Exploring Bottomonium Behaviour at Finite Temperatures Machine Learning and Lattice QCD

Benjamin Jäger
for the FASTSUM collaboration

UNIVERSITY OF SOUTHERN DENMARK

QUANTUM THEORY CENTER

D·IAS

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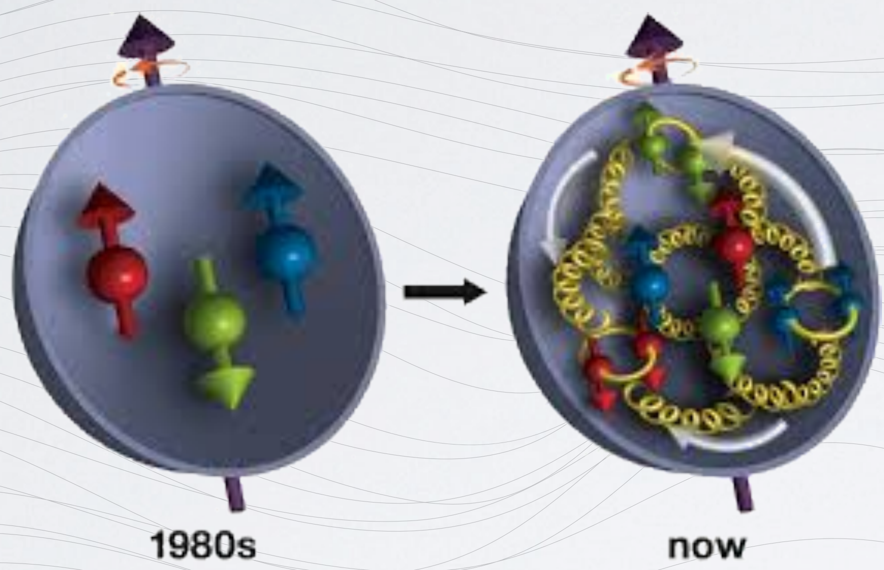
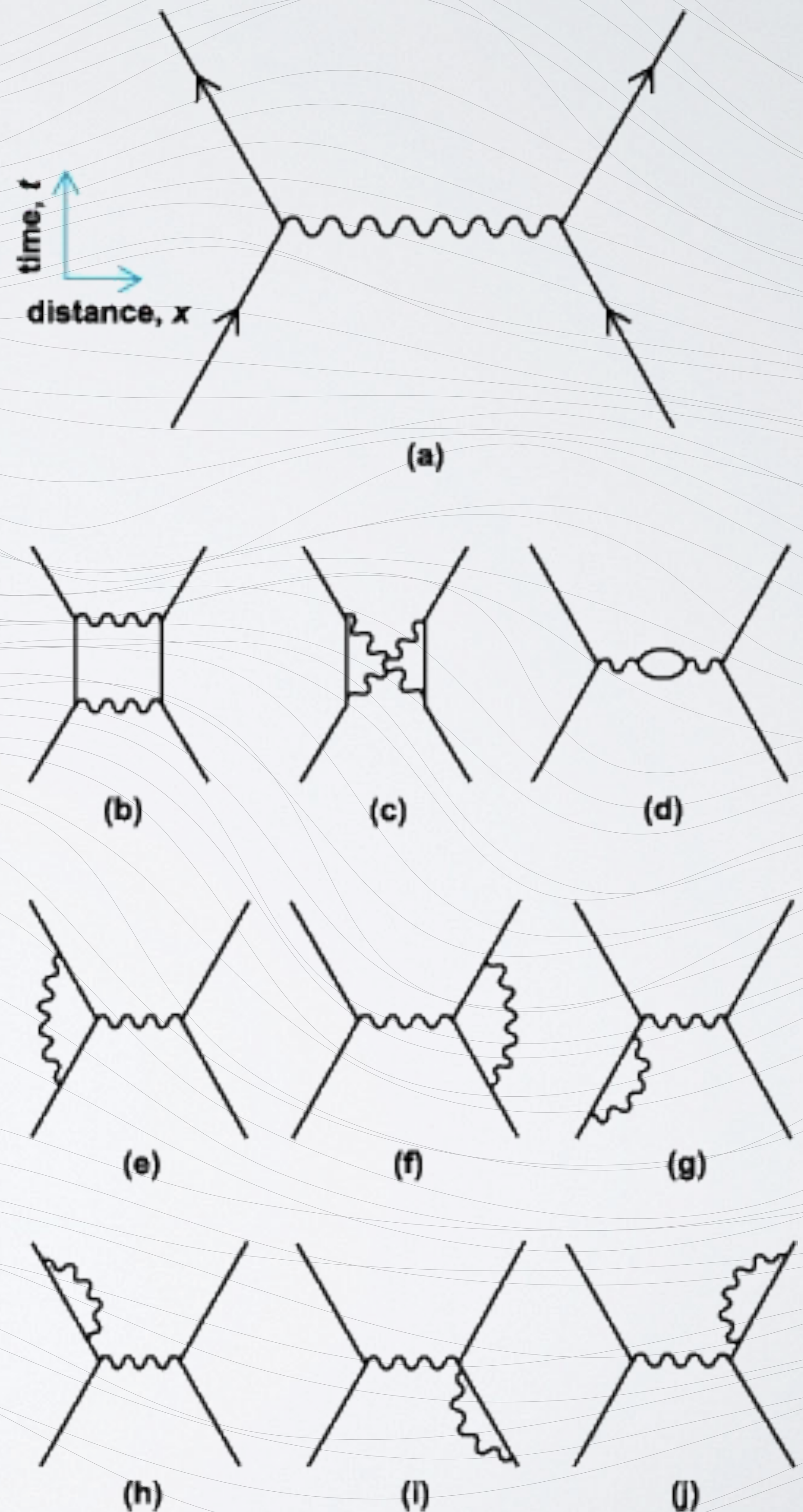
QCD @ small Energies?





- Usually: Expand observables

$$O = \sum_{n=0}^{\infty} c_n \, \alpha^n$$

- At low energies

$$\alpha \sim 1$$

LatticeQCD

Lattice simulations

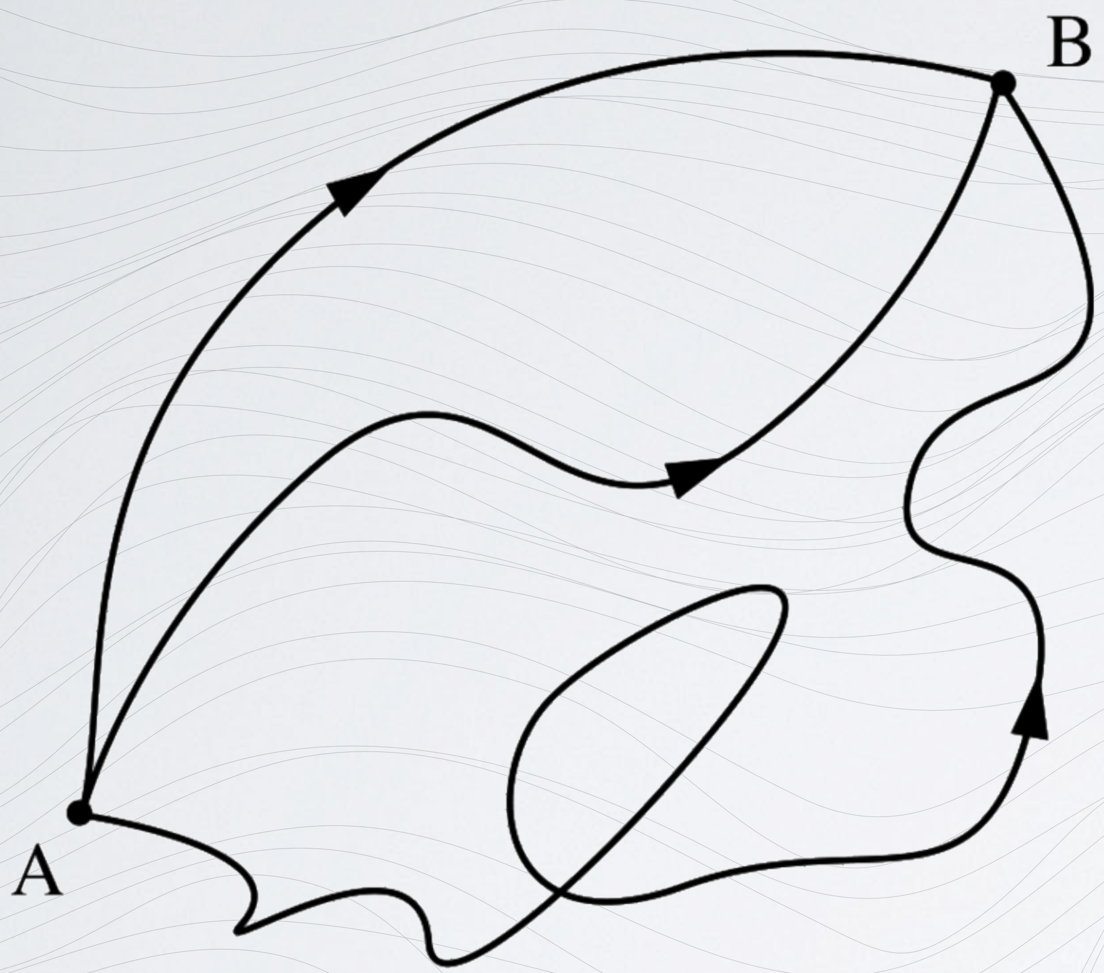
- Discretise space-time by a hyper cubic lattice
- Quantise QCD using Path Integrals
- Calculate observables using Monte Carlo techniques

$$Z = \int \mathcal{D}U \det(M)\, e^{-S_G[U]}$$

Systematically improvable

- Lattice spacing $a \to 0$
- Volume effects $V \to \infty$
- Monte Carlo method $\to$ Statistical uncertainty remains

Euclidean Path Integrals



Path integral formalism

- Integrate over all possible paths

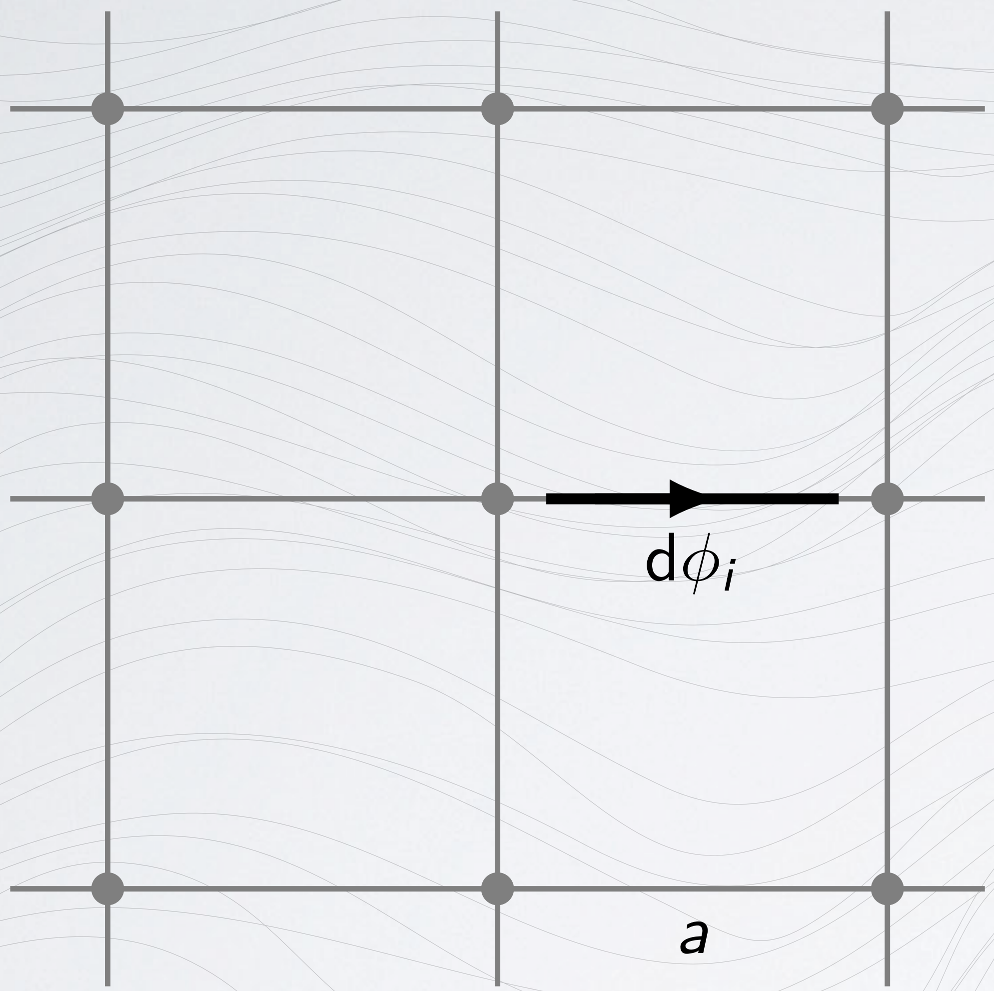
$$A \to B$$

$$Z = \int \mathcal{D}\phi \; \mathrm{e}^{-S[\phi]}$$

- Mathematically not well defined

$$\mathcal{D}\phi = \prod_{i=1}^{\infty} \mathrm{d}\phi_i$$

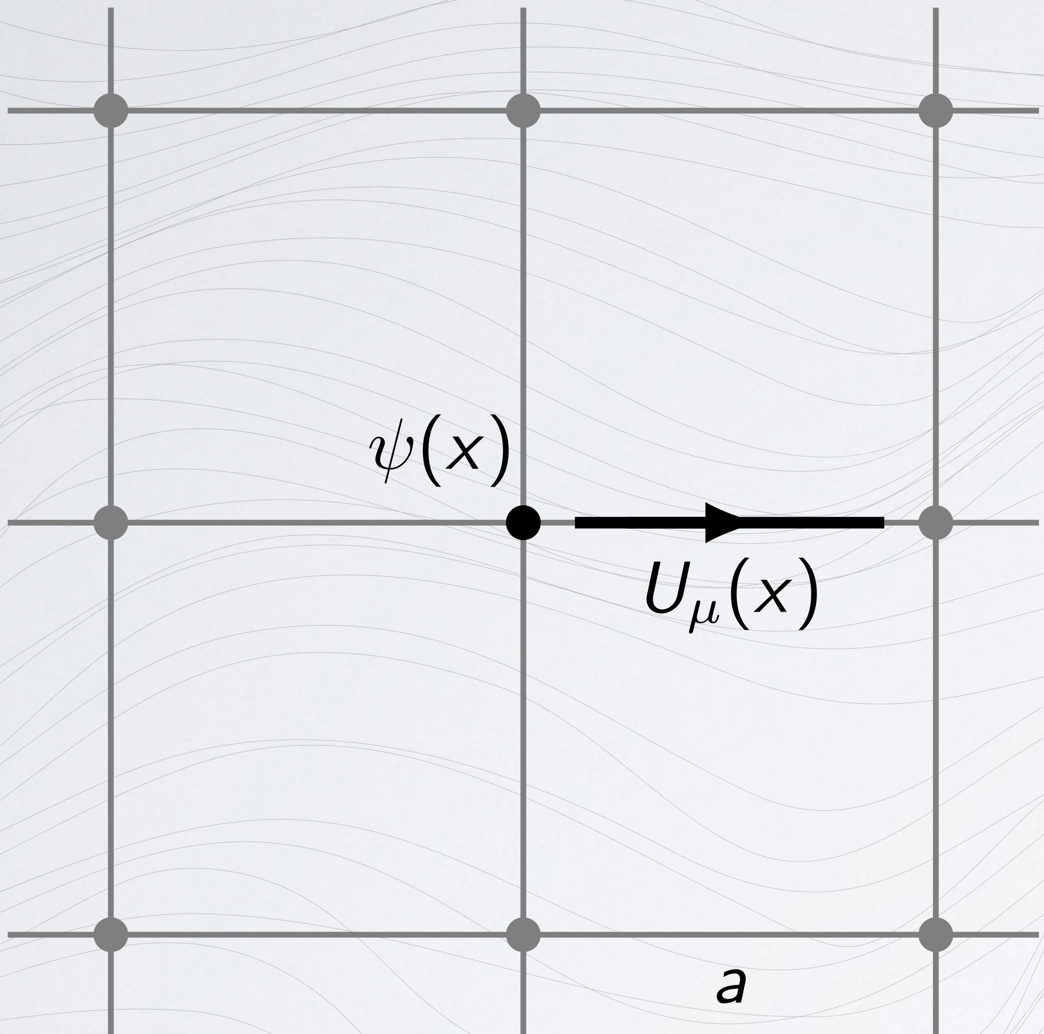
Lattice Simulations



- $4d$ - Lattice in space and time
- Path integration possible

$$Z = \int \mathrm{d}\phi_0 \ldots \mathrm{d}\phi_N \; \mathrm{e}^{-S[\phi]}$$

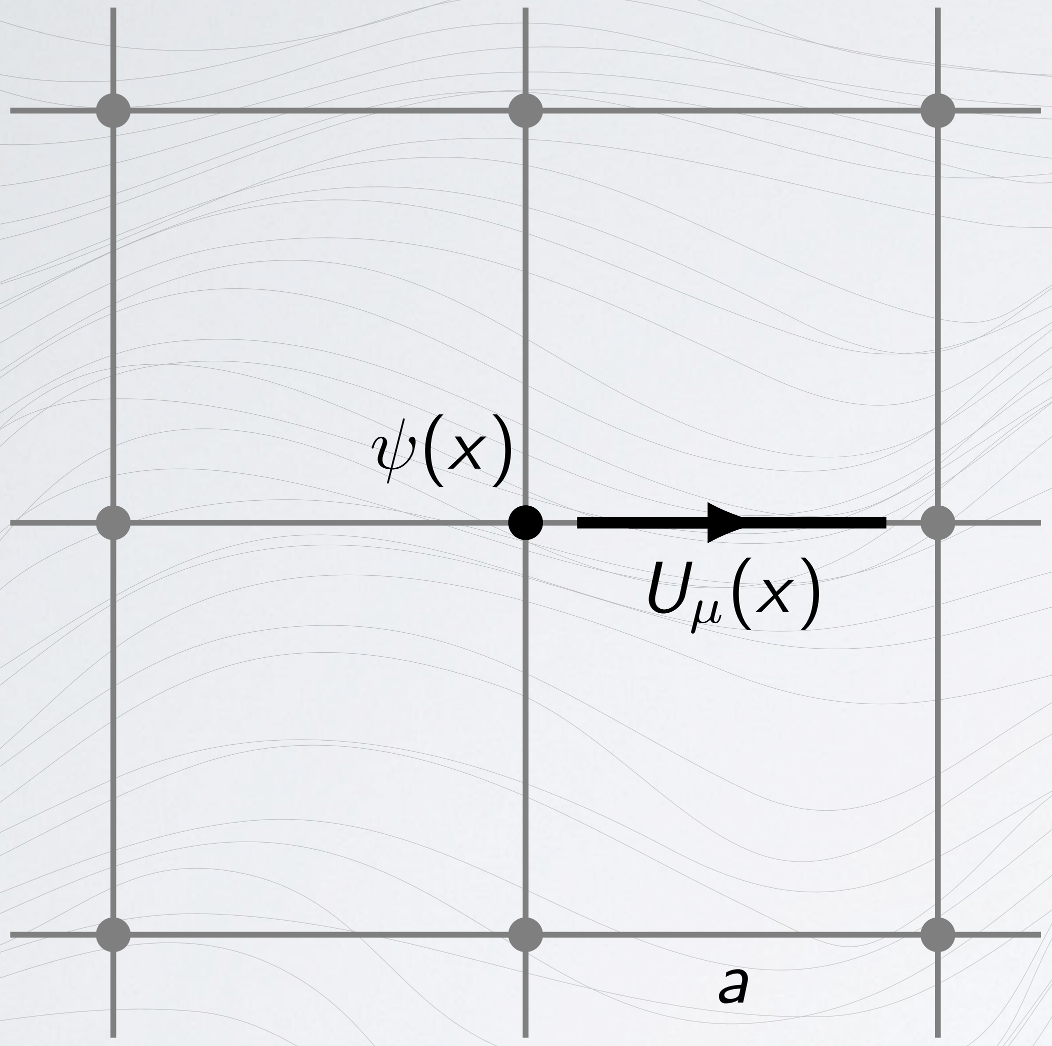
Lattice Simulations



- **QCD**
  - Quarks: ψ, $\overline{\psi}$
  - Gluons: $U_\mu(x) \quad \in SU(3)$

$$Z = \int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\overline{\psi}\,\mathrm{e}^{-S[U,\psi,\overline{\psi}]}$$

Lattice Simulations

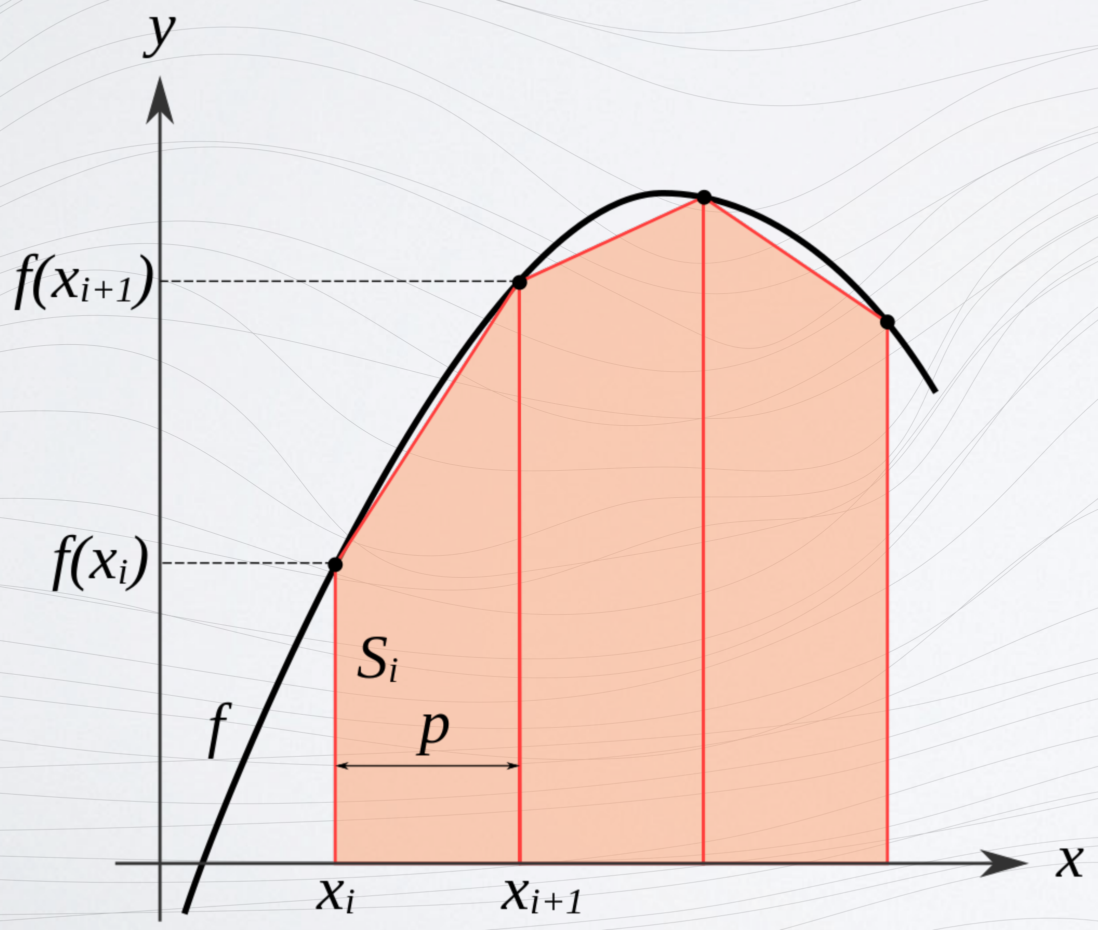


- **Quarks**: $\psi(x)$ anti-commute
  - Grassmann variables :(
  - Integrate out :)
- **Gluons** $(N_c = 3)$: $U_\mu(x)$
  - SU(3) Matrices $\rightarrow$ Haar-Measure $\mathcal{D}U$

$$Z = \int \mathcal{D}U \det(M)\, e^{-S_G[U]}$$

Numerical Integration

$$Z = \int \mathcal{D}U \det(M)\, \mathrm{e}^{-S_G[U]}$$



- **Dimension** (Integral)

$$D = 128 \cdot 64^3 \cdot 8 \cdot 4$$
$$= 1.073.741.824$$

- Trapezoidal rule **impossible**

$$4^{1.073.741.824} \sim 10^{646.456.993}$$

- **Monte Carlo**

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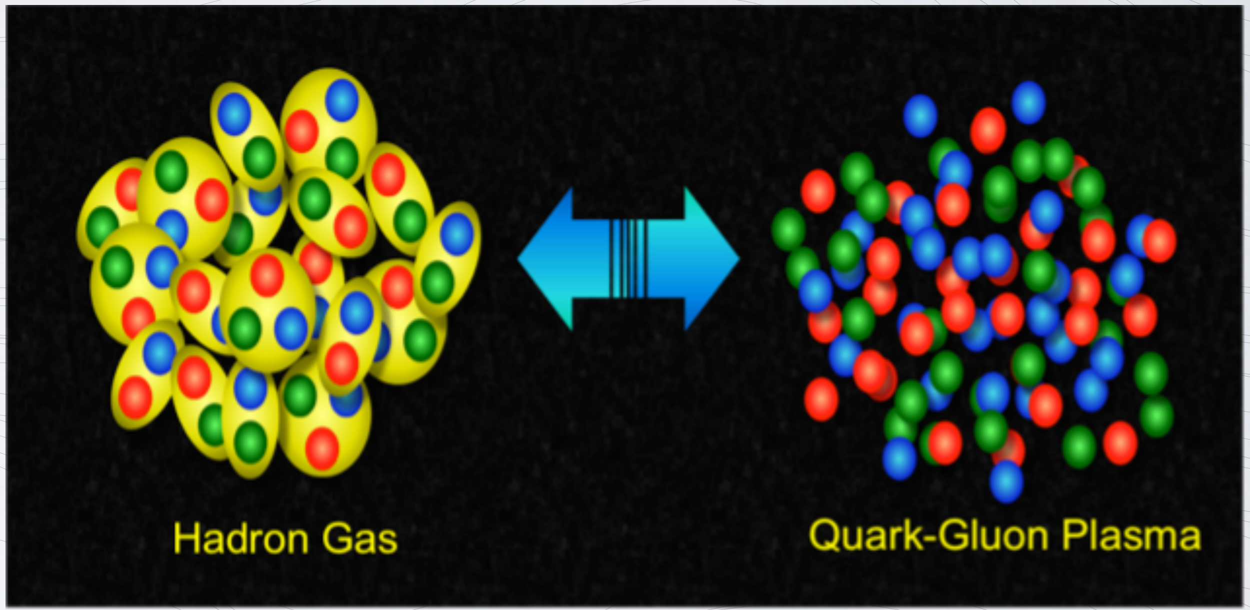
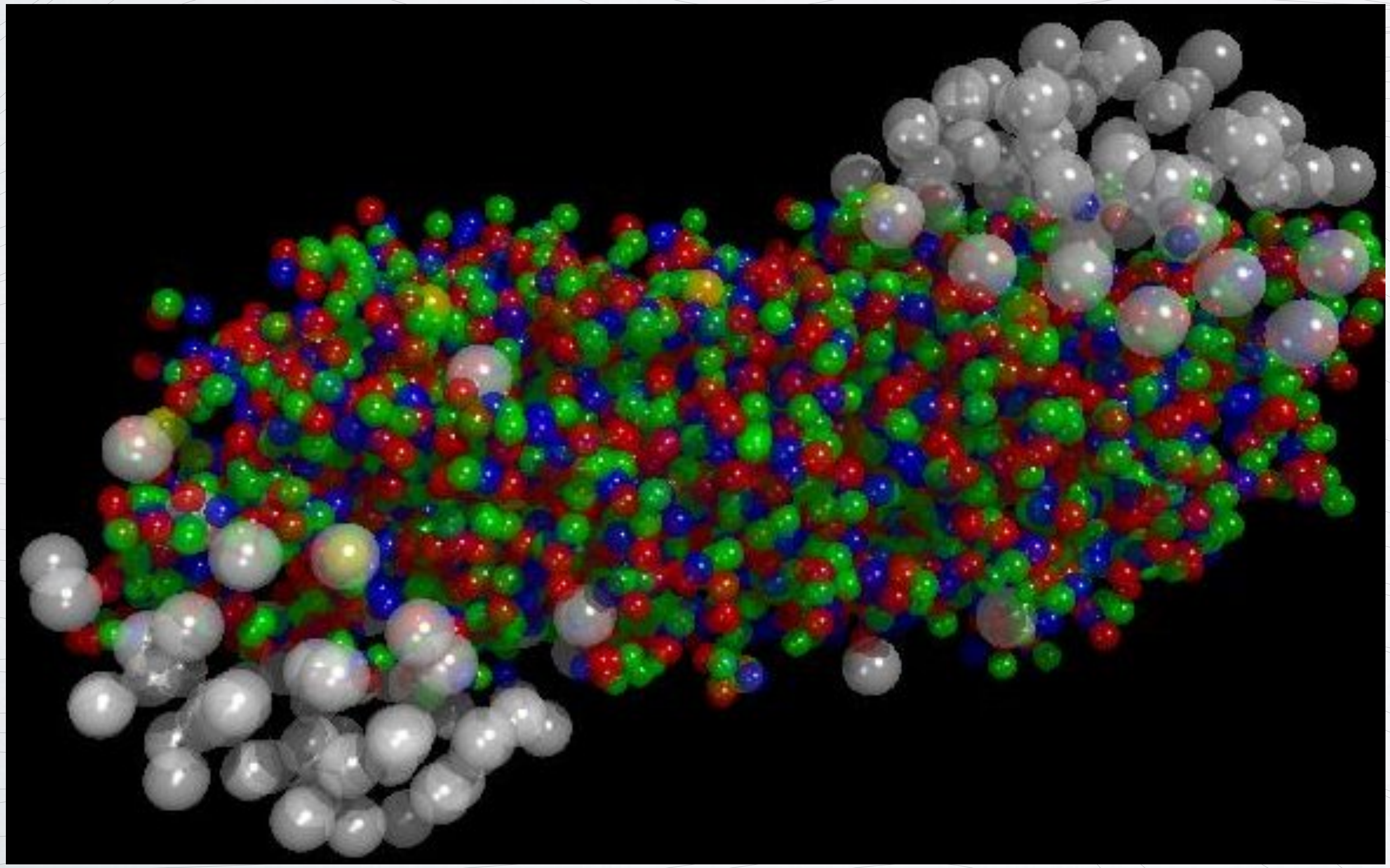
QCD under extreme conditions

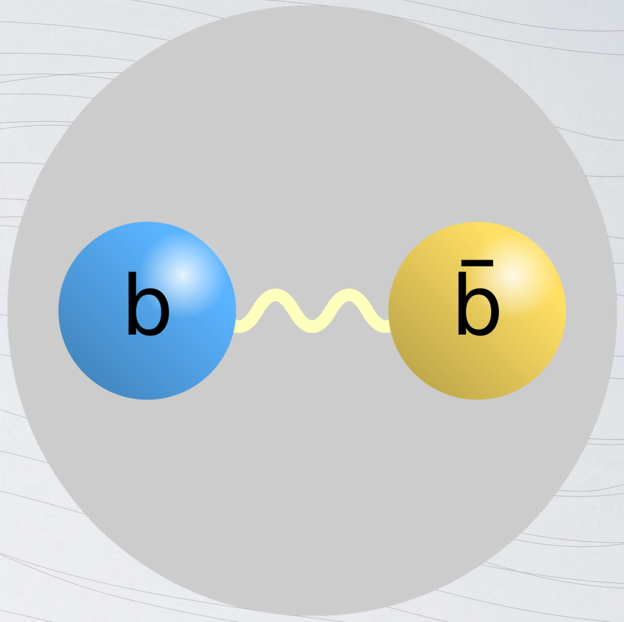
Understand the transition





Heavy-Ion
Collision
Experiments

Quarkonia



Why Bottomonium (or Charmonium)

- Υ meson
- Stongly bound state
- Mass ($m = 9.460\,\mathrm{GeV/c^2}$) is above critical $T_C \sim 151\,MeV$
- Lifetime $\tau = 10^{-20}s$
- Experimentally measurable
- Study properties and dissociation
- Allows to probe the Quark Gluon Plasma

Spectral Function

- **Problem:**
  - We **have** correlation function C_τ **(discrete)**
  - We **want** the spectral density $\rho(\omega)$ **(continuous)**

$$C_\tau = \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{2\pi} K(\tau, \omega)\, \rho(\omega)$$

with known kernel

$$K(\tau, \omega) = e^{-\omega\tau}$$

  - Physics is encoded in the spectral function

Inverse Problem

- **General problem (not just particle physics)**

$$C_\tau = \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{2\pi} K(\tau,\omega)\,\rho(\omega)$$

  - In general: discrete datapoints ↔ (continuous) function

- **Potential solutions**
  - Bayesian approaches
  - Maximum entropy method
  - Backus–Gilbert method
  - Machine Learning
  - …

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Inverse Problem

- **General problem (not just particle physics)**

$$C_\tau = \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{2\pi} K(\tau, \omega)\, \rho(\omega)$$

  - In general: discrete datapoints ↔ (continuous) function

- Make an **ansatz** for the spectral function (N_p number of par)

$$\rho(\omega) = \sum_{p=1}^{N_p} Z_p \exp\left(\frac{-(\omega - m_p)^2}{\Gamma_p} \right)$$

  - Gaussian with variable height, width and peak position

Kernel Ridge Regression (KRR)

- Kernel Ridge Regression: Similar to **linear regression**
- The data target y is a linear a set of functions $\phi(x)$

$$y = w^\top \phi(x)$$

where input data x and parameters w

- Minimize a "cost" function to find optimal parameters w

$$E = \frac{1}{2}\left[y - w^\top \phi(x)\right]^2$$

Kernel Ridge Regression (KRR)

- Kernel Ridge Regression: **Kernel method + Ridge regression**
- **Kernel** function G_{ij} (not unique)

$$G_{ij} = \exp\left(-\gamma \sum_{n_\tau} \left[C_i(n_\tau) - C_j(n_\tau)\right]^2\right) \qquad Y = G\alpha$$

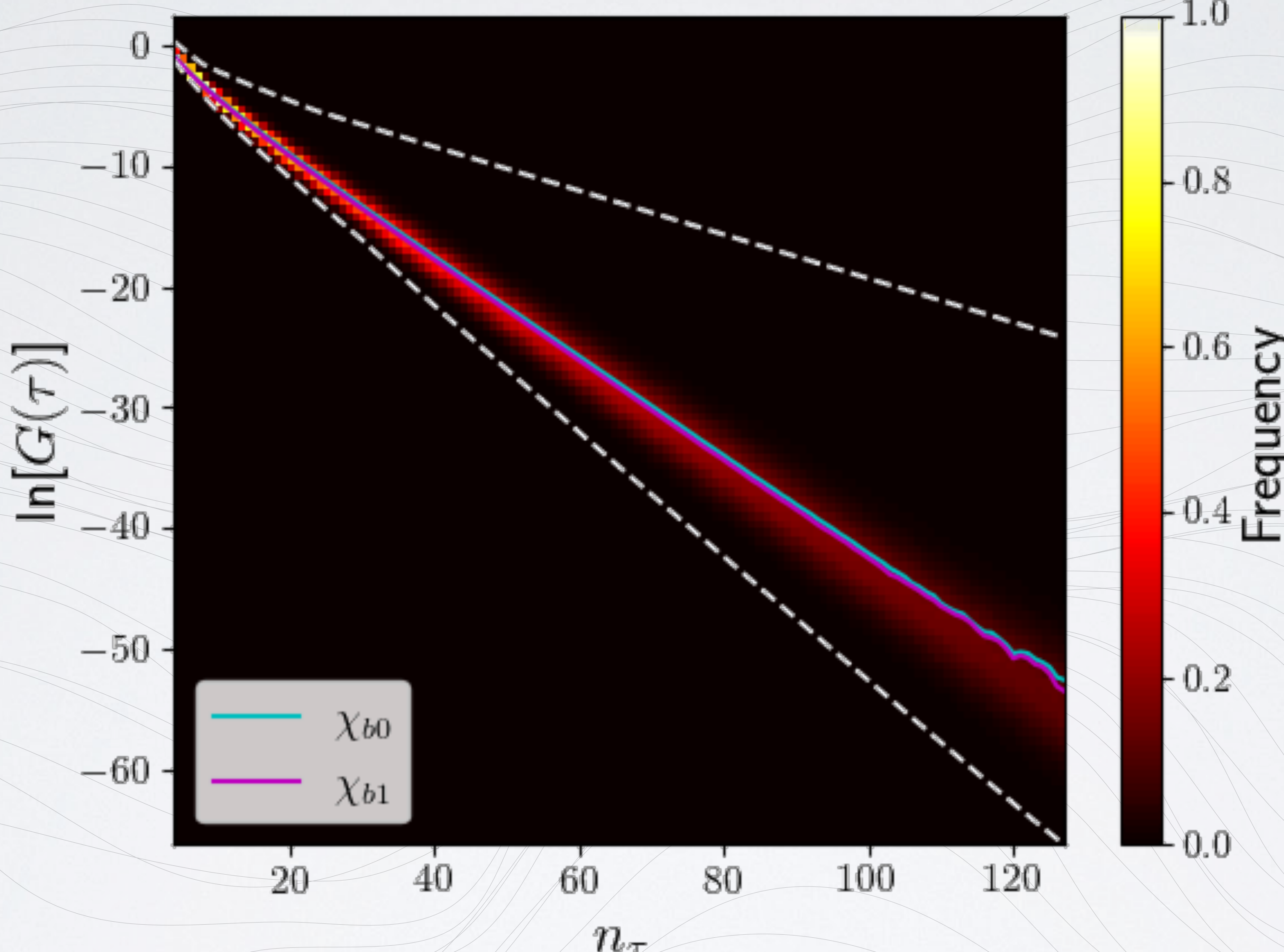
- **Ridge** regression (Regulator for parameters α)

$$\frac{1}{2}\lambda\,\alpha^\top G\alpha$$

- **Cost function** E (to be minimised: Y data and $\hat{Y}$ prediction)

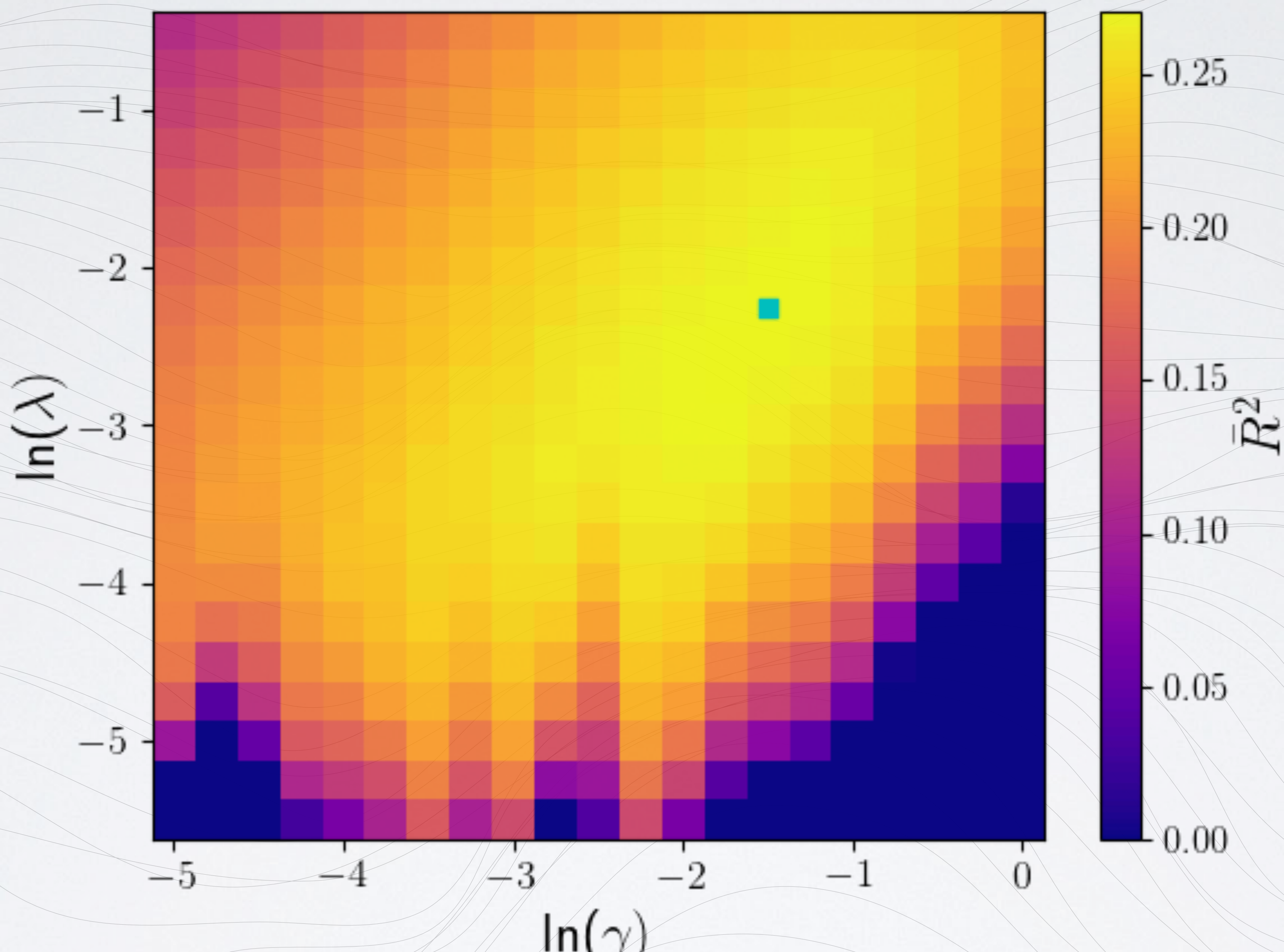
$$E = \sum\left(Y - \hat{Y}\right)^2 + \frac{1}{2}\lambda\,\alpha^\top G\alpha$$

Correlation function C_{n_τ}



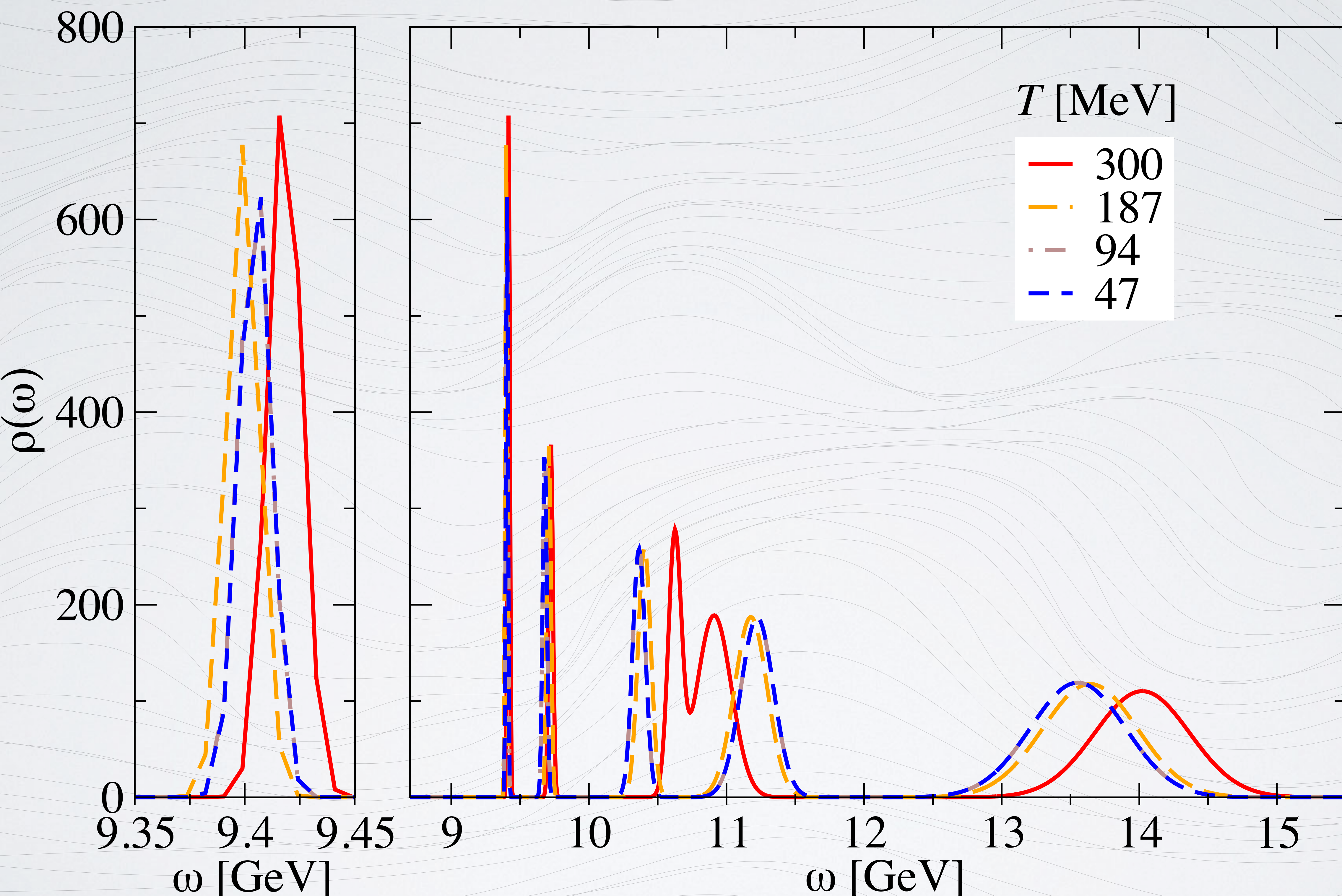
- Solid line is the data
- Heatmap shows the prediction from the ML model

Heatmap of parameters



- Find optimal parameter for spectral reconstruction
- Blue dot is the "best" choice

Result



800
600
400
200
0
ρ(ω)
9.35
9.4
9.45
ω [GeV]
9
10
11
12
13
14
15
ω [GeV]
T [MeV]
300
187
94
47

Limitations

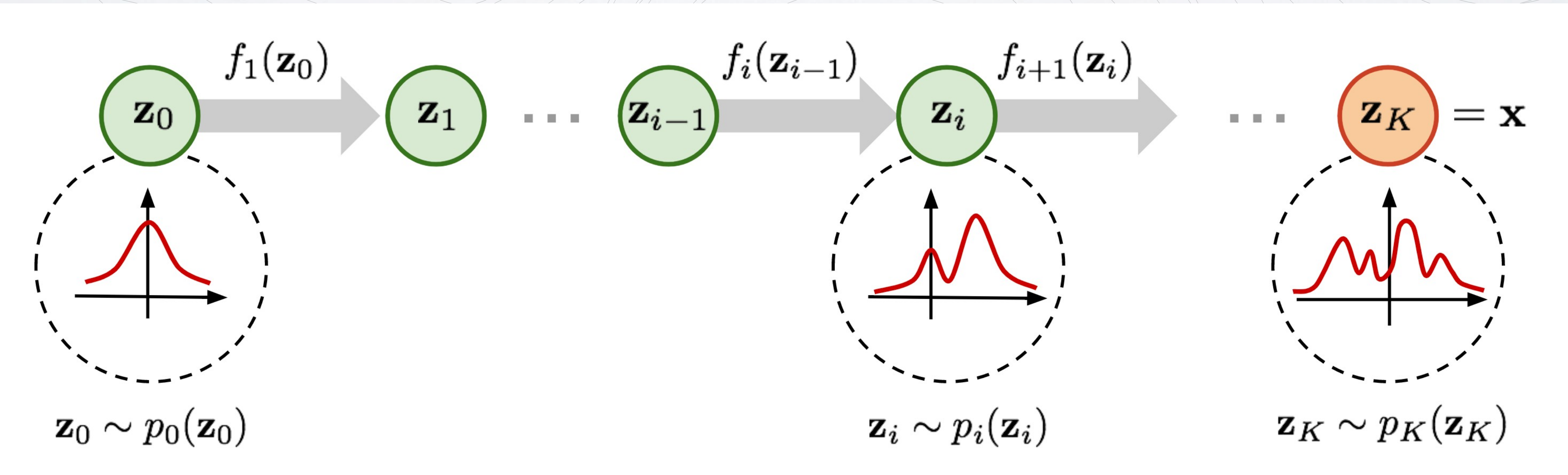
- **Resolution** is determined by **precision** on the data
- Good training data (Mock data) needed to train ML
- Doesn't solve the inverse problem, only a few distributions
- Minimisation can be tricky / **requires** data of **sufficient quality**
- Results are **compatible** to "standard" methods

What next?

- Other Machine Learning approaches
- Unknown spectral density : **Normalising Flows - RealNVP**
- Start with known distribution (fx. Gaussian) and transform

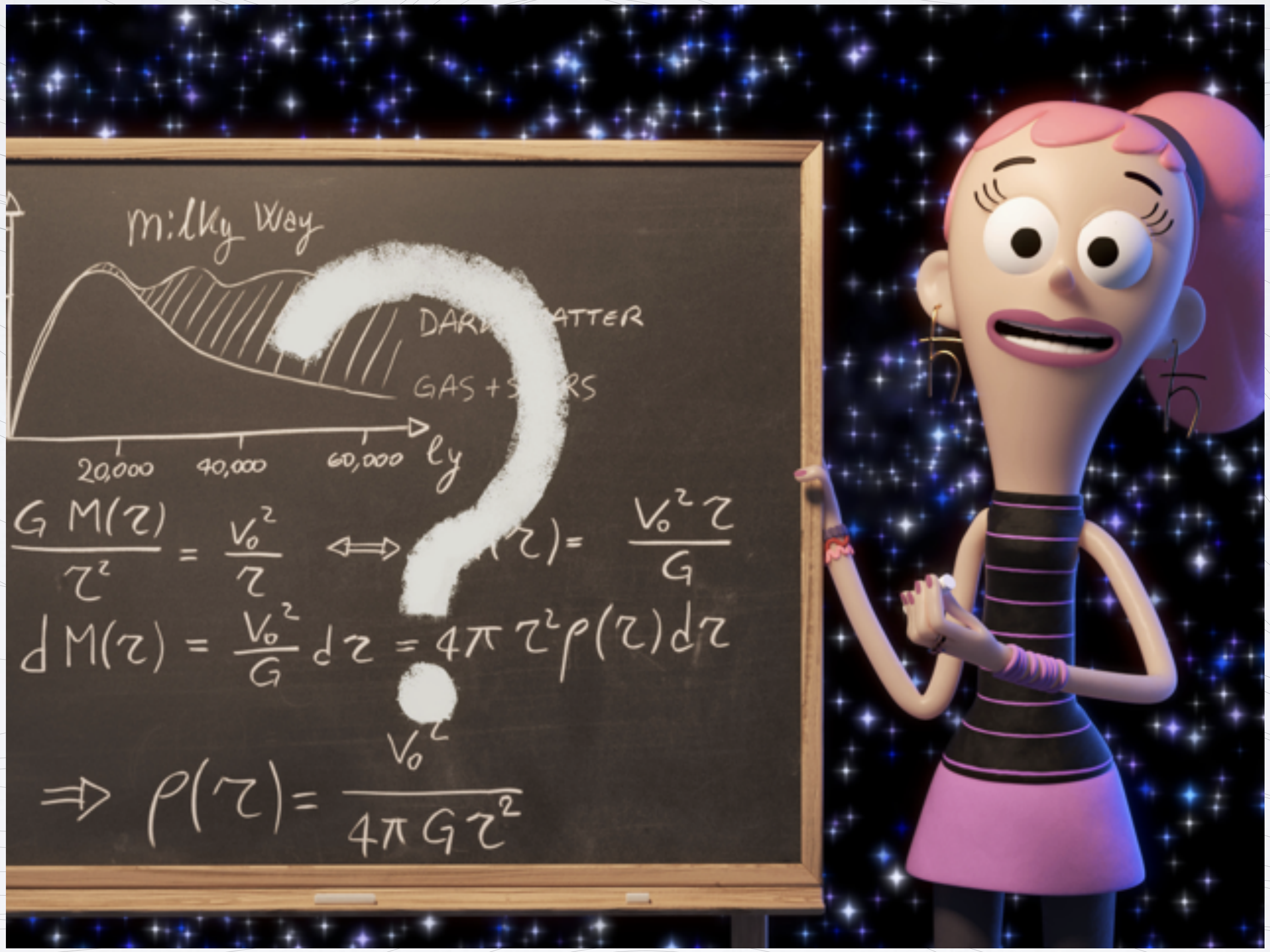
$$x_0 \rightarrow z_1 = x_0$$

$$x_2 \rightarrow z_2 = \mathrm{NN}(x2) + \exp(s_1) \cdot x1$$



Questions?

Thank you for your attention!



Quantum Kate (orig. Kvante Karina): CP3 Outreach http://www.kvantebanditter.dk/en