IV International PhD Summer School on Neutrinos: Here, There & Everywhere.

Exercises Neutrino Theory & Phenomenology - Mariam Tórtola

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1. **How difficult is it to observe a neutrino?**

   Already since 1934, after the first calculations by Bethe and Peierls, it is known that the neutrino interaction cross sections are very small compared to other processes. For instance, the inverse beta decay process
   $$\overline{\nu}_e \, p \to n \, e^+ \,,$$
   used as target reaction in reactor experiments, has a cross section of the order of 10^{-43} cm^2 for a 2 MeV neutrino.

   a) Estimate the neutrino mean free path in water for the process above. The mean free path is the average distance traveled by a particle in a given medium between two successive interactions, and can be calculated through the expression
   $$\lambda = (n\sigma)^{-1} \,.$$
   Here σ is the cross-section of the process and n is the number density of target particles per unit volume. Consider only the free protons in the water molecule as target particles.

   b) The event number in a neutrino experiment is obtained (at first approximation) from the convolution of the initial neutrino flux at the detector, the cross section of the process under study, the number of target particles in the detector and the exposure time of the experiment.
   For a neutrino process with cross section around 10^{-43} cm^2, estimate the size of the detector required to observe few neutrino events per day in a solar and a reactor neutrino experiment. Consider a solar neutrino flux of 10^{10} cm^{-2}s^{-1}. For the reactor experiment, consider an antineutrino production rate of 10^{20} s^{-1} and a detector located at 1 km from the reactor core.

2. **Dirac and Majorana neutrinos**

   a) Before detecting solar neutrinos at the Homestake experiment, Raymond Davis tried to observe reactor antineutrinos with the same radiochemical technique he used for neutrinos, namely
   $$\nu_e +^{37}\mathrm{Cl} \to^{37}\mathrm{Ar} + e^- \qquad (1)$$
   Why he did not succeed? Does it mean that neutrino is different from antineutrino and therefore it is a particle of Dirac-type?

   b) If $0\nu\beta\beta$ is not observed by the new generation of experiments, does it necessarily mean that neutrinos are Dirac particles?

   c) In the minimal seesaw model, one can explain the light neutrino masses by introducing very heavy right-handed neutrinos. Estimate the scale of the Majorana mass of right-handed neutrinos required to obtain light neutrino masses, of the order of the current cosmological bounds. Assume only one neutrino flavour and consider Dirac masses of the order of the electroweak scale (Y $\sim$ 1).

3. **Neutrino mixing**

   a) How many angles and phases (Dirac and Majorana) do we need to describe a unitary neutrino mixing matrix of dimension N? For N=3?

b) Some models of neutrino masses introduce new fermionic states that naturally mix with the three light neutrinos. In that case, neutrino mixing will be described by a unitary NxN matrix and, therefore, the 3x3 matrix will not be unitary in general. How many extra parameters do we need to describe a 3x3 non-unitary mixing matrix?

4. **The neutrino oscillation probability in vacuum**

Neutrino flavor states and neutrino mass states are related through the following relation

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle \ , \tag{2}$$

where both bases are orthonormal

$$\langle \nu_\alpha | \nu_\beta \rangle = \delta_{\alpha\beta} \ , \langle \nu_i | \nu_j \rangle = \delta_{ij} \ . \tag{3}$$

The massive eigenstates are eigenvectors of the free Hamiltonian

$$H |\nu_k\rangle = E_k |\nu_k\rangle \ , \tag{4}$$

with the eigenvalues given by $E_k = \sqrt{p^2 + m^2}$. With this information one can derive the amplitude for the transition $\nu_\alpha \to \nu_\beta$: $\langle \nu_\beta(t) | \nu_\alpha(t_0) \rangle$. Considering the unitarity conditions of the neutrino mixing matrix U one can obtain the neutrino oscillation probability:

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{k>j} \mathcal{R} \left(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \right) \sin^2 \left(\frac{\Delta m^2_{kj} L}{4E} \right) + 2 \sum_{k>j} \mathcal{I} \left(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \right) \sin \left(\frac{\Delta m^2_{kj} L}{2E} \right) . \tag{5}$$

5. **Neutrino oscillations in vacuum and matter**

a) We have now plenty of evidences for neutrino oscillations from several sources. Would it be possible to observe flavor oscillations of charged leptons?

b) According to the oscillation probability expression, do neutrino oscillations preserve CP invariance? Is it preserved in 2-neutrino oscillations? Is there any difference between appearance and disappearance experiments?

c) Neutrino oscillation experiments have measured very accurately the solar and atmospheric mass splittings. Reactor experiments have proven to be a very powerful tool, since they can be sensitive to the two splittings, with a different choice for the baseline between reactor and detector. Estimate the value of these baselines.

d) Neutrino interactions with matter in the Sun affect solar neutrino oscillation probabilities. Using the two–neutrino survival probability given by Parke's formula

$$P(\nu_e \to \nu_e) = \frac{1}{2} \left[1 + \cos 2\theta \cos 2\theta_m \right] \ ,$$

show why the oscillation probability is different for *pp* neutrinos (E$\sim$ 0.3 MeV) and for ^8B neutrinos (E$\sim$ 5 MeV). In the formula, θ corresponds to the vacuum mixing angle, while θ_m is the effective mixing angle at the neutrino production point in the Sun. Consider an electron density of 100 mol/cm^3 at the center of the Sun.