SDU Syddansk Universitet

QUANTUM THEORY CENTER

D·IAS DANISH INSTITUTE FOR ADVANCED STUDY

Machine Learning and QFT

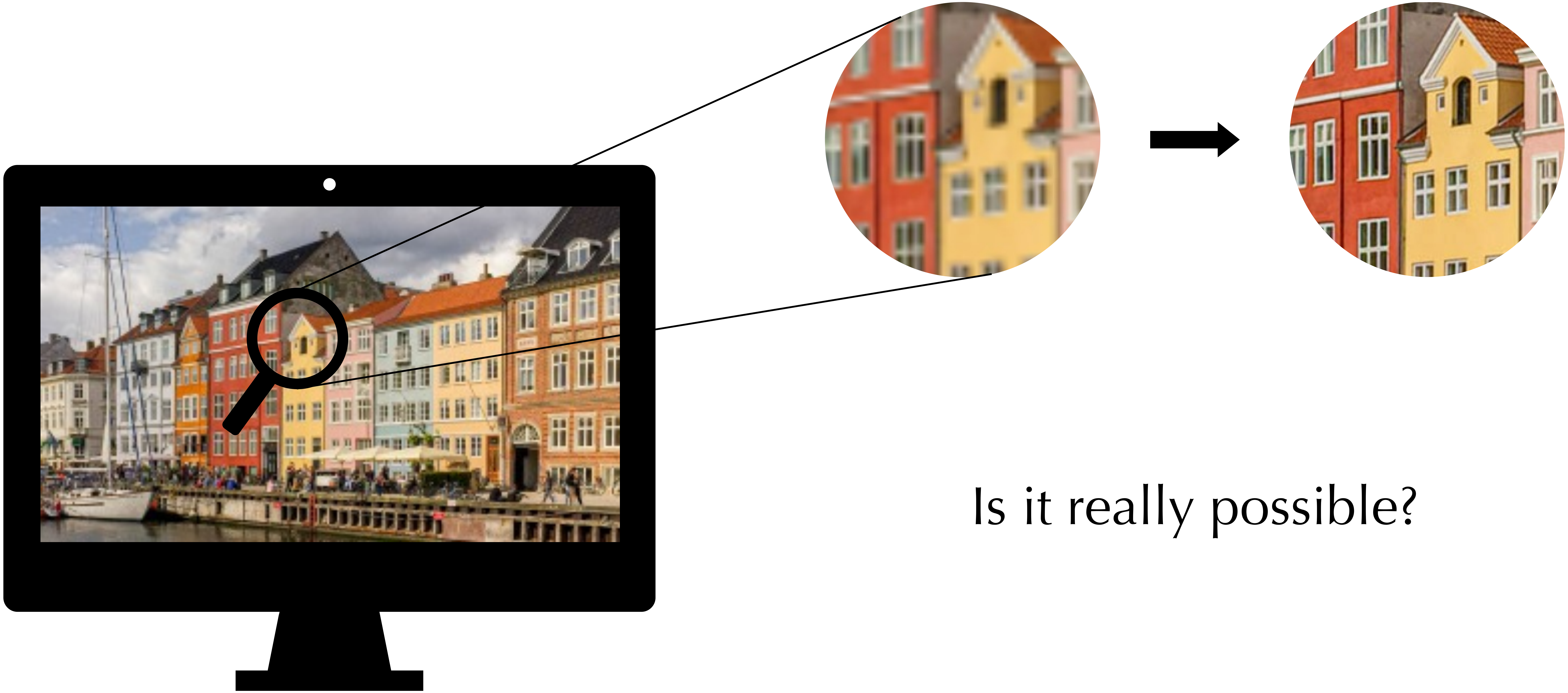
A two-way dialogue

Pietro Butti

HAMLET-PHYSICS, August 2025, København

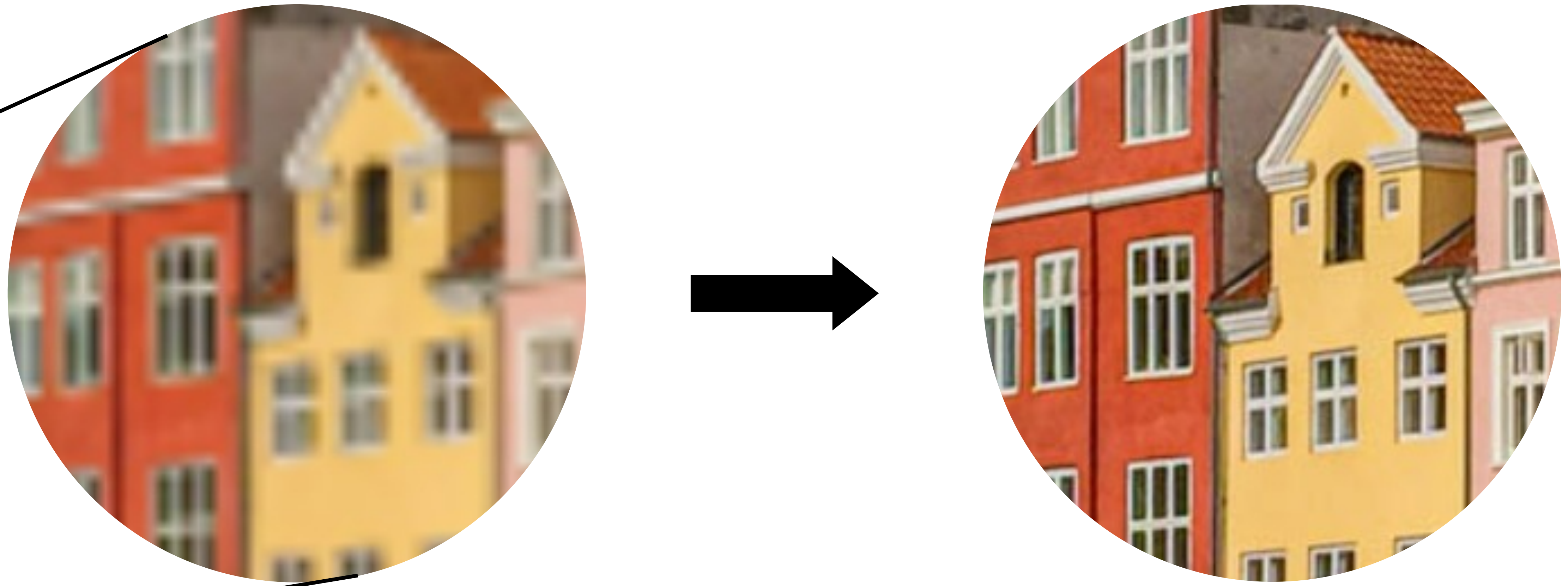
SDU

ħQTC

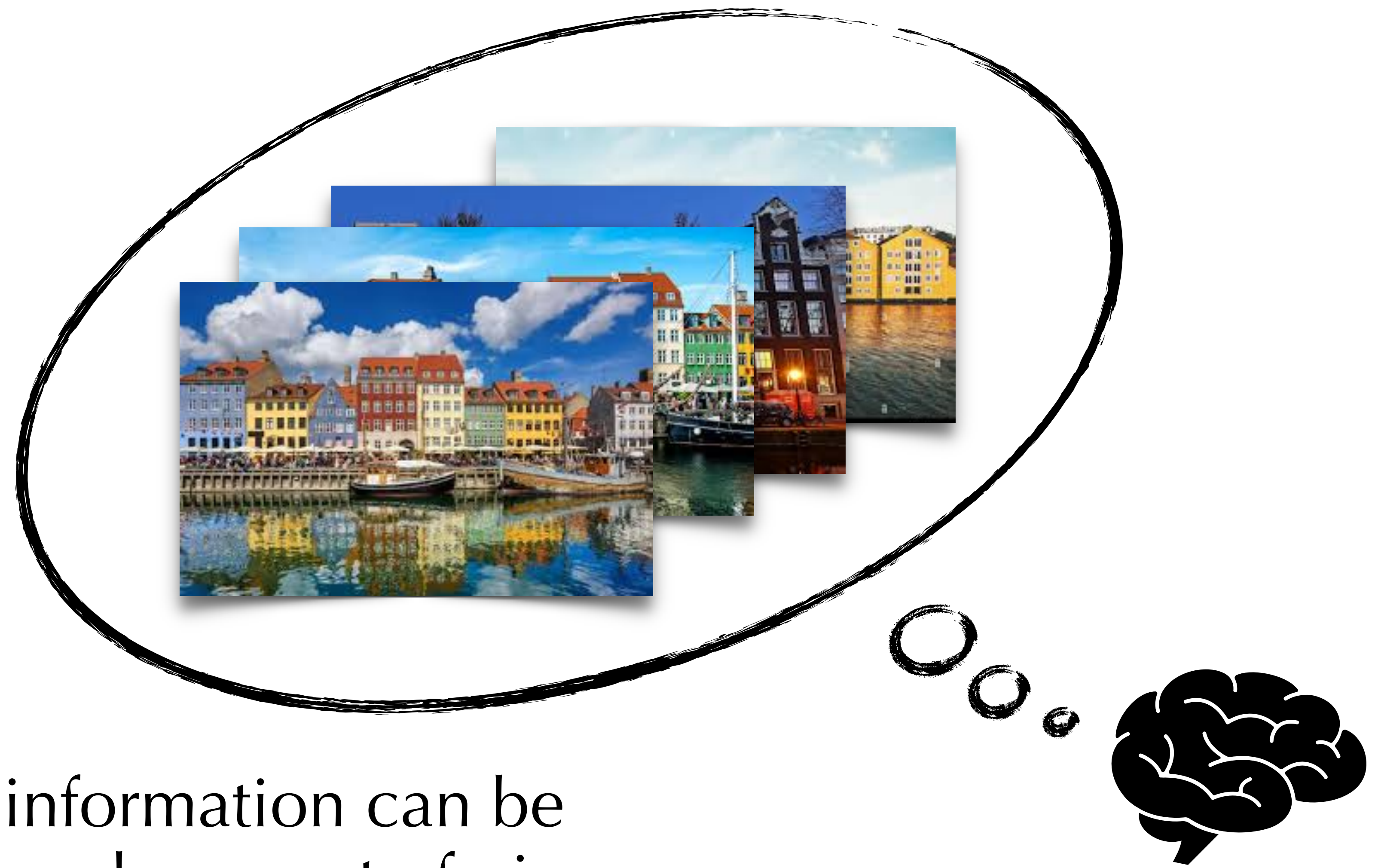


Is it really possible?

Super Resolution (SR)



Learn a **prior** over a dataset



Mission information can be
learned from a larger set of pics…

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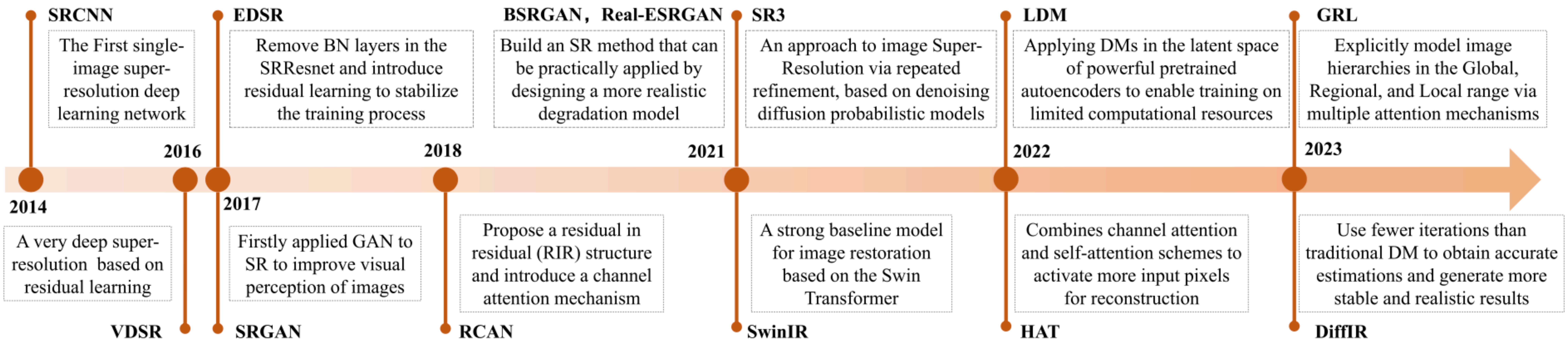

IET Image Process. 18, 2215–2243 (2024)

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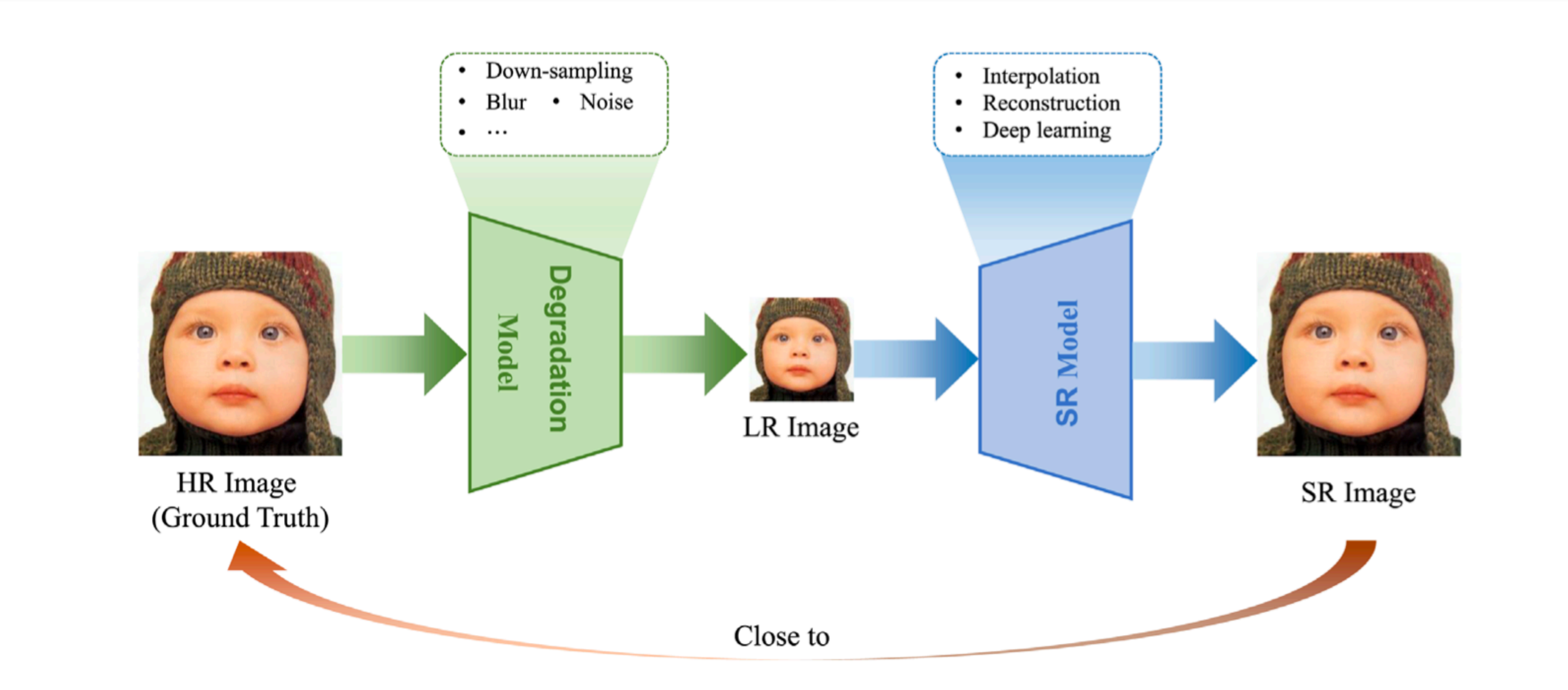

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Distributional flows

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Unknown probability distribution
of every single pixel ϕ_i

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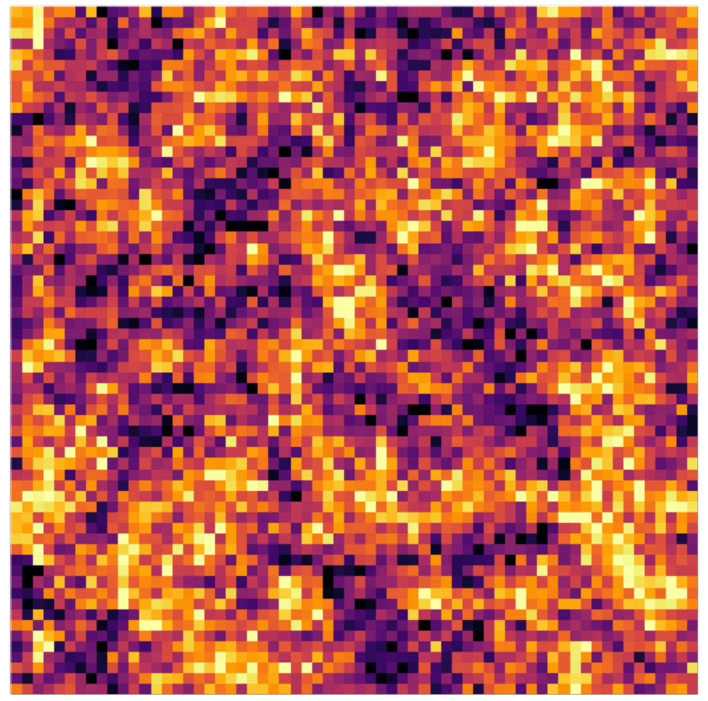
QFT as "energy"-based model for (quantum) pictures…

$$p(\phi) \sim e^{-S[\phi]}$$

$$S = \int \mathrm{d}^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \lambda \phi^4 \right]$$



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Blurring

Super-resolution
(Learn a prior over a dataset)

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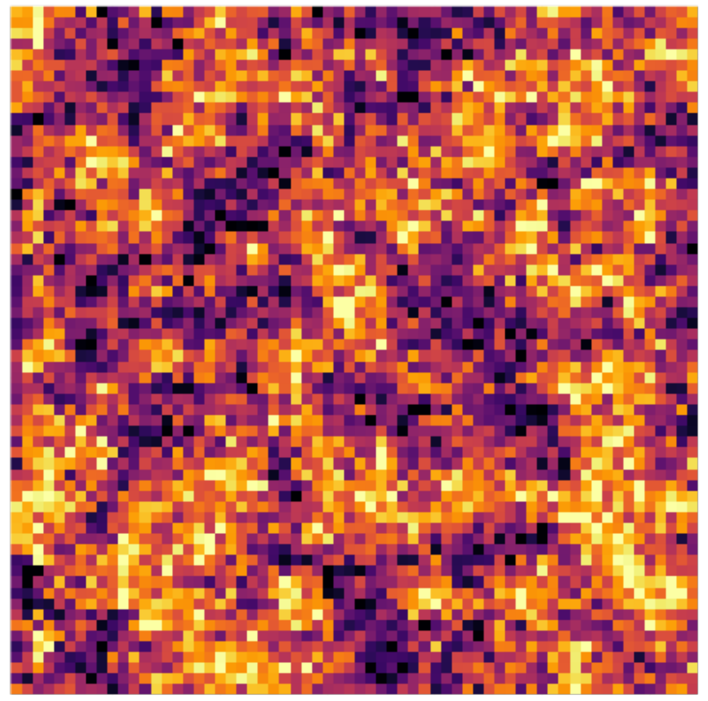

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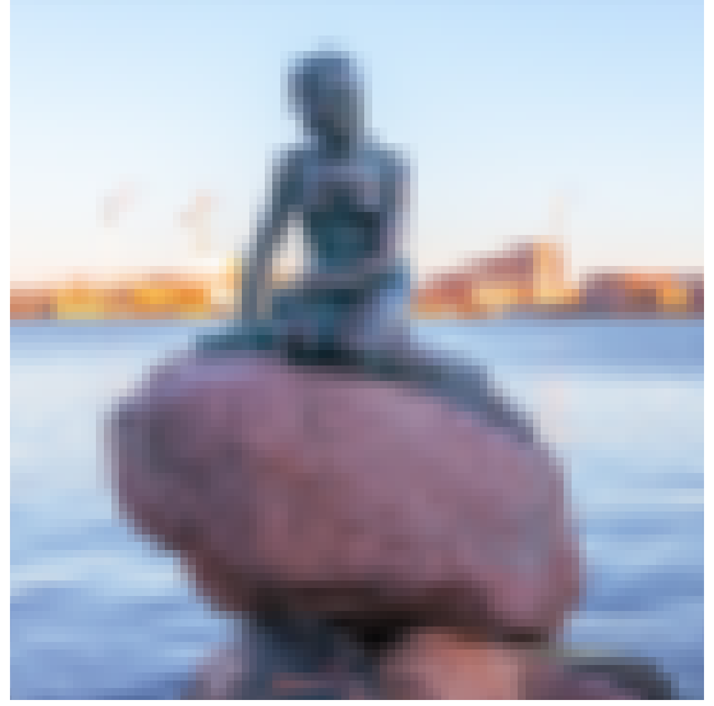

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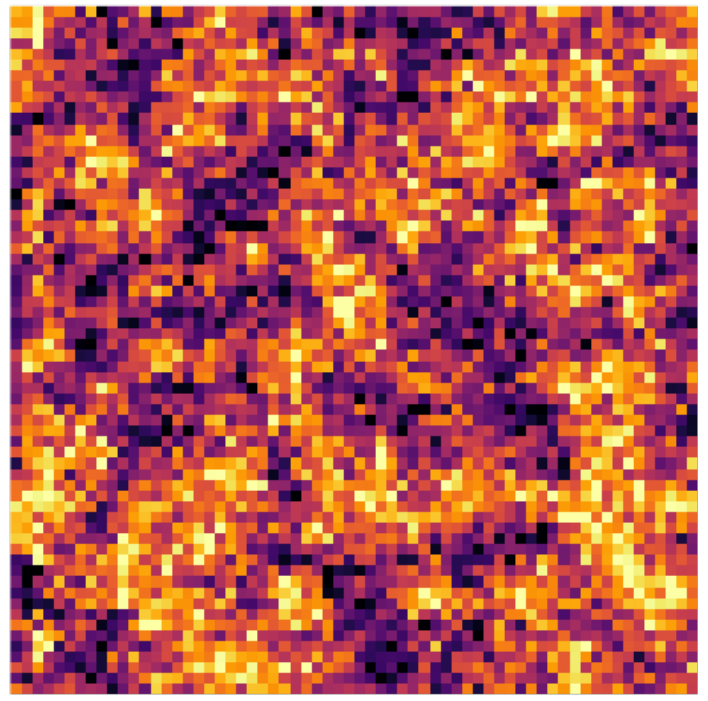
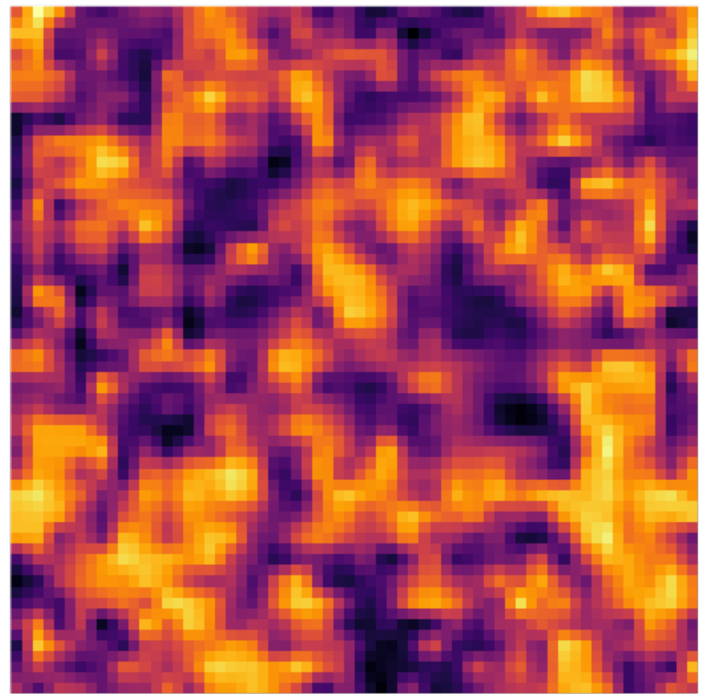


RG flow

Inverse RG



$$\sim e^{-\tilde{S}[\tilde{\phi}]}$$

$$\tilde{S} = \int \mathrm{d}^d x \left[\frac{1}{2}(\partial_\mu \tilde{\phi})^2 + \frac{\tilde{m}^2}{2}\tilde{\phi}^2 + \tilde{\lambda}\tilde{\phi}^4 + \ldots\right]$$

Lattice QFT

Spacetime is discretised in a square lattice

Bare parameters

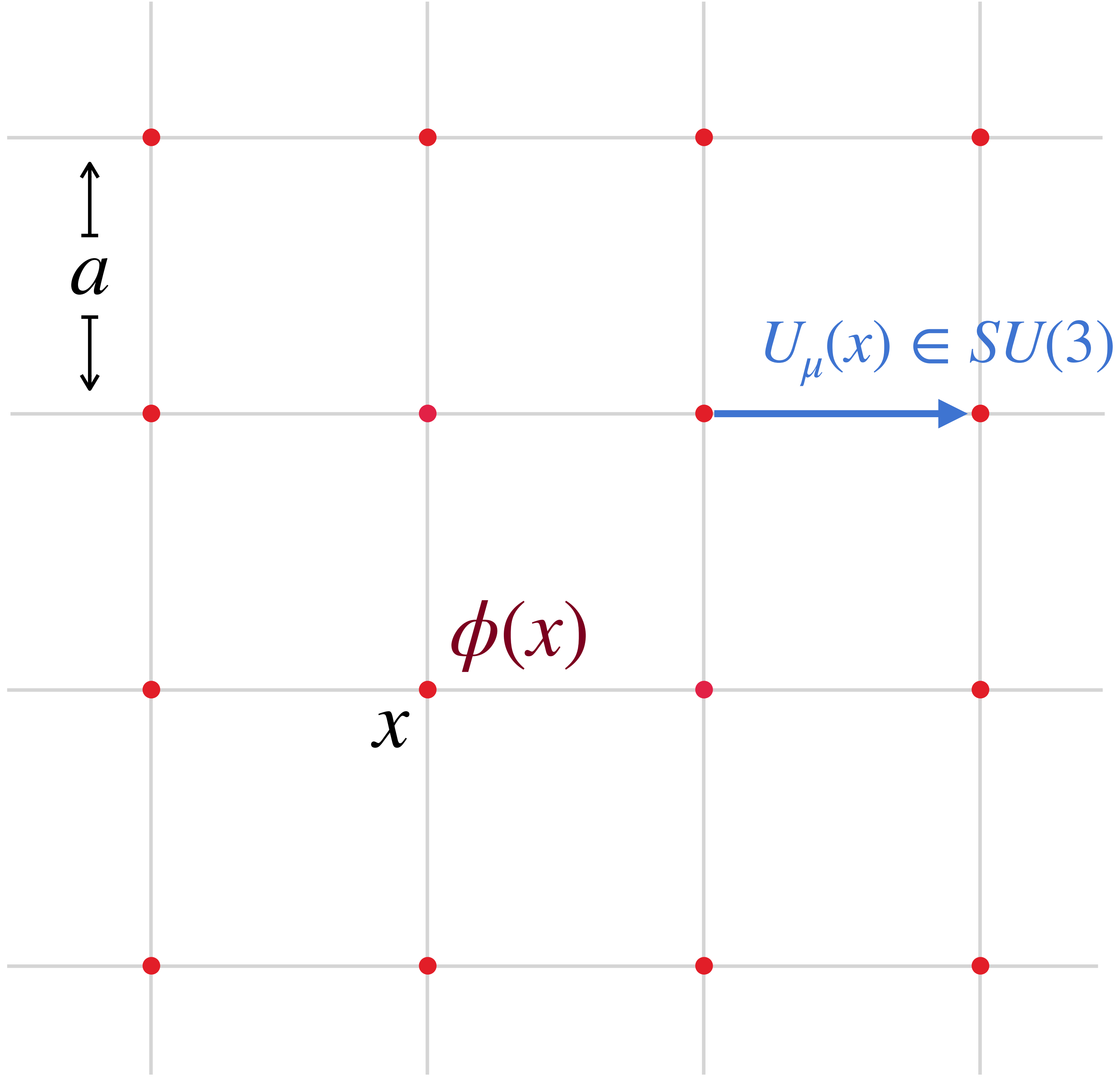
$$S = \sum_x \left[-2\kappa \sum_i \phi_x \phi_{x+i} + \phi_x^2 + \lambda \left(\phi_x^2 - 1 \right)^2 \right]$$

Euclidean path-integral becomes a sum over a thermal ensemble

$$Z = \sum_{\{\phi\}} e^{-S[\phi]} \qquad \langle \mathcal{O} \rangle_S = \sum_{\{\phi\}} \mathcal{O}[\phi] \frac{e^{-S[\phi]}}{Z}$$

Sum over all possible "pictures" ϕ

Numerically evaluate, then extrapolate $a \to 0$, keeping $\langle \mathcal{O} \rangle$ fixed

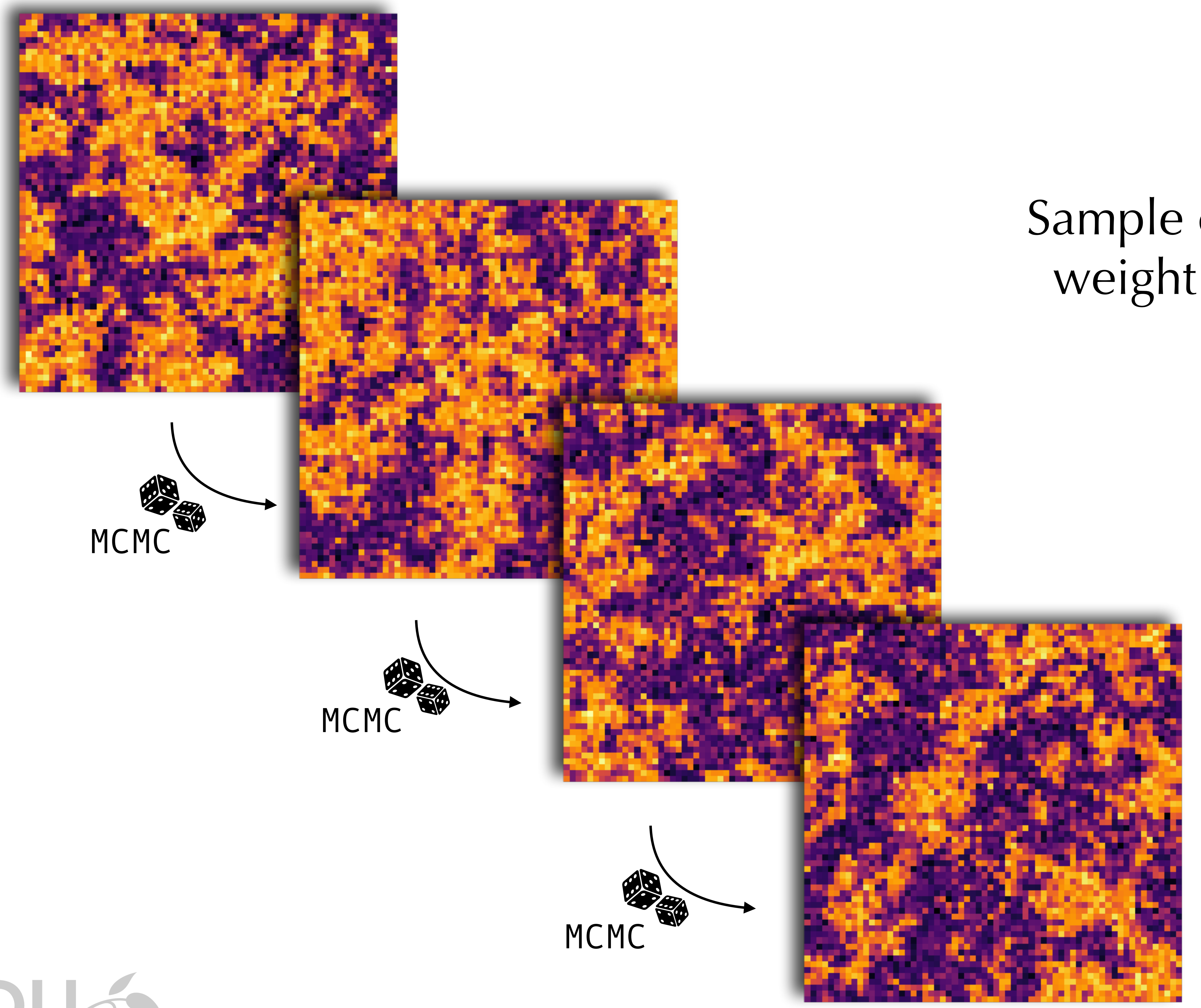


- Lattice spacing a cuts off all UV divergencies
- The value of a depends on bare parameters (κ, λ)

Lattice QFT

Exp. value in lattice QFT: $\langle \mathcal{O} \rangle_S = \frac{1}{Z} \sum_{\{\phi\}} e^{-S[\phi]} \mathcal{O}[\phi]$

every possible configuration



Sample configurations according to their Boltzmann weight with (κ, λ) via **Markov Chain Monte Carlo**

$$\phi_i \sim p(\phi) = \frac{e^{-S[\phi]}}{Z}$$

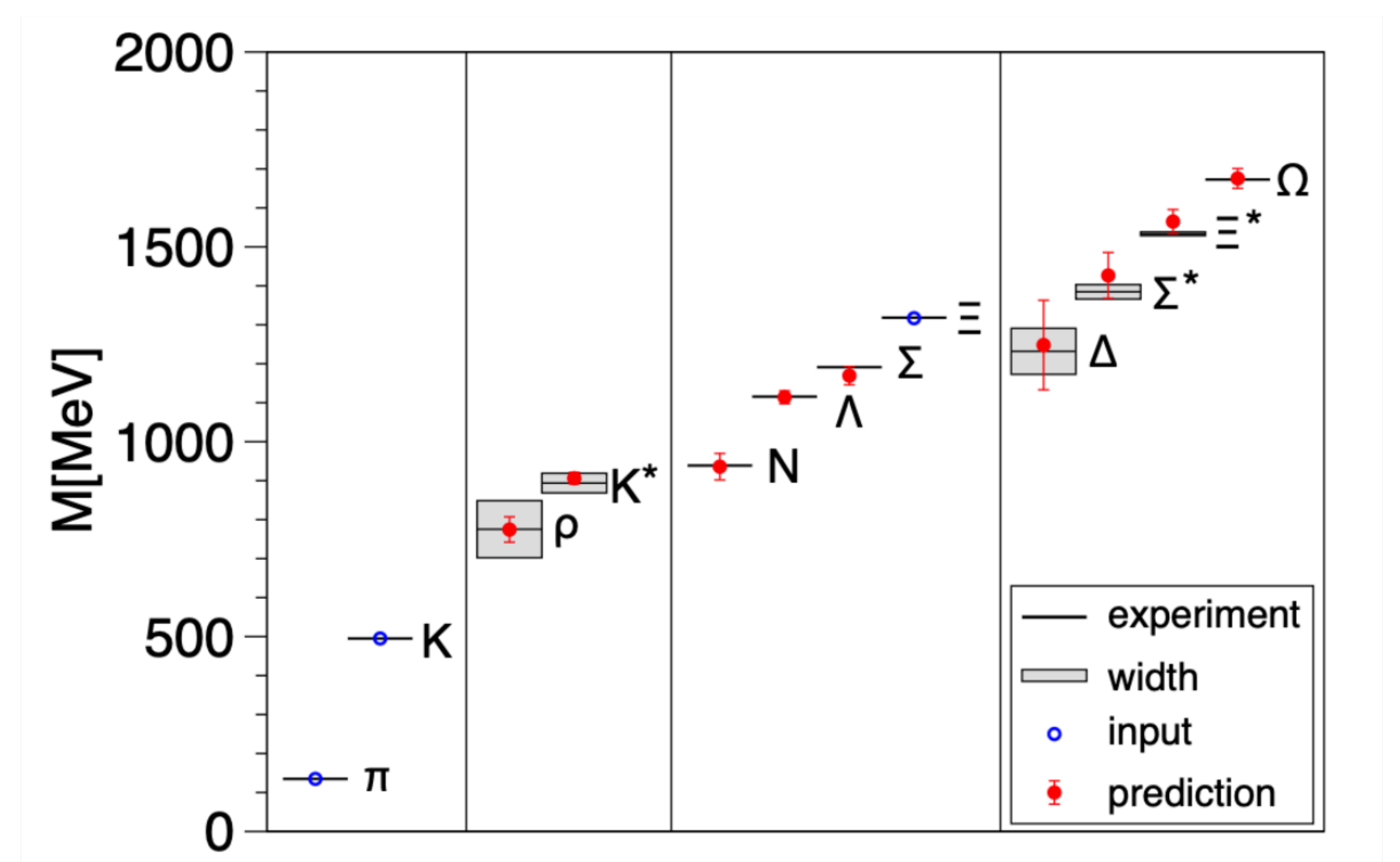
$$\downarrow$$

$$\langle \mathcal{O} \rangle_S \approx \frac{1}{n} \sum_{\{\phi_i\}} \mathcal{O}[\phi_i]$$

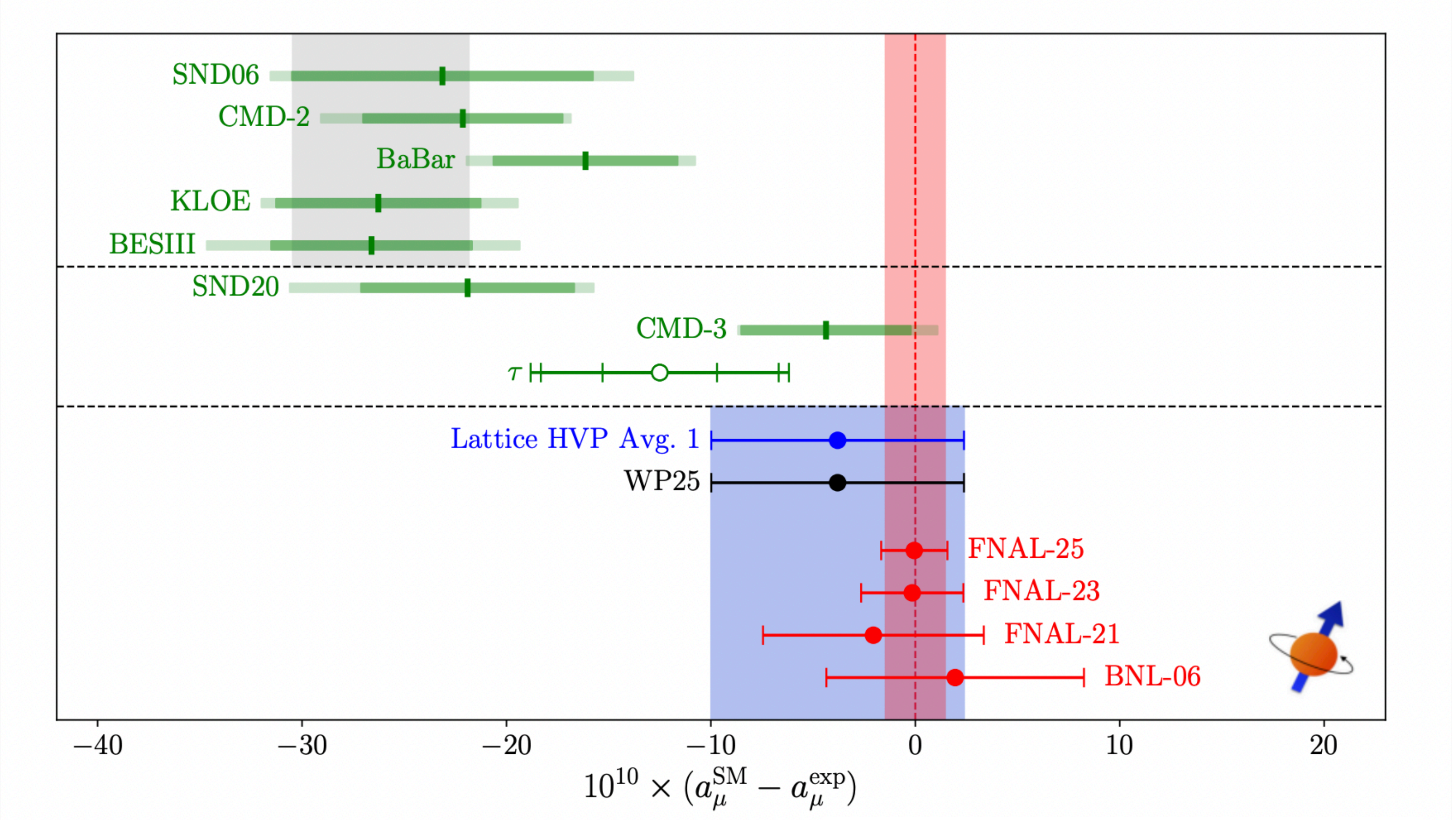
Sampled configuration

Lattice QCD

- Hadron spectrum
  - Heavy resonances
  - PDFs
  - Form factors
- QCD phase diagram
  - Equation of state
  - Critical point
- New physics in flavour ph.
  - Heavy mesons decays
  - Muon $g-2$
- …



[Fodor, Hoelbling] - Rev. Mod. Phys. **84**, 449



$g_\mu - 2$ white paper 25: hep-ph/2505.21476

SDU

hQTC

Lattice QFT: compute the spectrum

Large-time behaviour of 2-point functions contains infos about the spectrum of the theory

$$C(t) = \left\langle \phi(t)\phi(0) \right\rangle_S \xrightarrow{t \gg (\Delta E)^{-1}} Z_0 e^{-E_0 t}$$

Energy of the fundamental state generated by the quantum op. $\hat{\phi}$

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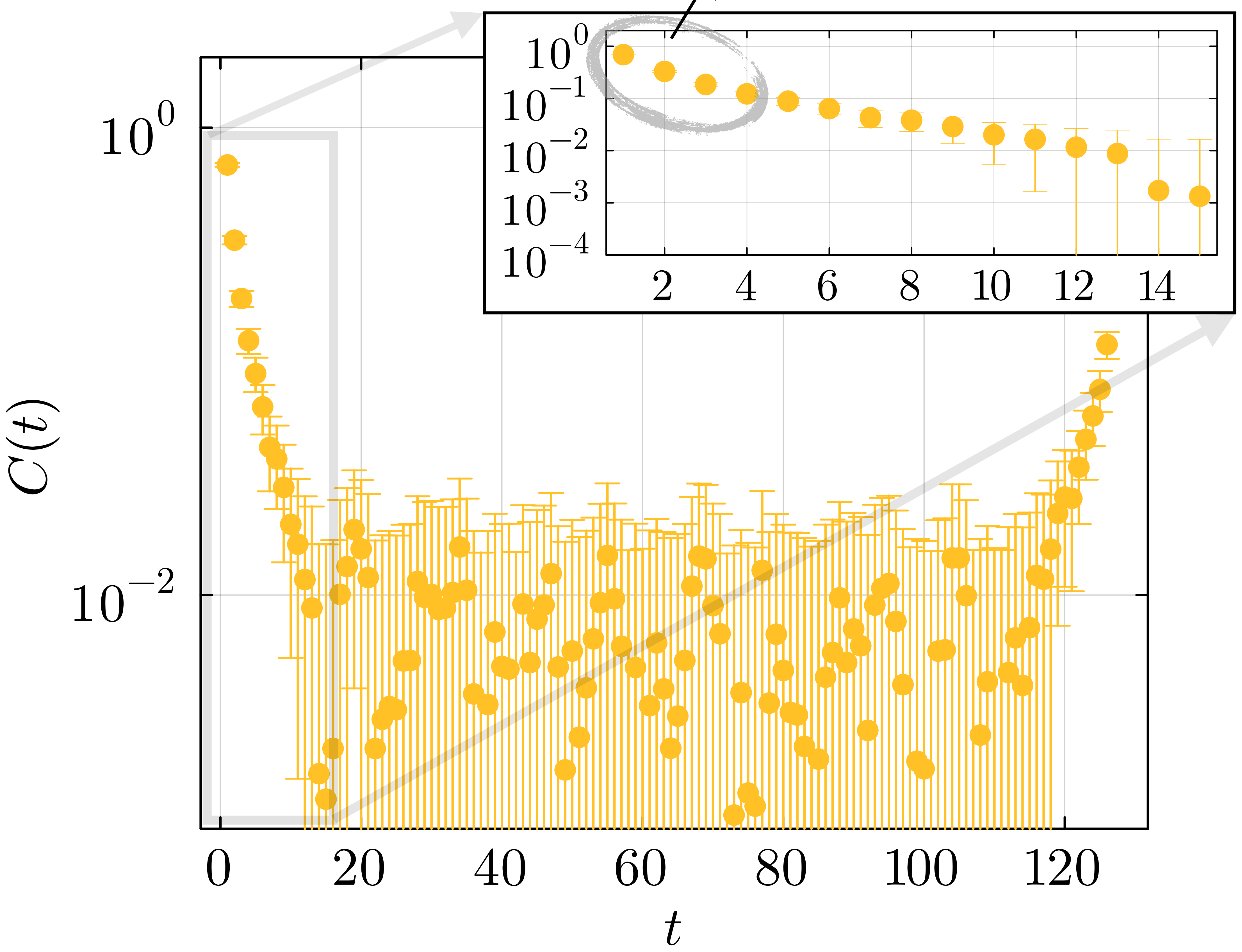
Signal dominated by excited states
$Z_0 e^{-E_0 t} + Z_1 e^{-\Delta E t} + \cdots$



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But…

$$\frac{\text{error}[C(t)]}{C(t)} \xrightarrow{t \gg (\Delta E)^{-1}} \frac{e^{mt}}{\sqrt{N}}$$

Signal dominated by excited states
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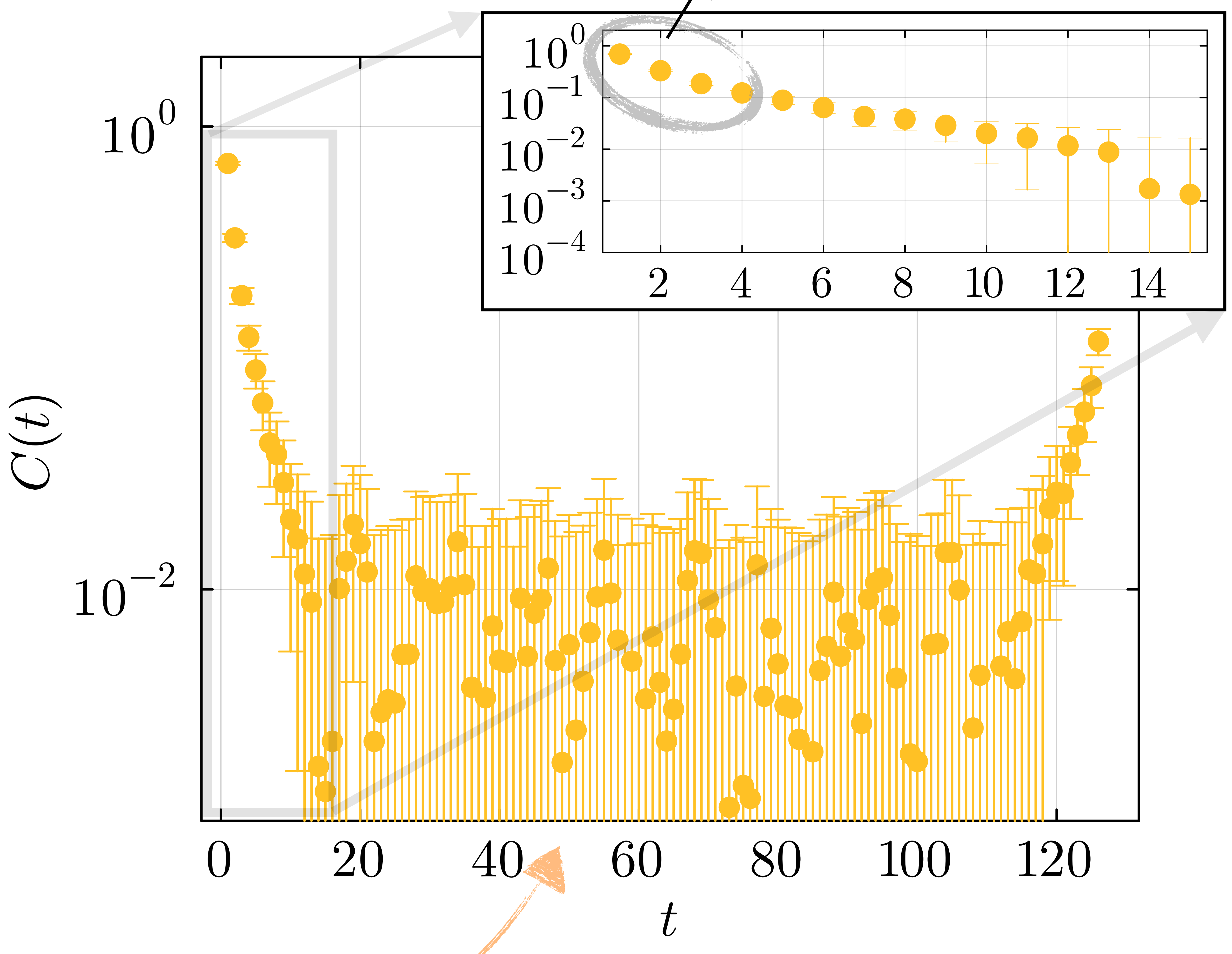

Super-resolving $\lambda\phi^4$

Inspired by
[Efthymiou, Beach, Melko] - Phys. Rev. B **99**, 075113
[Bacthis, Aarts, Di Renzo, Lucini] - Phys.Rev.Lett. 128 (2022) 8

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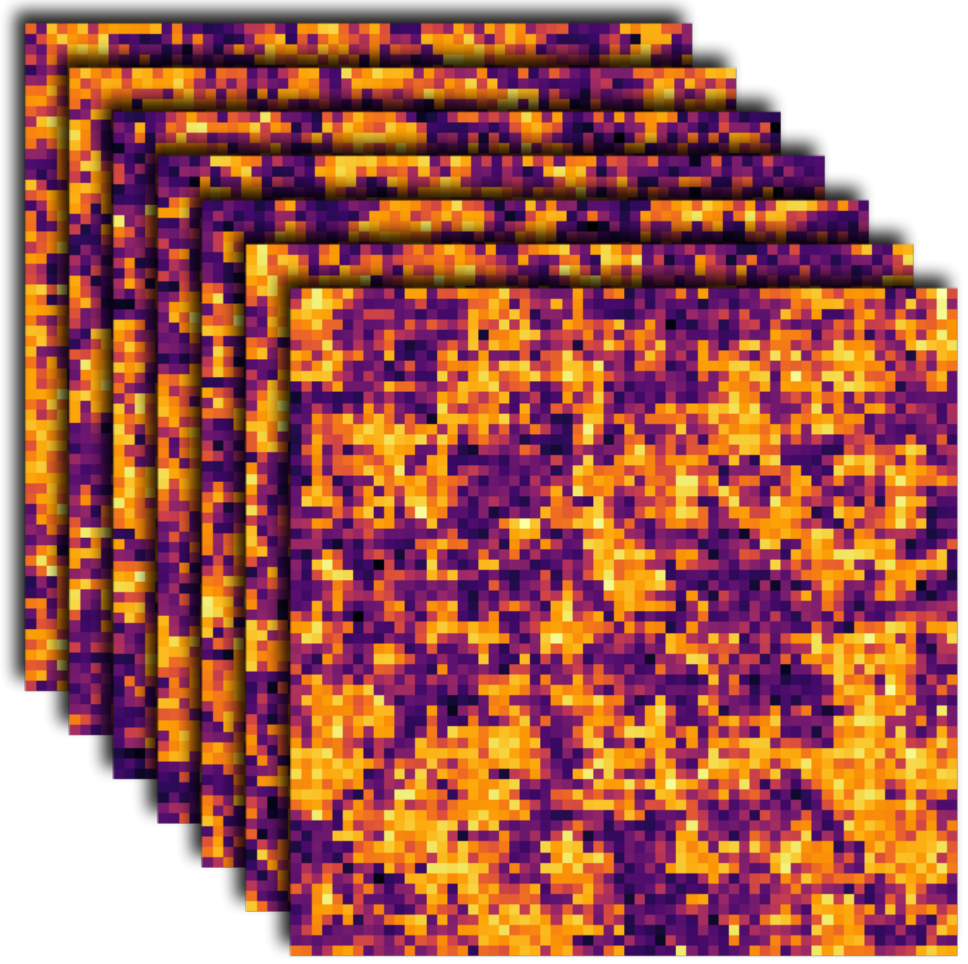
$(\lambda, \kappa) \rightarrow \xi$



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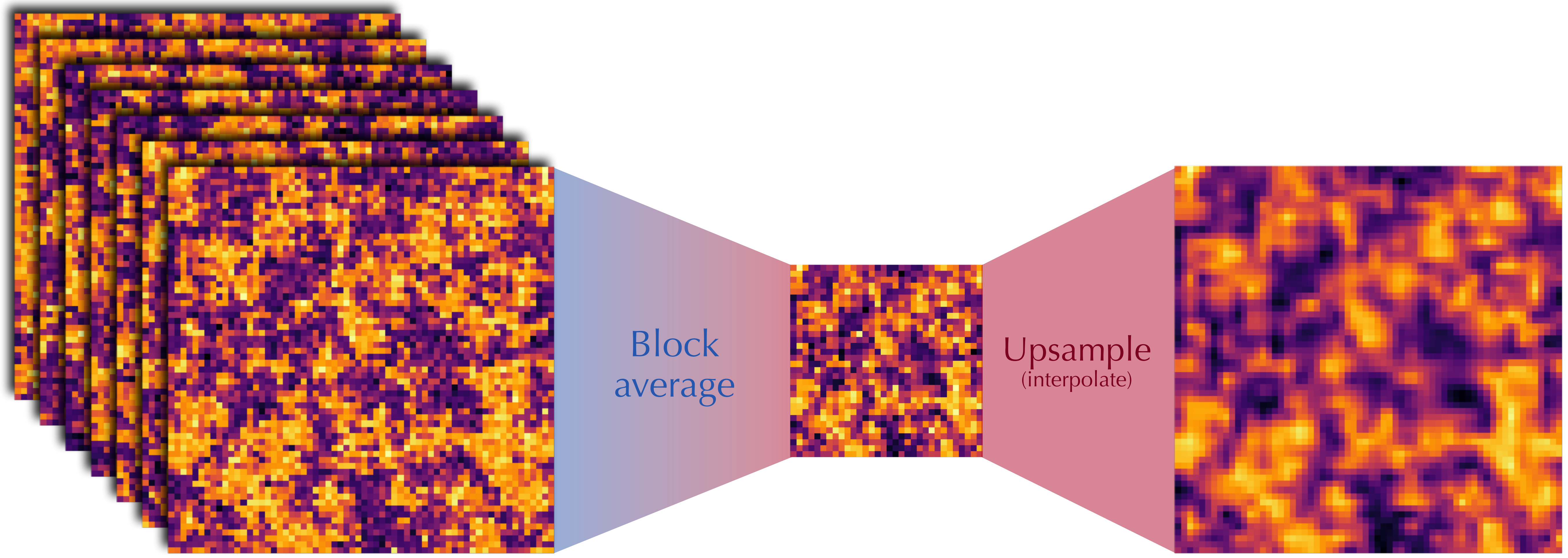

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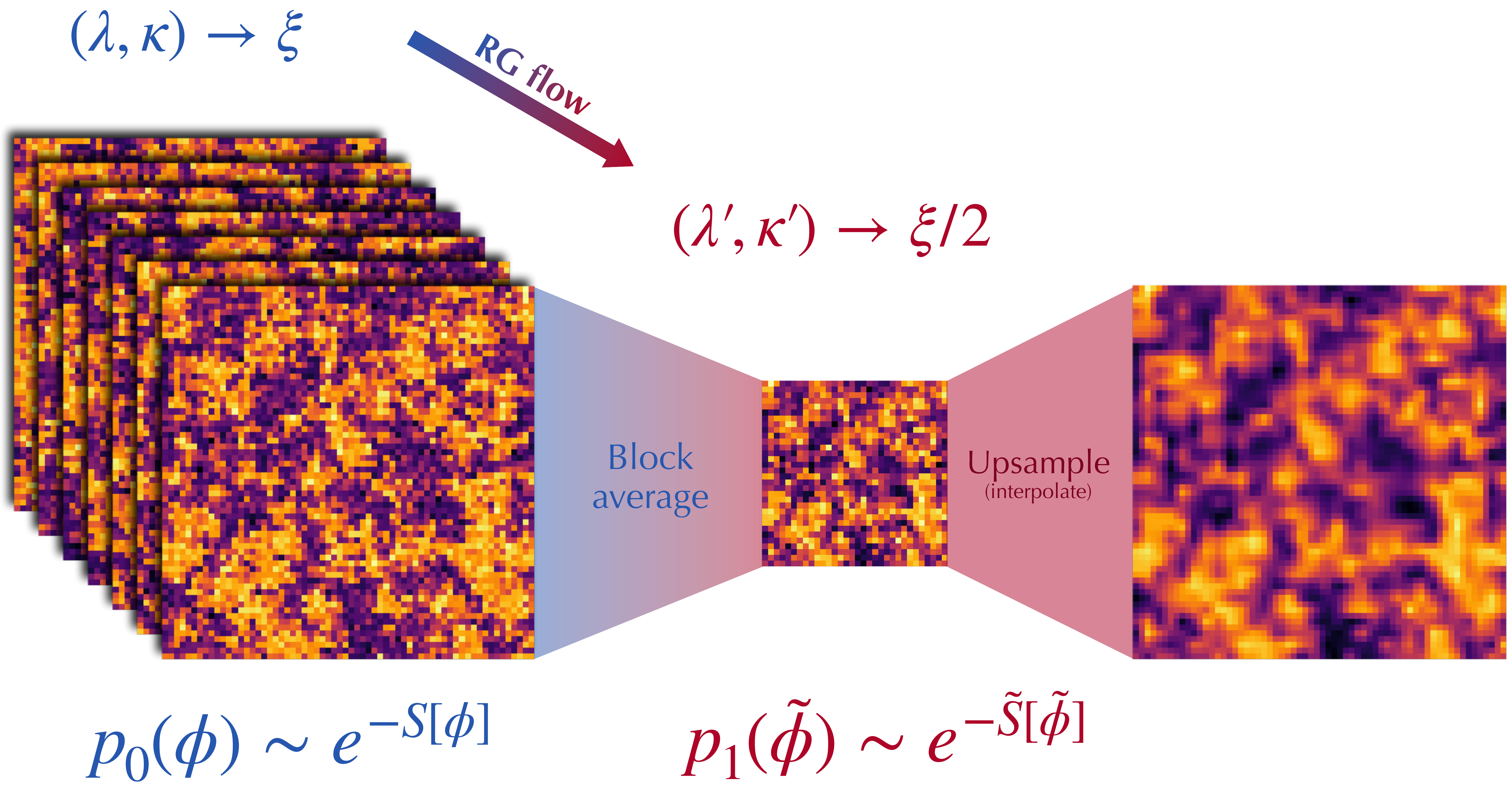

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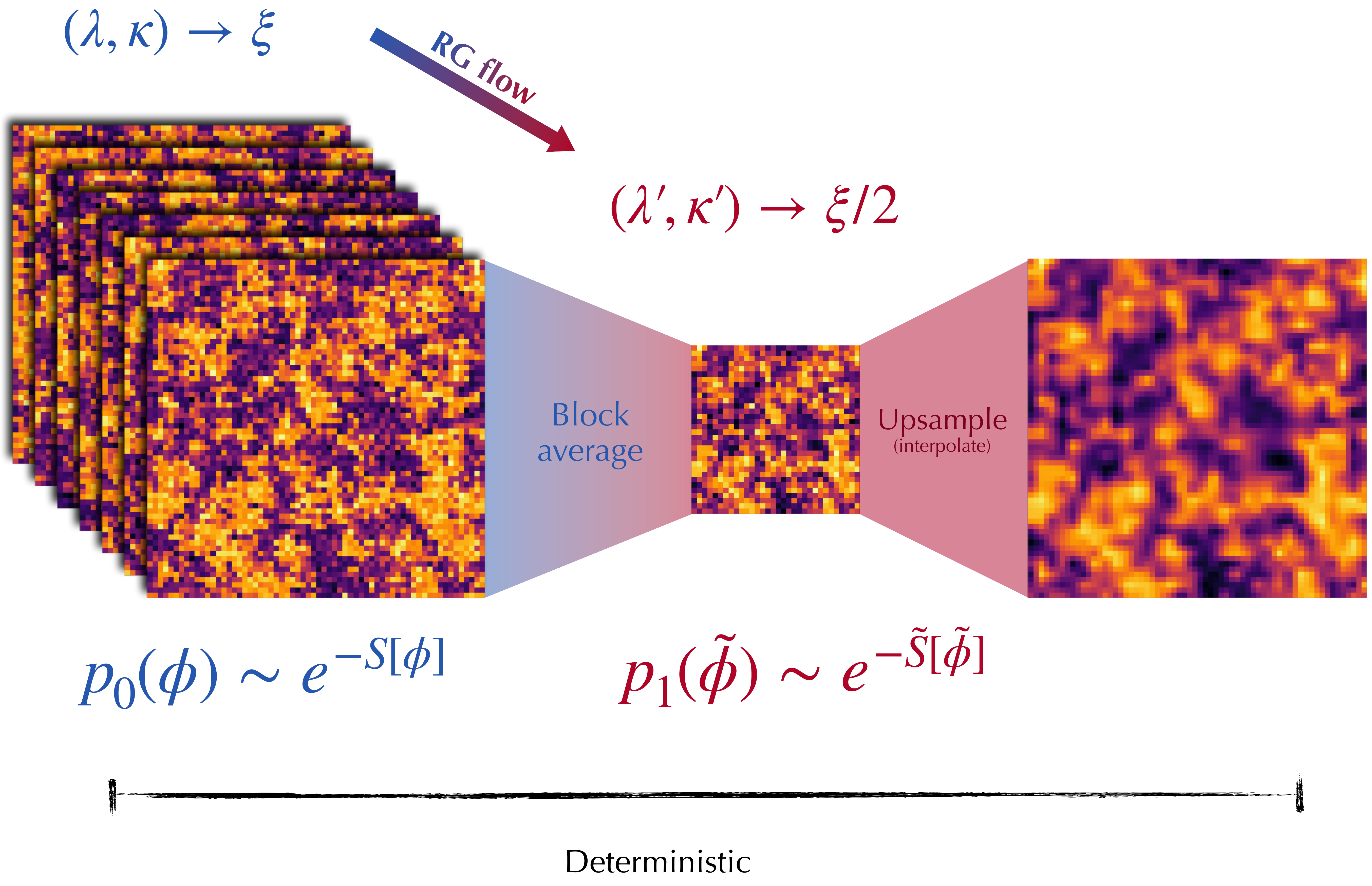

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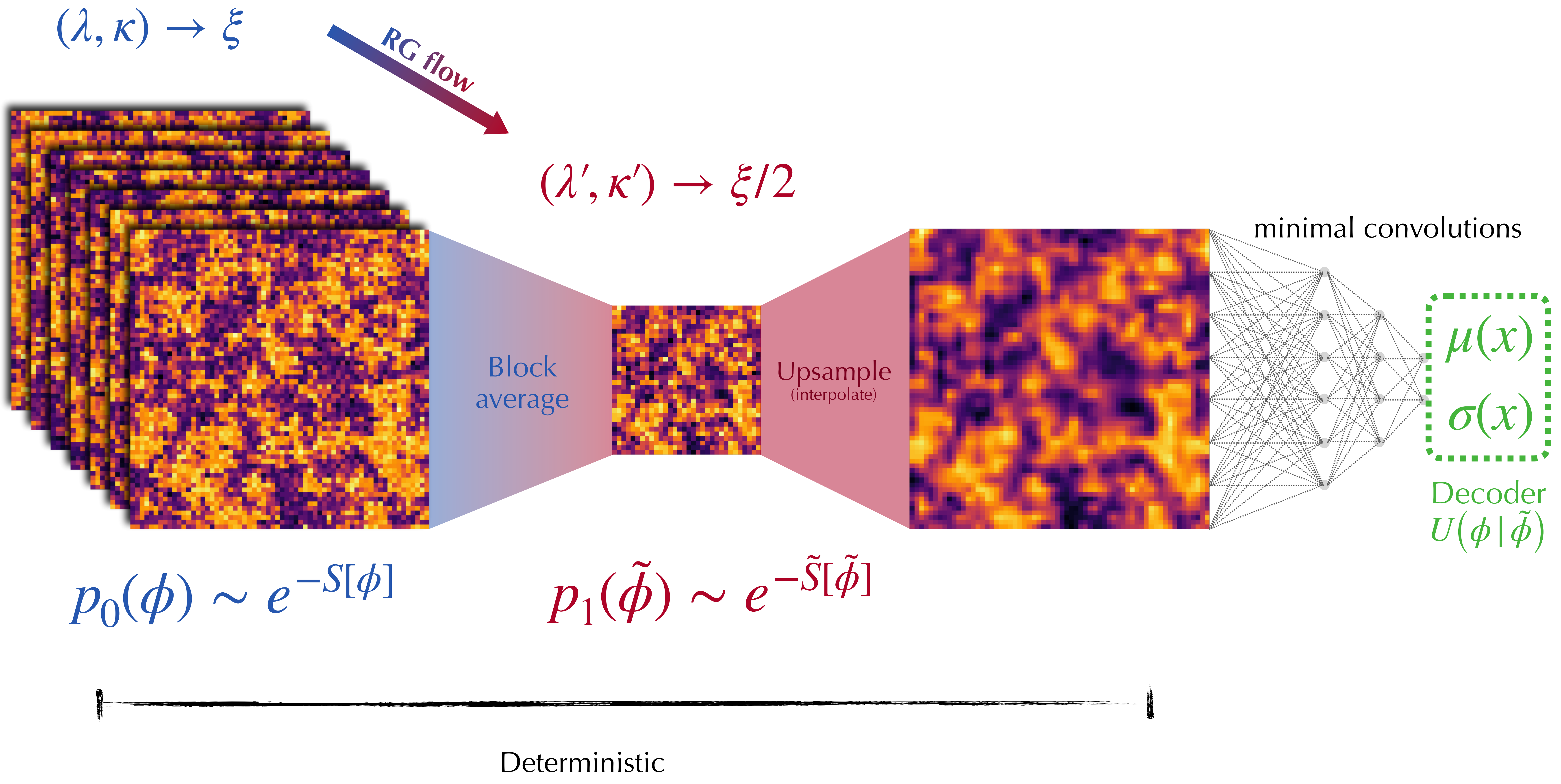

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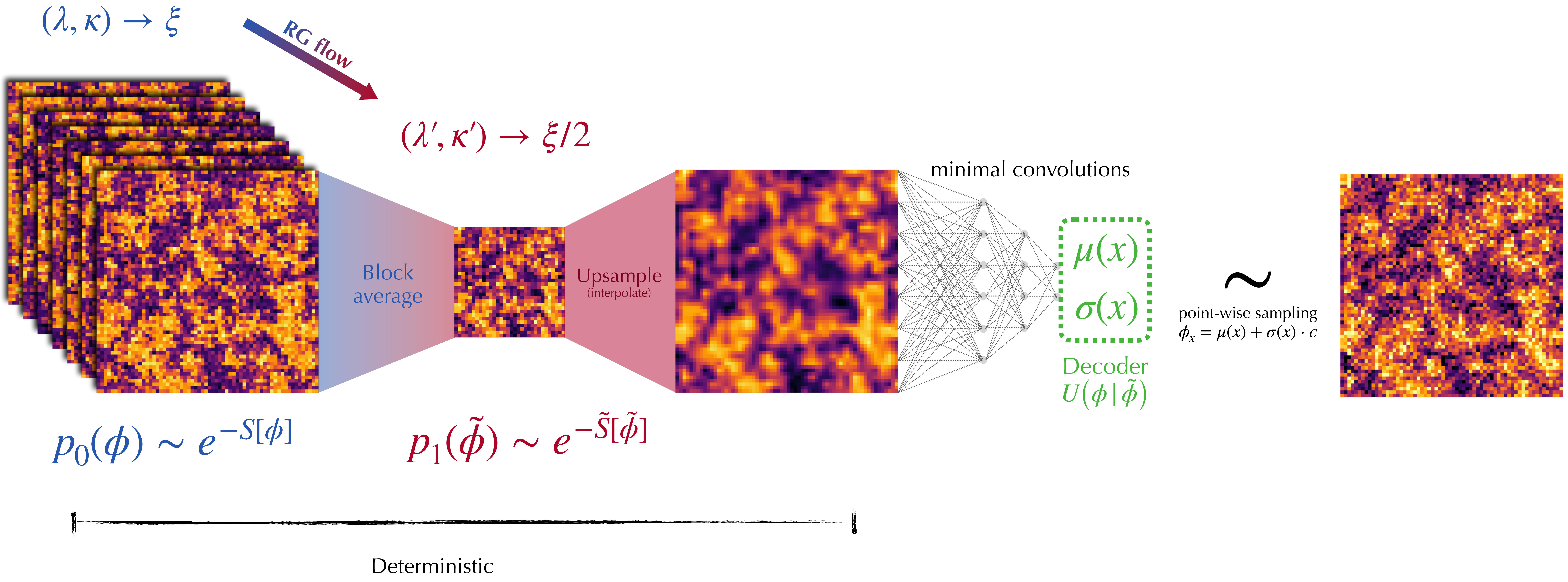

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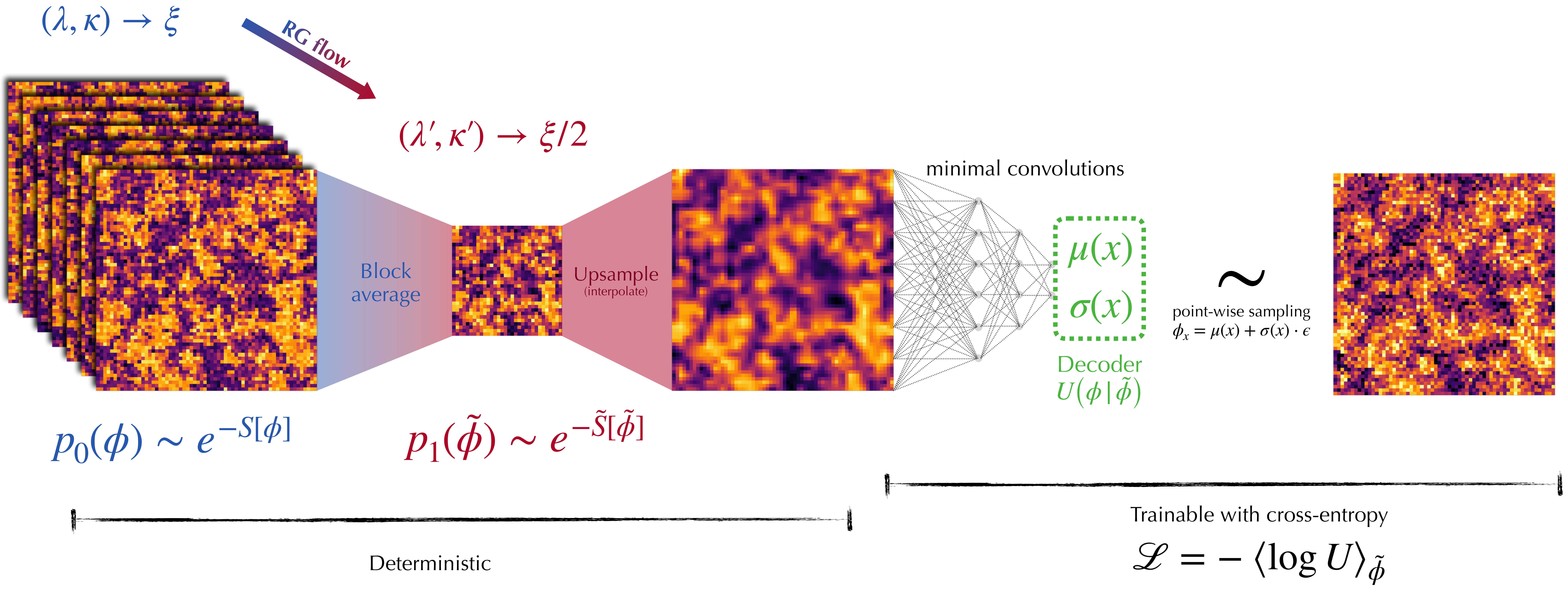

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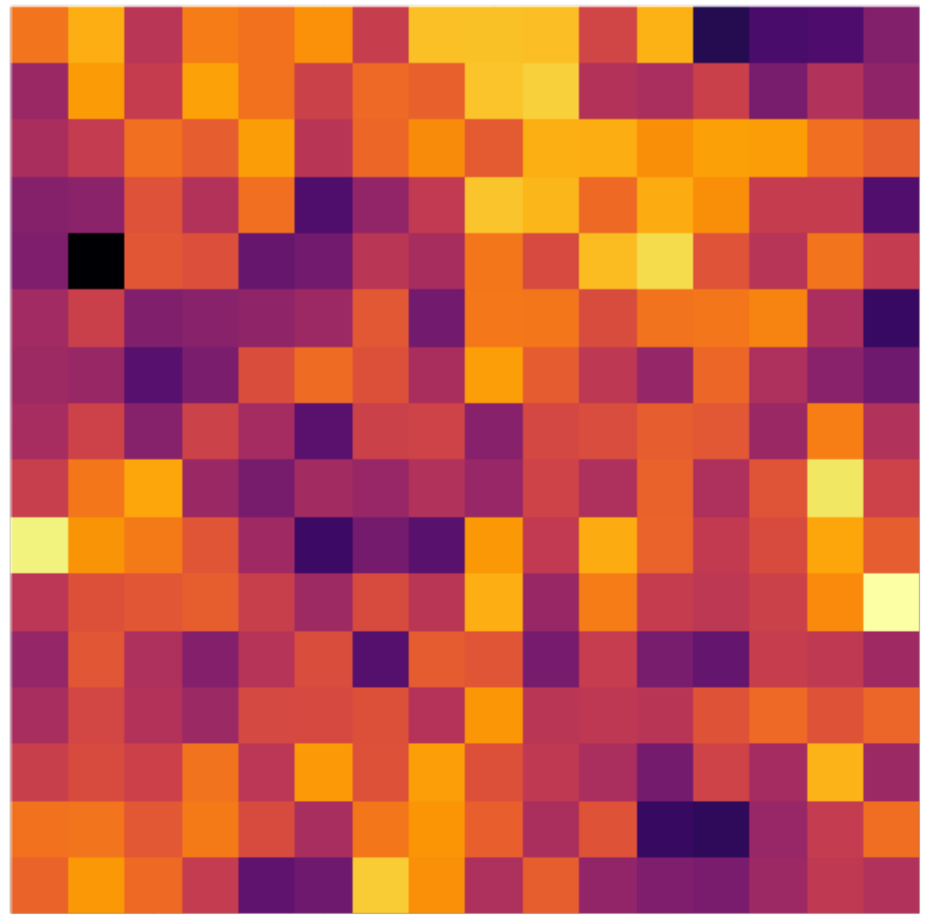
Training configuration



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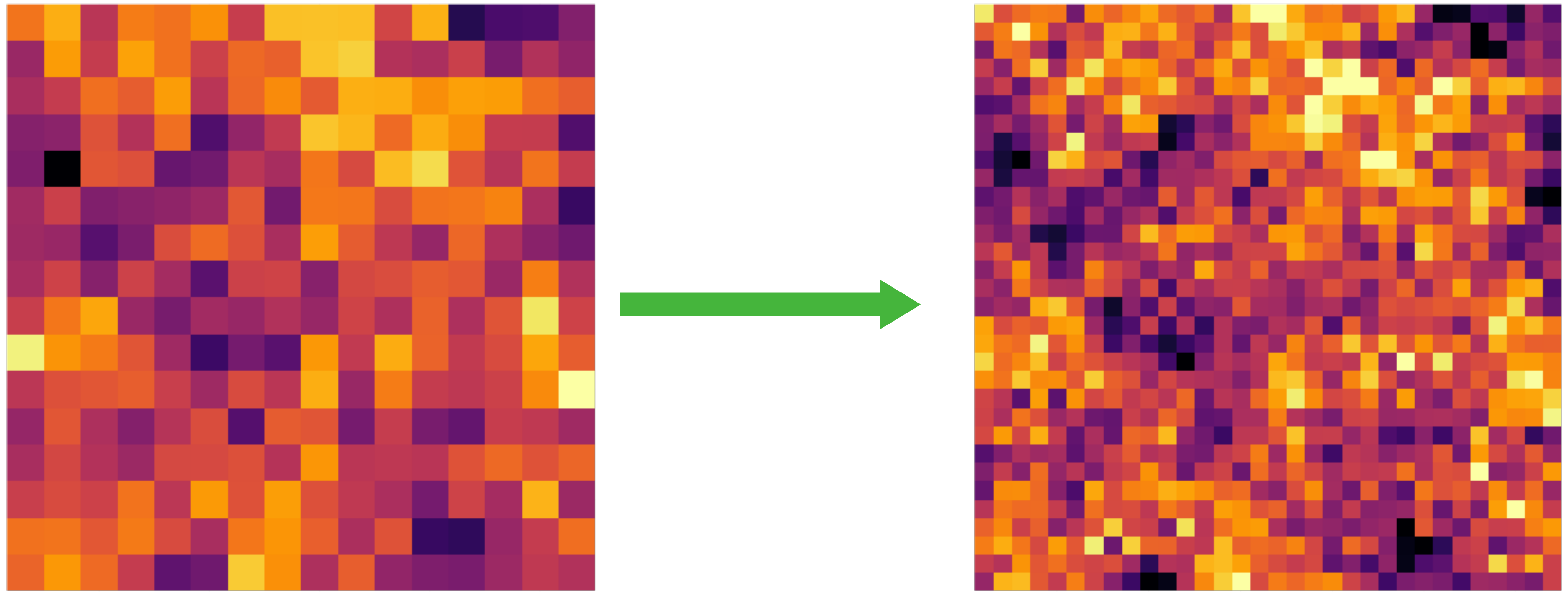

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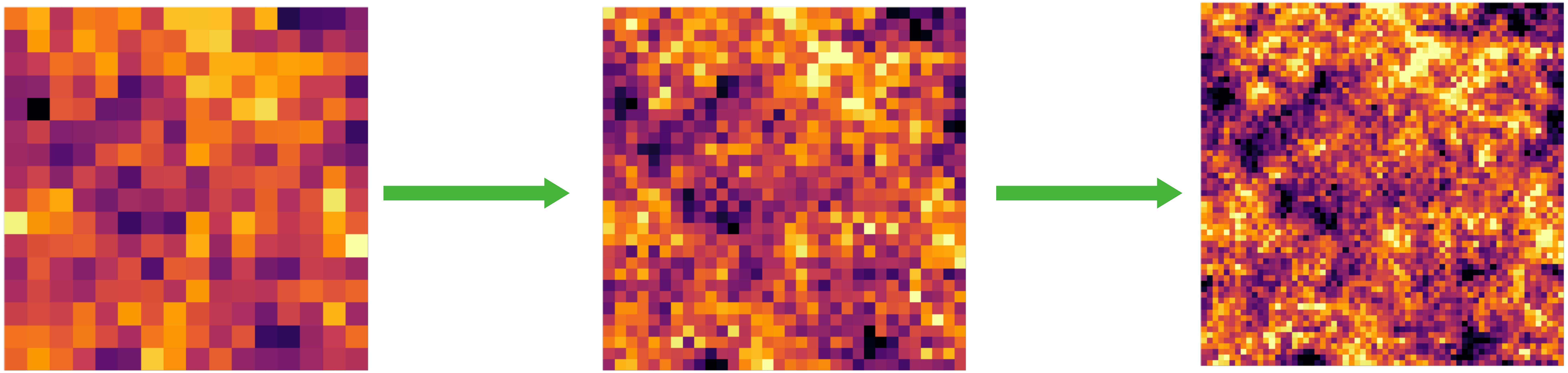

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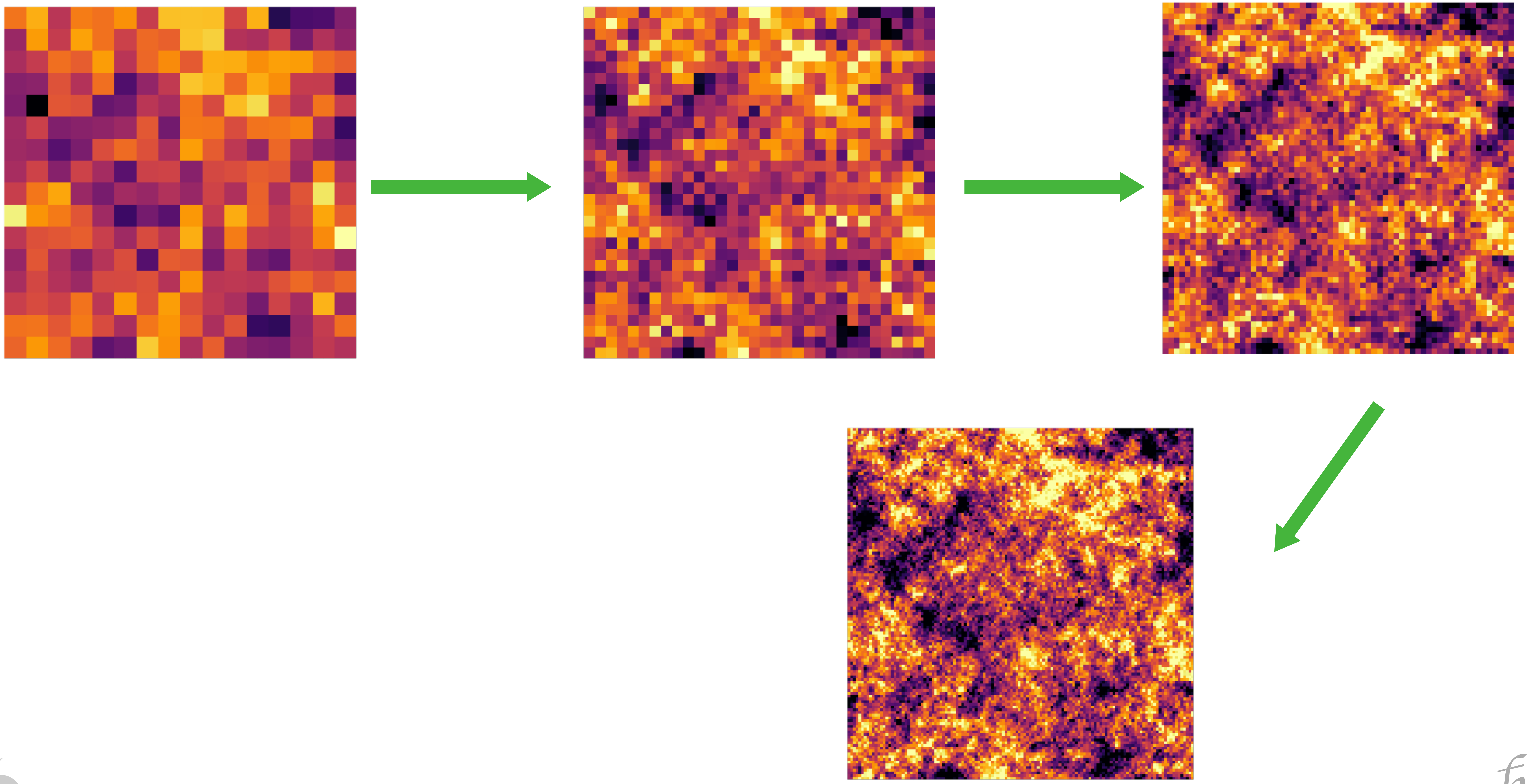

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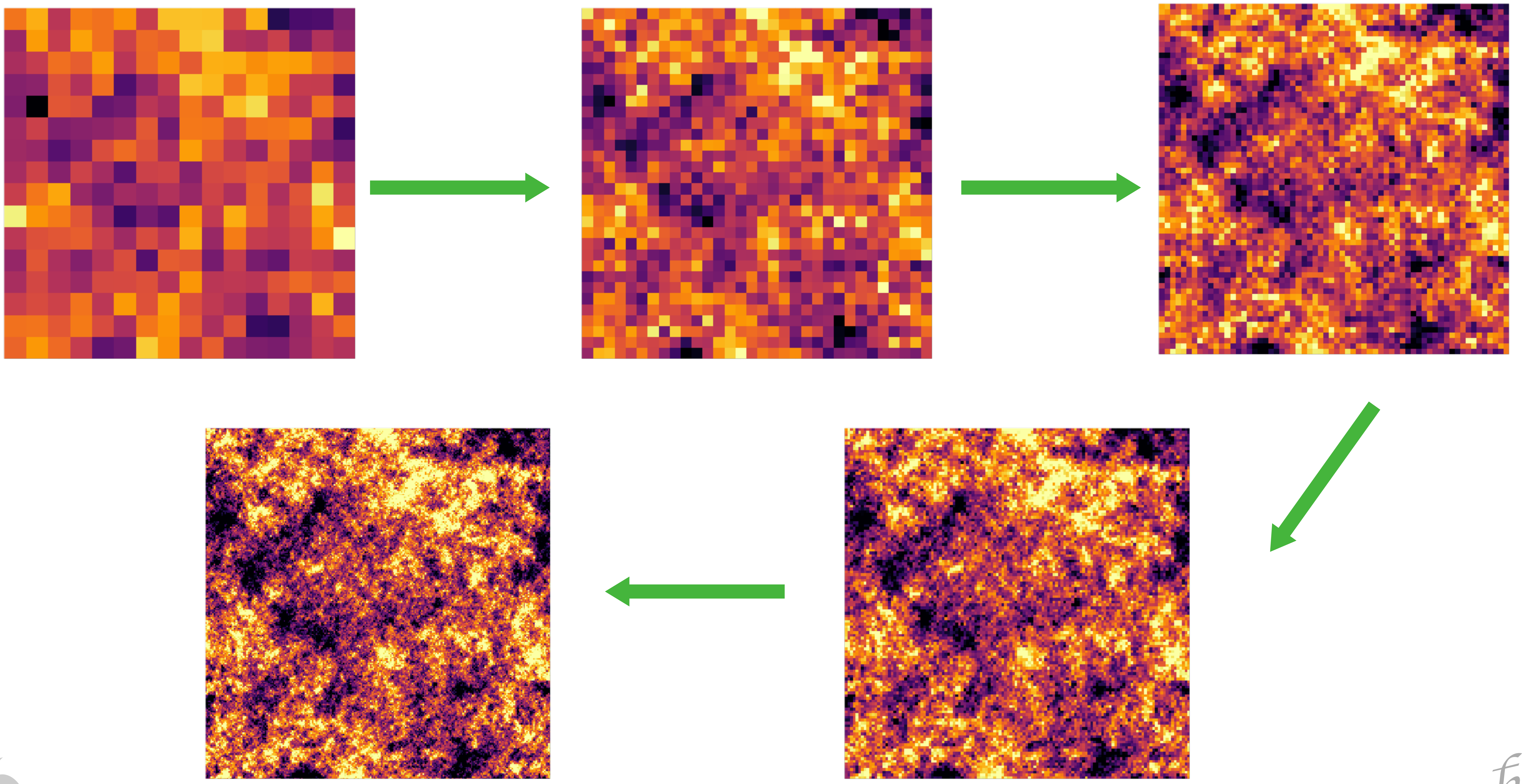
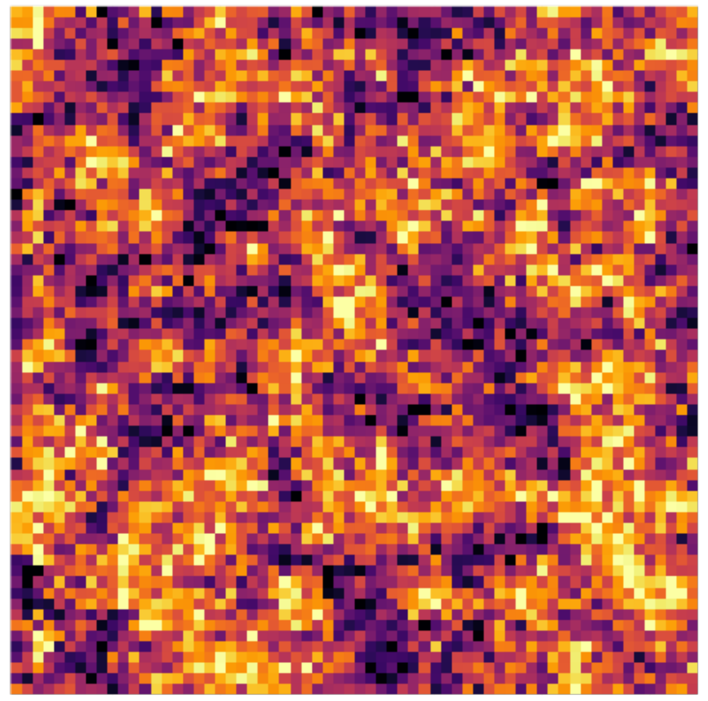


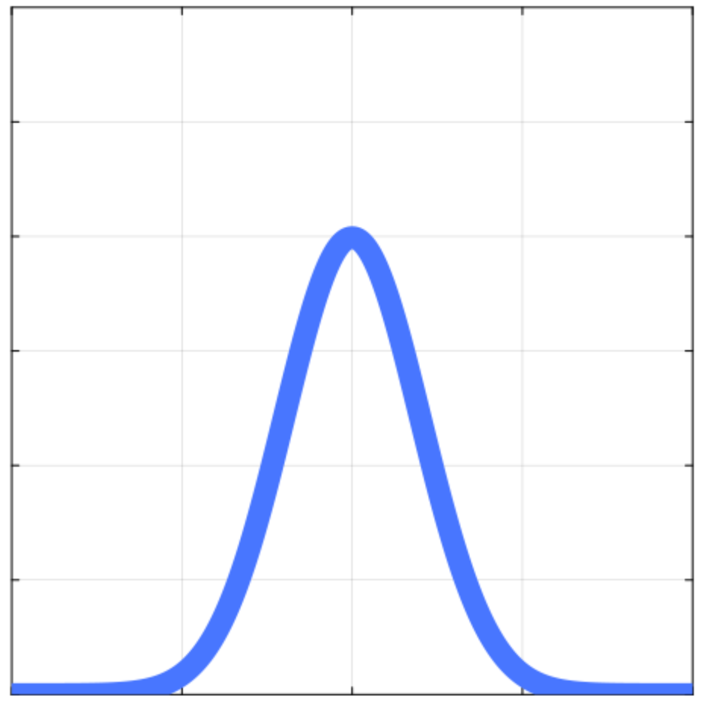
Distributional flows

Distributional flows

ϕ



$p(\phi) \sim e^{-S_{(\kappa,\lambda)}}$



Distributional flows

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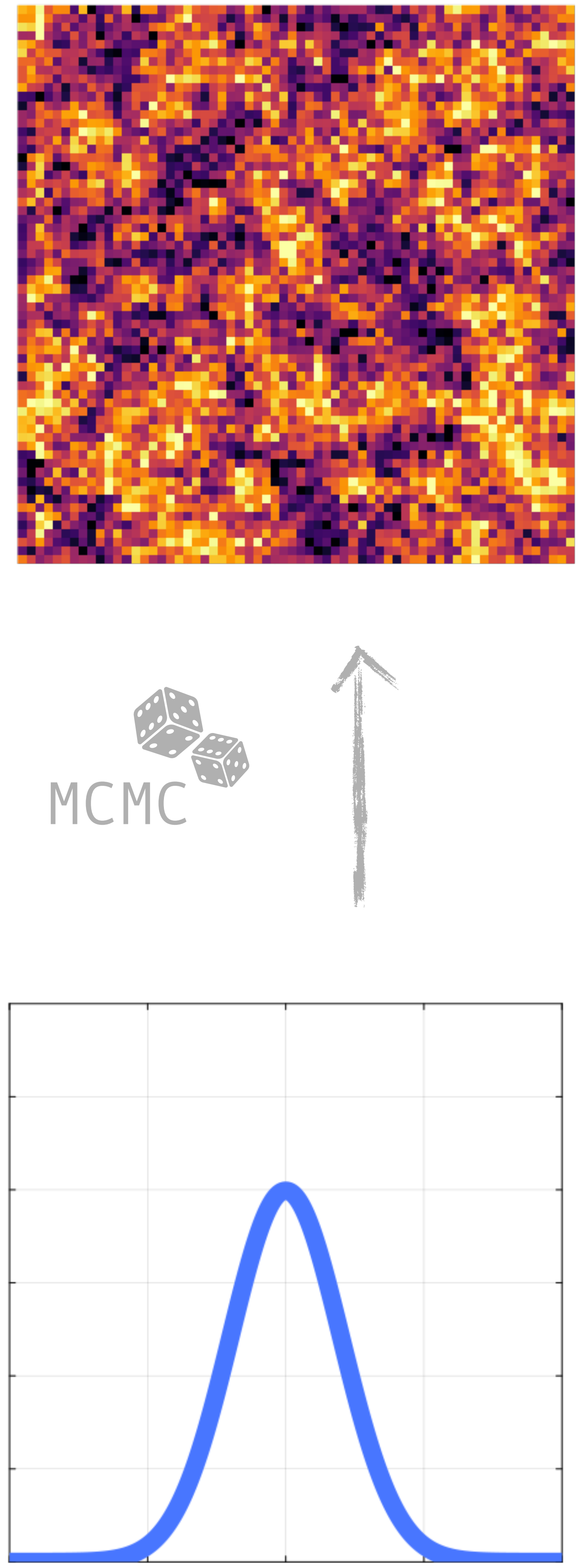

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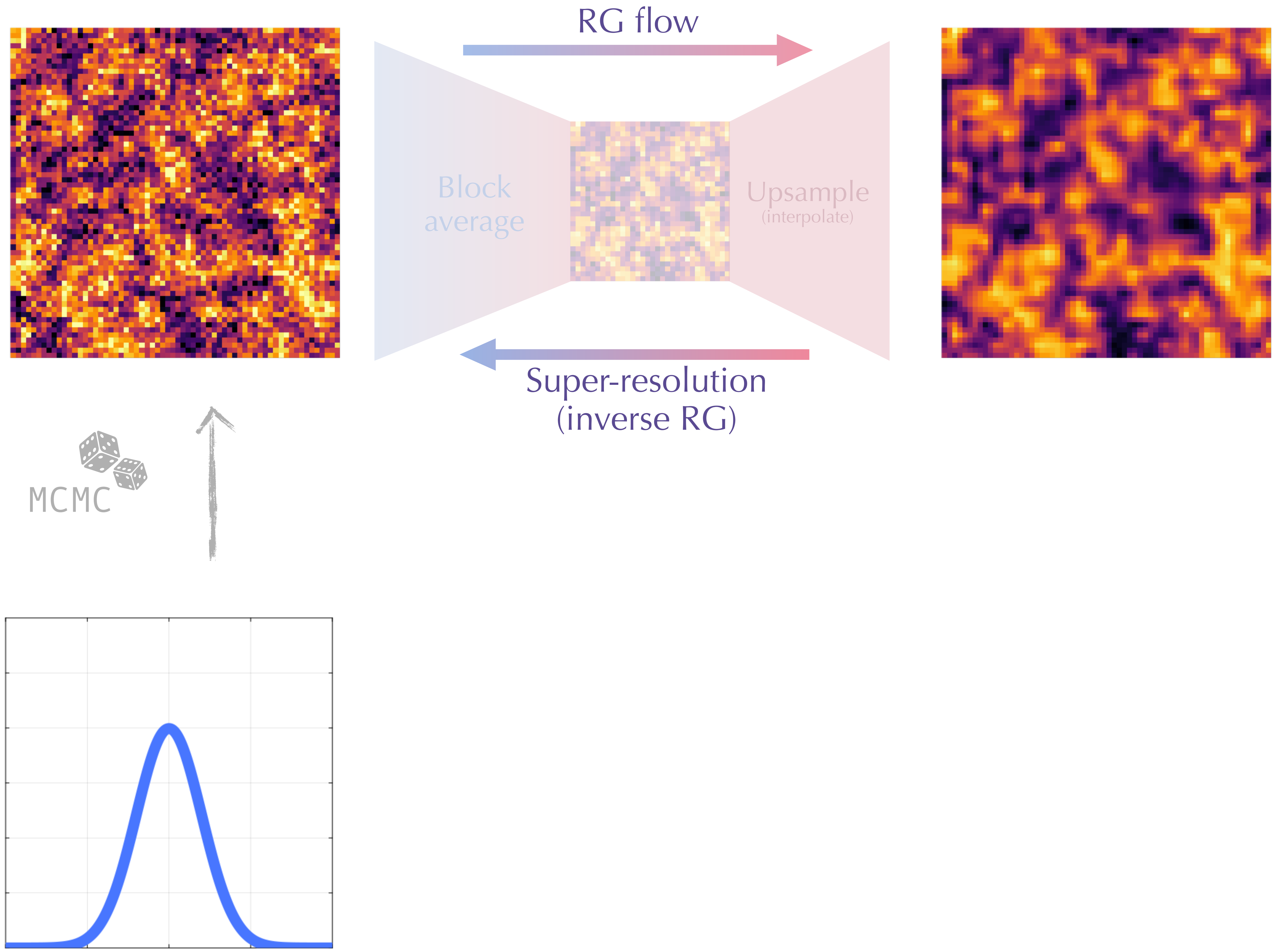


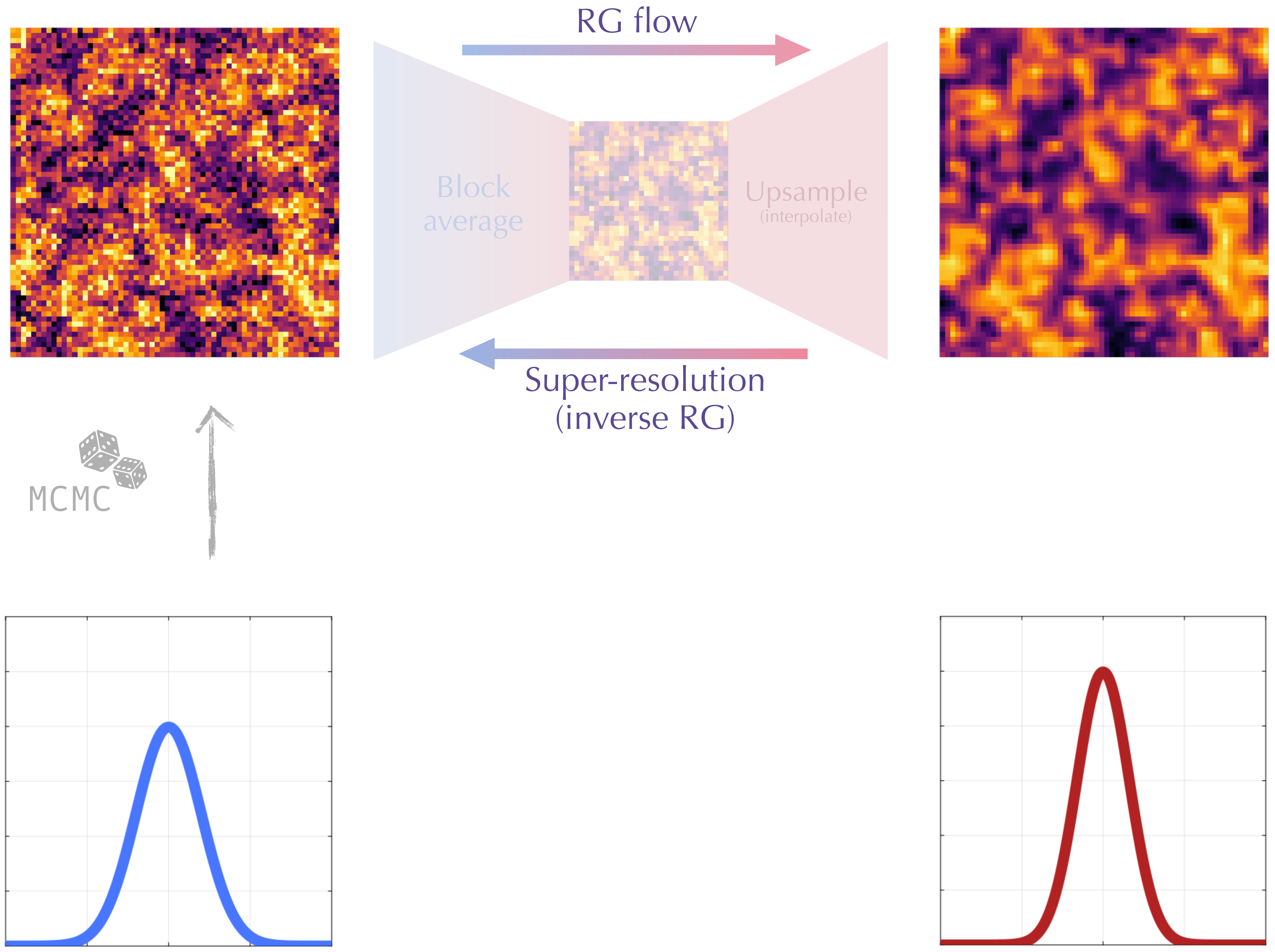
$\tilde{\phi}$

$p(\phi) \sim e^{-S_{(\kappa,\lambda)}}$

Distributional flows

ϕ

$\tilde{\phi}$

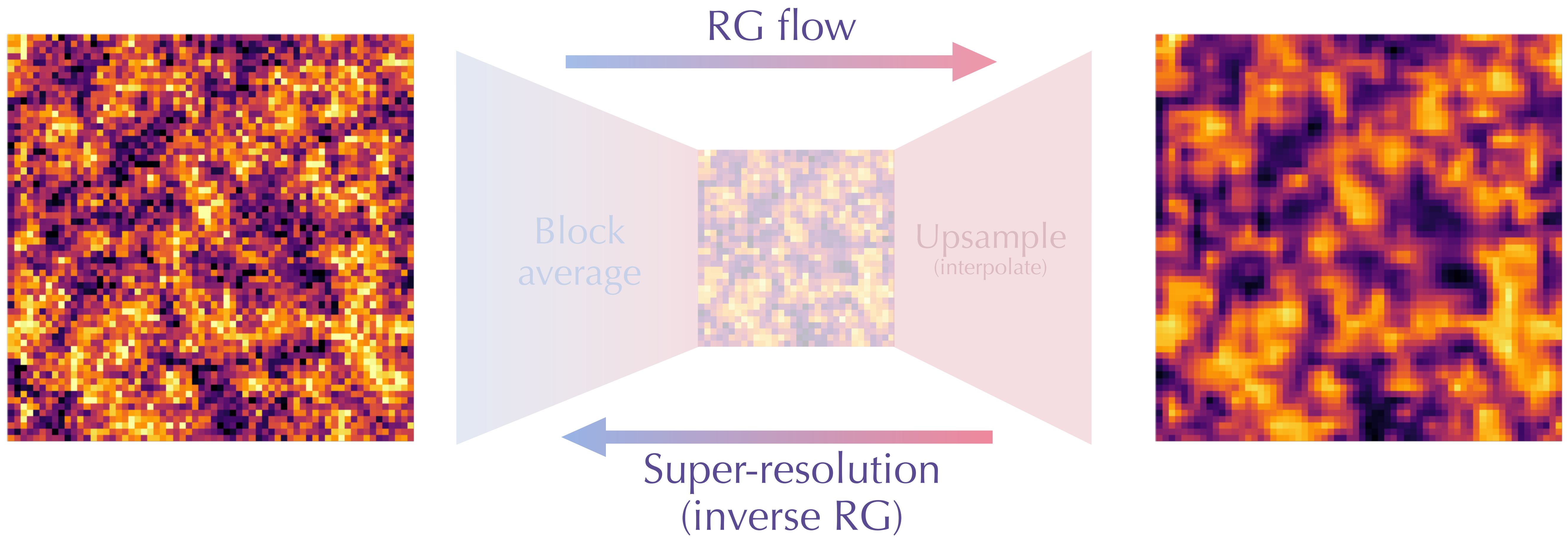


$p(\phi) \sim e^{-S_{(\kappa,\lambda)}}$

$p(\tilde{\phi}) \sim e^{-\tilde{S}_{\tilde{\kappa},\tilde{\lambda},\ldots}}$

Distributional flows

ϕ



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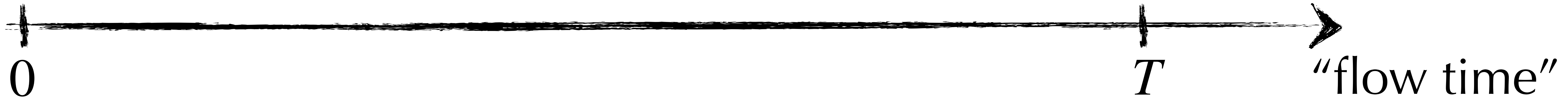

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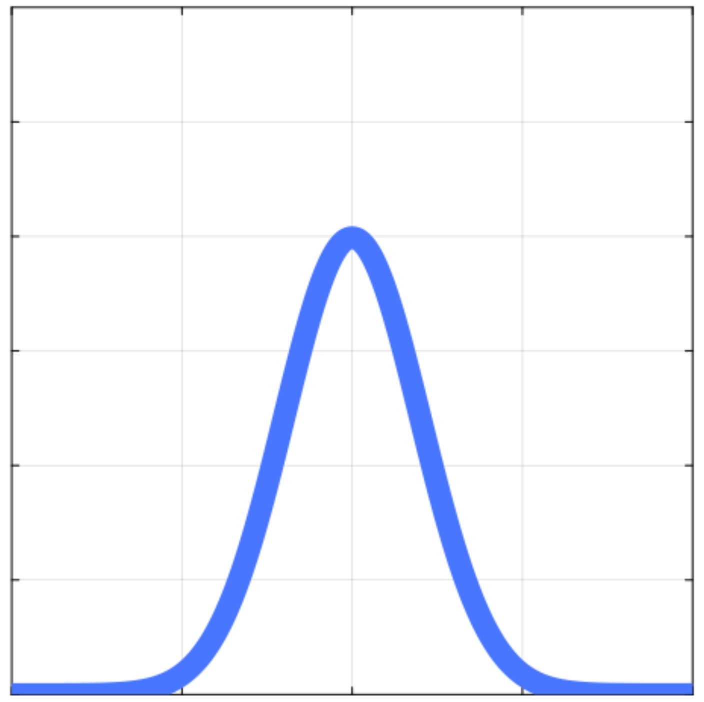
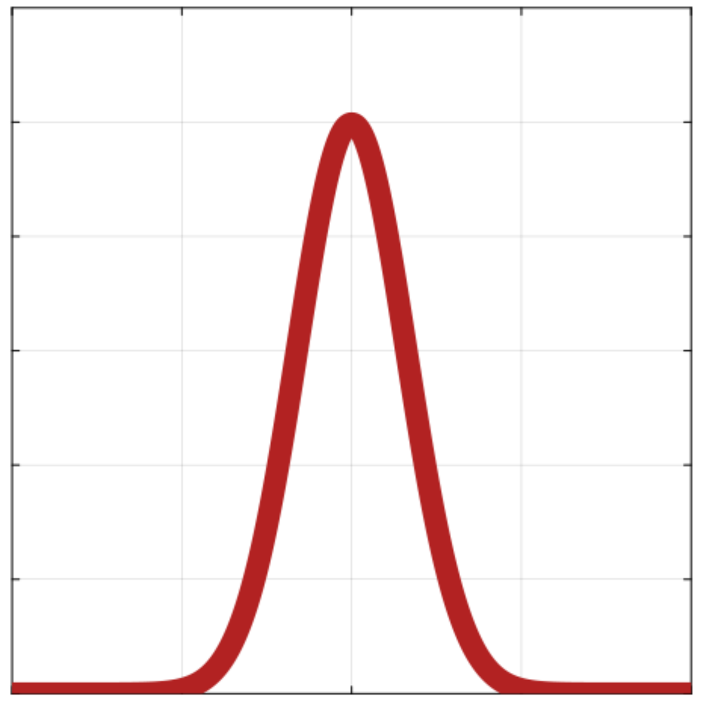
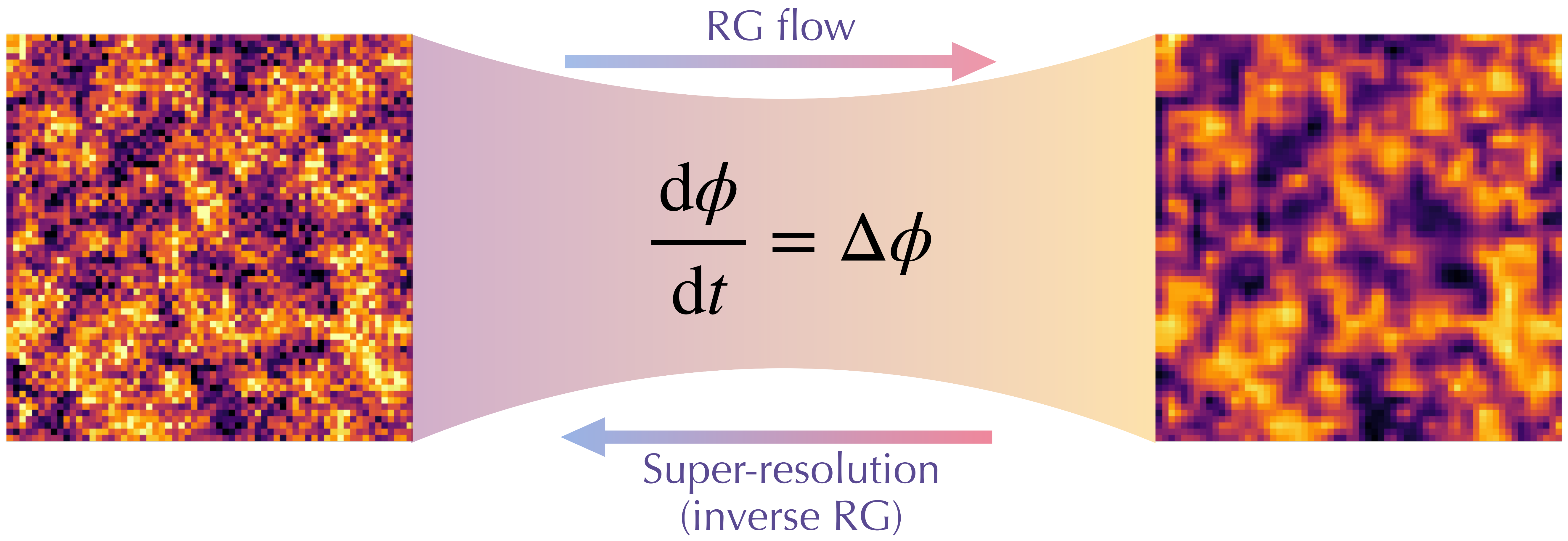




$p(\tilde{\phi}) \sim e^{-\tilde{S}_{\tilde{\kappa},\tilde{\lambda},\ldots}}$

Distributional flows

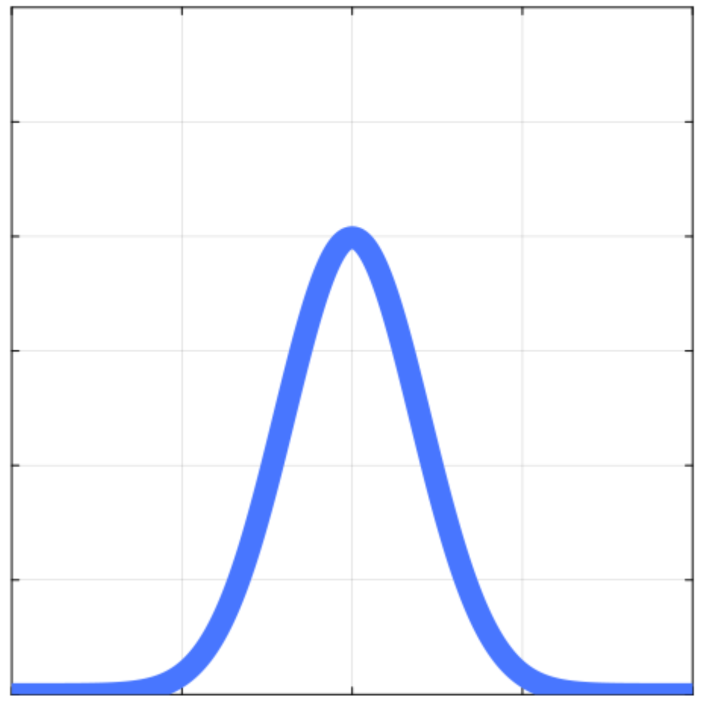
ϕ

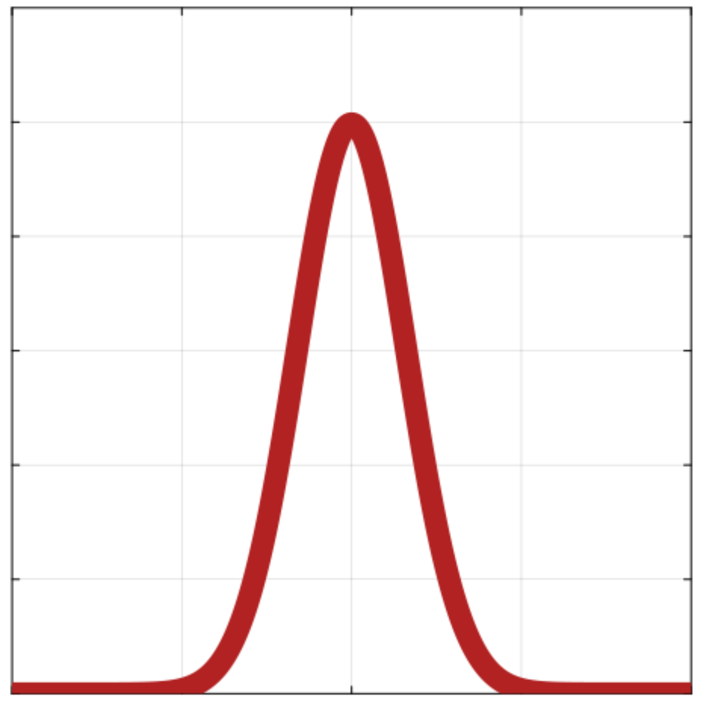


$\tilde{\phi}$

"flow time"

$p(\phi) \sim e^{-S_{(\kappa,\lambda)}}$





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Distributional flows

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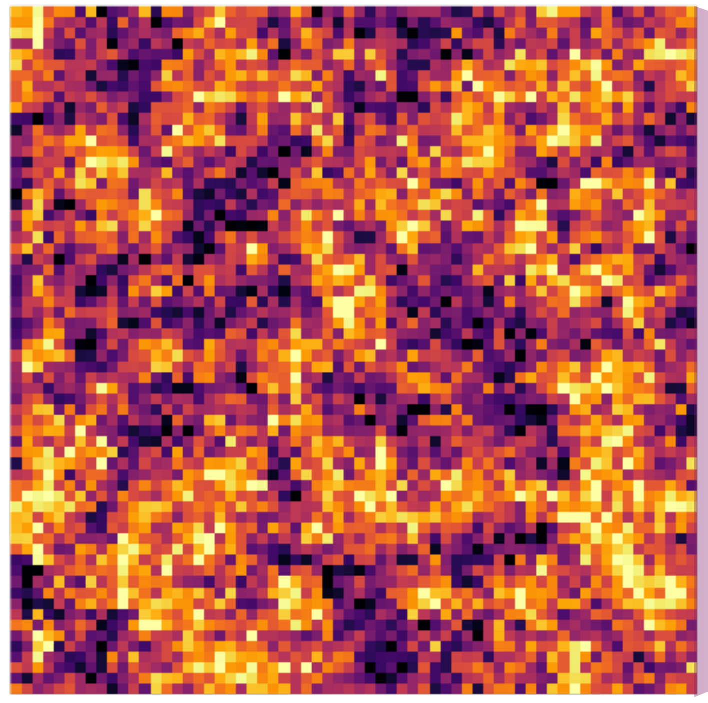
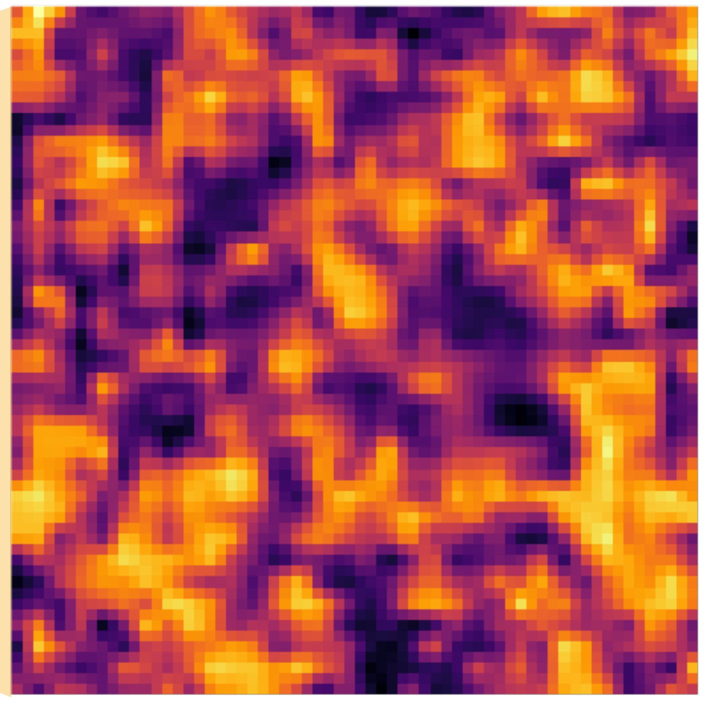


RG flow

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \Delta\phi$$

Super-resolution
(inverse RG)



$\tilde{\phi}$

0 — T — "flow time"

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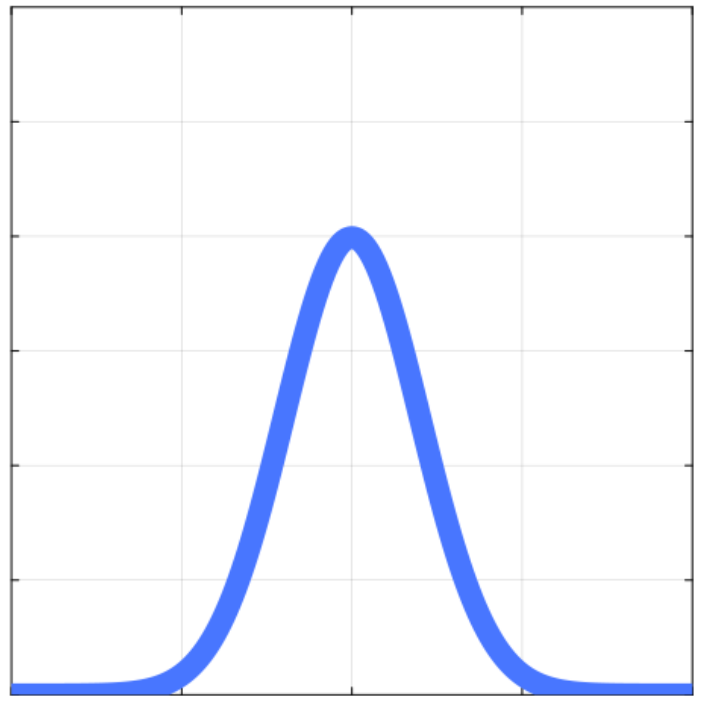
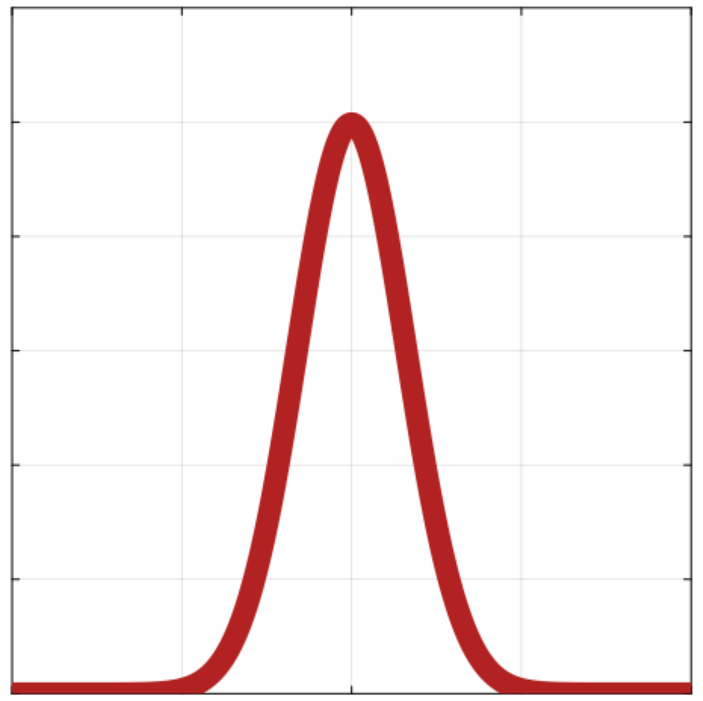


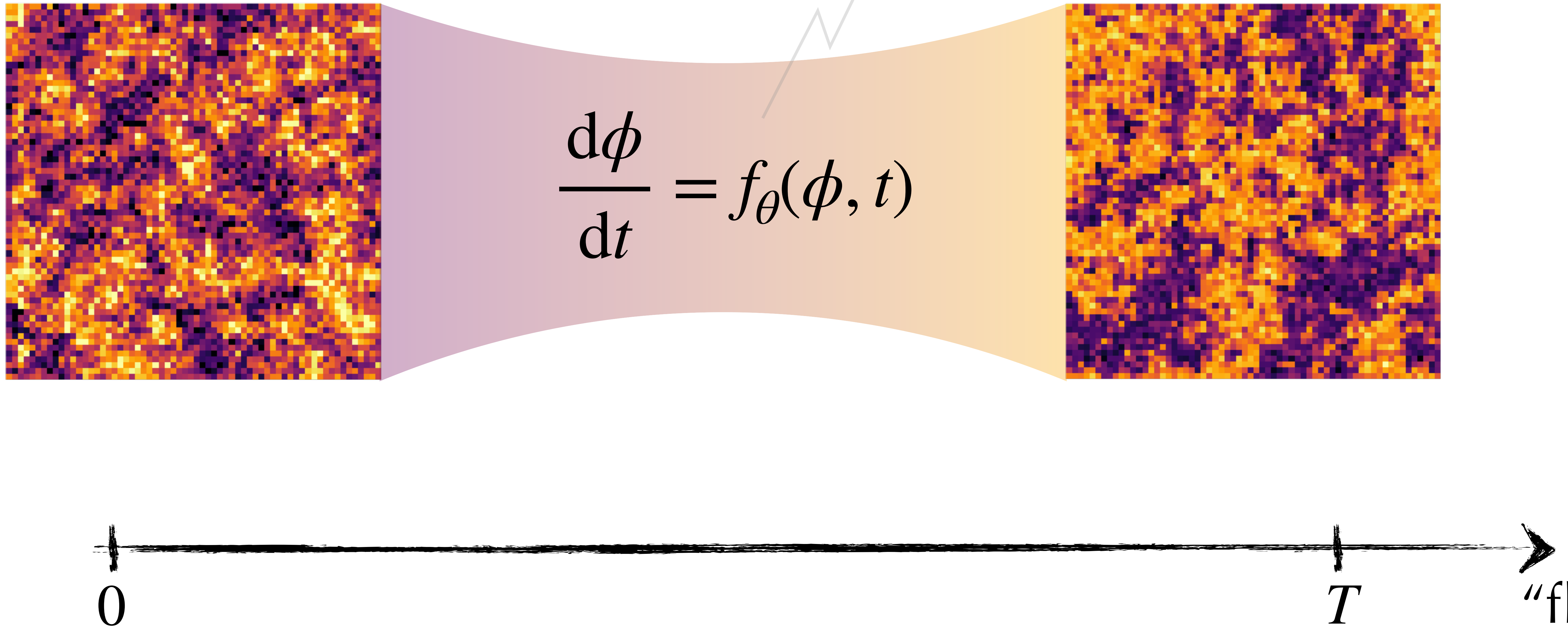


$p(\tilde{\phi}) \sim e^{-\tilde{S}_{\tilde{\kappa},\tilde{\lambda},\ldots}}$

Continuous normalizing flow

(equivariant)
Neural ODE

ϕ



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"flow time"

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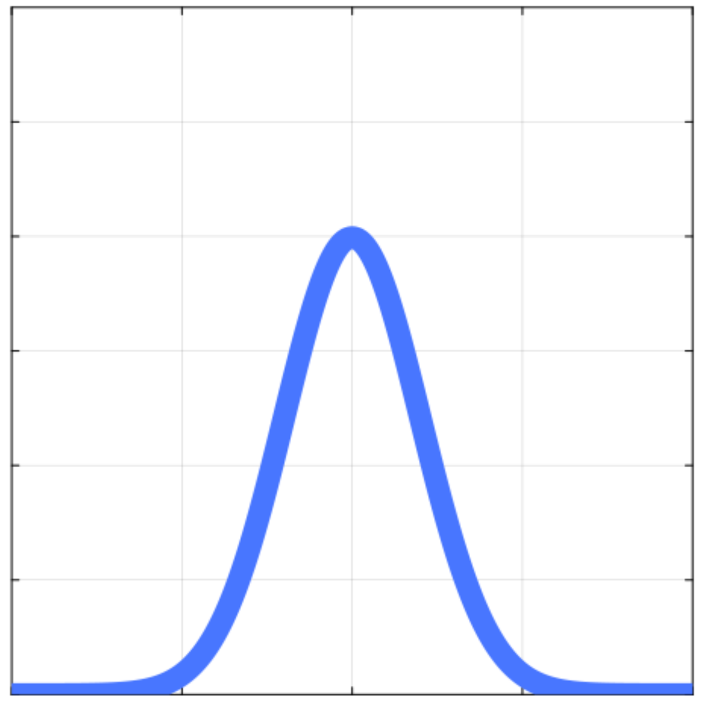

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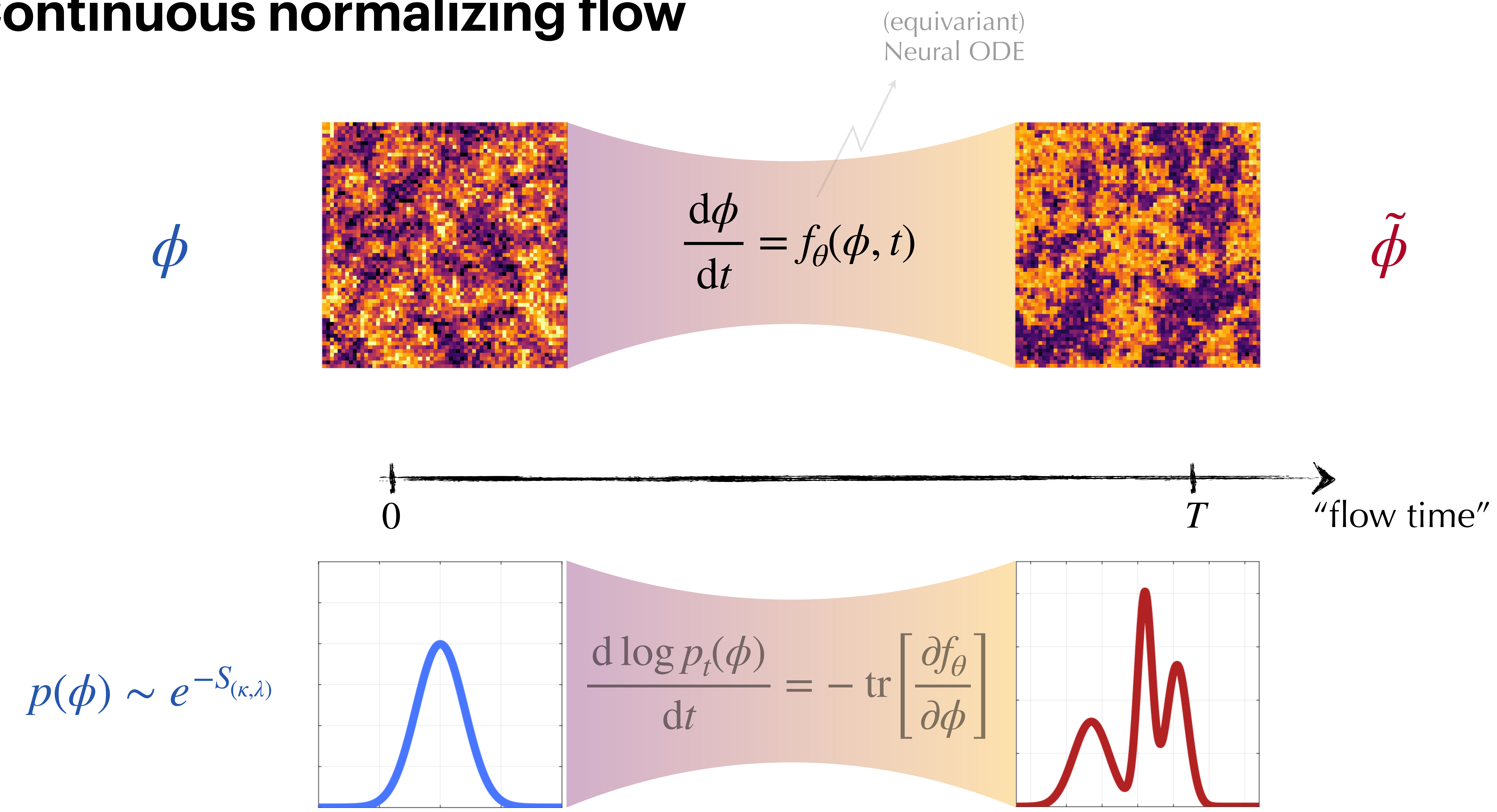
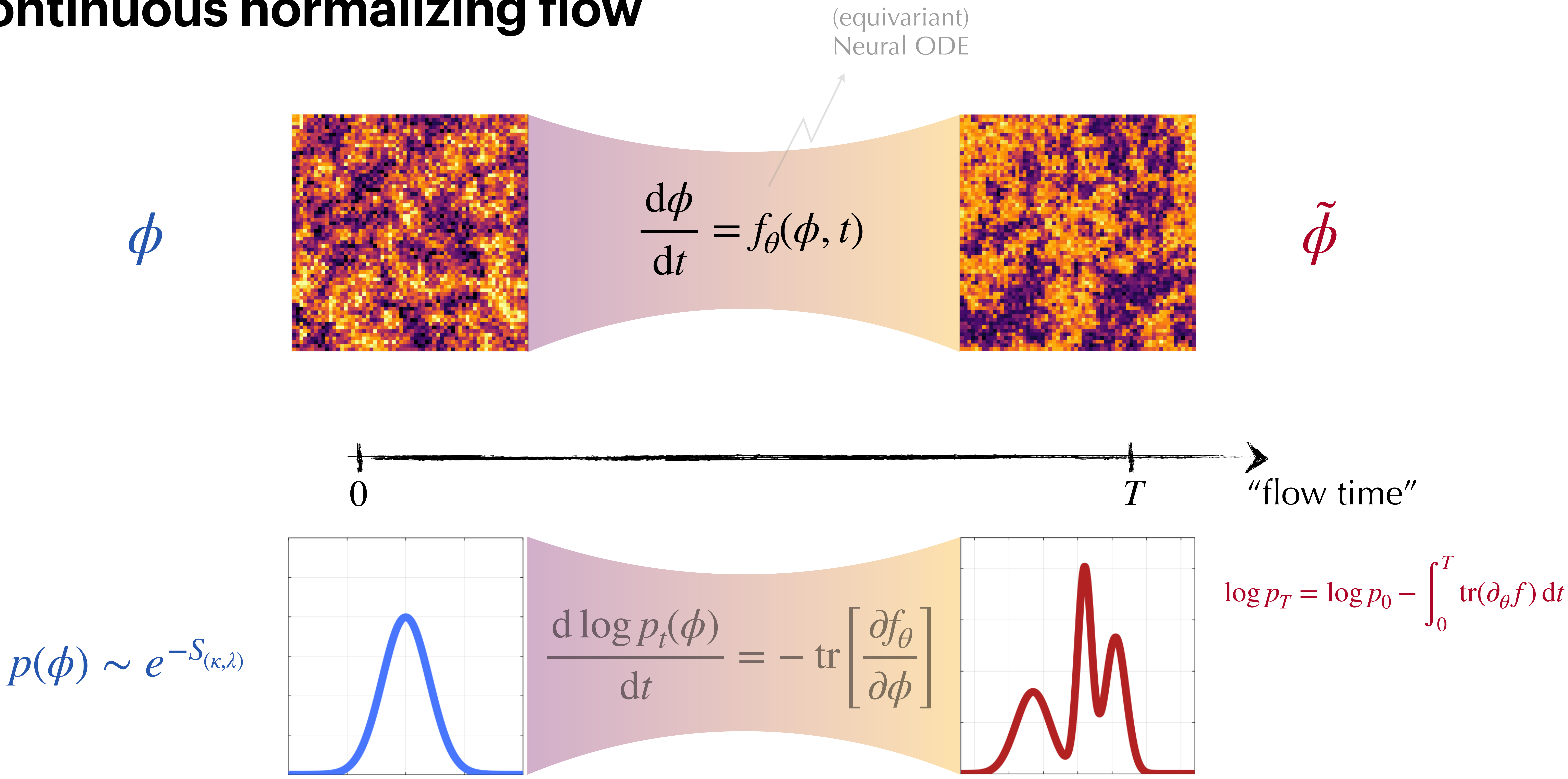


[Chen, Rubbanova, Bettencourt, Duvenaud]
- NeurIPS (2018)

Continuous normalizing flow

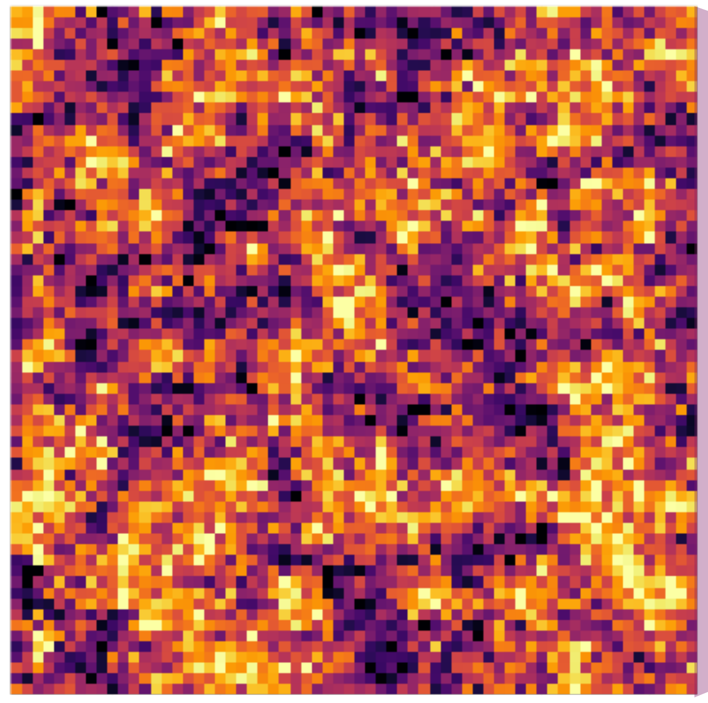


[Chen, Rubbanova, Bettencourt, Duvenaud]
- NeurIPS (2018)

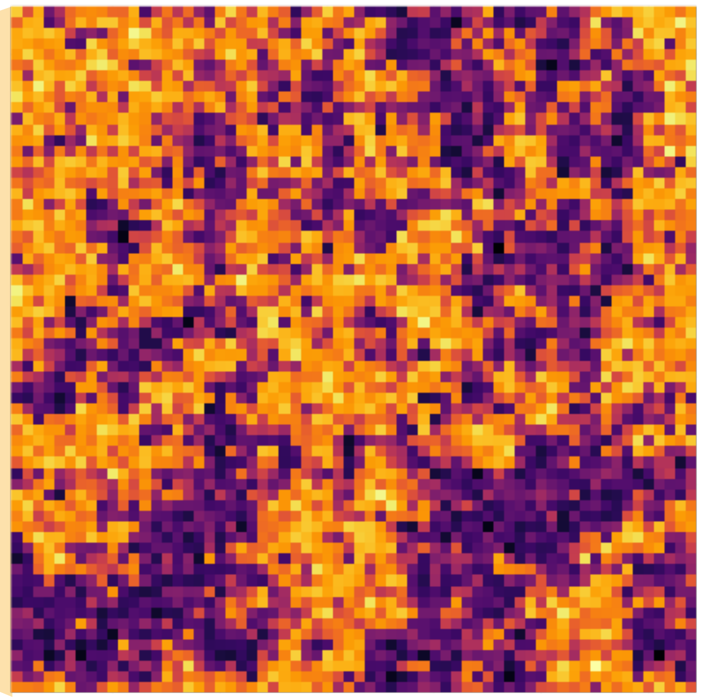
Continuous normalizing flow

(equivariant)
Neural ODE

ϕ



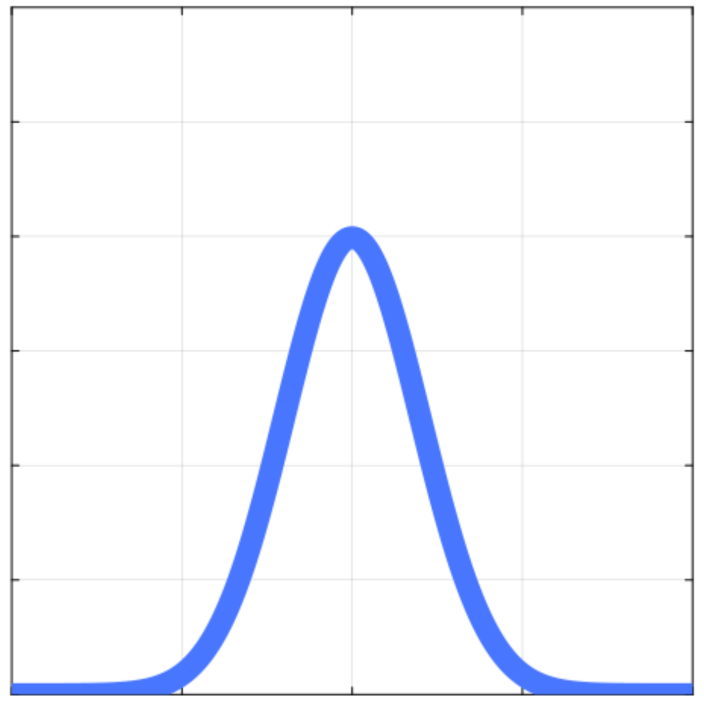
$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = f_\theta(\phi, t)$$



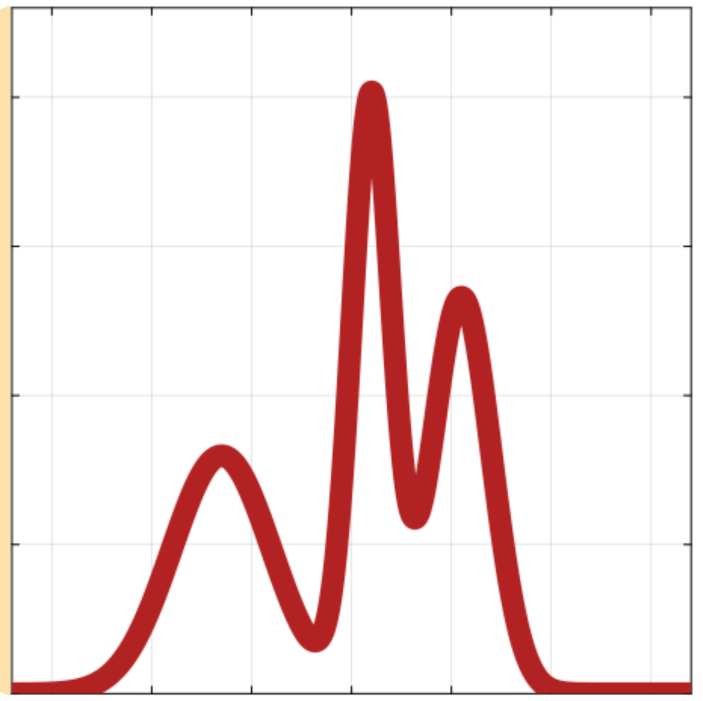
$\tilde{\phi}$

0 — T — "flow time"

$p(\phi) \sim e^{-S_{(\kappa,\lambda)}}$



$$\frac{\mathrm{d}\log p_t(\phi)}{\mathrm{d}t} = -\,\mathrm{tr}\left[\frac{\partial f_\theta}{\partial \phi}\right]$$



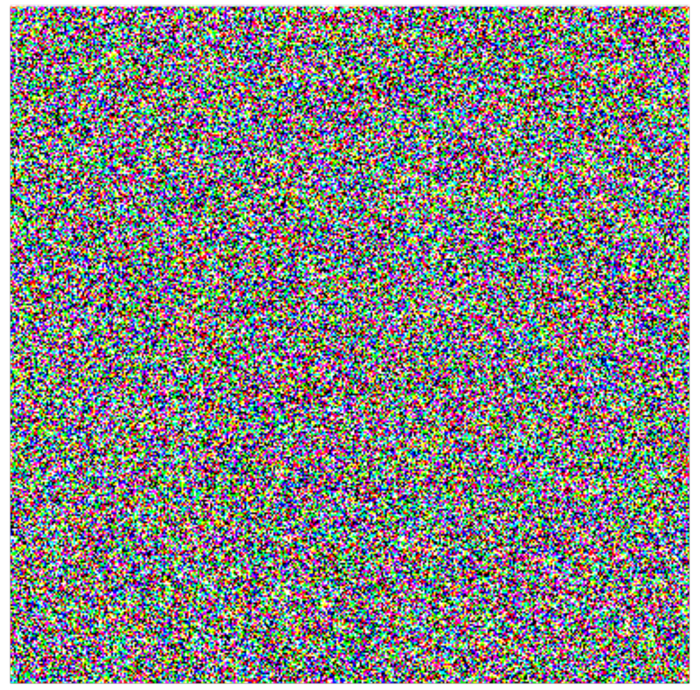
$$\log p_T = \log p_0 - \int_0^T \mathrm{tr}(\partial_\theta f)\,\mathrm{d}t$$

Use f_θ to parameterise a target a density from data!

[Chen, Rubbanova, Bettencourt, Duvenaud] - NeurIPS (2018)

Continuous NF: application 1

Generate lattice configuration
from (gaussian) scratch

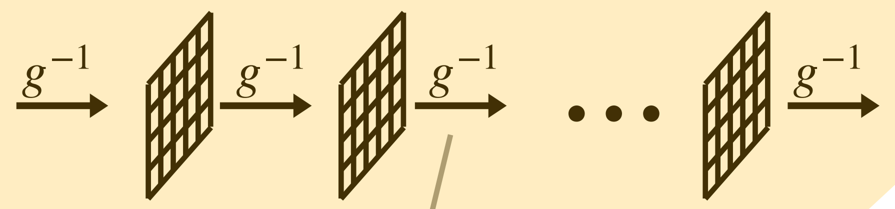


Random noise
$\sim \mathcal{N}(0,1)$

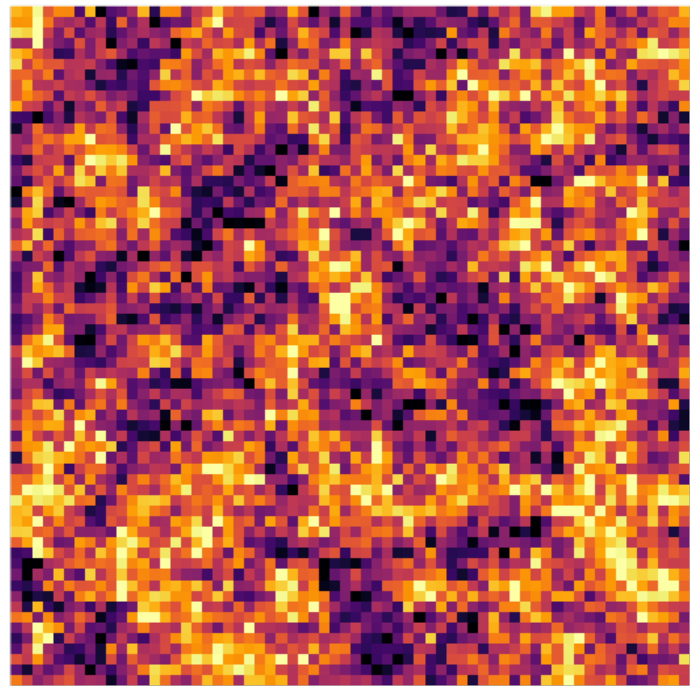
Integrate neural ODE

$$\phi_{t+1} = \phi_t + h f_\theta(\phi, t)$$

…or…



each layer is a **diffeomorphism** with **tractable Jacobian**



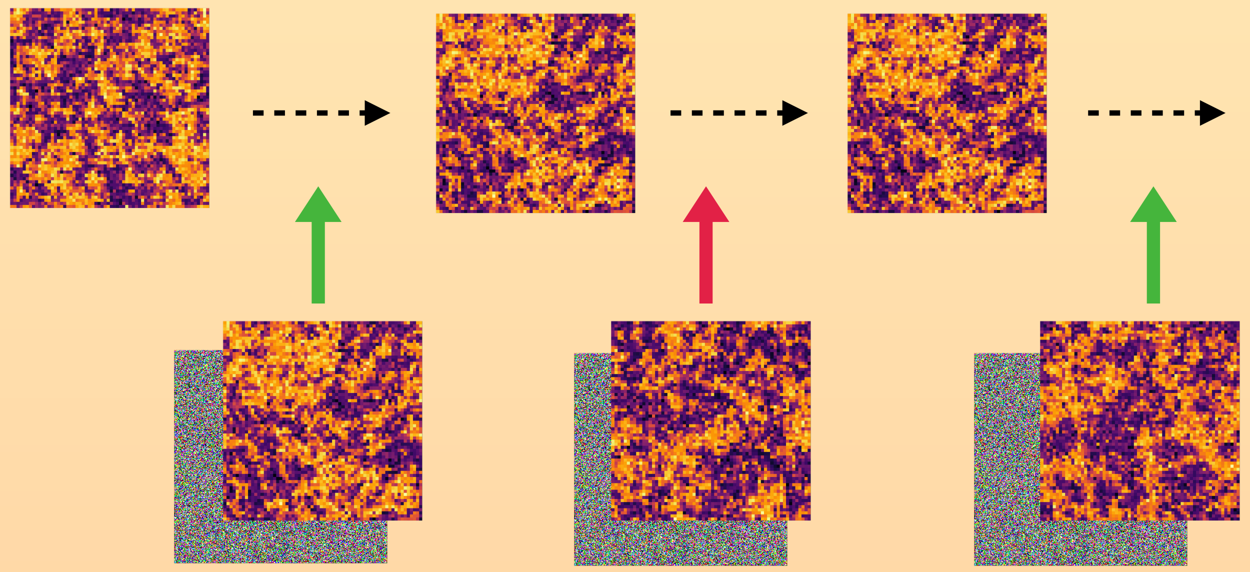
Typical lattice config. $\sim e^{-S}$

[Albergo, Kanwar, Shanahan]
Phys.Rev.D **100** (2019)

- Self-training minimising KL divergence

$$KL\left(p(\phi) \| e^{-S}\right)$$

- Employ gauge-equivariant network to deploy action symmetries

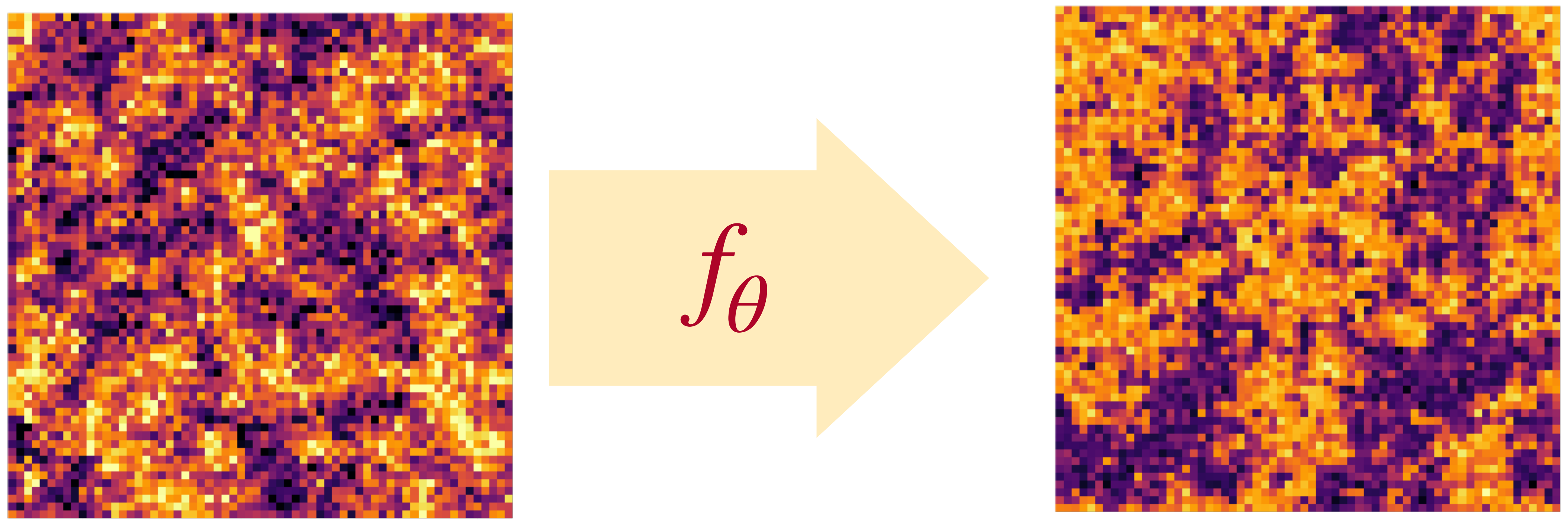


- Exactness through Metropolis accept/reject step (or reweighing)
- Independent sampling: no autocorrelations!
- Tradeoff with large training cost close to criticality

Continuous NF: application 2

Continuous NF: application 2

Generate configurations
from an **action with a source**



lattice config.
from MCMC
$\phi \sim e^{-S}$

Target:
$\tilde{\phi} \sim e^{-(S+J\phi)}$

Continuous NF: application 2

$$C(t) = \frac{\partial}{\partial J}\bigg|_{J=0} \langle \tilde{\phi}(x) \rangle_{S+J\phi} \equiv \langle \phi(x)\phi(0) \rangle_S$$

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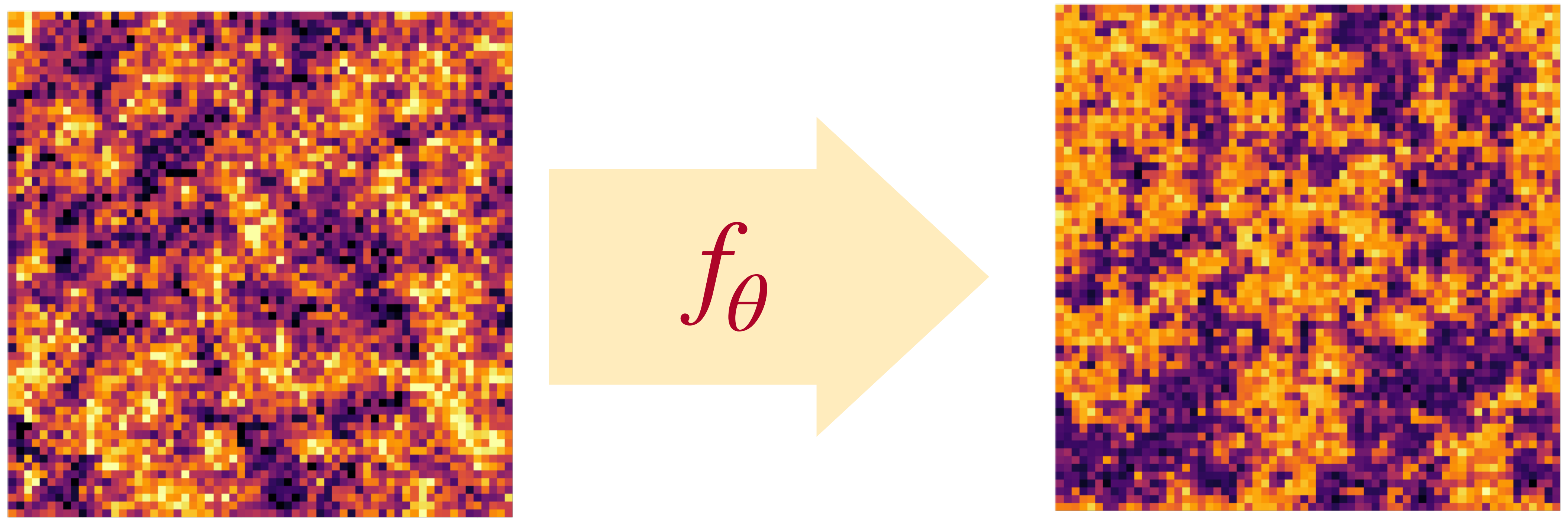

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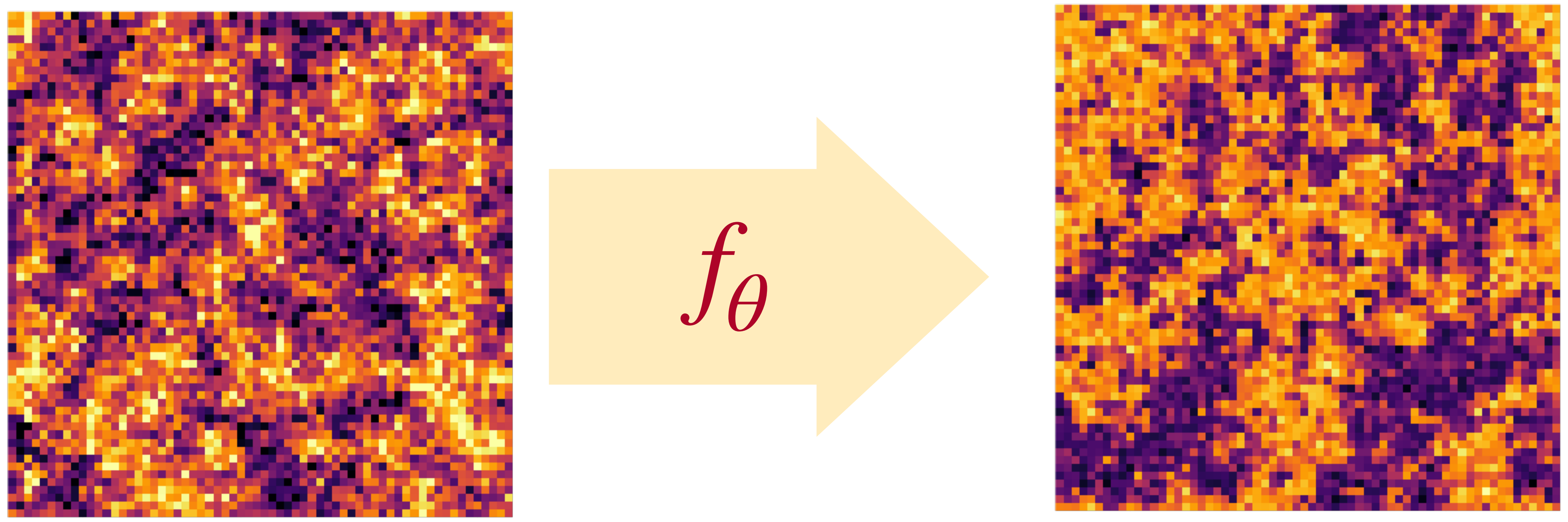

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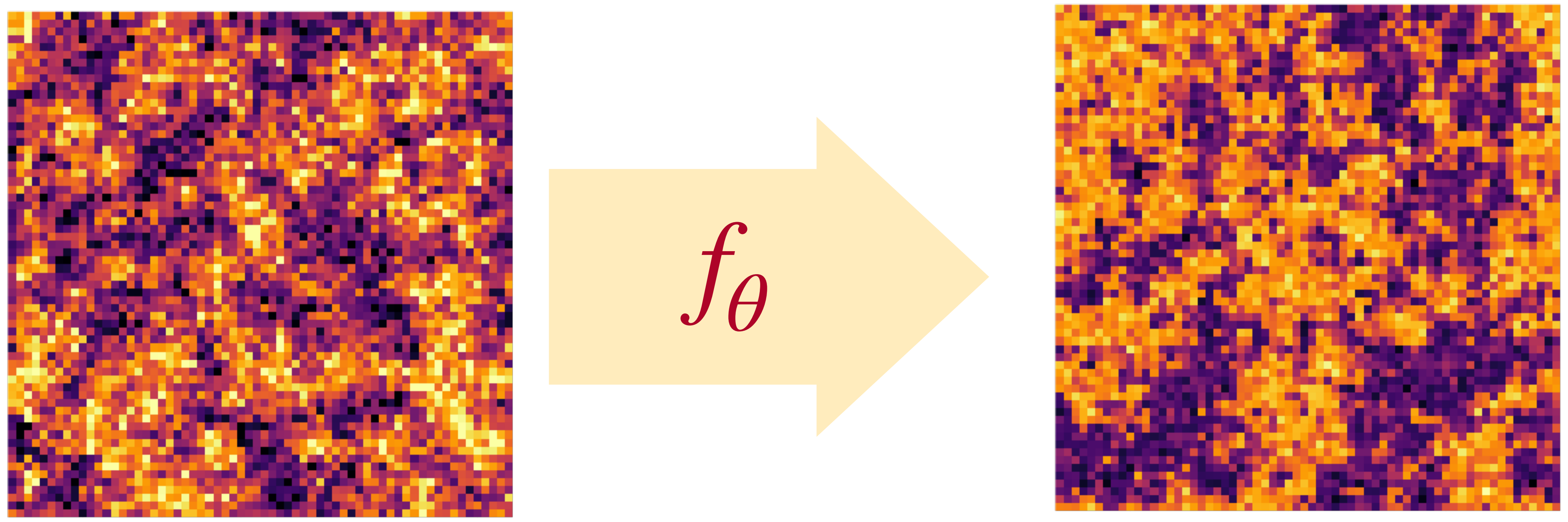

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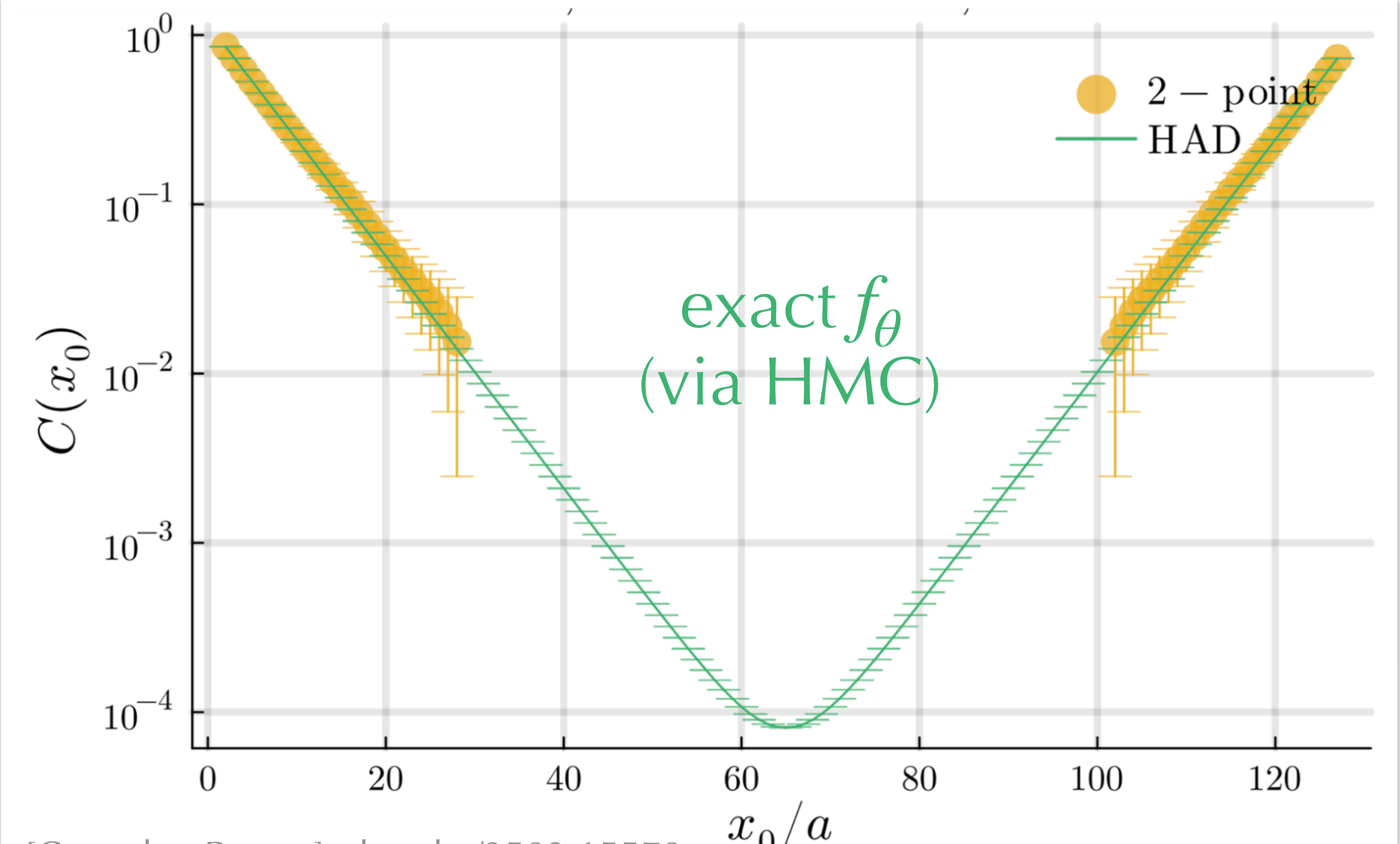


[Catumba, Ramos] - hep-lat/2502.15570

Conclusions

- ML algorithms for pics and QFT shares deep analogies
- The parallelism between blurring and RG flow can be used to super-resolve local theories
- (Continuous) normalizing flows can target distribution and used as generative tools and to drastically treat signal-to-noise ratio problem
- Keep exploring!

BACKUP

Hidden states and deepness

"The distribution of observational data
constrained to average observations,
maximises Shannon's entropy"

Given observations $\{\mathbf{x}_n\}$, their MaxEnt distribution p is

$$\max_p \langle -\log p(\mathbf{x}) \rangle \qquad \Rightarrow \qquad p(\mathbf{x}) = \frac{e^{-\beta E(\mathbf{x})}}{Z}$$

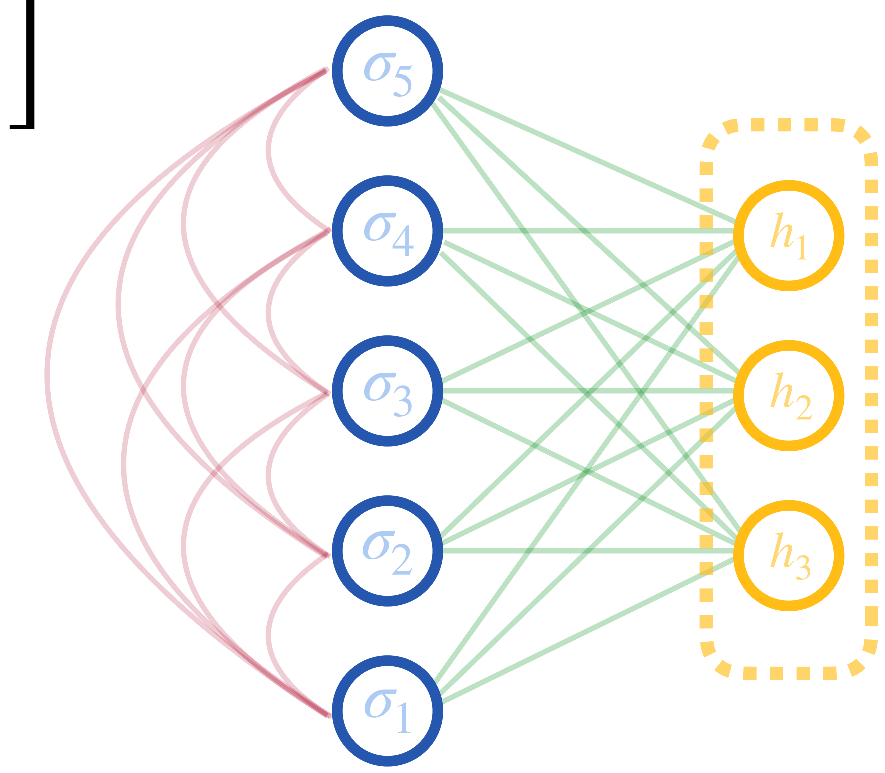
$$\langle E \rangle_p \equiv \frac{1}{N} \sum_n^{data} E(\mathbf{x}_n)$$

What is MaxEnt distribution of observed samples
that matches empirical mean $\langle \sigma_i \rangle_{\text{data}}$ and correlations $\langle \sigma_i \sigma_j \rangle_{\text{data}}$?

$$p(\sigma) = \frac{1}{Z} \exp\left[-\sum_i a_i \sigma_i - \sum_{i,j} \sigma_i J_{ij} \sigma_j \right]$$

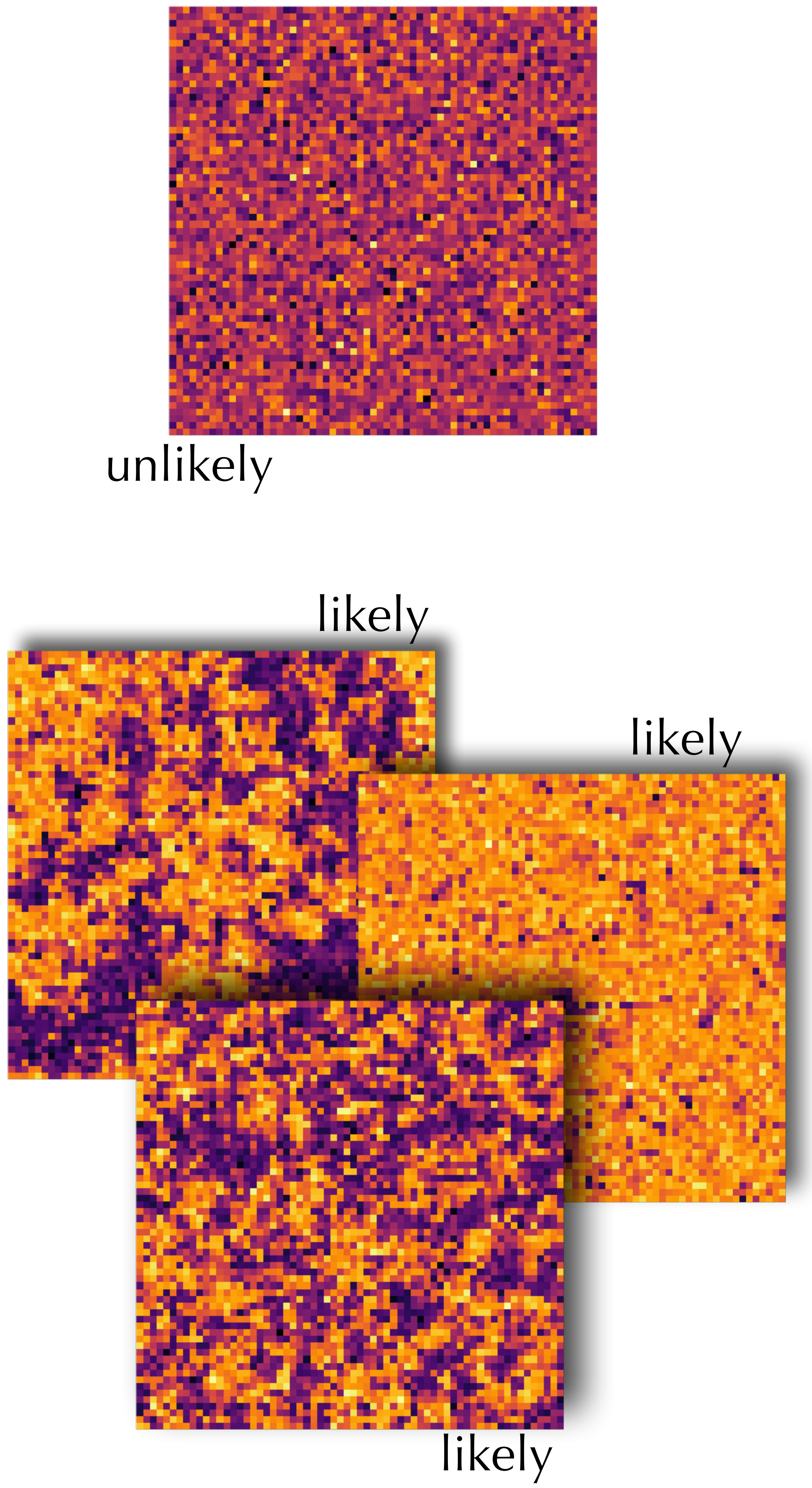
$$e^{\frac{\mathbf{x}M\mathbf{x}}{2}} = \int [d\mathbf{h}] e^{-\frac{1}{2}\mathbf{h}^2 + \mathbf{v}\sqrt{M}\mathbf{h}}$$

$$p(\sigma) = \frac{1}{Z} \int d\mathbf{h} \, \exp\left[-\sum_i a_i \sigma_i + \frac{1}{2} \sum_\mu h_\mu^2 - \sum_{i\mu} \sigma_i h_\mu W_{i\mu} \right]$$

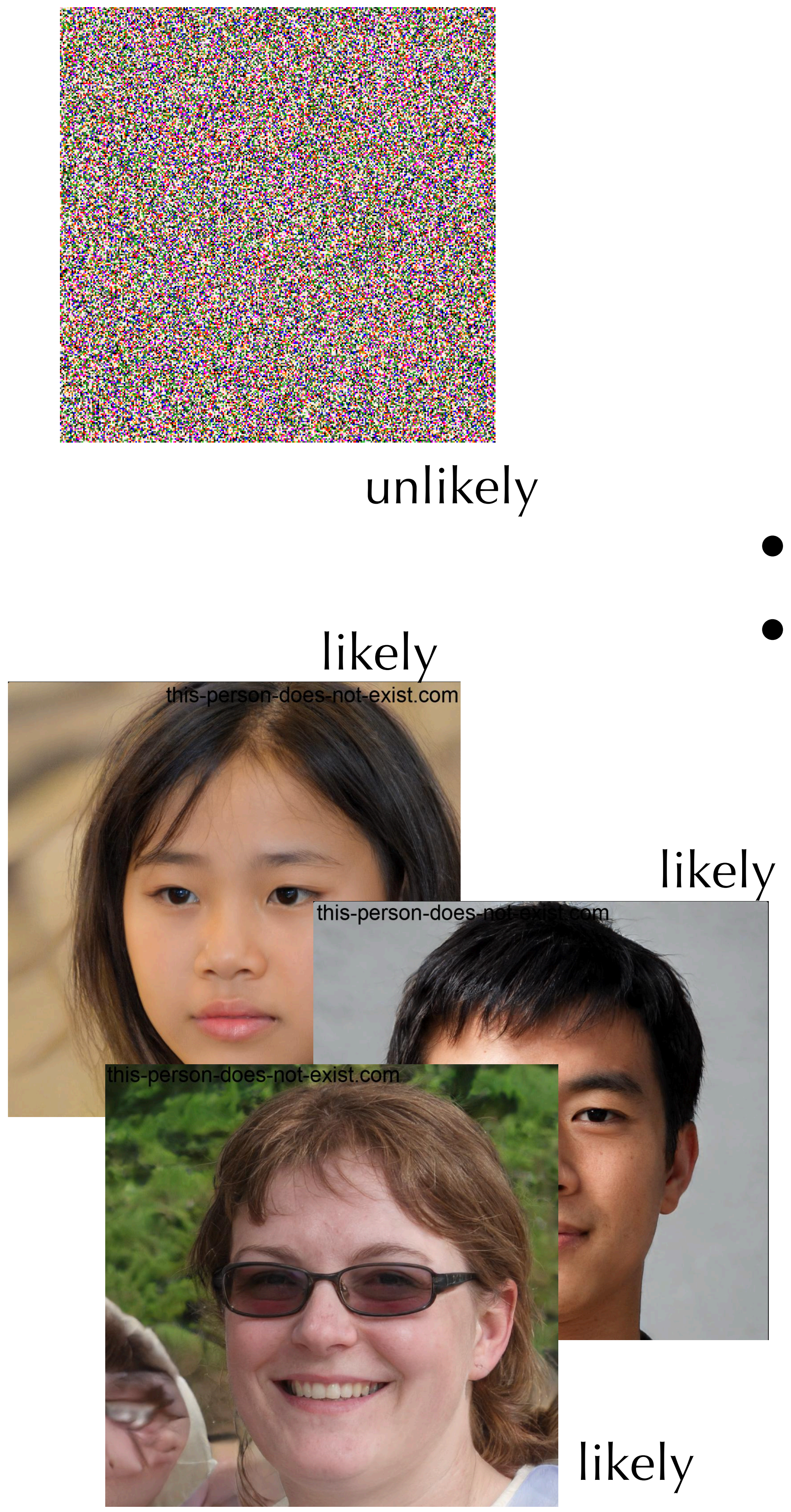


Lattice configs vs pics

- Boltzmann distribution known
- Lots of “pixel” - few samples
- Known physical symmetries



vs



- Unknown probability distribution
- Few pixel - lots of samples

Can we “inflate” the volume?

Lattice QFT: compute the spectrum

An alternative (novel) approach…

$$C(t) = \frac{\partial}{\partial J}\bigg|_{J=0} \frac{\langle e^{-J\phi}\phi(x)\rangle}{\langle e^{-J\phi}\rangle}$$

use Automatic Differentiation!

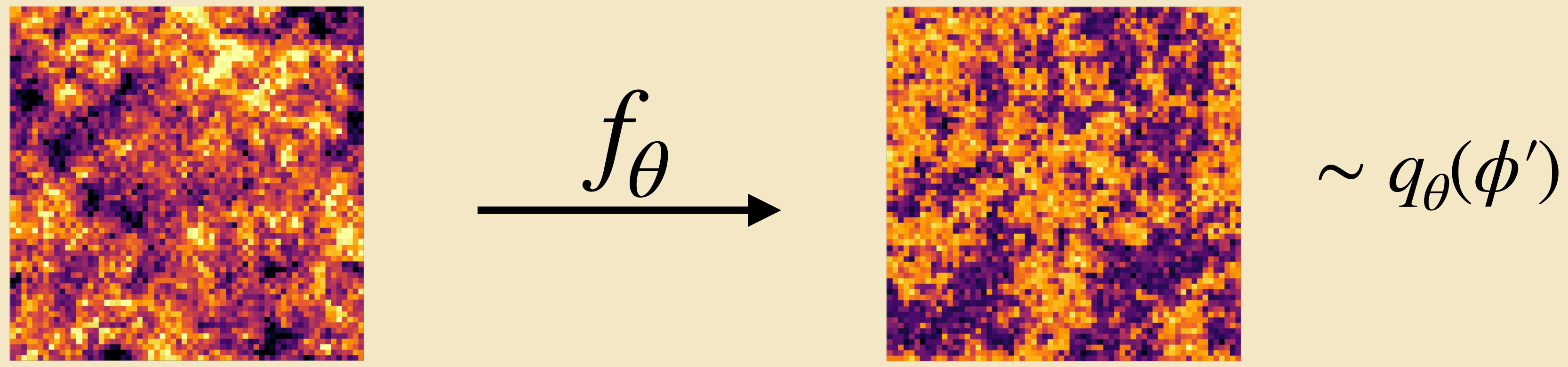
reweight $\langle \phi(x) \rangle_S$

$S \to S_J \equiv S + J\phi$

All the noise comes from weights

$$w_J[\phi] = e^{-J\phi}$$

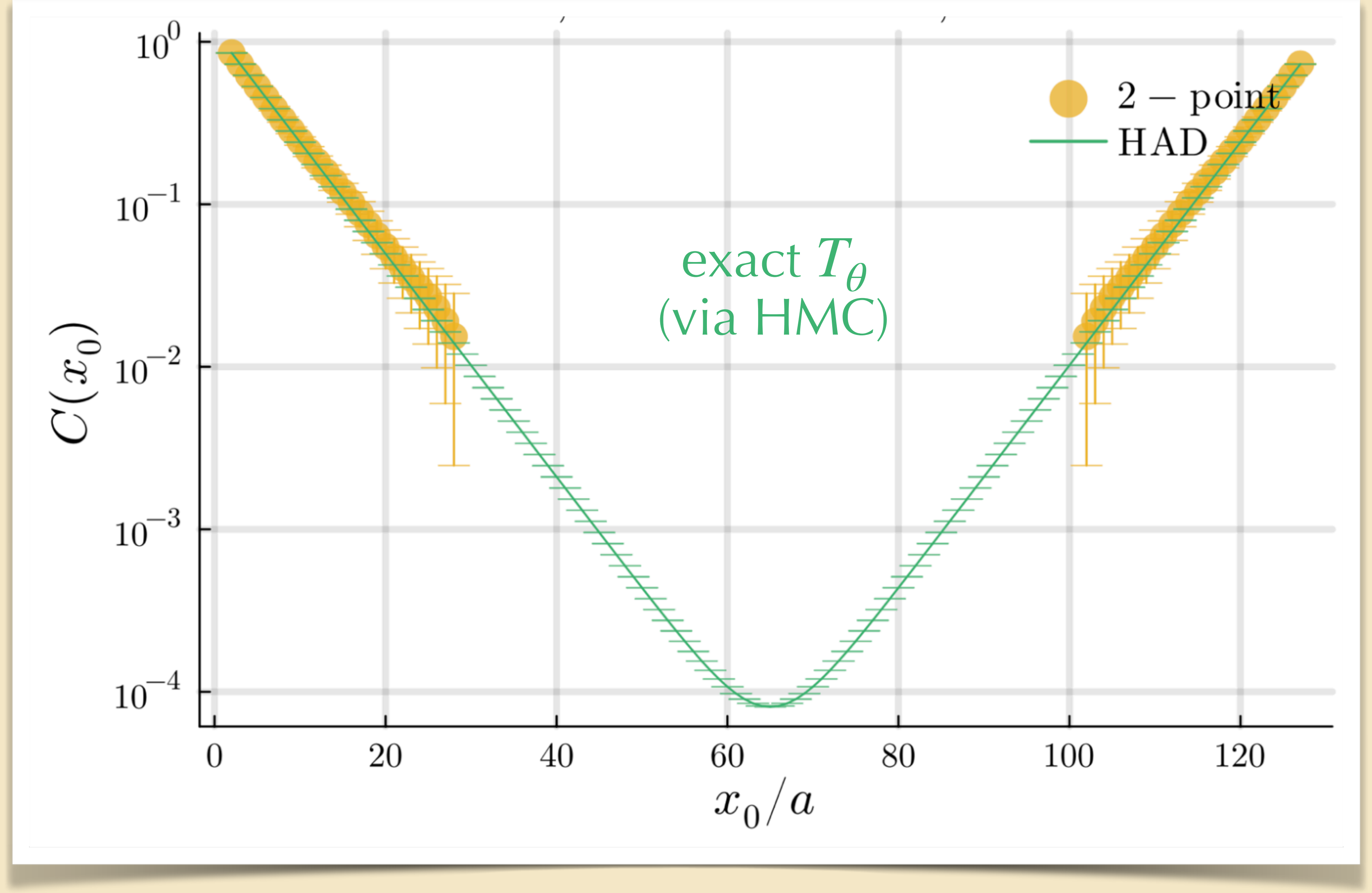
Find field transformation (flow) $f_\theta : p(\phi) \to q_\theta(\phi')$



Goal

$q_\theta \approx e^{-(S+J\phi)}$ $\Rightarrow$ $w_J \approx 1$

flat weights
(No fluctuations)



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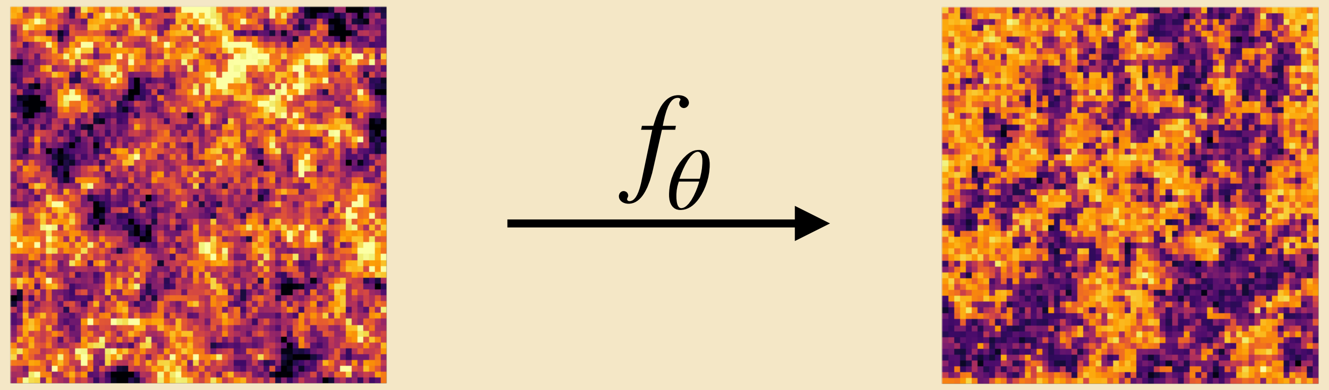


Goal

$q_\theta \approx e^{-(S+J\phi)}$ $\Rightarrow$ $w_J \approx 1$

flat weights
(No fluctuations)

…or machine learn it…

Requirements for f_θ

- Invertible
- Tractable Jacobian
- Respecting symmetries

Continuous NF

$$\log q_\theta = \log p - \int_0^T \mathrm{tr}(\partial_\theta f)\, dt$$

+

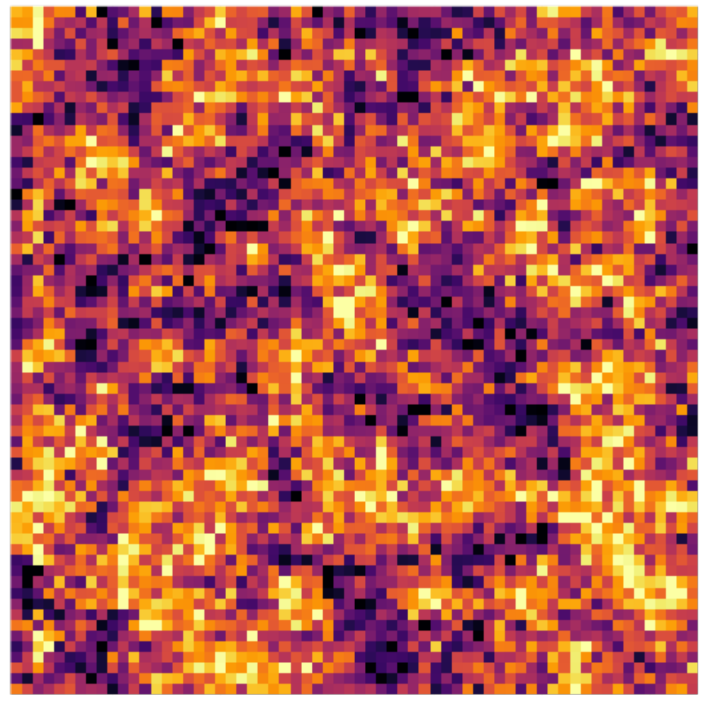
minimize

$$\mathrm{KL}\left(q_\theta \| e^{-(S+J\phi)}\right)$$

Distributional flows 2/2

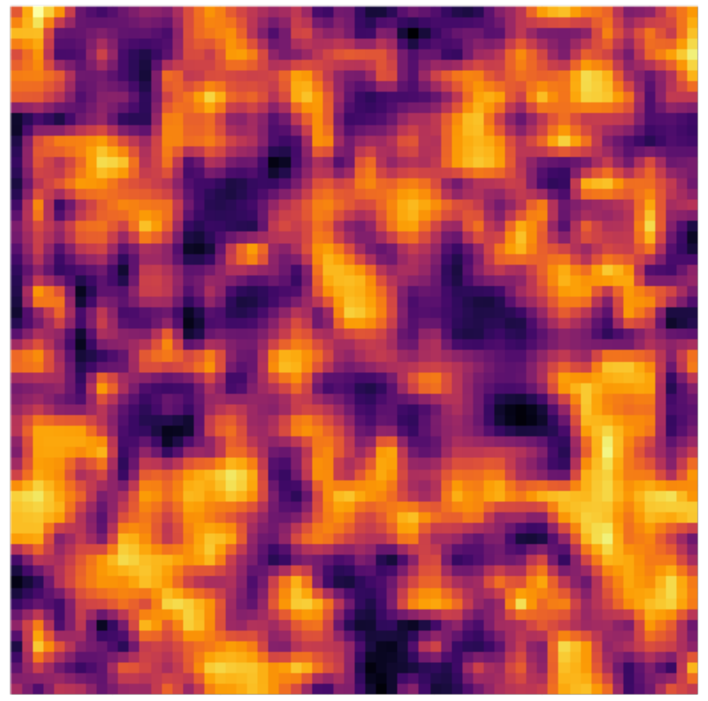
$\frac{e^{-S[\phi]}}{Z}$

(κ, λ)



RG flow →

← Super-resolution (inverse RG)



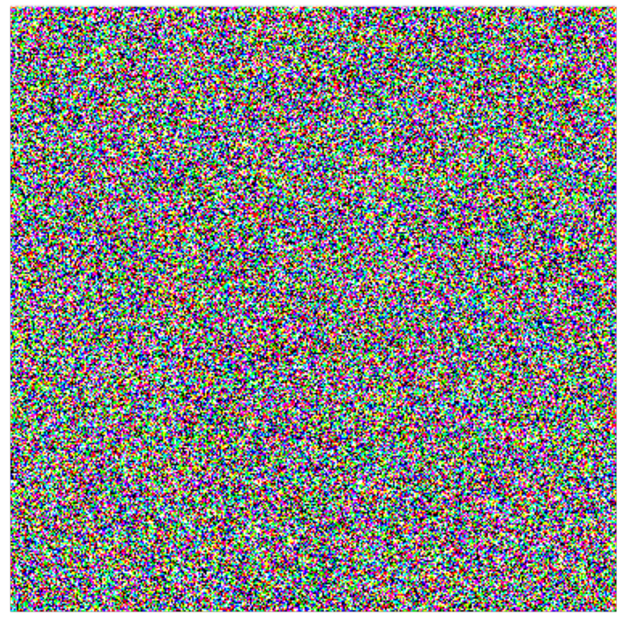
$\frac{e^{-\tilde{S}[\tilde{\phi}]}}{\tilde{Z}}$

$(\kappa', \lambda', \ldots)$

Another example…

Diffusion →

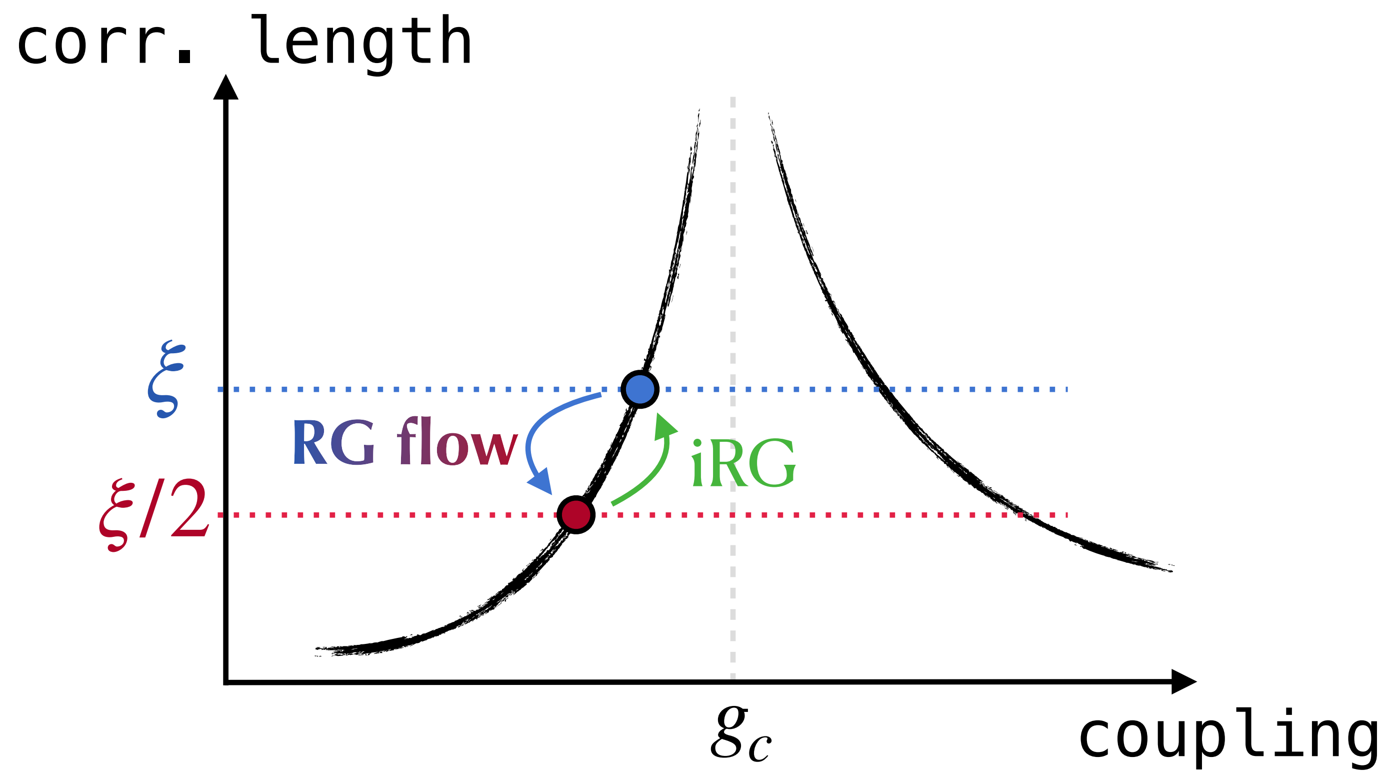
unknown target distribution

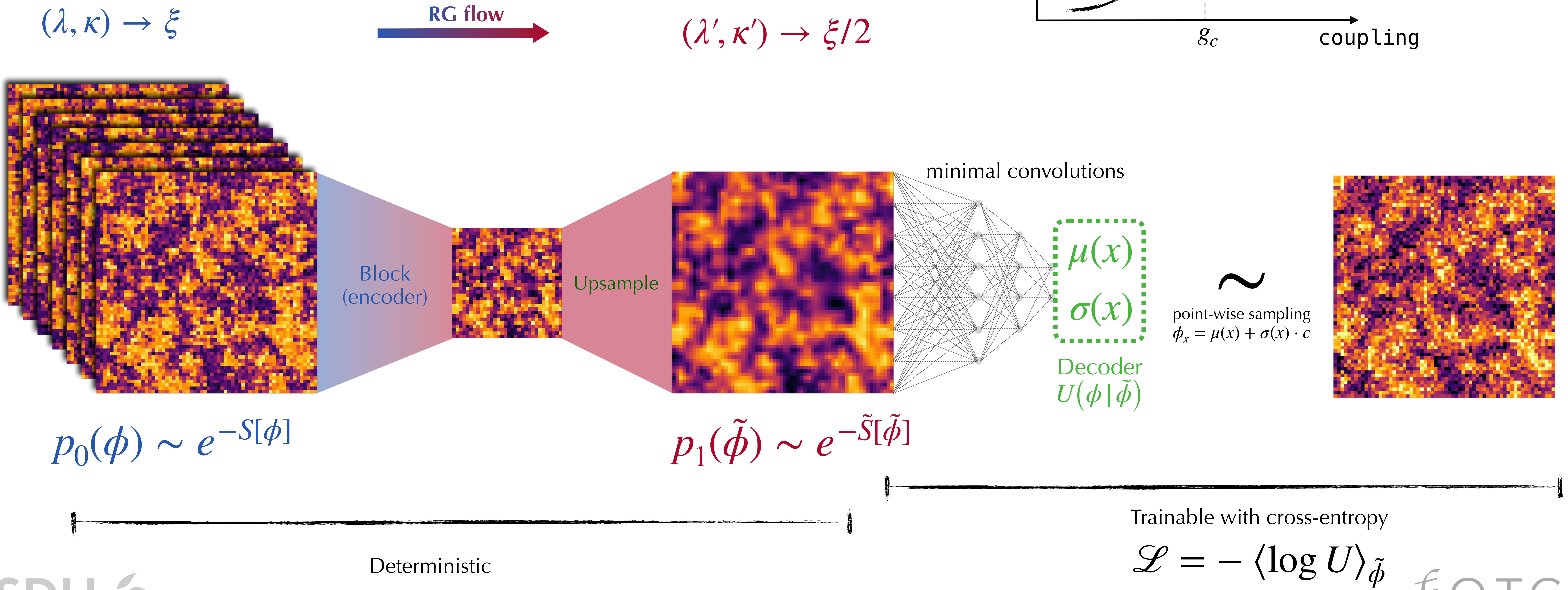


Gaussian prior

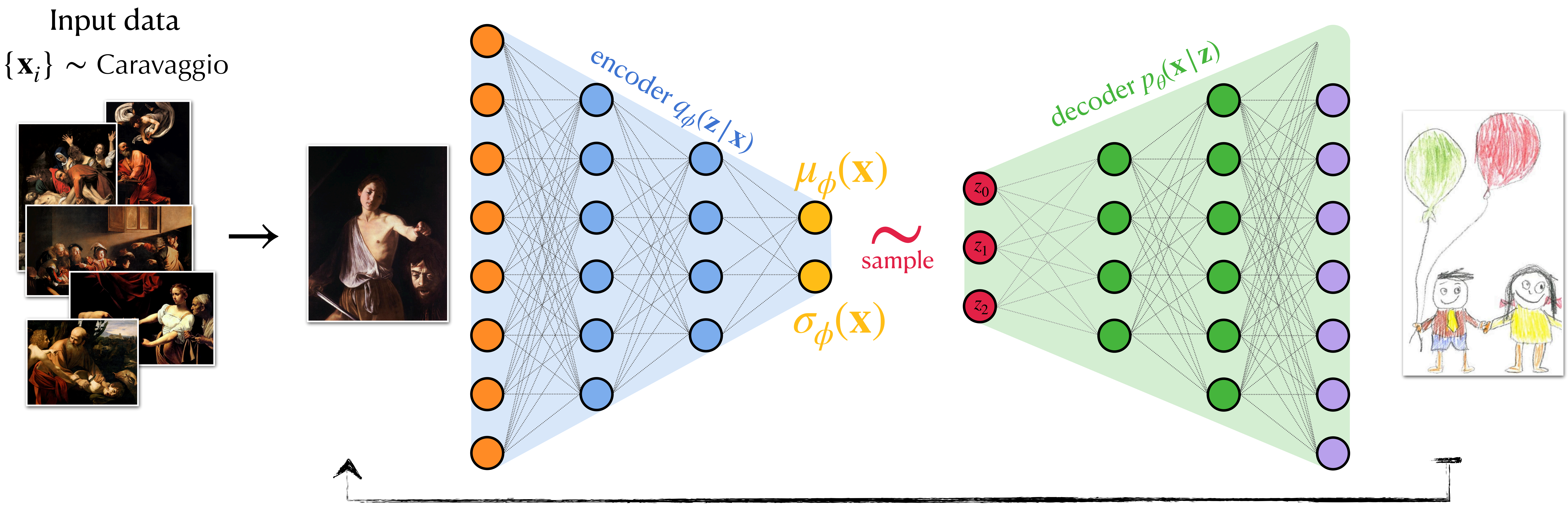
← de-noising (Normalising flow)

Super-resolving $\lambda\phi^4$





Variational autoencoders



$$\text{ELBO} = \left\langle p_\theta(\mathbf{x}|\mathbf{z}) \right\rangle_{q_\phi} - D_{\text{KL}}\left[q_\phi(\mathbf{x}|\mathbf{z}) \| p(\mathbf{z}) \right]$$