

# Exploring the Impact of Pseudospectra on the Stability of Echo State Networks\*

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## Abstract

The standard Echo State Network (ESN) model has gained wide recognition for its success in modeling nonlinear auto-regressive (NAR) dynamical systems. For example, it is suitable for modeling the Mackey-Glass system, which is used for analyzing bifurcations in physiological applications. It is also employed to analyze the Lorenz system, which models several physical phenomena such as the dynamics of electro-rotation, thermohaline ocean flow circulation, the Malkus waterwheel, and weather transition dynamics. Furthermore, the ESN model is particularly interesting in a quantum computing context. The recurrent topology of the ESN model ensures that the non-linear transformation of the input history is stored in internal states. The system operates in a regime that can be either stable or can exhibit chaotic behavior. Over the past few years, several studies concerning the network stability have been carried out using Lyapunov exponents and the computation of the edge of chaos. These works have shown the significant impact of the eigenvalues modulus on the system stability.

Recently, a framework was introduced where a graph embedding technique and evolutionary algorithms were combined to fully train a recurrent network with the ESN's design. This neuroevolutionary approach for optimizing an ESN works well in practical applications, but it may be expensive due to the numerous computations of the spectral radius of a weight matrix. Computing the eigenvalues of a matrix at each iteration in iterative algorithms may be unstable and computationally expensive. In this work, we analyze the impact of the pseudo-spectrum on the stability of the ESN dynamics, and how the pseudo-spectrum can be used for improving the neuroevolutionary framework of ESNs. We explore the relationship between the pseudospectra and stability of the recurrent network on both MGS and Lorenz systems.

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