Bounds on Ultra Heavy HNLs

Neutrinos: Here, There & Everywhere 2025

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Based on [2206.04540], [2409.13412], and [250x.xxxxx]





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Standard Model

The Standard Model is the most precise scientific theory

- Electroweak interactions modeled in the 60s by Weinberg, Salam, and Glashow
- Describes the interactions between elementary particles
- No deviations so far...



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However the SM can't explain

- Neutrino oscillations/masses
- Baryon asymmetry of the universe
- Dark matter
- CP strong problem
- Hierarchy problem
- Anomalies (?)



Mikowski (1977), Gell-Mann, et al. (1979), Mohapatra and Senjanović (1979), Yanagida (1980), Glashow (1980), ... Simplest way of adding neutrino masses: add right-handed neutrinos.

New terms in the Lagrangian with right-handed neutrinos

$$\mathcal{L} = \mathcal{L}_{ ext{SM}} + i ar{N}_R \, {\partial \!\!\!/} N_R - ar{L}_L \cdot ilde{H} \, Y \, N_R - rac{1}{2} ar{N}^C_R \, M_M \, N_R + ext{H.c.} \, ,$$

After SSB, *N* and ν mix in their mass terms

$$\mathcal{L}_{ ext{mass}} = -rac{1}{2} egin{pmatrix} ar{
u}_L & ar{N}_R^C \end{pmatrix} egin{pmatrix} 0 & M_D \ M_D^T & M_M \end{pmatrix} egin{pmatrix}
u_L^C \ N_R \end{pmatrix} + ext{H.c.} \,.$$



Diagonalization gives mass spectrum

$$M_
u \simeq -M_D^T rac{1}{M_N} M_D\,, \qquad \qquad M_N \simeq M_M\,.$$

We expect $M_N > \mathcal{O}(\text{GeV})$. This gives us additional particles that are heavy, that are neutral, and that are leptons. *HNLs*

HNLs interact with SM particles proportionally to a mixing angle $\Theta_e, \Theta_\mu, \Theta_\tau$.



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How to search for heavier HNLs?

- Not feasible to directly search for heavy HNLs, can only place bounds indirectly
- $\bullet\,$ HNLs can mediate cLFV processes that are not allowed in the SM, such as
 - $\mu \to e \gamma$
 - $\mu \rightarrow eee$
 - $\mu \rightarrow e$ conversion in nucleus
- The non-observation of such processes places bounds on HNL parameters
- Not a new idea, decay rates have been known for years

Petcov (1976), Bilenky, et al (1977), Marciano and Sanda (1977), Minkowski (1977), Cheng and Li (1980), Lim and Inami (1981), Langacker and London (1988), Pilaftsis (1992), Ilakovac and Pilaftsis (1994), Chang, et al. (1994), Pilaftsis (1998), Ioannisian and Pilaftsis (1999), Illana, et al. (1999), Illana and Riemann (2000), Pascoli, et al. (2003), Pascoli, et al. (2003), Pilaftsis and Underwood (2005), Deppisch, et al. (2005), ...

How to search for heavier HNLs?



Non-decoupling and new bounds

However, in presenting these bounds, the recent literature have not properly taken into consideration the effect of non-decoupling diagrams. The shape of the decay width of some cLFV should be

$$\Gamma \propto \left|\Theta^2 + \Theta^4 \left(rac{M_N}{M_W}
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Intermezzo (work in progress)

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Work in progress

cLFV processes involving mesons can also provide competitive bounds. Processes like $\tau \rightarrow \ell_{\alpha}M, \tau \rightarrow \ell_{\alpha}MM$.

Strongest ones are $\tau \to \rho^0 \, \ell_{\alpha}$ and $\tau \to \phi^0 \, \ell_{\alpha}$.



Bounds on Ultra Heavy HNLs

Perturbativity line?

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Where does the perturbativity line come from?

We can get a measure of the perturbativity of a theory by using perturbative unitarity. The unitarity condition of the S matrix, brings certain condition to scattering amplitudes

$$\mathcal{M} = 16\pi \sum_{J} (2J+1) d^{J}_{\mu_{i},\mu_{f}} a^{J},$$

 a^J are the partial waves (or the scattering amplitude with transferred J angular momentum). On $2\to 2$ elastic scatterings, unitarity demands the inequalities

$$\left|a^{J}
ight|\leq1\,, \qquad \quad 0\leq\left|\mathrm{Im}(a^{J})
ight|\leq1\,, \qquad \quad \left|\mathrm{Re}(a^{J})
ight|\leqrac{1}{2}\,.$$

Any scattering amplitude should automatically satisfy it. However, for tree-level computations alone cannot properly satisfy them for all coupling constants. For J = 0, we have the elastic scatterings

$$egin{aligned} &N_\pm\,\ell_\pm^\pm\leftrightarrow N_\pm\,\ell_\pm^\pm\,,\ &N_\pm\,
u_\pm\leftrightarrow N_\pm\,
u_\pm\,. \end{aligned}$$

Both processes have the same partial wave

$$a^{J=0} = -\frac{\left|Y_{\rm tot}\right|^2}{16\pi}\,,$$

for the unitarity of the S matrix to be maintained, we demand that

$$\left|Y_{\mathrm{tot}}\right|^2 \leq 8\pi$$
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This replicates a result widely used in different literature (up to a factor of 2)

$$rac{\Gamma_N}{M_N} \leq rac{1}{2} \implies |Y_{
m tot}|^2 \leq 4\pi$$

 $\left\{N_{-}N_{+},\nu_{-}\nu_{+},\ell_{-}^{-}\ell_{+}^{+},\phi_{0}^{0}\phi_{0}^{0*},\phi_{0}^{+}\phi_{0}^{-}\right\}\leftrightarrow\left\{N_{-}N_{+},\nu_{-}\nu_{+},\ell_{-}^{-}\ell_{+}^{+},\phi_{0}^{0}\phi_{0}^{0*},\phi_{0}^{+}\phi_{0}^{-}\right\}\,,$

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$$\mathbf{a}^{J=1} = \frac{\left|Y_{\text{tot}}\right|^2}{32\,\pi} \left($$

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$$egin{aligned} & N_- \, N_+ \ &
u_- \,
u_+ \ & \ell_-^0 \, \ell_+^+ \ & {
m Initial} \ & {
m states} \ & \phi_0^0 \, \phi_0^{0*} \ & \phi_0^+ \, \phi_0^- \ \end{aligned}$$

J = 1 results

For J = 1 we can have the set of scatterings

 $\left\{N_{-}N_{+},\nu_{-}\nu_{+},\ell_{-}^{-}\ell_{+}^{+},\phi_{0}^{0}\phi_{0}^{0*},\phi_{0}^{+}\phi_{0}^{-}\right\}\leftrightarrow\left\{N_{-}N_{+},\nu_{-}\nu_{+},\ell_{-}^{-}\ell_{+}^{+},\phi_{0}^{0}\phi_{0}^{0*},\phi_{0}^{+}\phi_{0}^{-}\right\}\,,$

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$$\mathbf{a}^{J=1} = \frac{|Y_{\text{tot}}|^2}{32\pi} \begin{pmatrix} 0 & 1 & 1 & -\sqrt{2} & -\sqrt{2} \\ 1 & 0 & 0 & -\sqrt{2} & 0 \\ 1 & 0 & 0 & 0 & -\sqrt{2} \\ -\sqrt{2} & -\sqrt{2} & 0 & 0 \\ -\sqrt{2} & 0 & -\sqrt{2} & 0 & 0 \\ -\sqrt{2} & 0 & -\sqrt{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} N_- N_+ \\ \nu_- \nu_+ \\ \ell_-^- \ell_+^+ \\ \phi_0^0 \phi_0^{0*} \\ \phi_0^+ \phi_0^- \end{pmatrix}$$

 $\left\{ N_{-} N_{+}, \nu_{-} \nu_{+}, \ell_{-}^{-} \ell_{+}^{+}, \phi_{0}^{0} \phi_{0}^{0*}, \phi_{0}^{+} \phi_{0}^{-} \right\} \leftrightarrow \left\{ N_{-} N_{+}, \nu_{-} \nu_{+}, \ell_{-}^{-} \ell_{+}^{+}, \phi_{0}^{0} \phi_{0}^{0*}, \phi_{0}^{+} \phi_{0}^{-} \right\} ,$ we can write all the partial amplitudes in a matrix

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We can get bounds by diagonalizing the matrix. Strongest bound comes from the largest eigenvalue.

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$$\mathbf{a}^{J=1} = \frac{|Y_{\text{tot}}|^2}{32 \pi} \begin{pmatrix} 1 + \sqrt{5} & & & \\ & 1 - \sqrt{5} & & & \\ & & \sqrt{2} & & \\ & & & -\sqrt{2} & \\ & & & & -2 \end{pmatrix}$$

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Best bound:

$$|Y_{ ext{tot}}|^2 \leq rac{8\pi}{arphi}\,.$$

J = 1 results

For J = 1 we can have the set of scatterings



$$\left|Y_{\mathrm{tot}}\right|^2 \leq rac{8\pi}{arphi} \, .$$

- Charged lepton flavour violating processes allow us to probe HNLs with masses that experiments will never be capable of probing
- These are further enhanced by the non-decoupling behaviour of the processes, which makes the bounds more sensitive to heavier HNL masses
- Perturbative unitarity tells us that $|Y_{\rm tot}|^2 \le 8\pi/\varphi$ as long as we want tree-level unitarity to hold

Bounds on Ultra Heavy HNLs

Backup slides

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Interactions between N and the rest of SM particles is proportional to the mixing angle Θ

$$\Theta = M_D \, rac{1}{M_N} \, .$$

Naively, the mixing angle should be proportional to $\Theta \propto \sqrt{m_{\nu}/M_N}$. However, we can choose parametrizations of that preserve small m_{ν} and large Θ .

Casas-Ibarra parametrization [Casas and Ibarra (2001)]

$$\Theta = i\,V^{
m PMNS}\,\sqrt{m_
u}\,O\,rac{1}{\sqrt{M_N}}\,,$$

where O is an arbitrary (semi-)orthogonal matrix.



Bounds on Ultra Heavy HNLs

Diagrams



We shall do the same analysis on the minimal type-I seesaw model. There are a few theoretical caveats

• In the limit $s \to \infty$, we can take advantage of the *Goldstone equivalence* theorem, and only consider interactions with scalars

$$\mathcal{M}(W_L^{\pm}, Z_L, \dots) = (i C)^n \mathcal{M}(\phi^{\pm}, \phi_Z, \dots)$$

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Only interactions that matter:

$$\mathcal{L}_{ ext{int.}} = -ar{
u}_lpha \, Y_{lpha,i} \, P_R \, N_i \, (h - i \phi^Z) + ar{\ell}_lpha \, Y_{lpha,i} \, P_R \, N_i \, \phi^- + ext{H.c.}$$

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• Multiple flavors of leptons and generations of HNLs complicate things old w

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$$\mathcal{M}(W_L^{\pm}, Z_L, \dots) = (i C)^n \mathcal{M}(\phi^{\pm}, \phi_Z, \dots)$$

 $\mathcal{L}_{\text{int.}} = -\bar{\nu} Y_{\text{tot.}} P_R N \phi^{0*} + \bar{\ell} Y_{\text{tot.}} P_R N \phi^- + \text{H.c.}$



• Multiple flavors of leptons and generations of HNLs complicate things 😦

However, choosing a parametrization of the Yukawa that is rank-one, makes interactions as if only one HNL and one lepton flavor interact \bigcirc

<u>Yukawa is rank-one?</u>

Casas-Ibarra parametrization

$$Y = i \, rac{\sqrt{2}}{v} \, V^{ ext{PMNS}} \sqrt{m_
u} \, O \sqrt{M_N} \, .$$

for 2 HNLs, and in the case of normal ordering

$$O = \begin{pmatrix} 0 & 0\\ \cos \omega & \sin \omega\\ -\sin \omega & \cos \omega \end{pmatrix} \simeq e^{-i\omega} \begin{pmatrix} 0 & 0\\ 1 & -i\\ i & 1 \end{pmatrix}$$

if $\operatorname{Im}(\omega) \gg 1$.

Other popular parametrization for 3 HNLs:

$$Y = egin{pmatrix} Y_e & i \, Y_e & 0 \ Y_e & i \, Y_\mu & 0 \ Y_e & i \, Y_ au & 0 \end{pmatrix} \,,$$

is also rank-one.

<u>Results at the Seesaw line</u>

We can do the same analysis at the seesaw line, it is interesting since it gives us an "upper-bound of the HNL mass". At the seesaw line:

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previous bounds are not valid, Yukawa matrix is not rank-one.



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	$M_N < \dots \; [{ m GeV}]$	
	Normal ordering	Inverted ordering
J = 0	1.30×10^{16}	8.01×10^{15}
$J=rac{1}{2}$	1.52×10^{16}	1.59×10^{16}
J = 1	9.33×10^{15}	7.60×10^{15}

Results for general shape of Yukawa

For J = 0 the results hold for any shape of the Yukawa matrix. This is because the partial wave matrix will have the shape

$$\mathbf{a}^{J=0} = -\frac{1}{16\pi} \begin{pmatrix} Y_{e1}^* \\ Y_{\mu1}^* \\ \vdots \\ Y_{\tau\mathcal{N}}^* \end{pmatrix} \begin{pmatrix} Y_{e1} & Y_{\mu1} & \cdots & Y_{\tau\mathcal{N}} \end{pmatrix},$$

is rank-one. Only non-zero eigenvalue is the trace.

 $J = \frac{1}{2}$ general results give

$$\mathbf{a}^{J=\frac{1}{2}} = -\frac{1}{16\pi} Y Y^{\dagger},$$

whose eigenvalues in general do not have a nice shape. However, regardless of the number of additional HNLs, the matrix only has three non-zero eigenvalues.

J = 1 general results

For ${\mathcal N}$ HNLs, the J=1 matrix becomes a $({\mathcal N}^2+20)\times ({\mathcal N}^2+20)$ matrix

$$\mathbf{a}^{J=1} = \frac{1}{32\pi} \begin{pmatrix} 0 & \mathbf{Y} & \mathbf{Y} & -\sqrt{2}\,\mathcal{Y} & -\sqrt{2}\,\mathcal{Y} \\ \mathbf{Y}^{\dagger} & 0 & 0 & -\sqrt{2}\,\tilde{\mathcal{Y}} & 0 \\ \mathbf{Y}^{\dagger} & 0 & 0 & 0 & -\sqrt{2}\,\tilde{\mathcal{Y}} \\ -\sqrt{2}\,\mathcal{Y}^{\dagger} & -\sqrt{2}\,\tilde{\mathcal{Y}}^{\dagger} & 0 & 0 & 0 \\ -\sqrt{2}\,\mathcal{Y}^{\dagger} & 0 & -\sqrt{2}\,\tilde{\mathcal{Y}}^{\dagger} & 0 & 0 \end{pmatrix},$$

with

$$\mathbf{Y} = \begin{pmatrix} |Y_{e1}|^2 & Y_{e1} Y_{\mu 1}^* & Y_{e1} Y_{\tau 1}^* & |Y_{\mu 1}|^2 & Y_{\mu 1} Y_{e1}^* & Y_{\mu 1} Y_{\tau 1}^* & |Y_{\tau 1}|^2 & Y_{\tau 1} Y_{\tau 1} \\ \vdots & \vdots \\ Y_{e\mathcal{N}} Y_{e1}^* & Y_{e\mathcal{N}} Y_{\mu 1}^* & Y_{e\mathcal{N}} Y_{\tau 1}^* & Y_{\mu \mathcal{N}} Y_{\mu 1}^* & \cdots & \cdots & \cdots \\ Y_{e1} Y_{e2}^* & Y_{e1} Y_{\mu 2}^* & Y_{e1} Y_{\tau 2}^* & Y_{\mu 1} Y_{\mu 2}^* & \cdots & \cdots & \cdots \\ \vdots & \vdots \\ |Y_{e\mathcal{N}}|^2 & Y_{e\mathcal{N}} Y_{\mu \mathcal{N}}^* & Y_{e\mathcal{N}} Y_{\tau \mathcal{N}}^* & |Y_{\mu \mathcal{N}}|^2 & \cdots & \cdots & \cdots \\ \end{pmatrix}$$

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with

For J=0, we have HNLs in the final and initial state. Conditions on partial waves change

$$\left|a^{J}
ight|\leqrac{\sqrt{s}/2}{\left|ec{p}_{f}
ight|}\,,\qquad 0\leq\mathrm{Im}[a^{J}]\leqrac{\sqrt{s}/2}{\left|ec{p}_{f}
ight|}\,,\qquad \left|\mathrm{Re}[a^{J}]
ight|\leqrac{1}{2}\,rac{\sqrt{s}/2}{\left|ec{p}_{f}
ight|}\,.$$

for $J = \frac{1}{2}$ the bounds change because we have a resonance.

J = 1 states have both HNLs in the final and initial state, as well as resonances. Not clear how to proceed.

