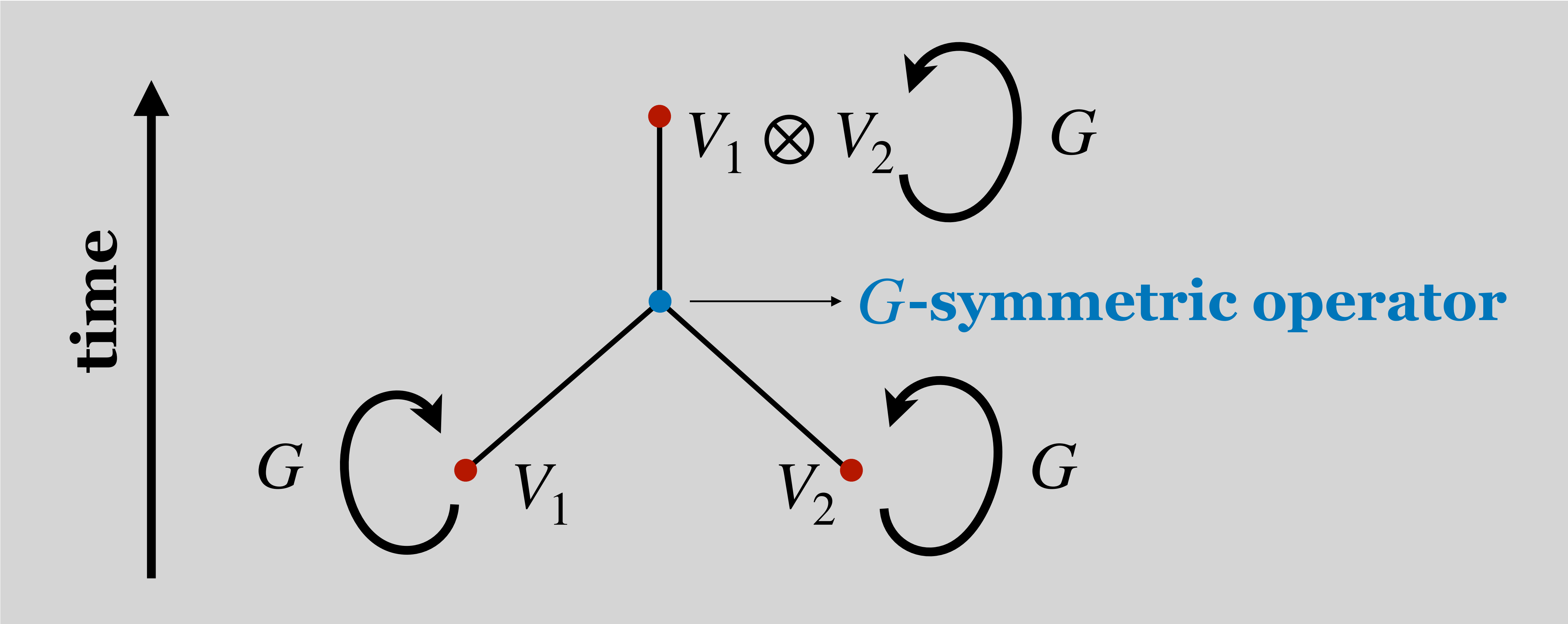
Examples of Fusion 1-categories

- **Vec(G) :**



- **Rep(G) :**





Conventional G symmetry

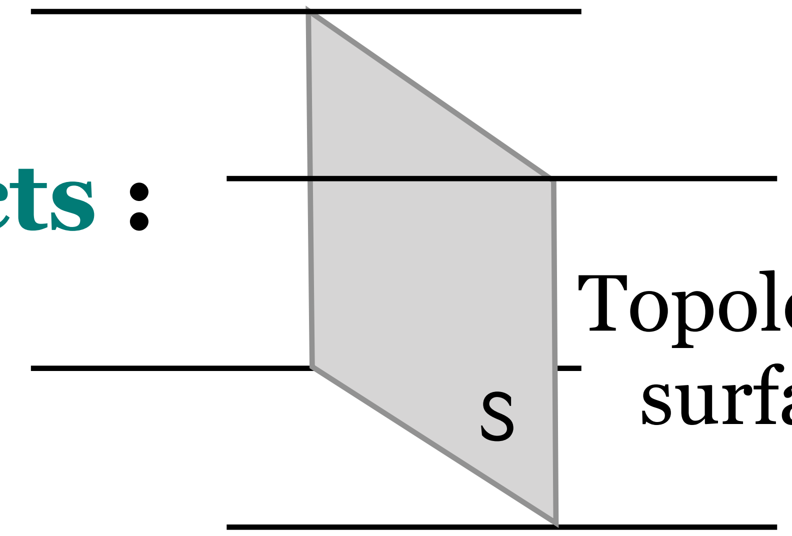
Rep(G) symmetry

≃

G-symmetric (topological) quantum mechanics

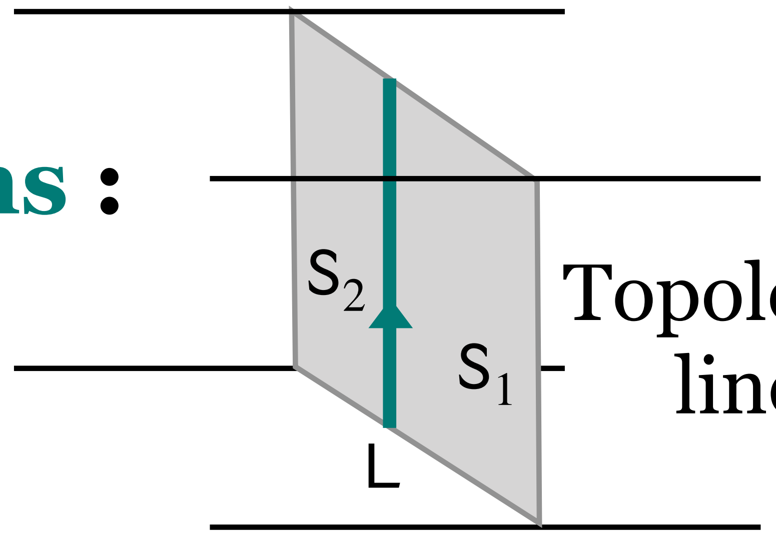
Fusion 2-categories: Symmetries in 2+1d

➡ **Objects :**



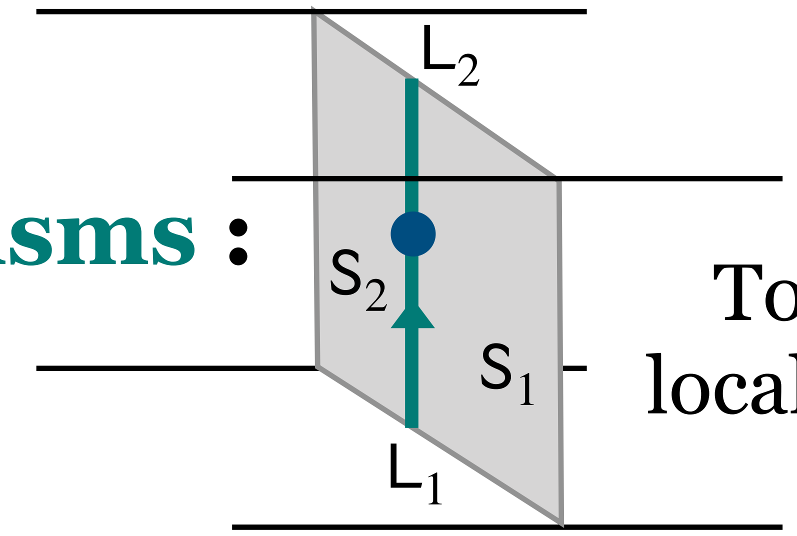
Topological surfaces

1-morphisms :



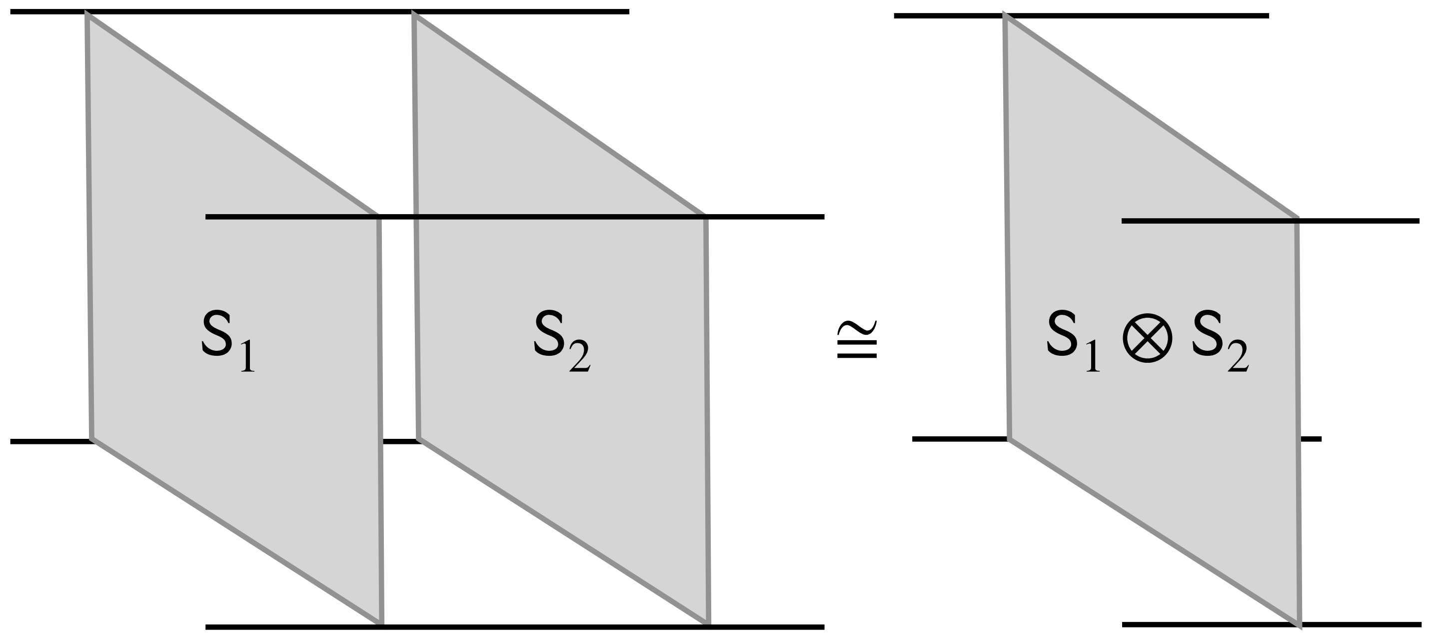
Topological lines

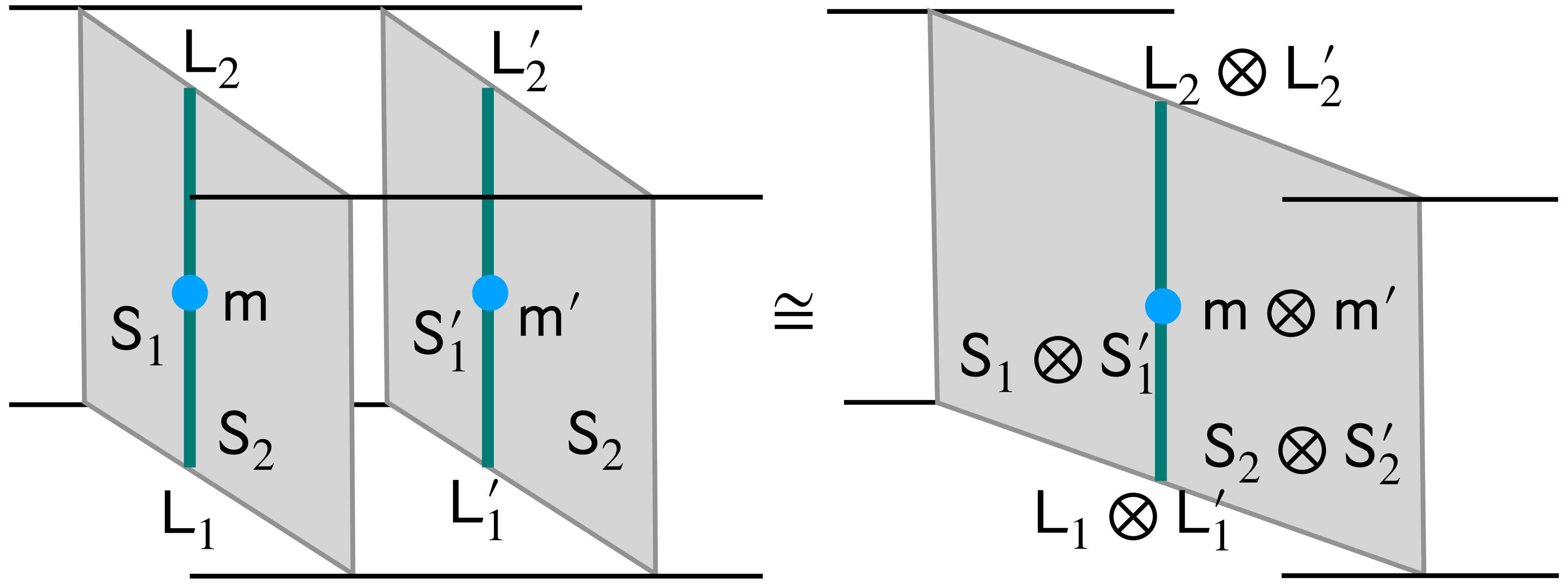
2-morphisms :



Topological local operators

➡ **Fusions and compositions:**



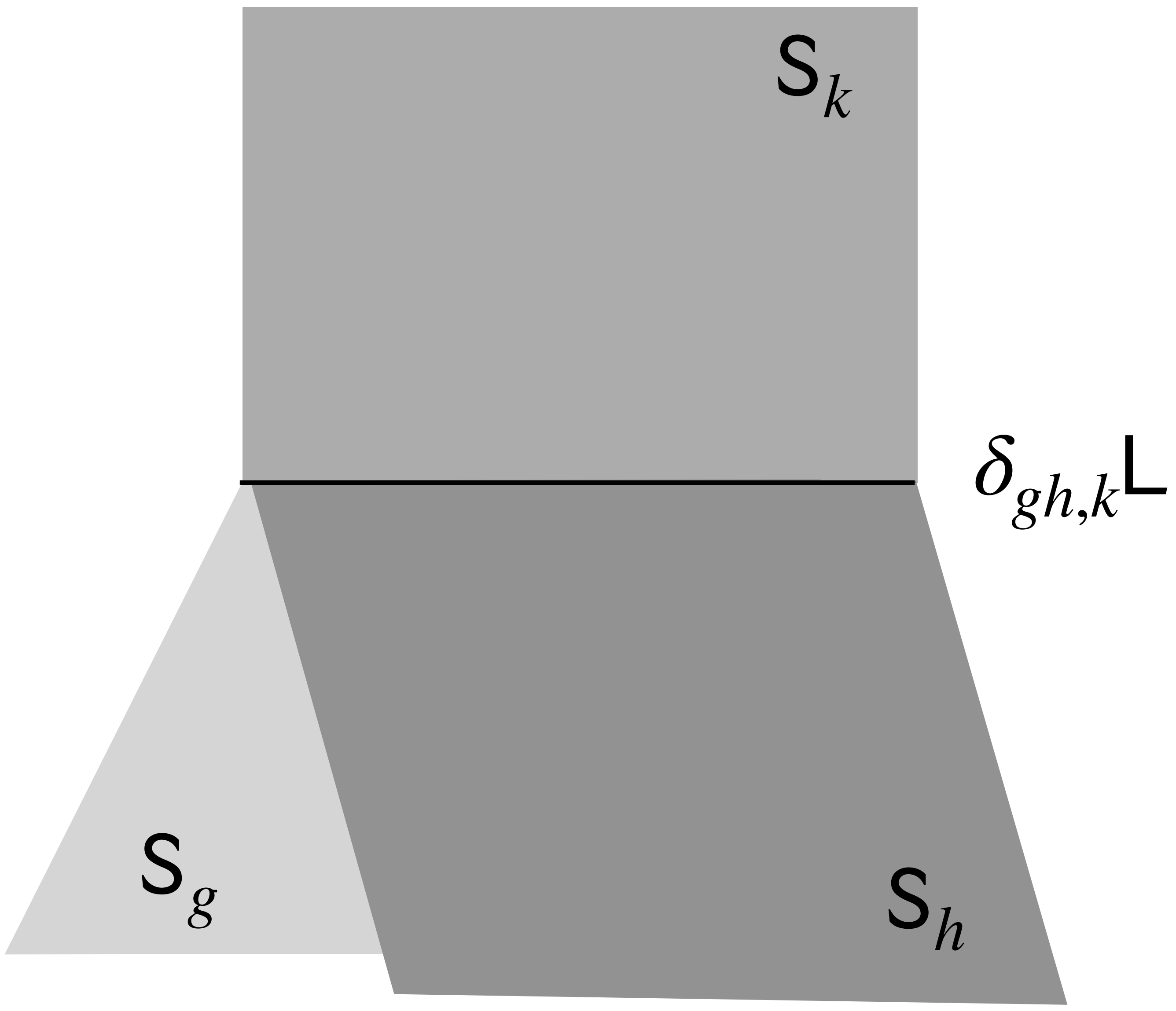


➡ **Many compatibility conditions** [Douglas-Reutter '18]

[Douglas Reutter], [Cui],[Fuchs-Valentino-Schweigert '02], [Gaiotto-Johnson Freyd], [Kong], [Bhardwaj-Bottini-Nameki-Tiwari]

Examples of fusion 2-categories

I. **Group 2-categories** (2VecG):



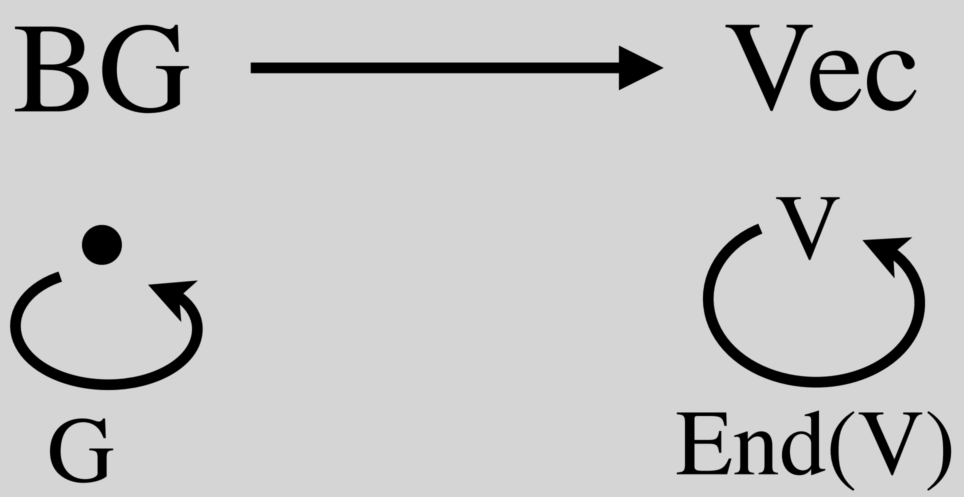
Conventional G symmetry

Examples of fusion 2-categories

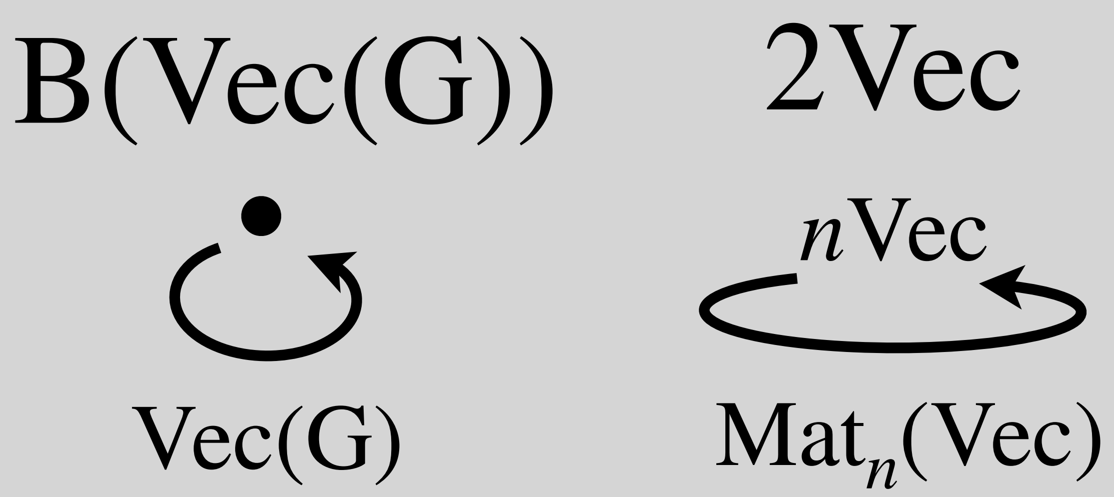
II. **Group representation 2-categories** (2Rep(G)):

Recall a representation on V **is a map** $\rho : G \to \mathrm{End}(V)$ **.**

Rep(G) is the 1-category of functors.



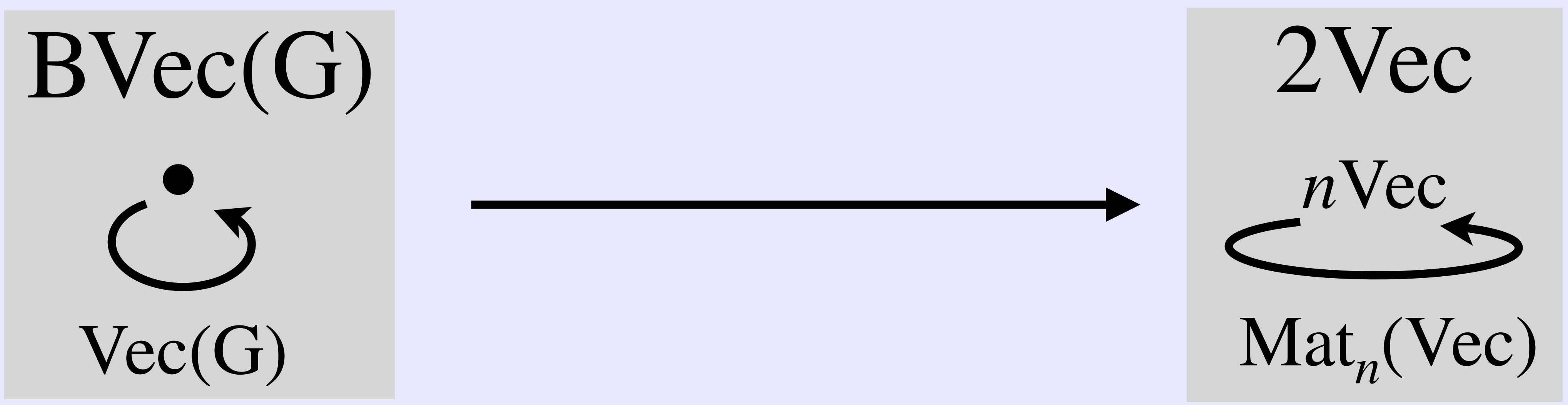
2Rep(G) is the categorification:



A 2-**representation is a monodical functor :** $\mathrm{Vec}(G) \longrightarrow \mathrm{Mat}_n(\mathrm{Vec})$

Examples of fusion 2-categories

2Rep(G) is the 2-category of functors :



2-category of 2d TQFTs

2d TQFT with n vacua/ground states

1-category of lines

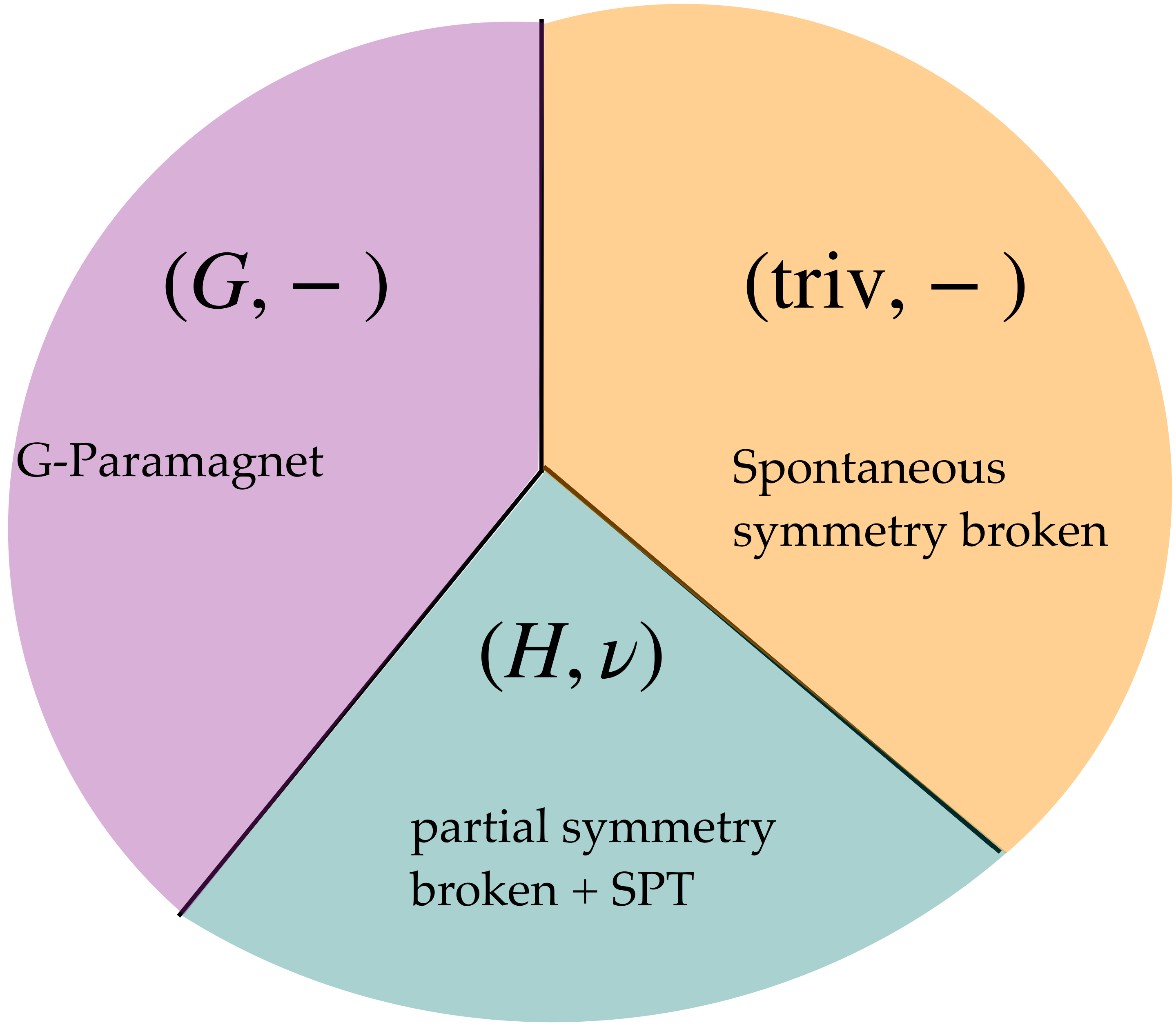
A 2-**representation is a monodical functor :** Vec(G) $\longrightarrow$ Mat$_n$(Vec)

$\simeq$

1+1d gapped phase with G-symmetry and n vacua/GSs.

Examples of fusion 2-categories

II. **Group representation 2-categories** (2Rep(G)):



Physics interpretation

Objects: Gapped phases of matter with G symmetry.

1-Morphisms: Gapped boundaries between gapped phases.

2-Morphisms: Gapped junctions.

$\mathrm{H} \subset \mathrm{G}$ **: Spontaneous symmetry breaking G ⟶ H**

$\nu \in \mathrm{H}^2(\mathrm{H}, \mathrm{U}(1))$ **: H symmetry protected topological phase**

[Ostrik], [Chen-Gu-Liu-Wen]

Group representation 3-categories (3Rep(G)):

3 category of monoidal functors from:

$$3C_{\mathrm{G}} \longrightarrow 3\mathrm{Vec}$$

(Single object,
1-endomorphisms $\simeq$ 2Vec(G))

(Category of 2+1d TQFTs
with gappable boundaries
~ (centers of) Fusion 1-categories)

Physically corresponds to possible ways in which non-anomalous topological orders can be made G-symmetric.

SPT and SSB phases are recovered by making the trivial topological order G-symmetric.

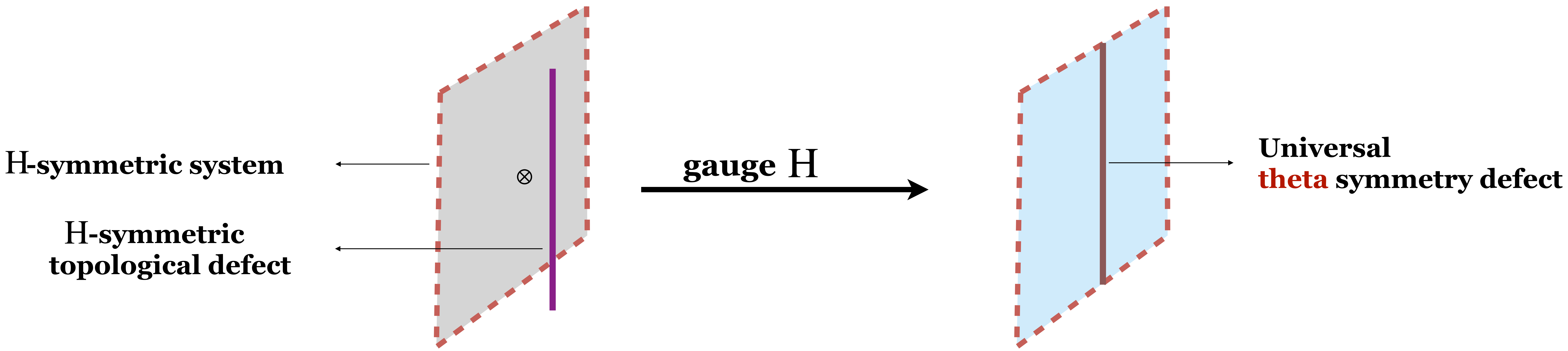
3Rep(G) = 3 category of invertible and non-invertible G-symmetric gapped phases of matter

Theta defects in (2+1) dimensions

In d+1 dimensions, when gauging a subgroup $H \subset G$

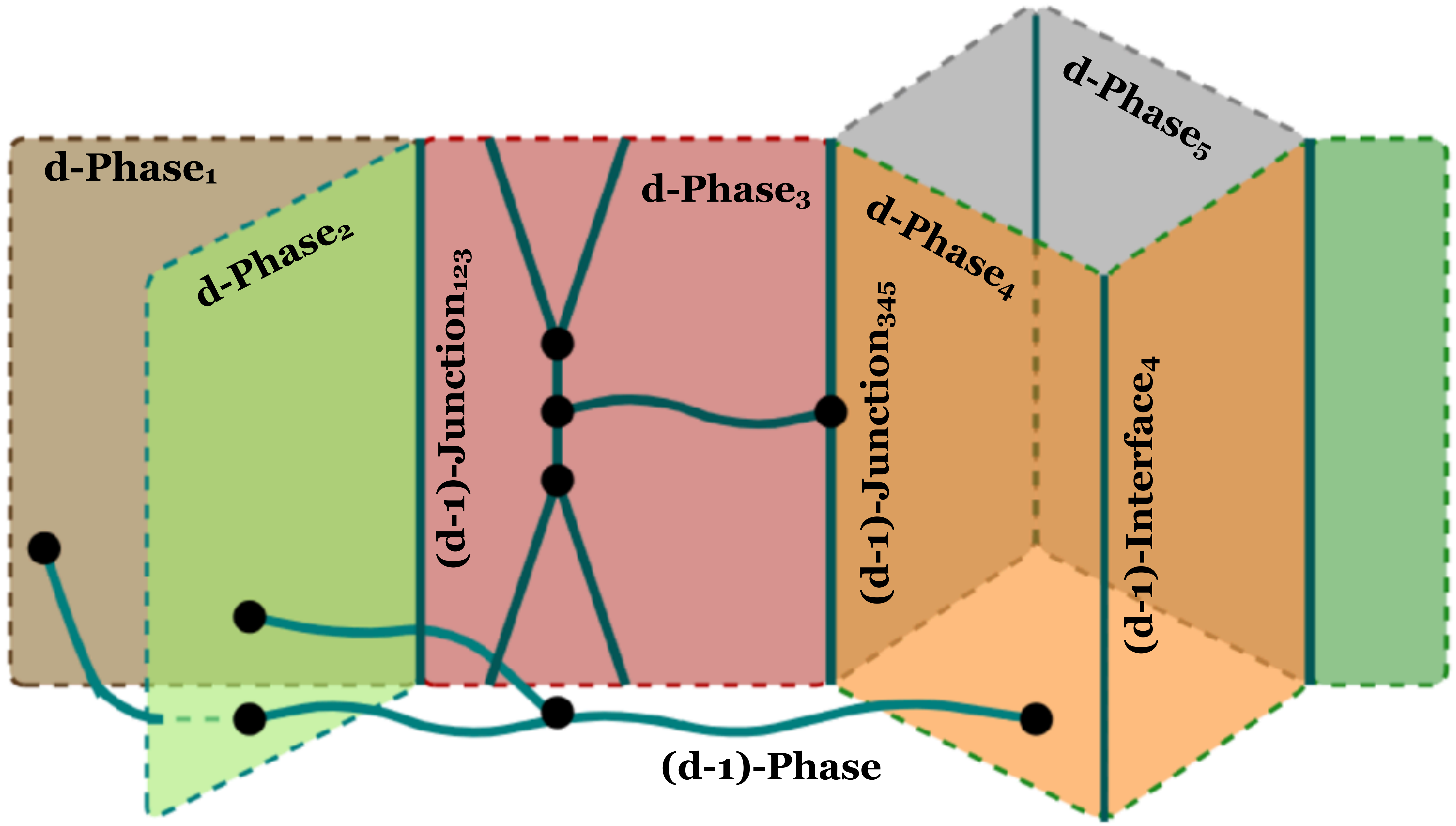
$$\mathcal{C}_G \xrightarrow{\textbf{gauge } H} \mathcal{C}_{G/H} \supset \mathrm{dRep}(H) \boxplus \ldots\ldots$$

Theta defects (dRep(H)) — **Twisted theta defects** (......)



Main messages

I. Any (d+1) dimensional quantum system obtainable by gauging an invertible symmetry $\mathcal{S}$ contains a universal d-Fusion category of symmetry defects/operators



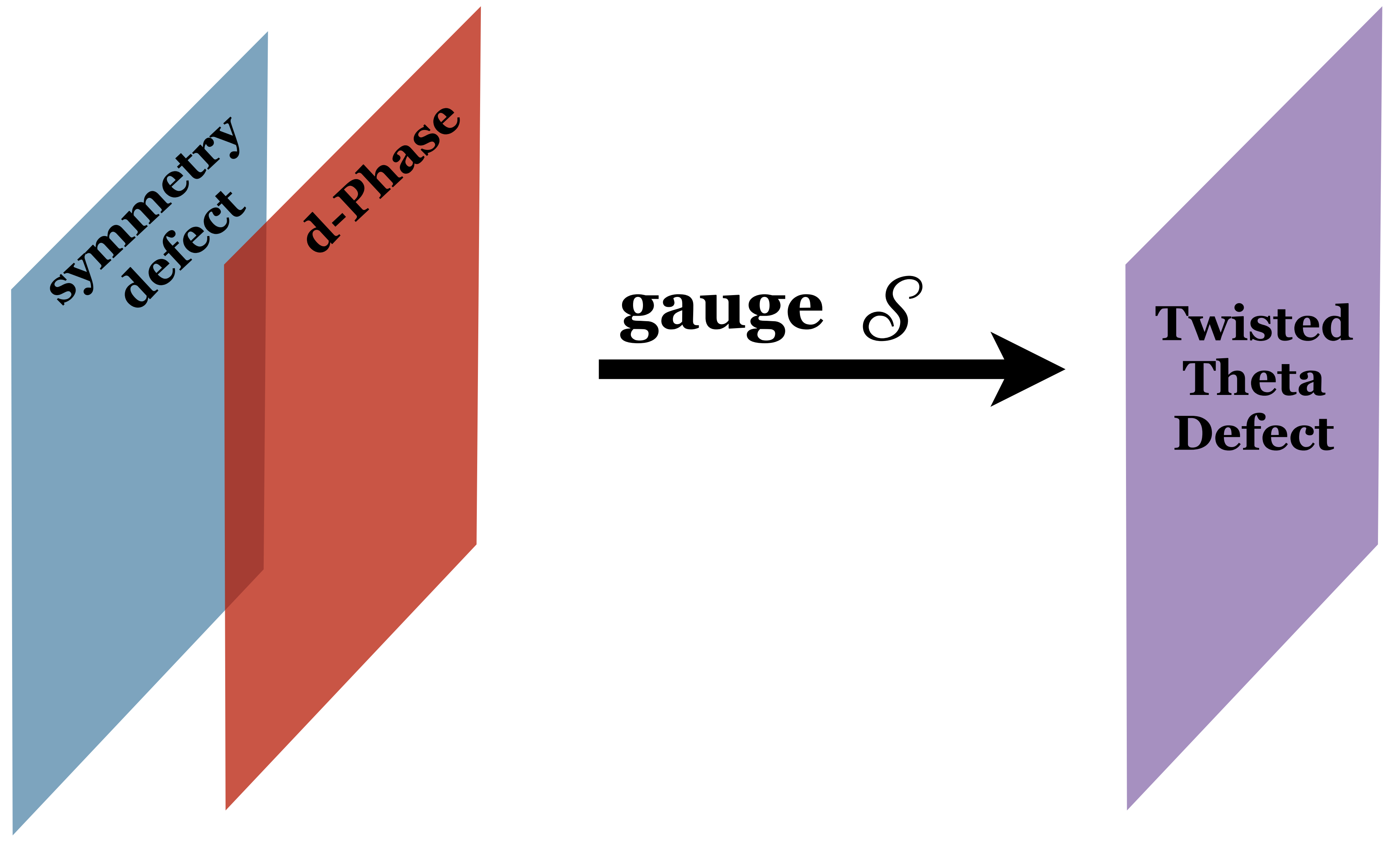
All phases, junctions and interfaces are gapped and $\mathcal{S}$-symmetric

These are Theta Defects

[Bhardwaj-Bottini-Nameki-Tiwari SciPost Phys. 15, 122 (2023), SciPost Phys. 15, 160 (2023)]

Main messages

II. Additionally there are non-universal Twisted Theta Defects, obtained by stacking the pre-gauged system with lower dimensional $\mathcal{S}$-symmetric phases and gauging



pre-gauged system **gauged system**

Directions

Classification of phases with generalized symmetries

Mechanisms for unconventional transitions
+
Generalized Landau paradigm

Consequences for dynamics/ excited states

Incorporating :

- Crystalline symmetries
- Fermionic symmetries
- Time reversal
- Continuous symmetries

Topological defects for topological quantum computation

Thank you!