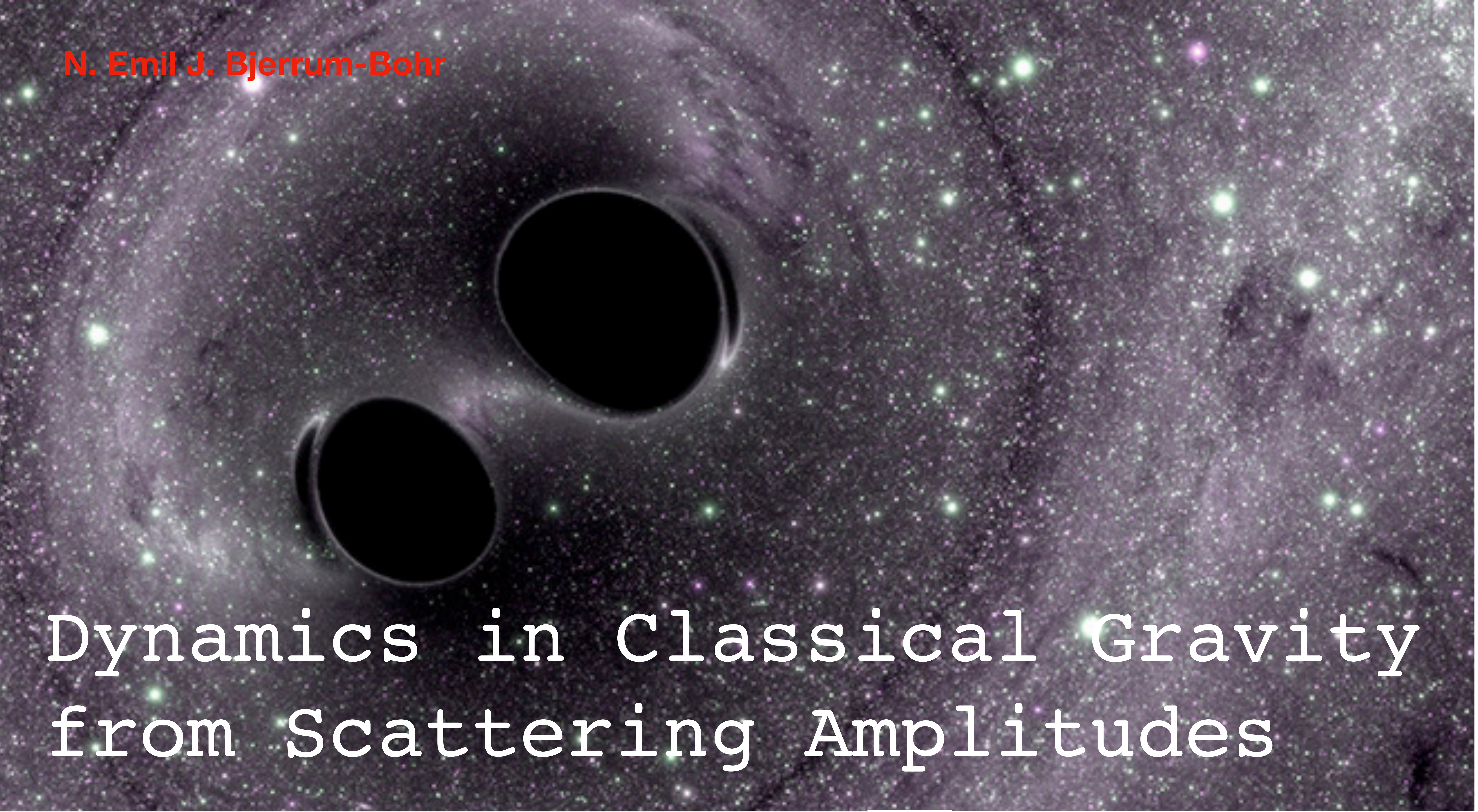
N. Emil J. Bjerrum-Bohr



Dynamics in Classical Gravity from Scattering Amplitudes

MARIE CURIE ACTIONS
DANMARKS FRIE FORSKNINGSFOND
THE CENTER OF GRAVITY
Danmarks Grundforskningsfond
Danish National Research Foundation
Niels Bohr Institutet
The Niels Bohr International Academy

General Relativity

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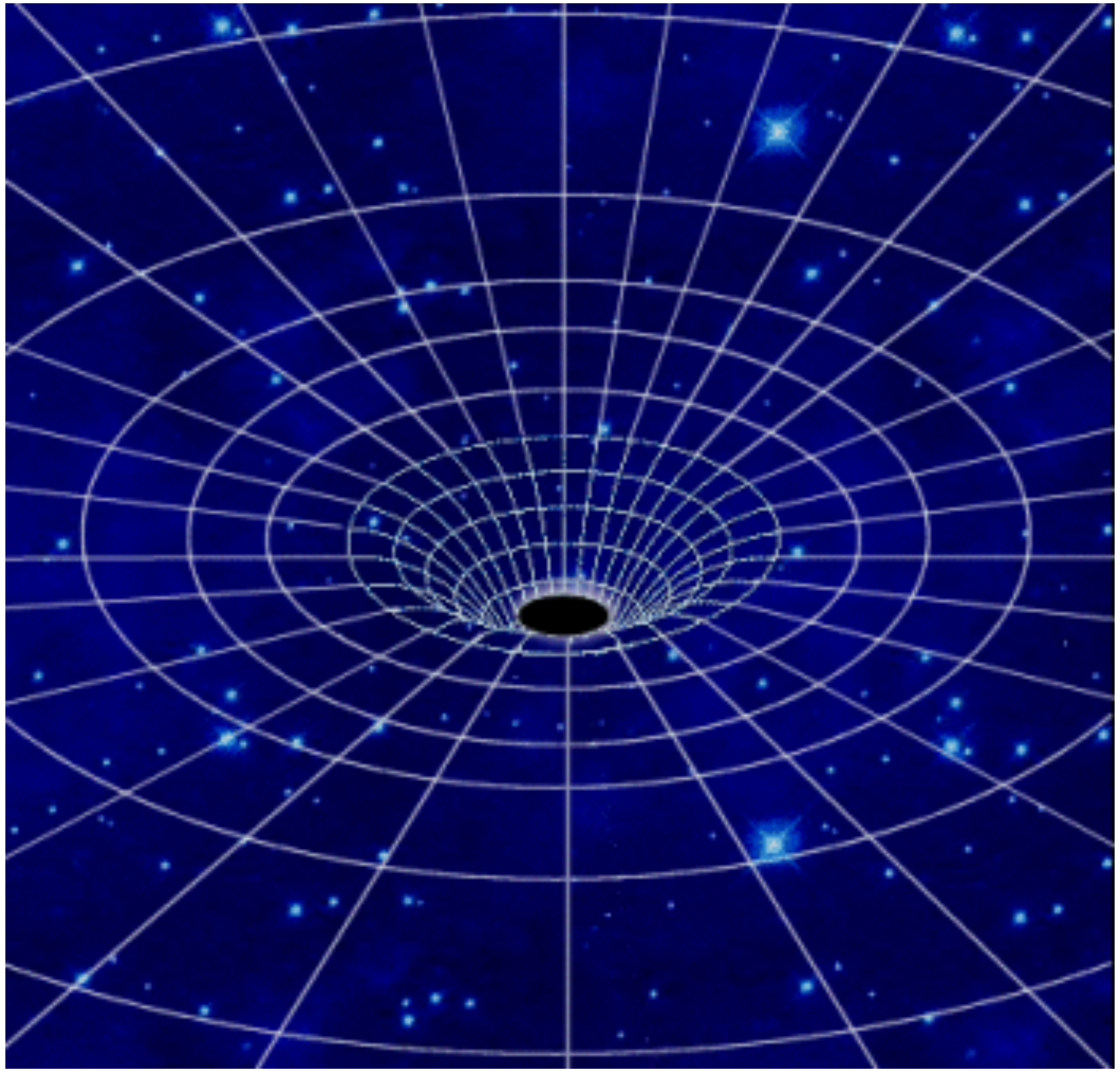

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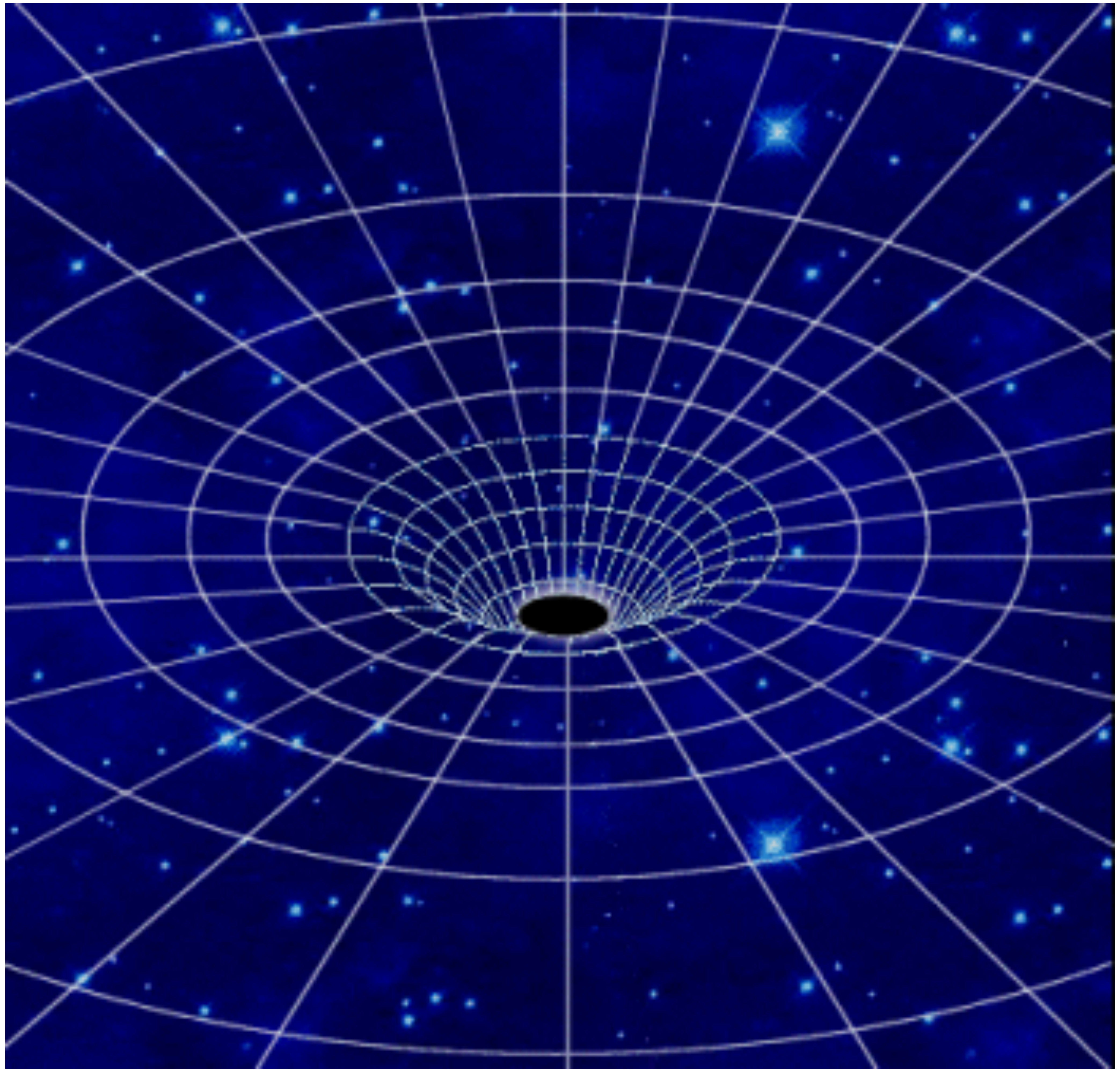

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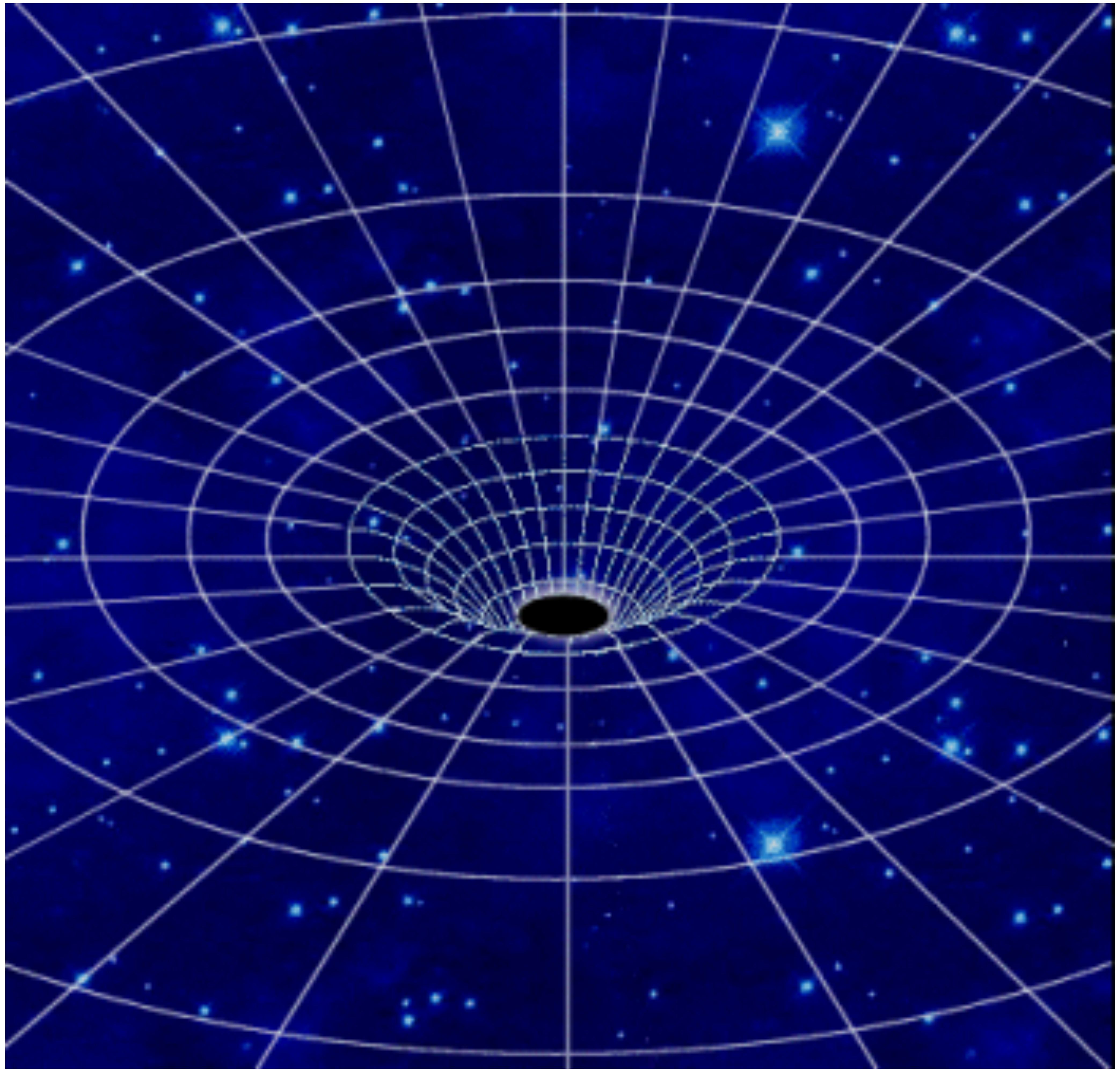

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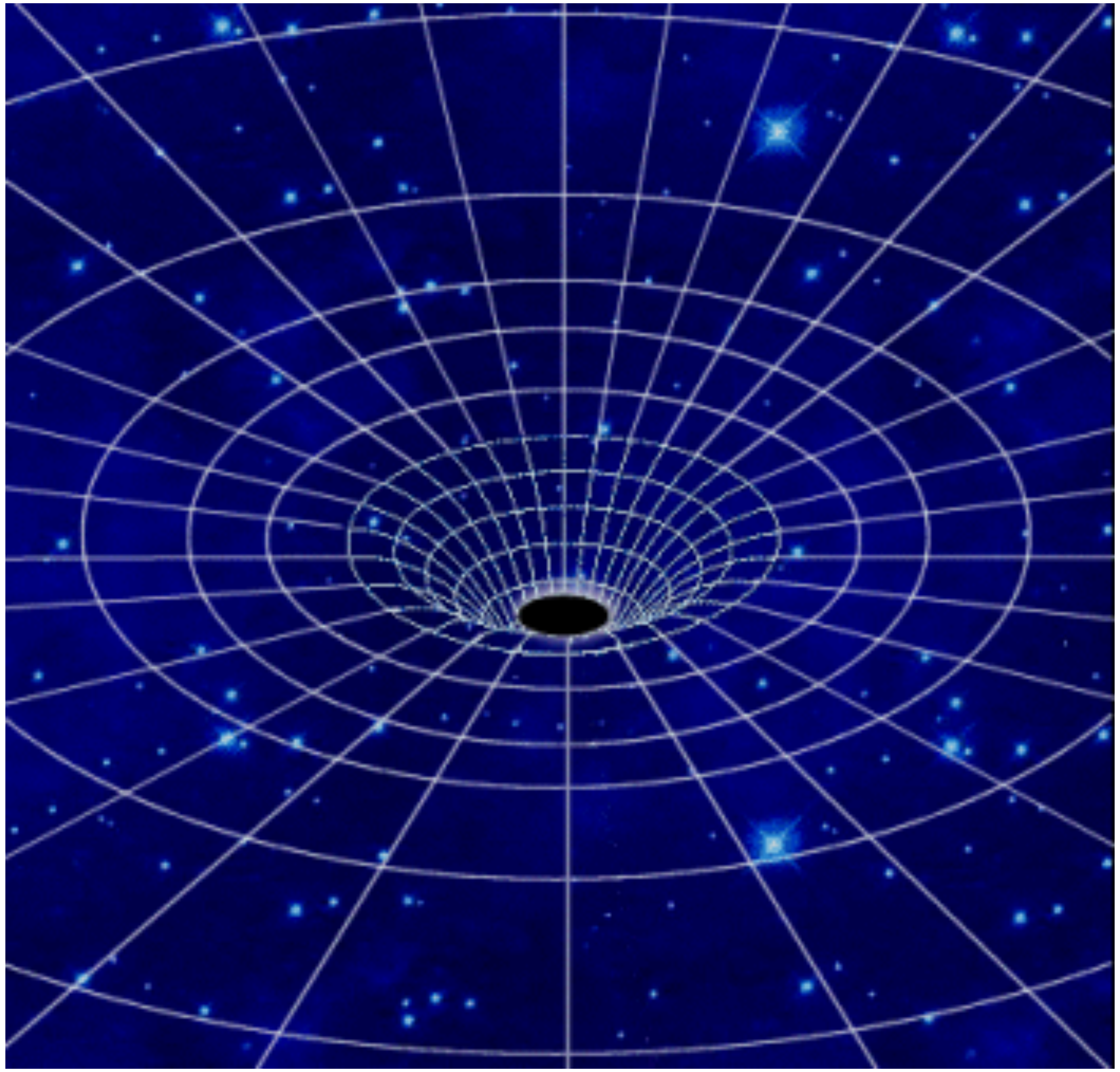

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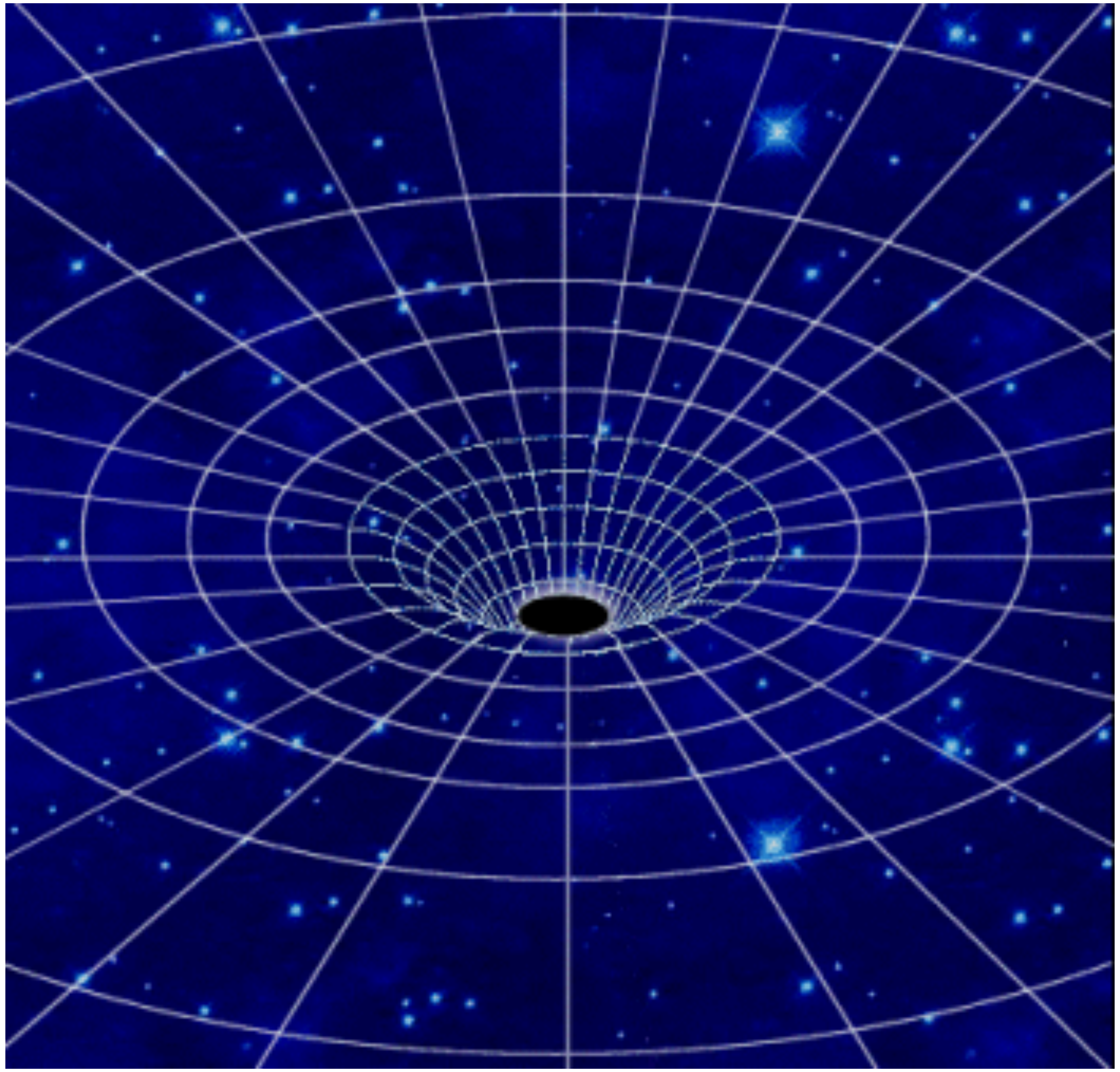

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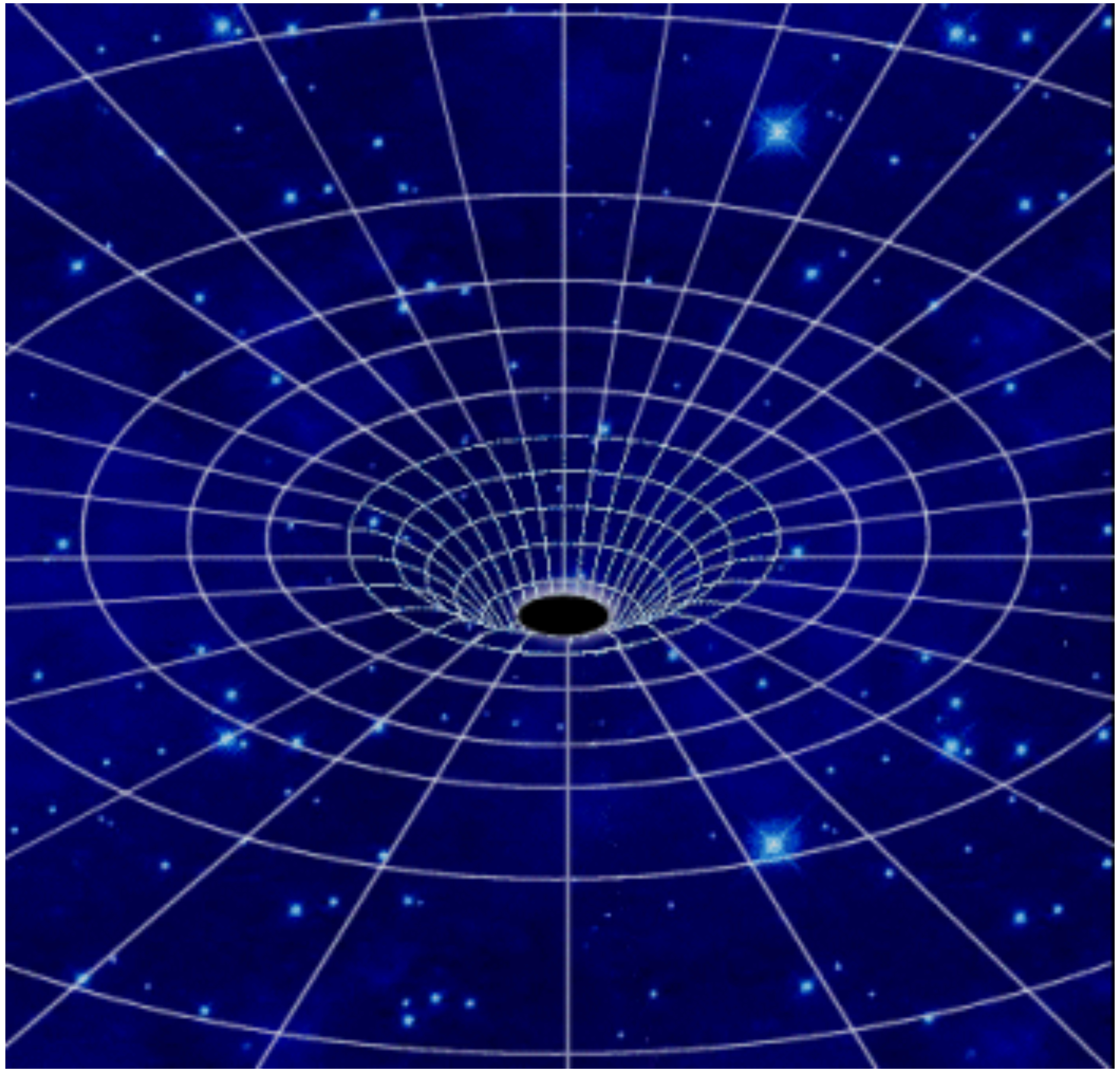

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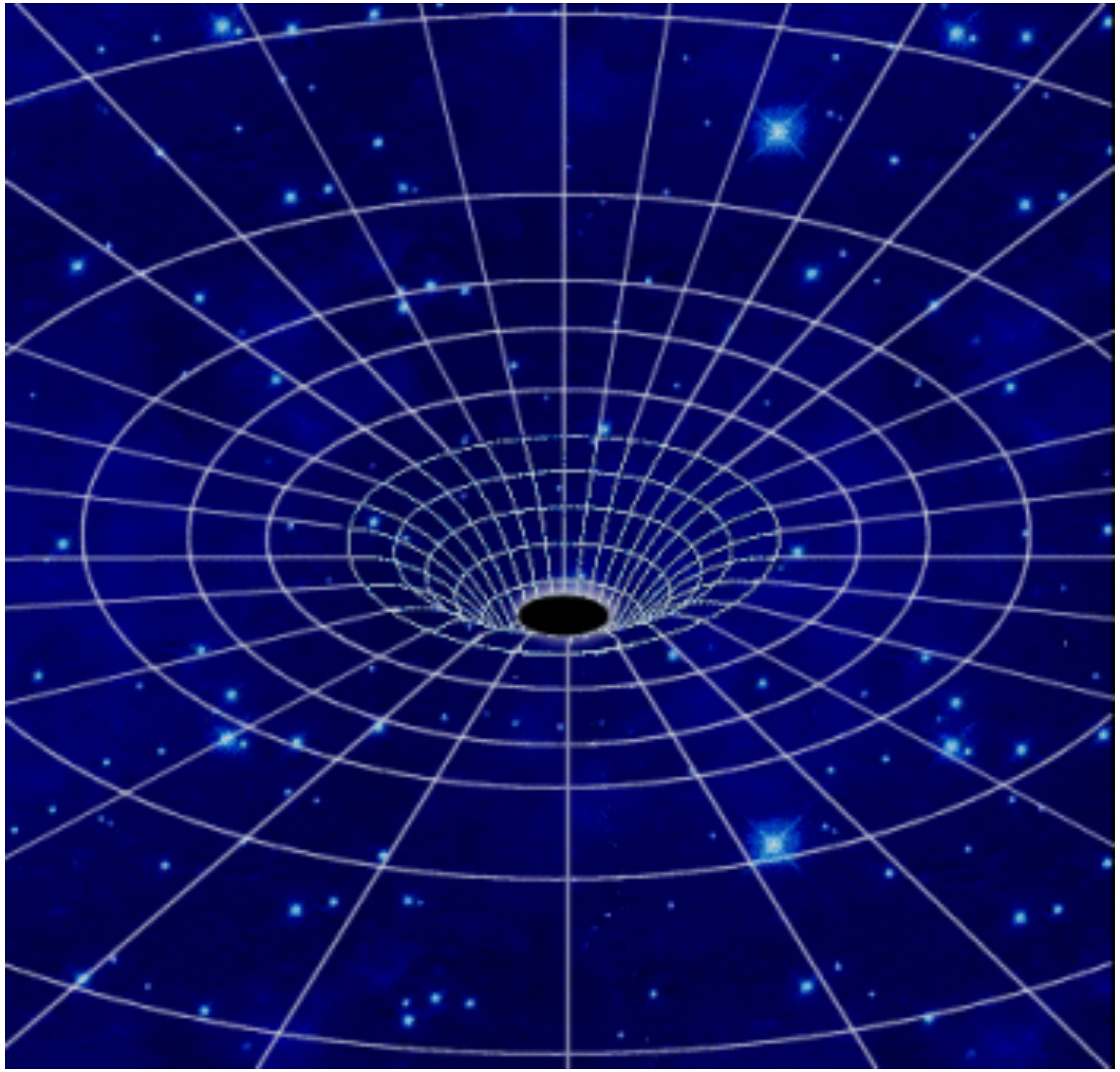

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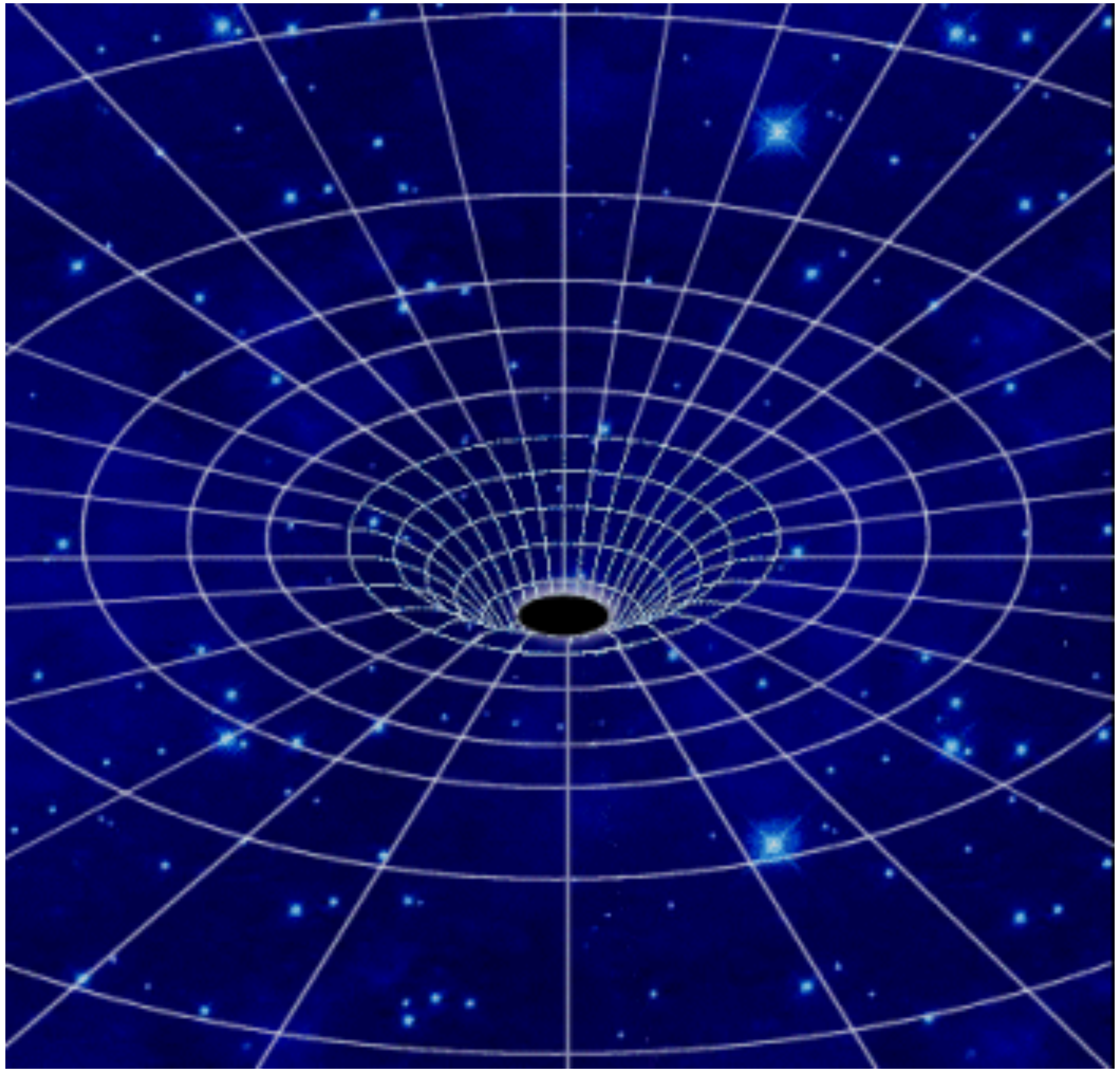

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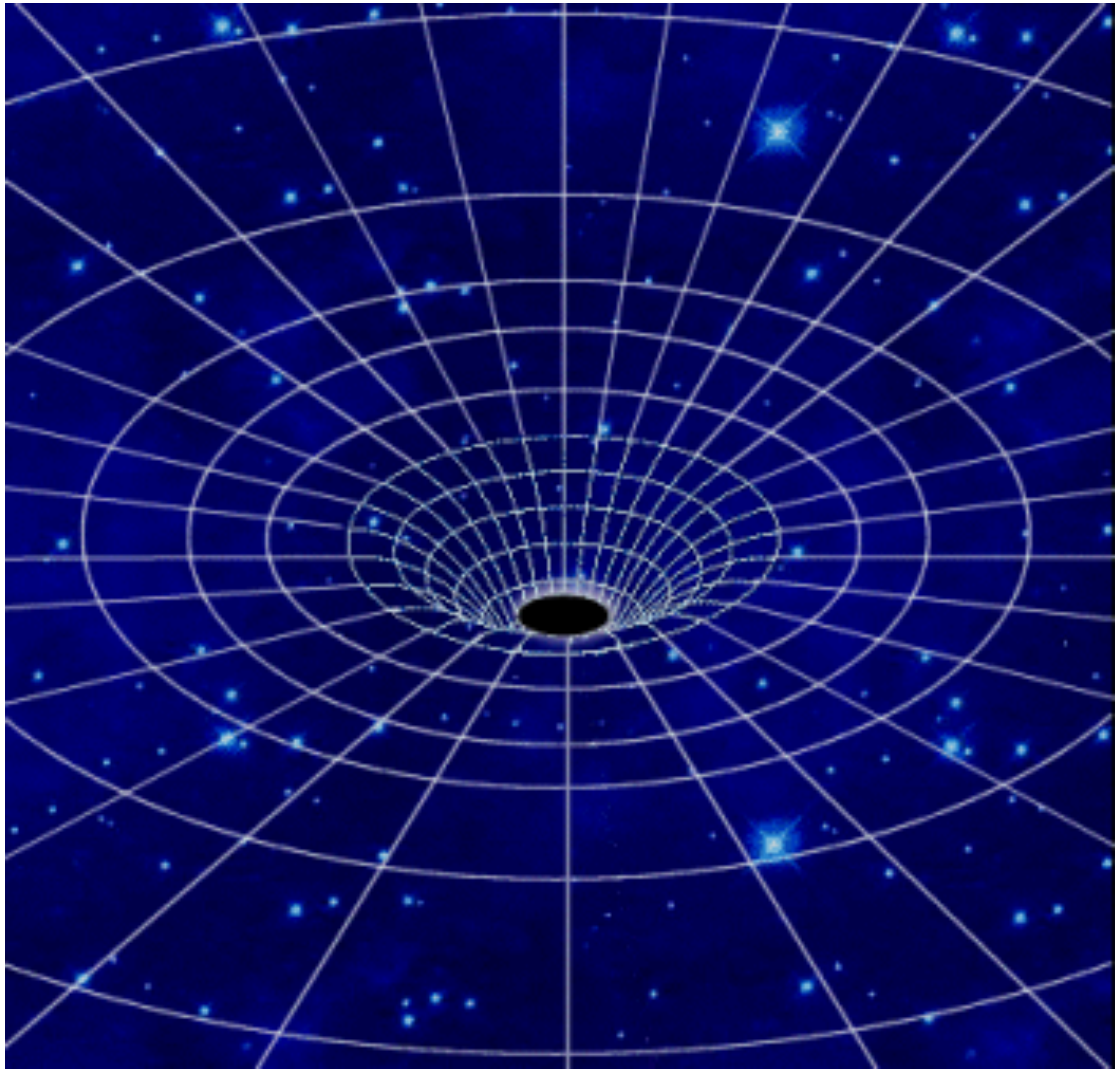

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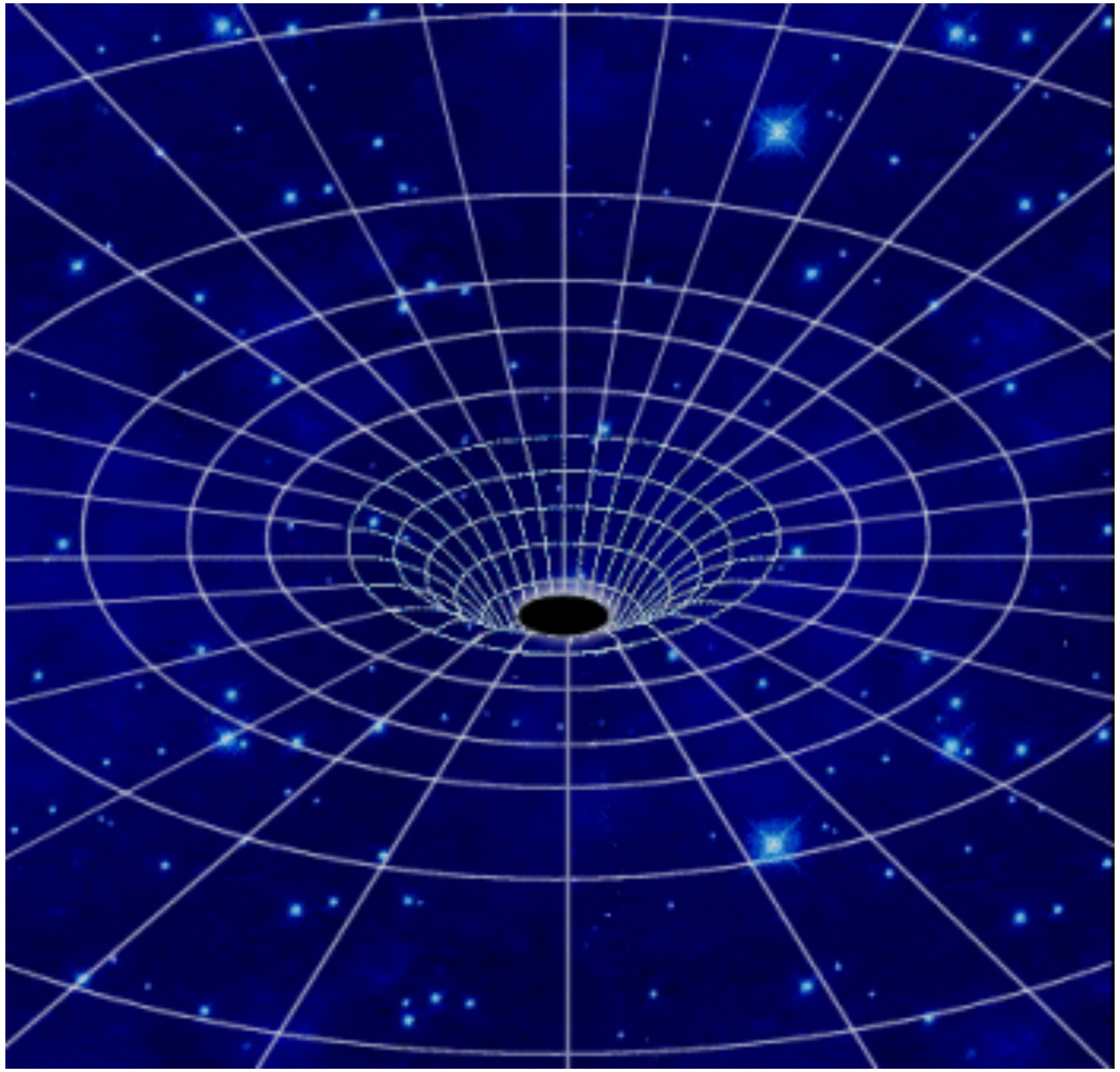
- Signs of string theory gravity….



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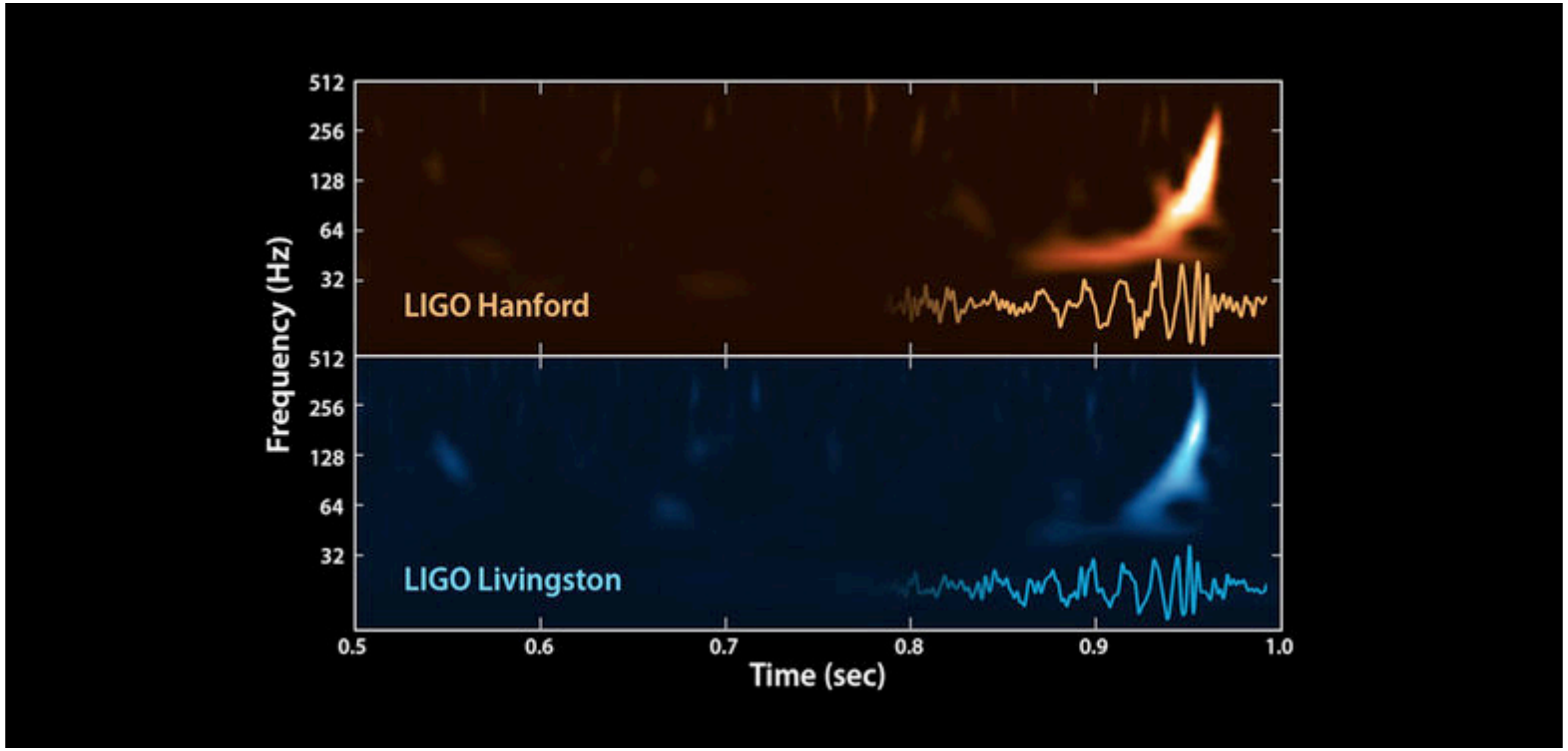

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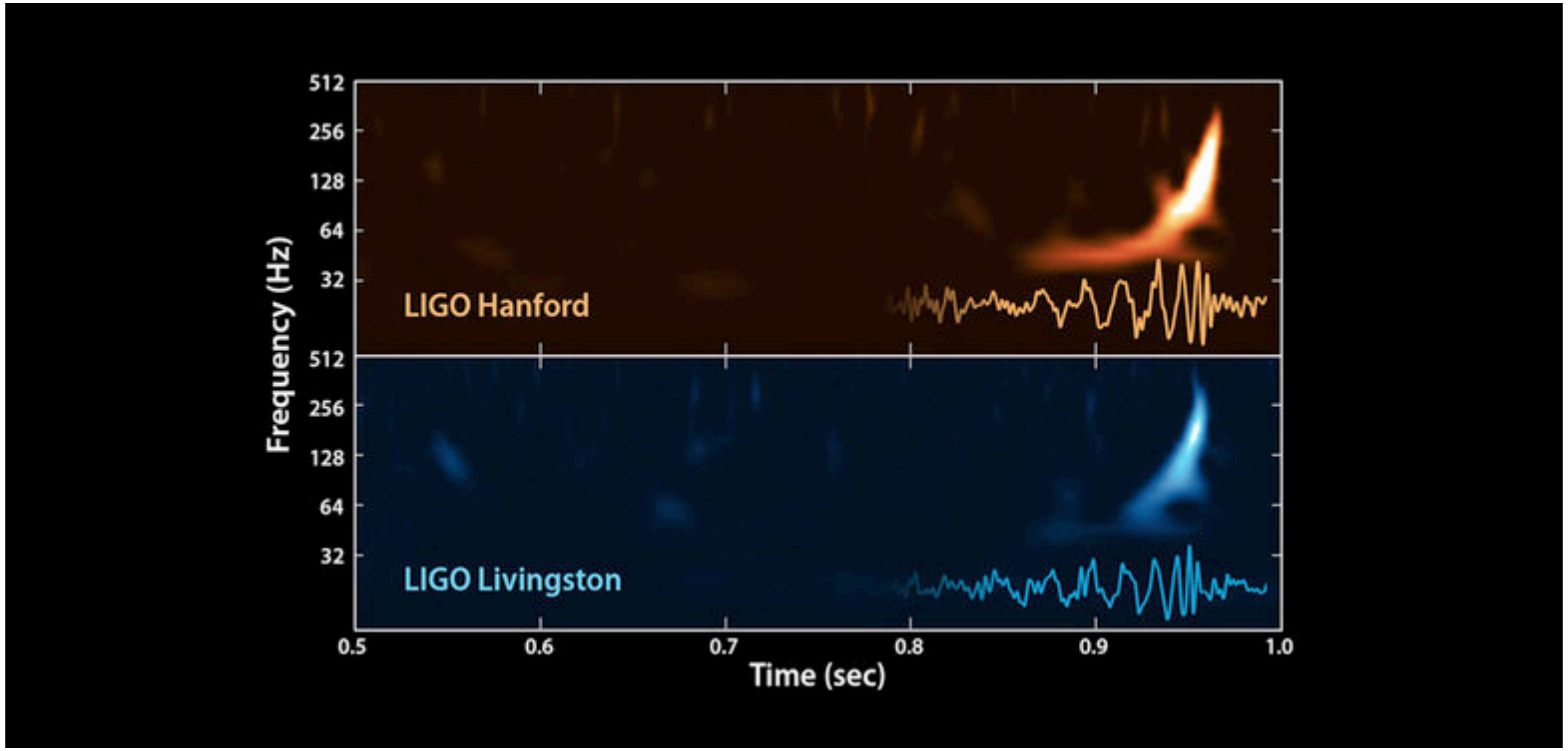

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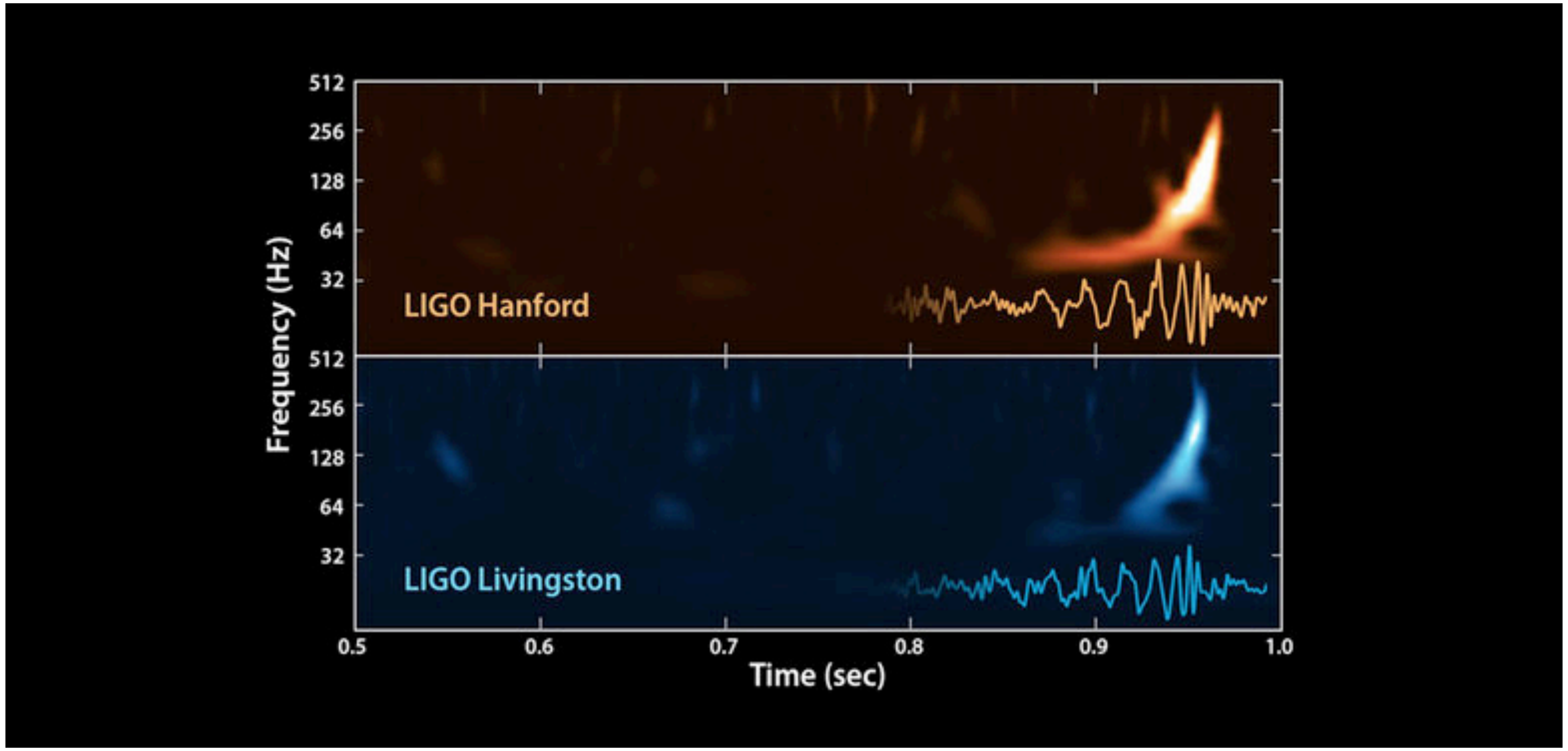

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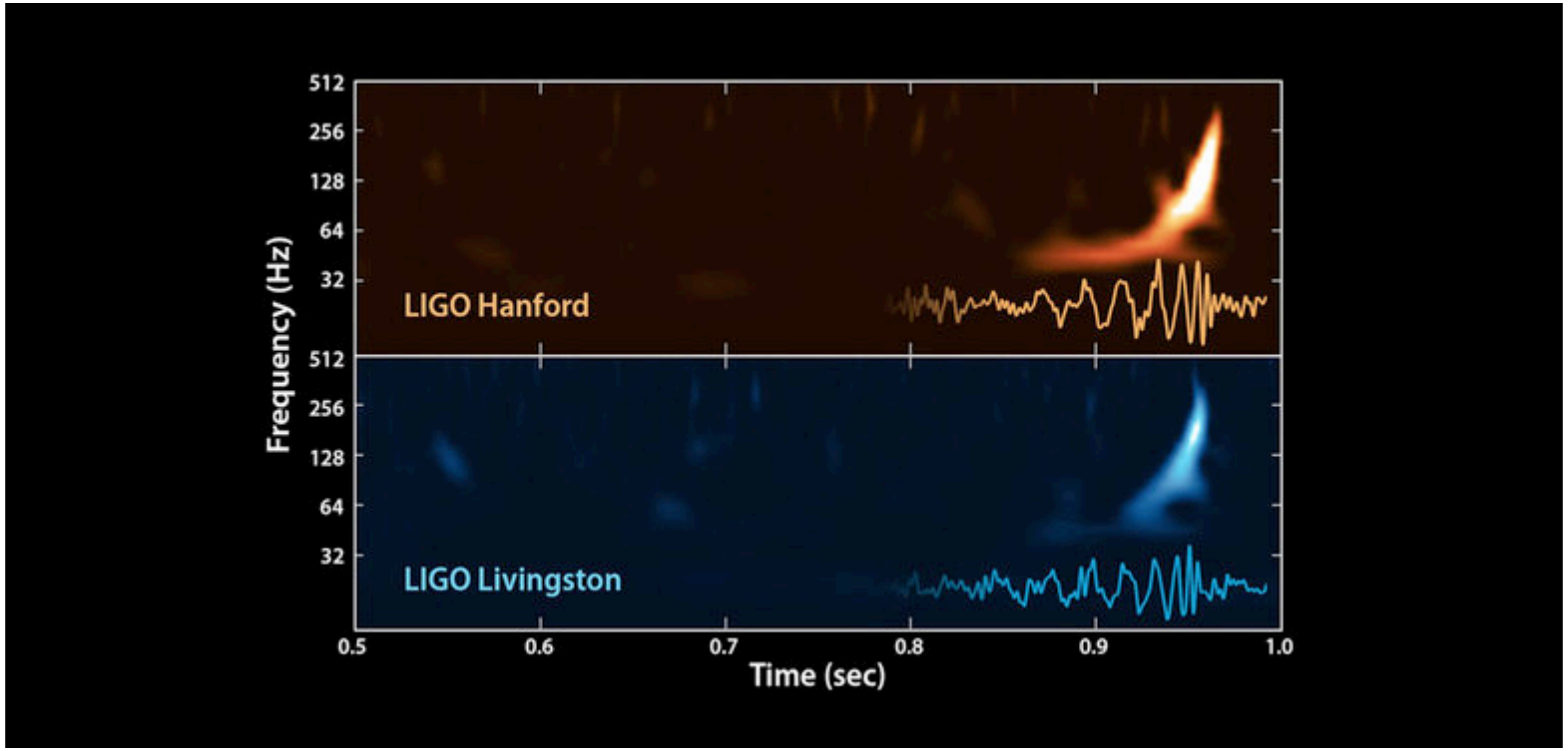

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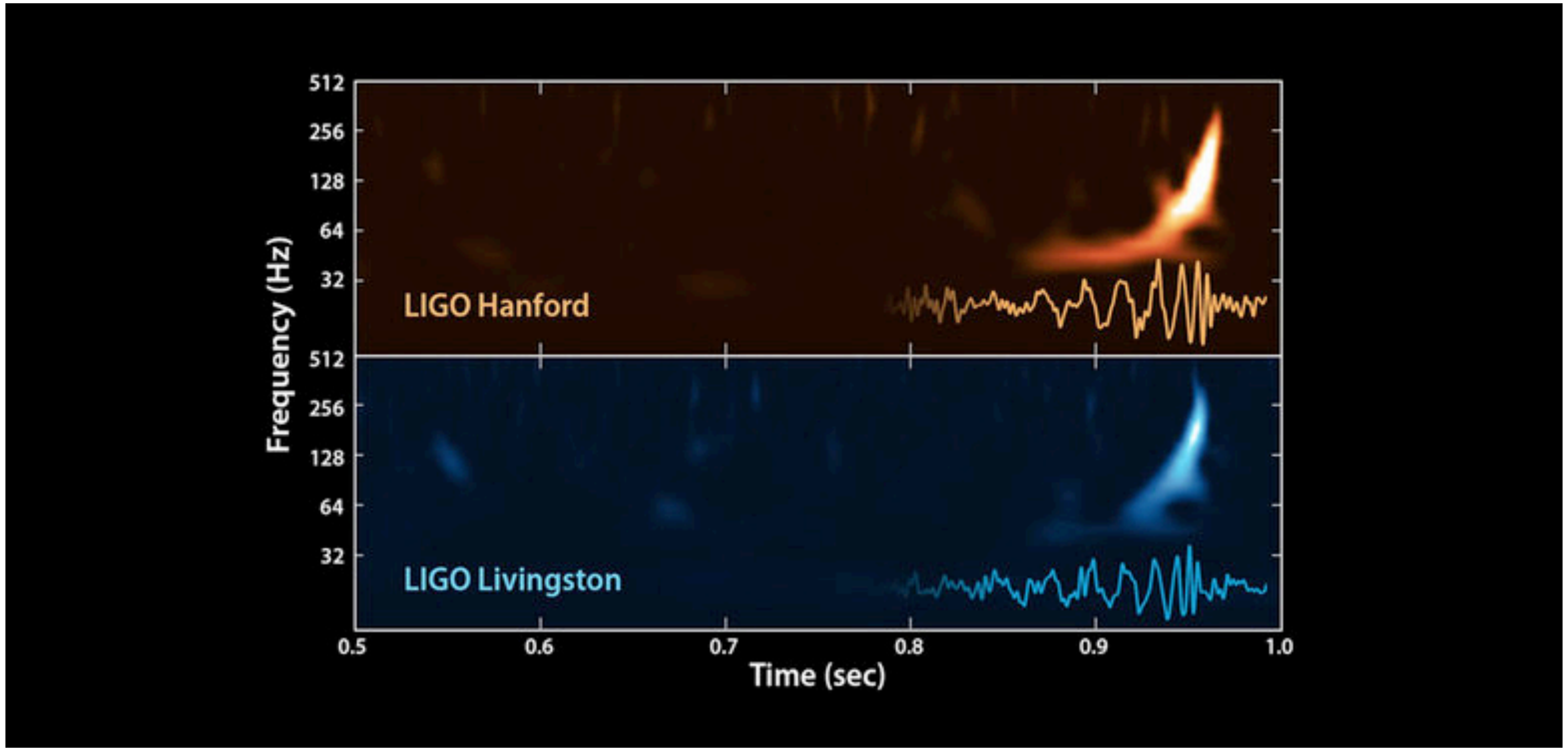

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A potential window to make new discoveries in gravity

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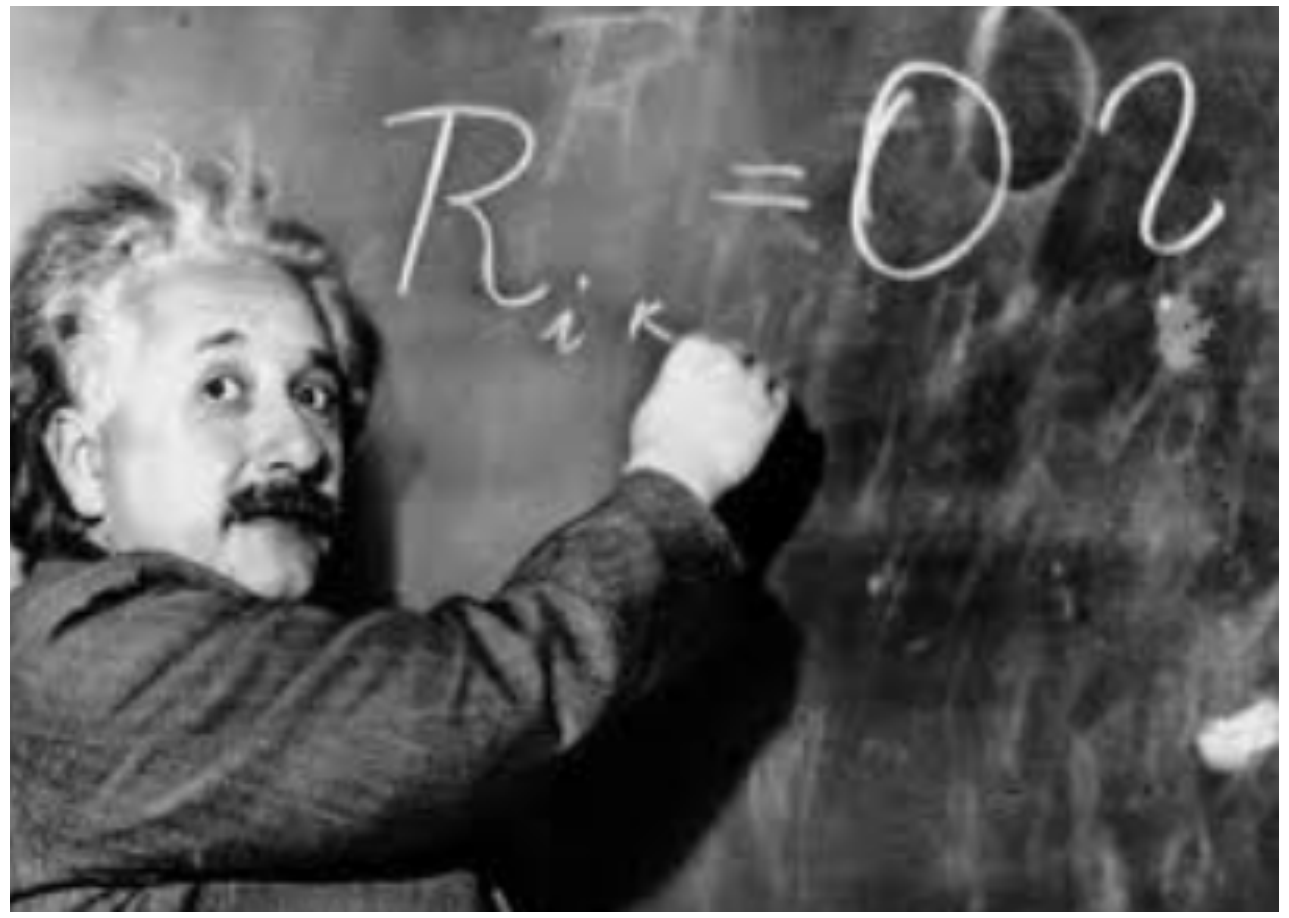

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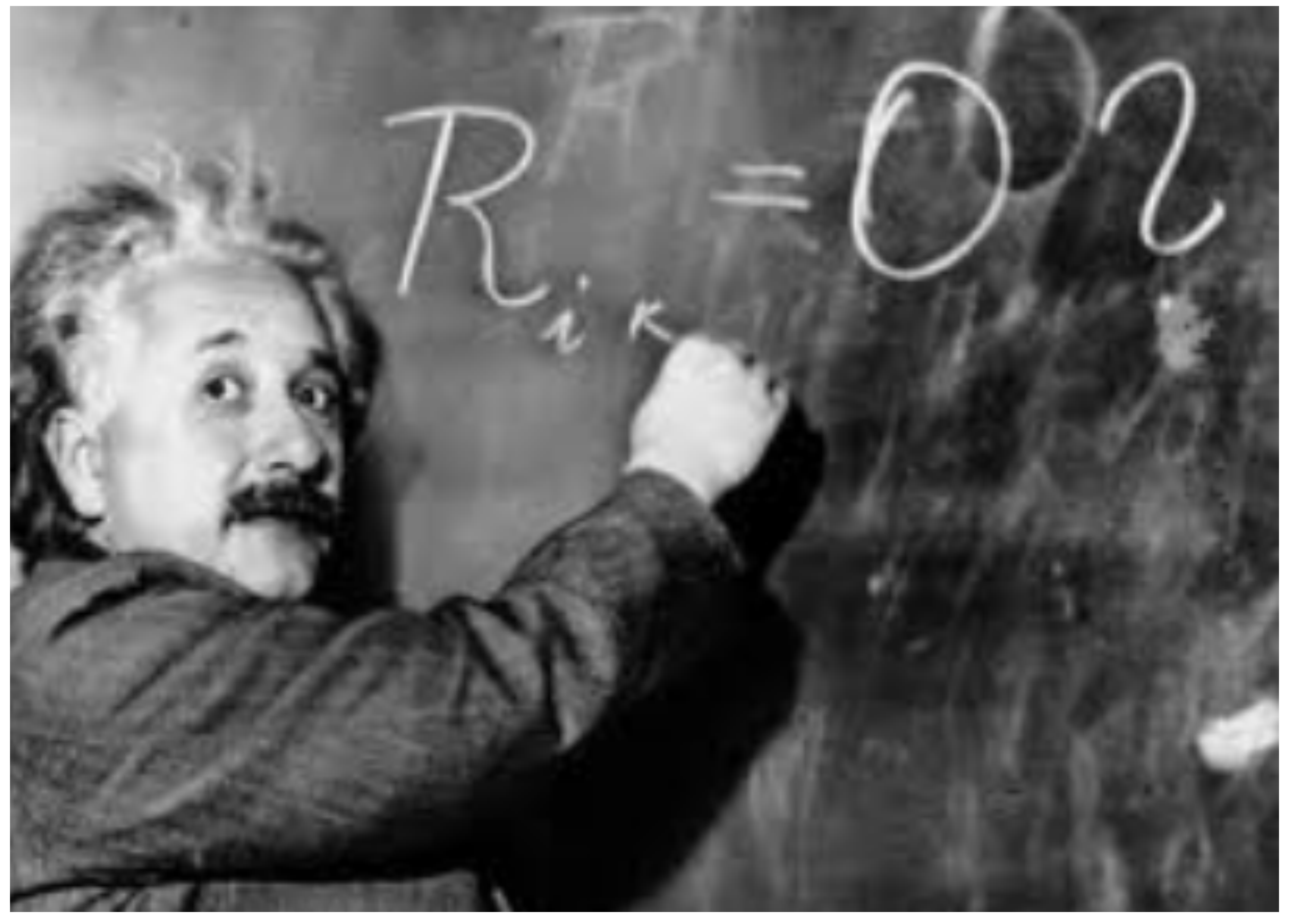

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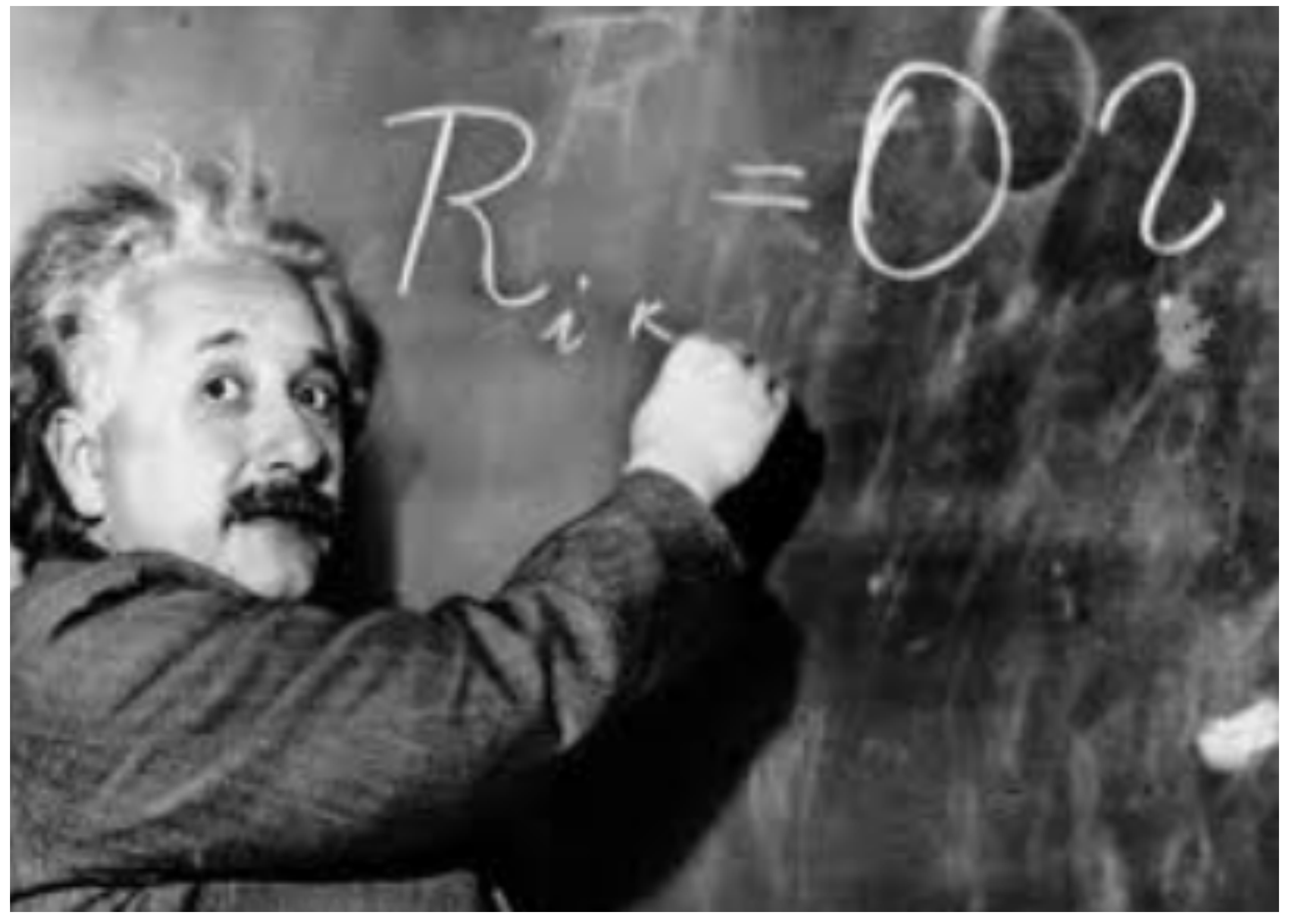

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Off-shell QFT methods: not very computationally efficient!

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Gravity as a theory with self-interactions

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3pt, 4pt, … n-pt self-interactions

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String theory can by introducing new length scales

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EFT -gravity EFT -matter

$$\mathscr{L}_{\mathrm{eff\ GR}} = \sqrt{-g}\left[\frac{2R}{16\pi G_N} + R^2 + R_{\mu\nu}^2 + \ldots + \mathscr{L}_{\mathrm{matter}} + \ldots\right]$$

Consistent quantum gravity at low energies long-distance (Donoghue; NEJBB, Donoghue, Holstein)

Advantages: Gravity as an EFT

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Universal consequences of underlying fundamental quantum theory ~~ link to low energy features, e.g., string and super-gravity theories

New on-shell toolbox for computations

Off-shell gravity amplitudes

Vertices: 3pt, 4pt, 5pt,..n-pt

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$$V^{(3)}_{\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) = \kappa\,\mathrm{sym}\Big[-\frac{1}{2}P_3(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \frac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma})$$

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45 terms + sym

(DeWitt;Sannan)

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Loop order: complicated tensor integrals

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Different representations of the Lorentz group $p_{a\dot a} = \sigma^\mu_{a\dot a}p_\mu$

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Momentum parts of amplitudes:

$$q_{a\dot a} = \mu_a\tilde\mu_{\dot a} \quad p_{a\dot a} = \lambda_a\tilde\lambda_{\dot a} \quad 2(p\cdot q) = s_{ij} = -\langle\lambda,\mu\rangle[\tilde\lambda,\tilde\mu]$$

Spin-2 polarisation tensors in terms of helicities,
(squares of those of YM):

$$\varepsilon^-_{a\dot a} = \frac{\lambda_a\tilde\mu_{\dot a}}{[\tilde\lambda,\tilde\mu]} \qquad \tilde\varepsilon^+_{a\dot a} = \frac{\mu_a\tilde\lambda_{\dot a}}{\langle\mu,\lambda\rangle}$$

$$\varepsilon^-\ \varepsilon^- \qquad \tilde\varepsilon^+\ \tilde\varepsilon^+$$

(Xu, Zhang, Chang)

Simplifications from Spinor-Helicity

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$$\begin{aligned}
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Vanish in spinor helicity formalism

Contractions

$$\varepsilon^-_{a\dot{a}} = \frac{\lambda_a \tilde{\mu}_{\dot{a}}}{[\tilde{\lambda}, \tilde{\mu}]} \qquad \tilde{\varepsilon}^+_{a\dot{a}} = \frac{\mu_a \tilde{\lambda}_{\dot{a}}}{\langle \mu, \lambda \rangle}$$

Gravity: $A_3(1^-, 2^-, 3^+)$

$\varepsilon^- \ \varepsilon^-$

$\tilde{\varepsilon}^+ \ \tilde{\varepsilon}^+$

$$= -i\frac{\langle 1\,2\rangle^6}{\langle 2\,3\rangle\,\langle 3\,1\rangle}$$

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Huge simplifications

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Yang-Mills MHV-amplitudes

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(n) same helicities vanishes

$$A^{tree}(1^+,2^+,3^+,4^+,..) = 0$$

(n-1) same helicities vanishes

$$A^{tree}(1^+,2^+,..,j^-,..) = 0$$

(n-2) same helicities:

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First non-trivial example,
(M)aximally
(H)elicity (V)iolating
(MHV) amplitudes

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$A^{tree\ MHV}$ Given by the formula (Parke and Taylor) and proven by (Berends and Giele)

First non-trivial example,
(M)aximally
(H)elicity (V)iolating
(MHV) amplitudes

One single term!!

$$i\frac{\langle j\,k\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}$$

Squaring relation for gravity

Gravity from (Yang-Mills)^2 (Kawai, Lewellen, Tye)

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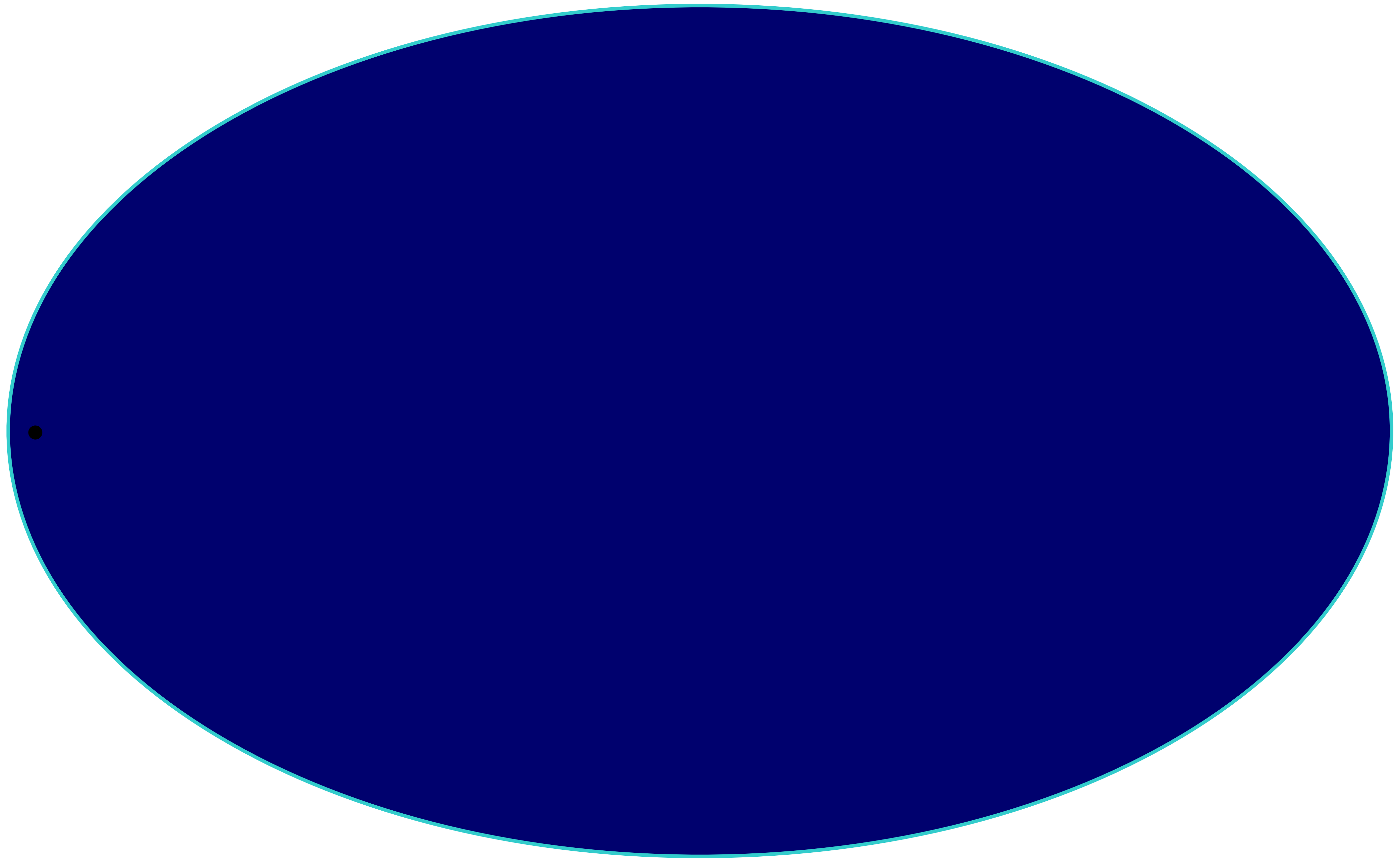

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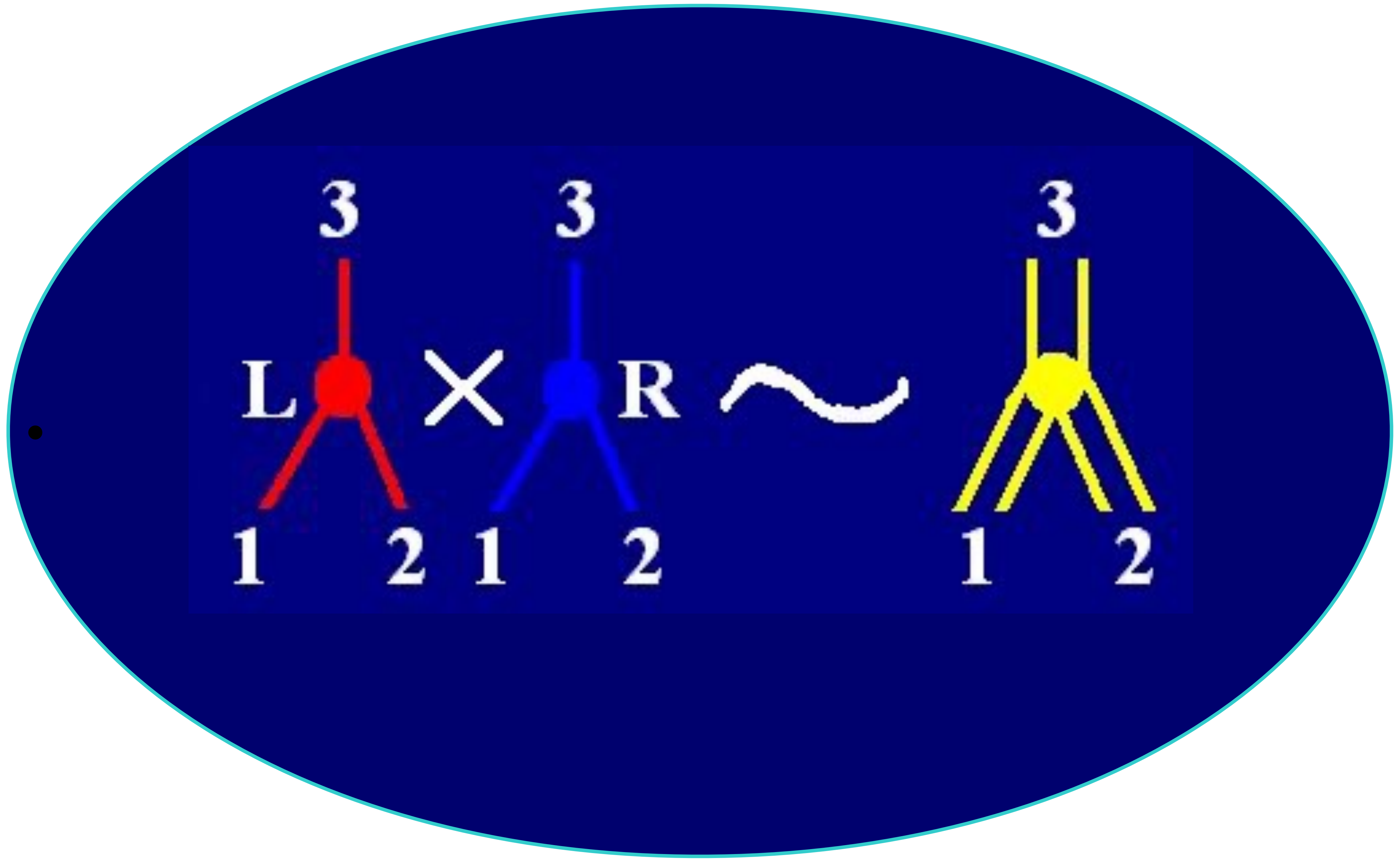

Gravity MHV amplitudes

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Can be generated from KLT via YM MHV amplitudes.

$$M_4^{\text{tree}}(1^-,2^-,3^+,4^+) = i\,\langle 1\,2\rangle^8 \frac{[1\,2]}{\langle 3\,4\rangle\; N(4)}$$

$$M_5^{\text{tree}}(1^-,2^-,3^+,4^+,5^+) = i\,\langle 1\,2\rangle^8 \frac{\varepsilon(1,2,3,4)}{N(5)}$$

Anti holomorphic
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$$M_n^{\text{tree}}(1^-,2^-,3^+,\cdots,n^+)$$
$$= -i\,\langle 1\,2\rangle^8 \times \left[\frac{[1\,2]\,[n-2\;n-1]}{\langle 1\;n-1\rangle\, N(n)} \left(\prod_{i=1}^{n-3}\prod_{j=i+2}^{n-1}\langle i\,j\rangle\right)\prod_{l=3}^{n-3}(-[n|K_{l+1,n-1}|l\rangle) + \mathcal{P}(2,3,\cdots,n-2)\right]$$

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(Berends-Giele-Kuijf) recursion formula

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Advantage that all poles are simple — no spurious poles!

Compact massive tree amplitudes

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CHY formalism leads to the following
very compact amplitudes

$$M_1^{\text{tree}}(p, \ell_2, -p') = i\, N_1(p, \ell_2, -p') A_1(p, \ell_2, -p') = i\, N_1(p, \ell_2, -p')^2,$$

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$$= \frac{iN_2(p, 2, 3, -p')^2}{(\ell_2+p)^2-m^2+i\varepsilon} + \frac{iN_2(p, 3, 2, -p')^2}{(\ell_3+p)^2-m^2+i\varepsilon} + \frac{i({N_2}^{[2,3]})^2}{(\ell_2+\ell_3)^2+i\varepsilon}$$

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Straightforward to compute any tree order needed with manifest color-kinematic numerators

- no double poles

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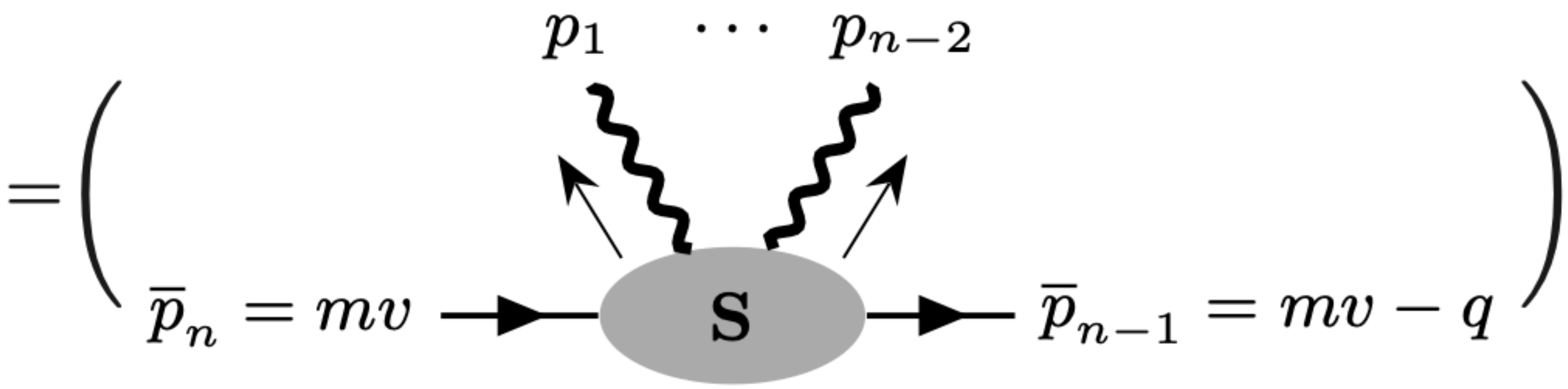

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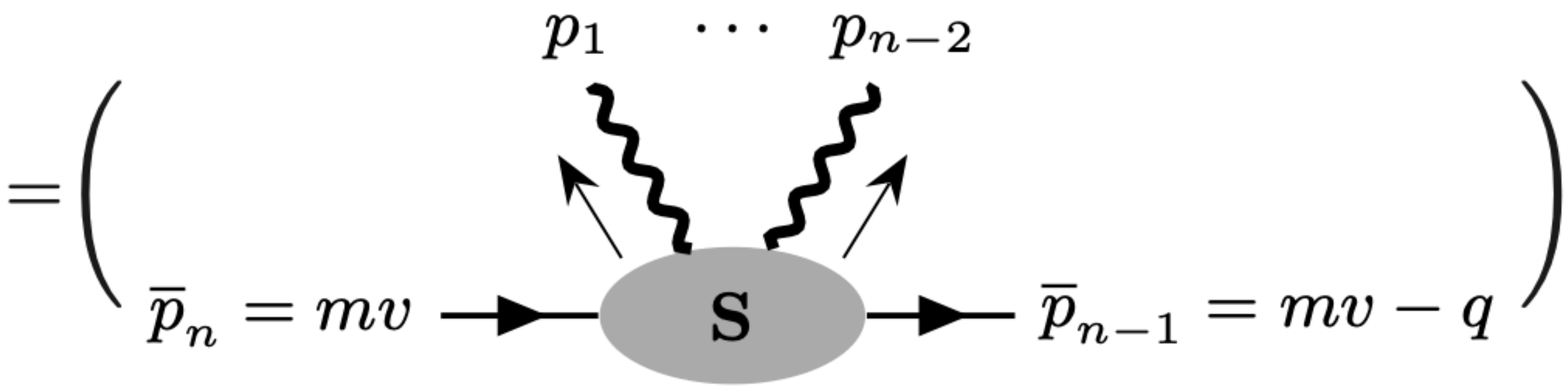

Covariant formalism that use a basis of entire functions

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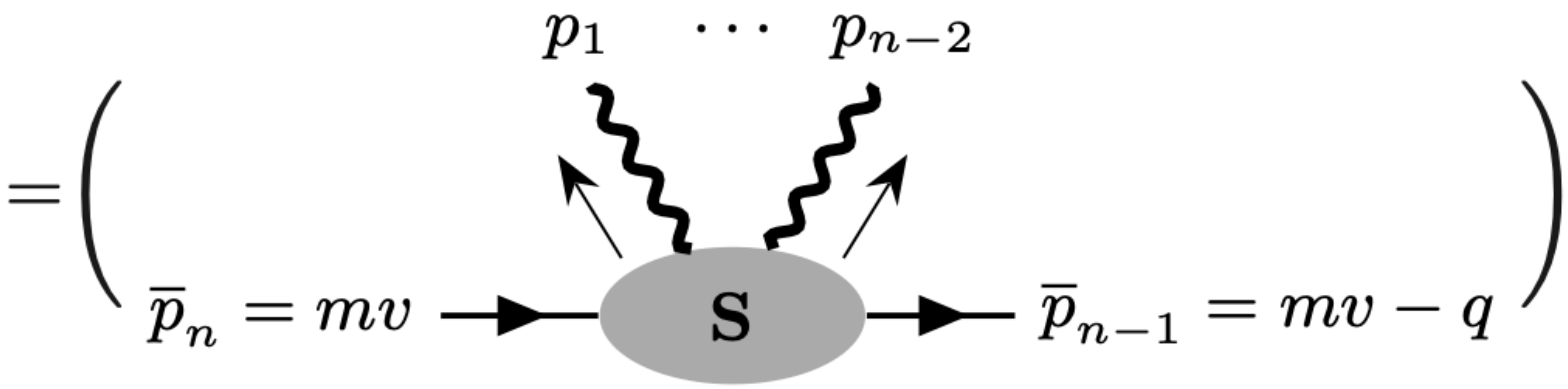

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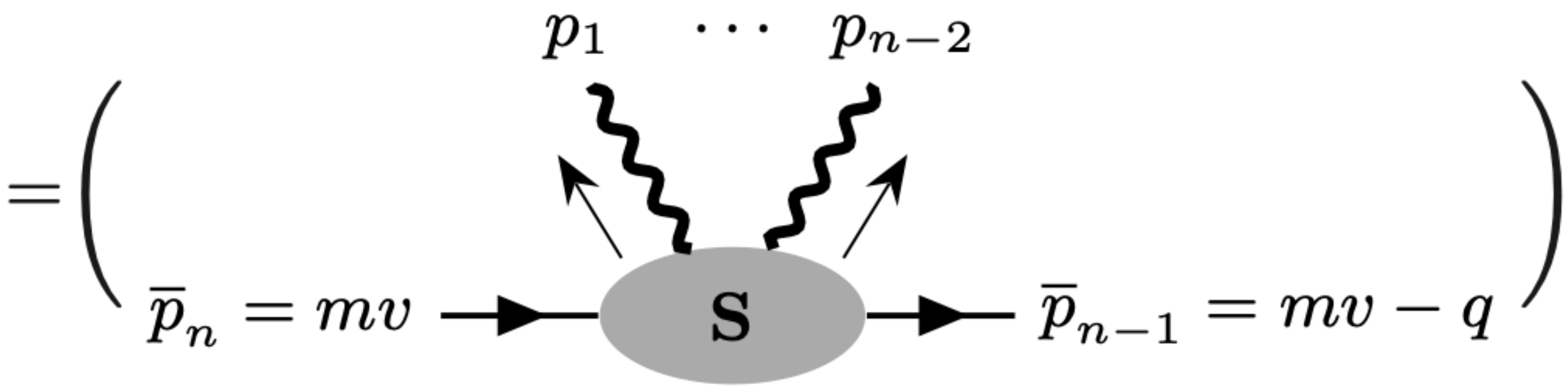

Covariant formalism that use a basis of entire functions

Match with result by (Cangemi, Chiodaroli, Johansson, Ochiov, Pichini, Skvortsov)

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Classical gravity

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Faster and more accurate theoretical breakdown of gravitational wave events!

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We work with heavy fields: Black holes as point particles in quantum field theory.

Gravity from quantum field theory

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We start with Einstein-Hilbert term

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + g^{\mu\nu} T_{\mu\nu} \right]$$

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Other possibilities of course possible

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Spin-less gravity from quantum field theory

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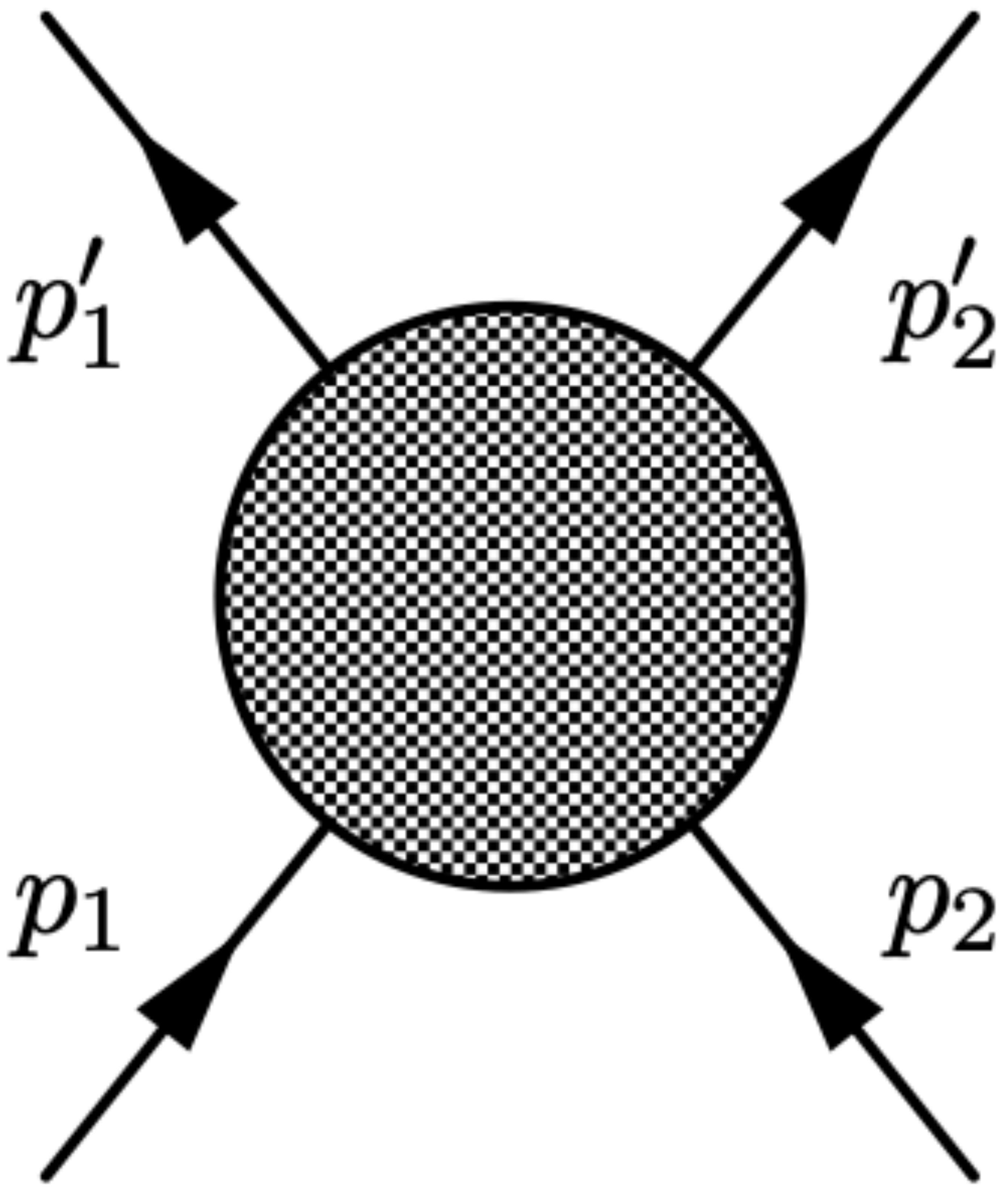
$$= \sum_{L=0}^{\infty} \mathcal{M}_L(p_1, p_2, p_1', p_2') =$$



Tree level — two ‘heavy’-masses

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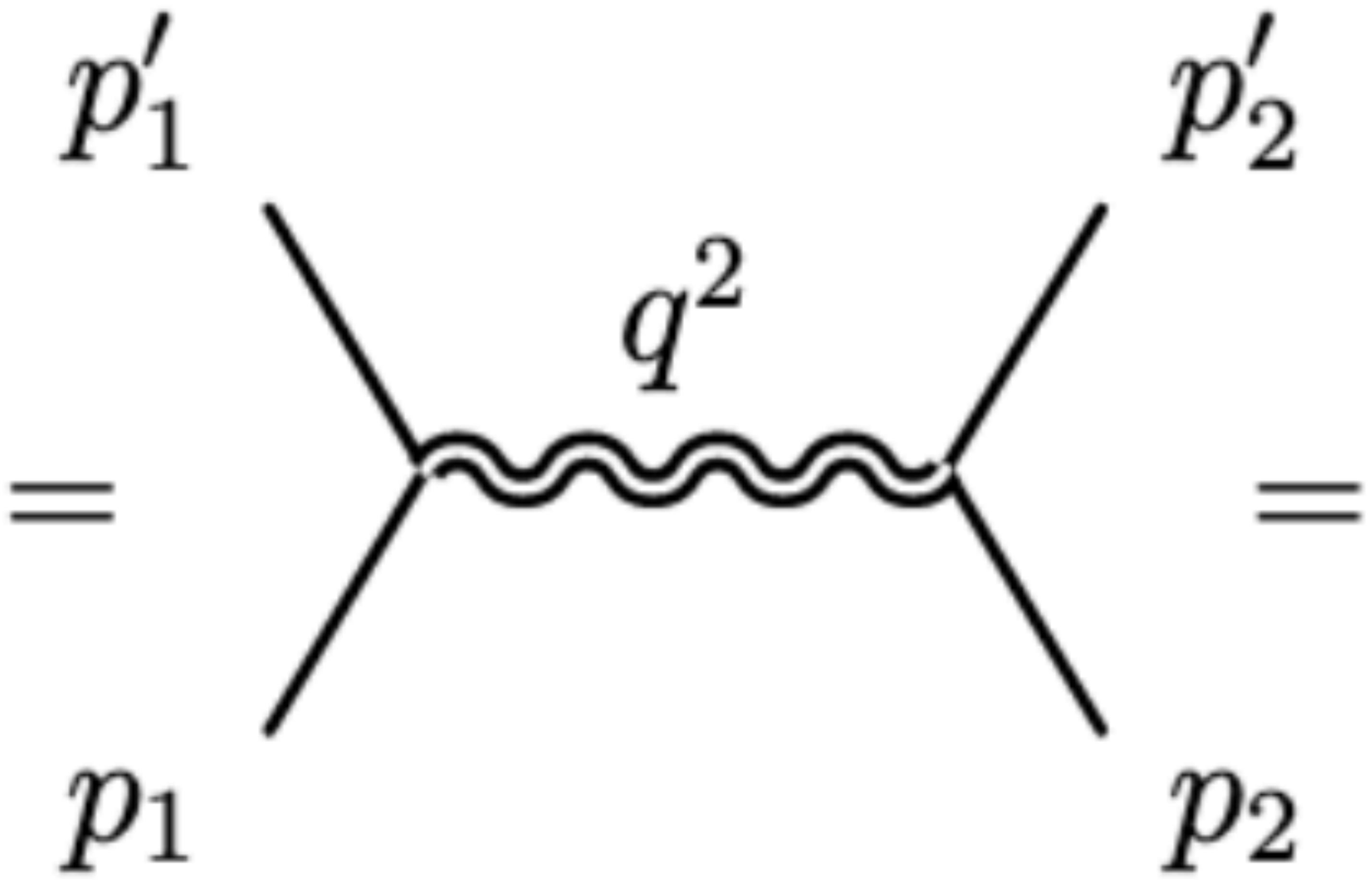
$$\mathcal{M}_0(\gamma, \underline{q}^2, \hbar) =$$



$$= \hbar \frac{2\pi m_1^2 m_2^2 G_N (2\gamma^2 - 1)}{|\underline{q}|^2} + O(\hbar^0)$$

Newton’s law through Fourier transform

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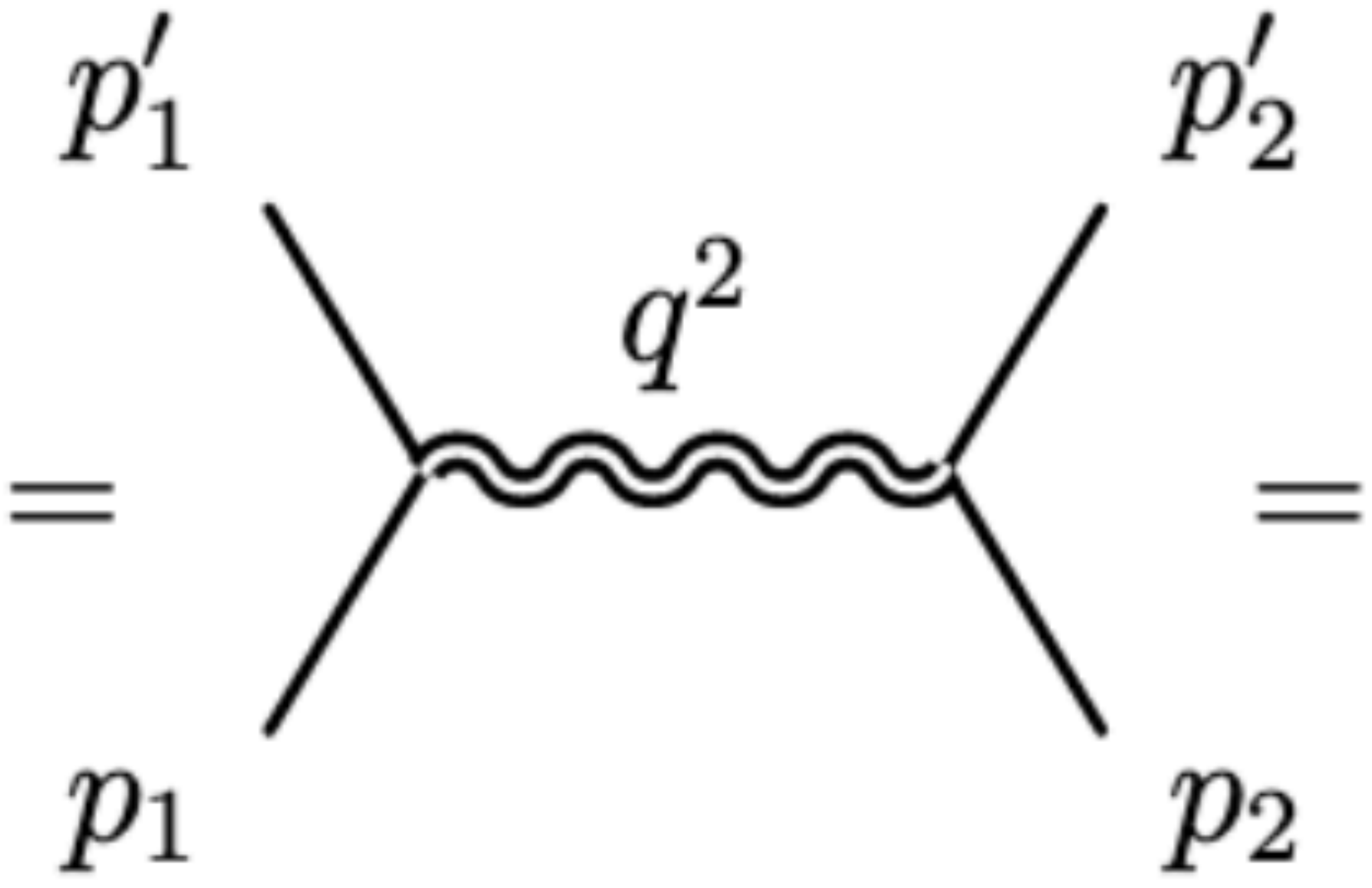

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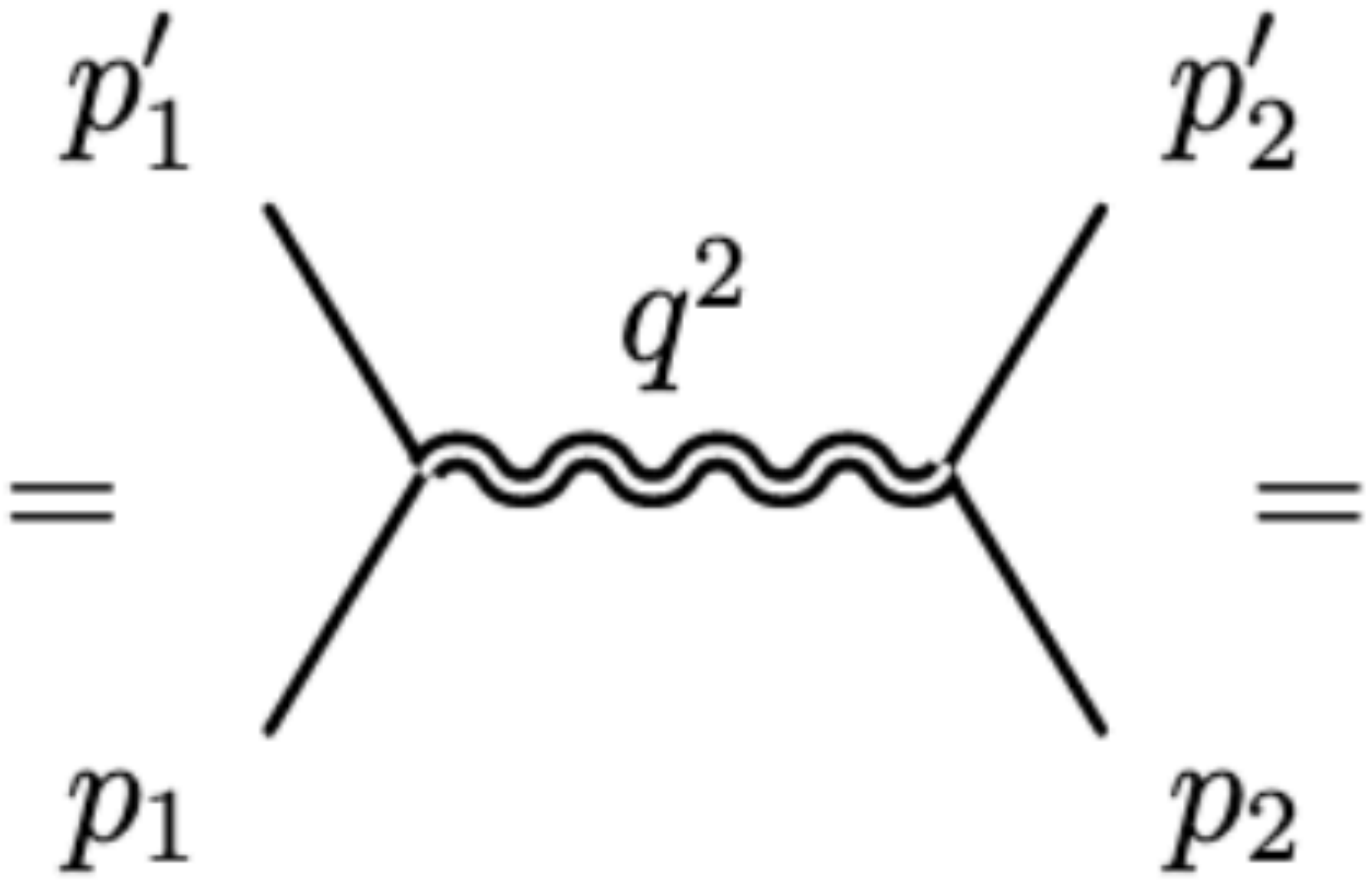

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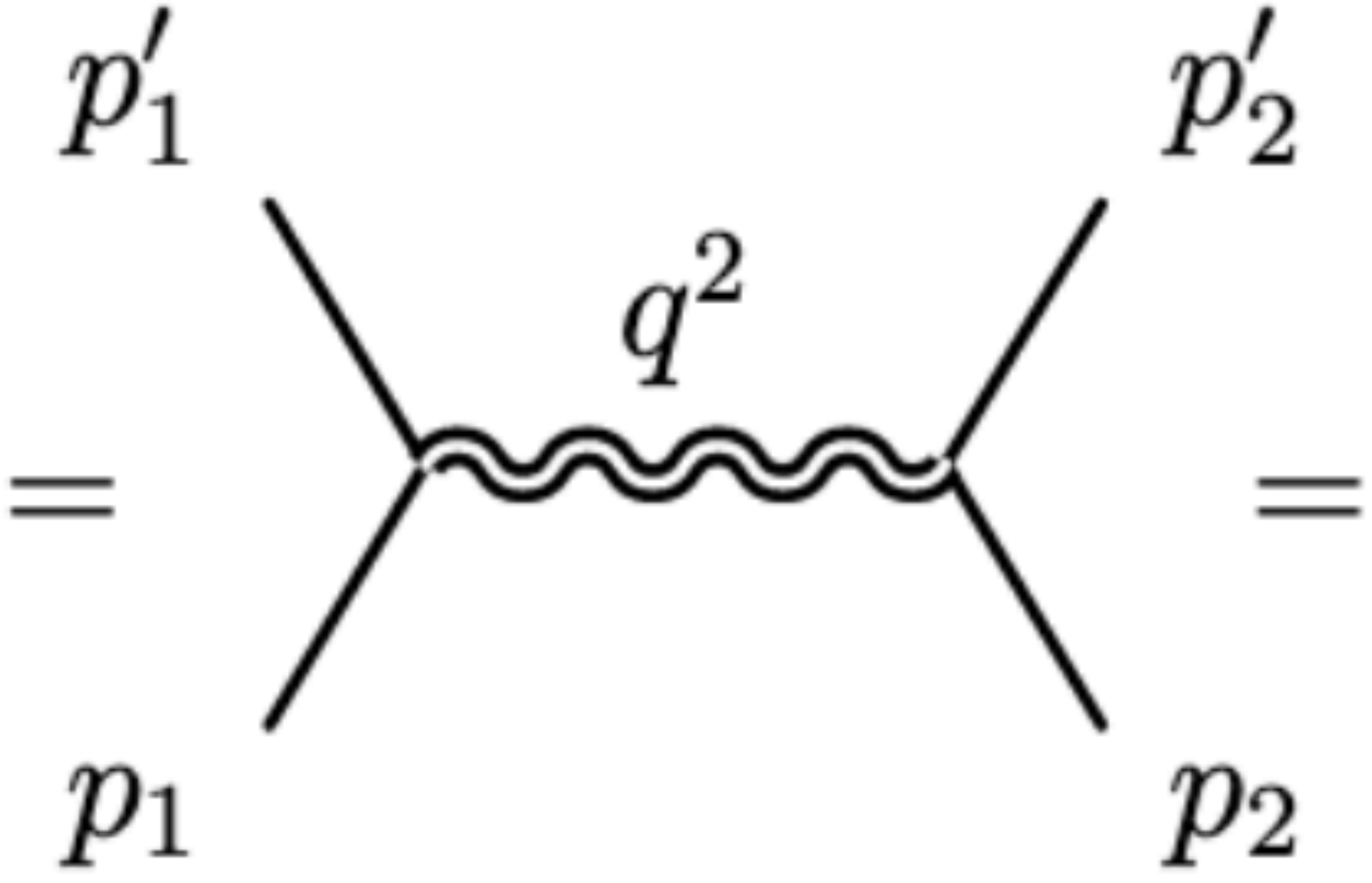

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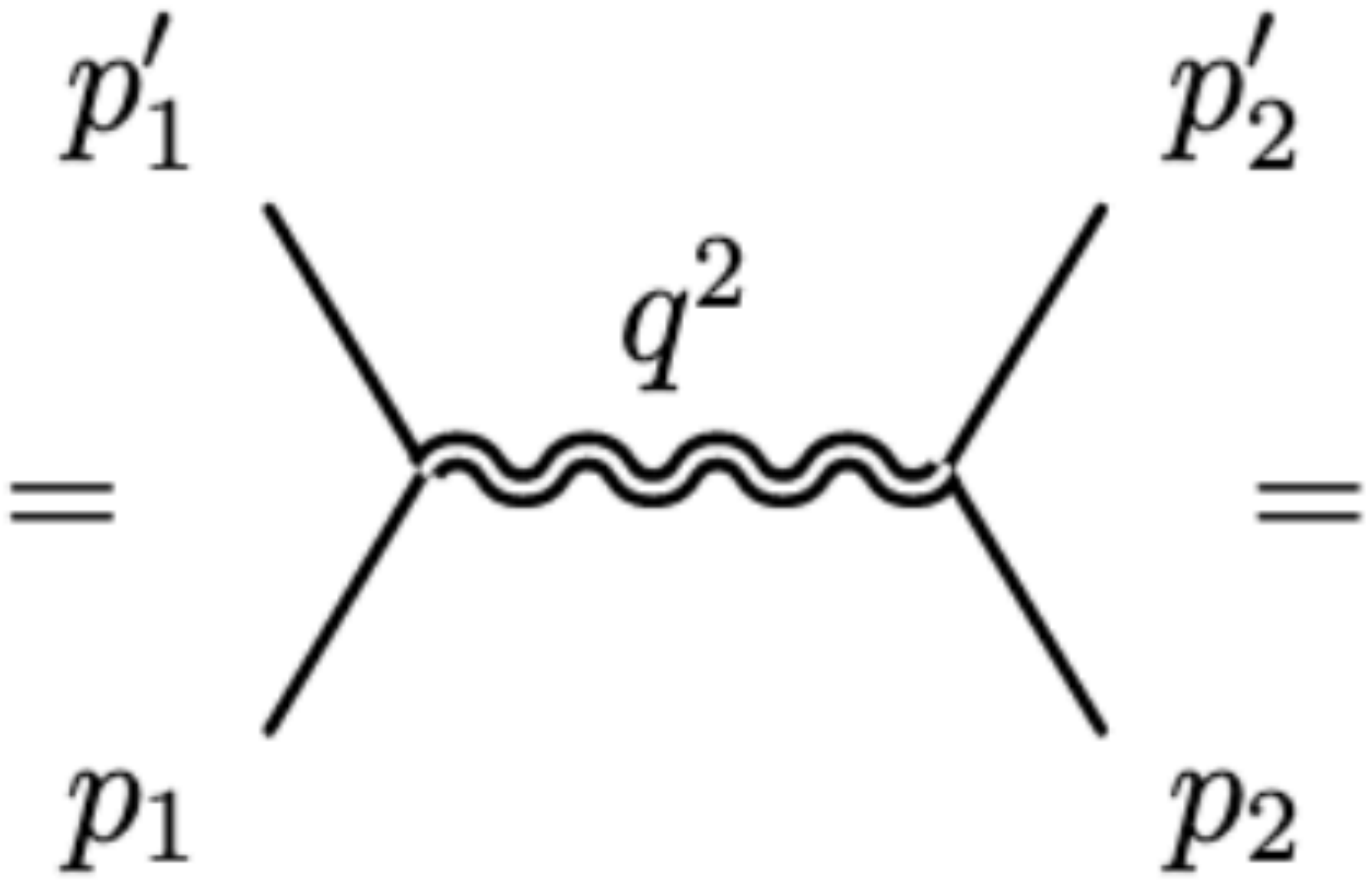

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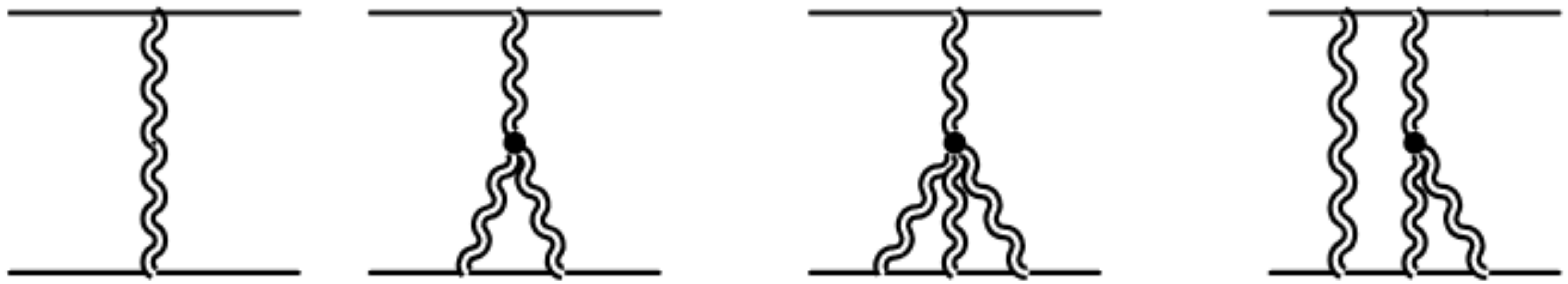

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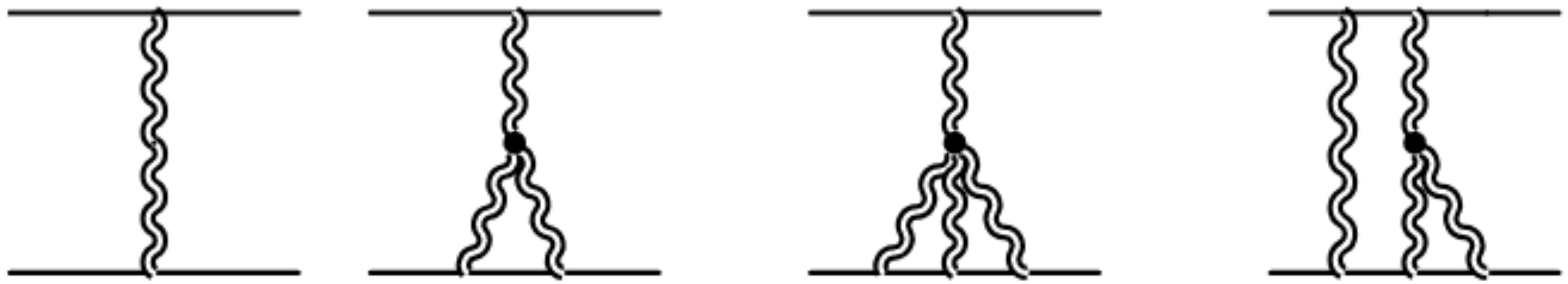

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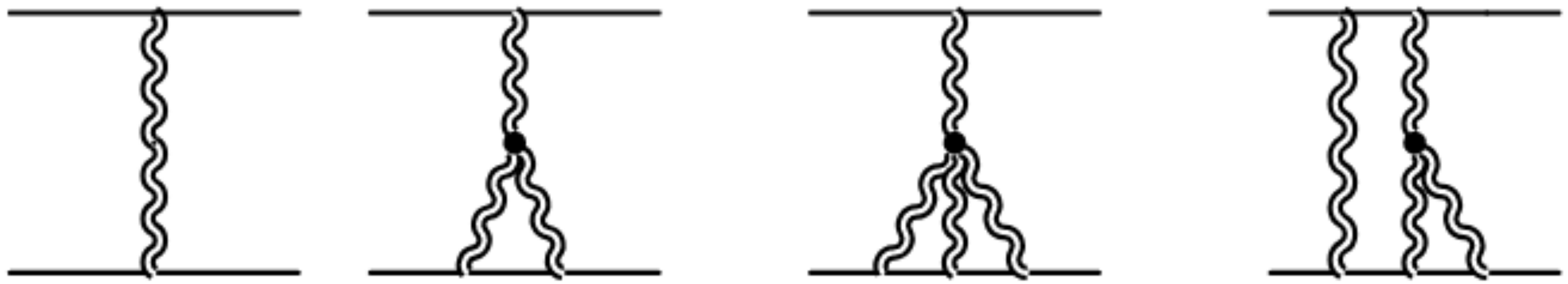
Normally we work with expressions where factors of $\hbar$ are hidden ($\hbar$ = c =1)



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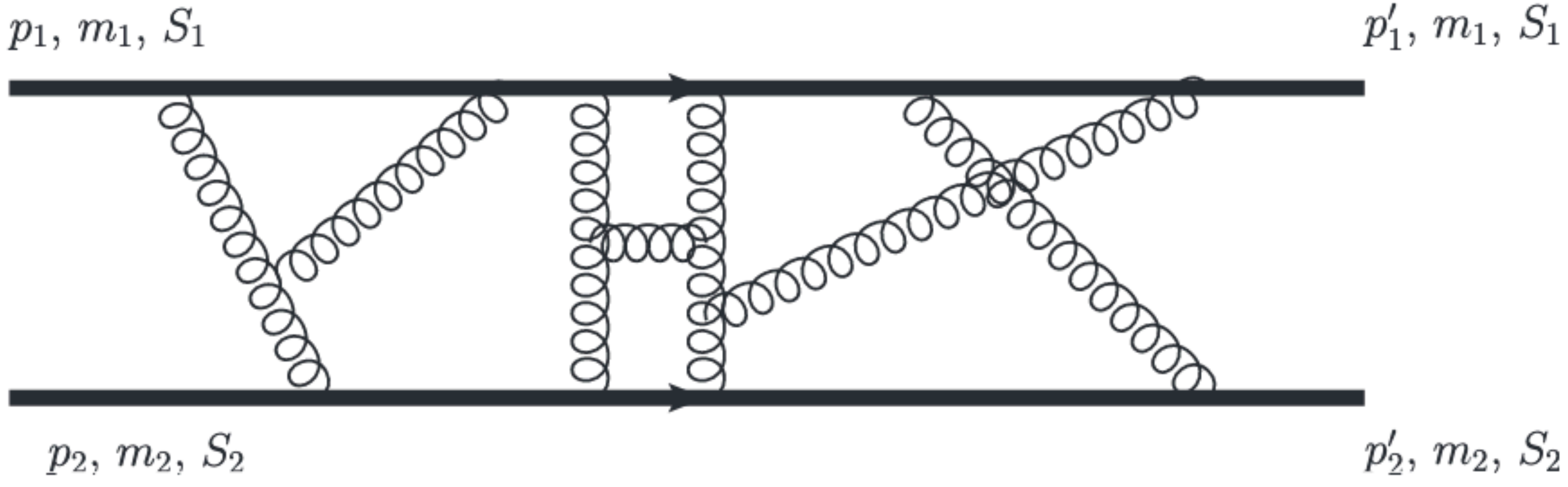

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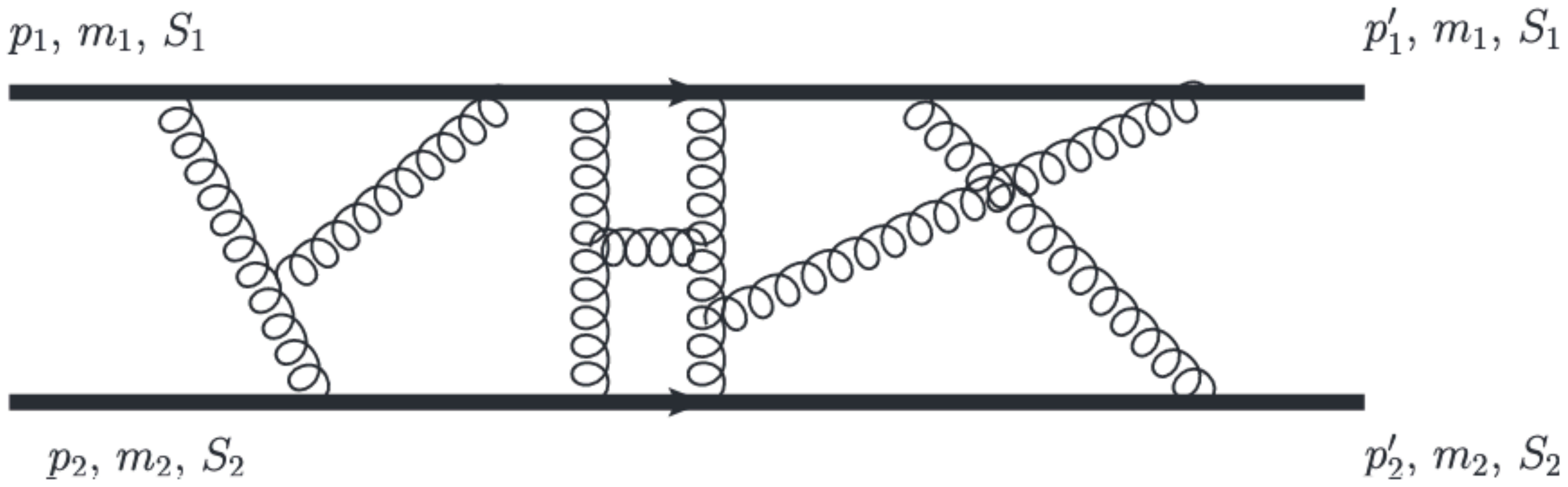

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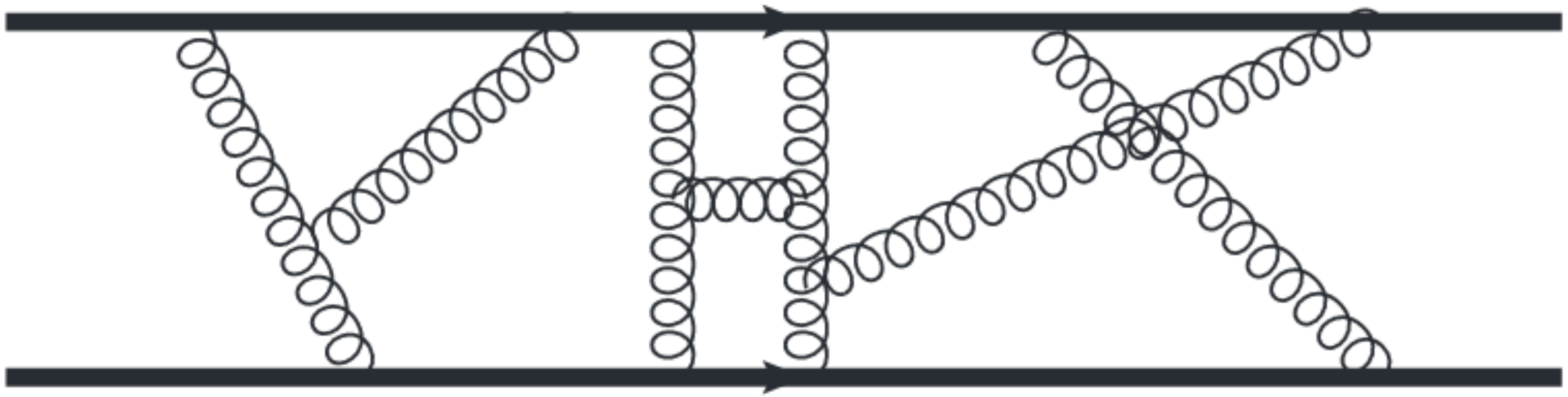

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$$\mathcal{M}_L(\gamma, \underline{q}^2, \hbar) = \frac{\mathcal{M}_L^{(-L-1)}(\gamma, \underline{q}^2)}{\hbar^{L+1}|\underline{q}|^{\frac{L(4-D)}{2}+2}} + \cdots + \frac{\mathcal{M}_L^{(-1)}(\gamma, \underline{q}^2)}{\hbar|\underline{q}|^{\frac{L(4-D)}{2}+2-L}} + O(\hbar^0) \qquad 1/\hbar$$

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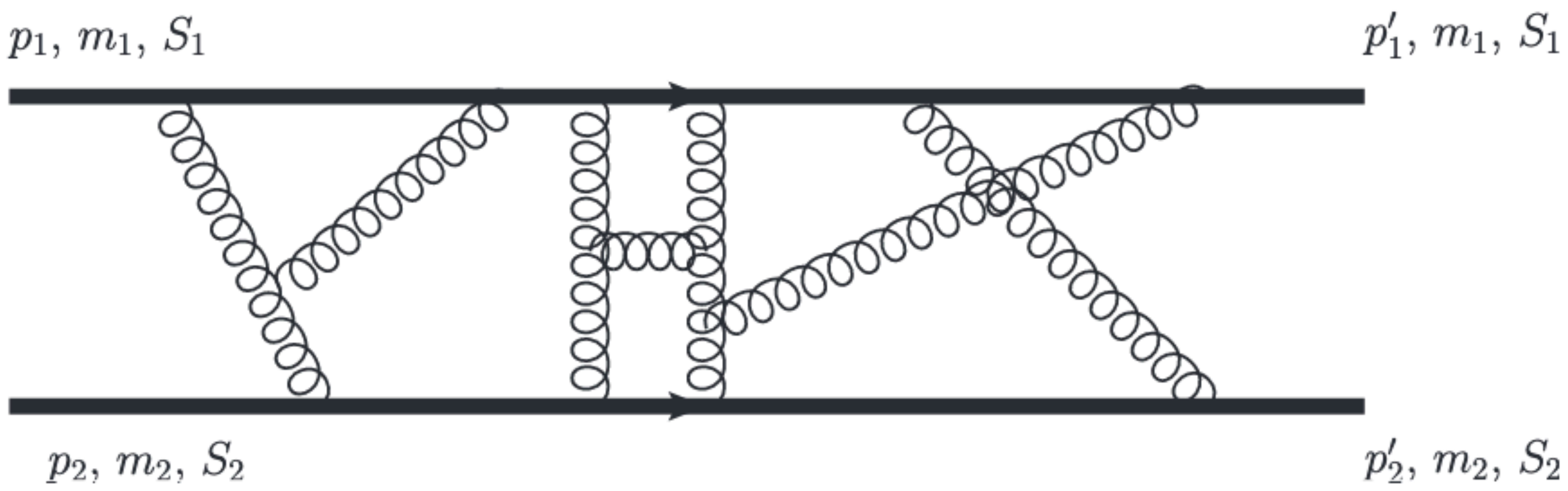

Reinstating $\hbar$ there is a difference between massive and massless propagation

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- For tests of Einstein’s theory we only need to retain leading classical terms (often with simplifications beyond expectations)
- For quantum effects one need to include subleading terms as well (much harder…)

Important simplification

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Important point: Long range behaviour can be captured from unitarity cuts

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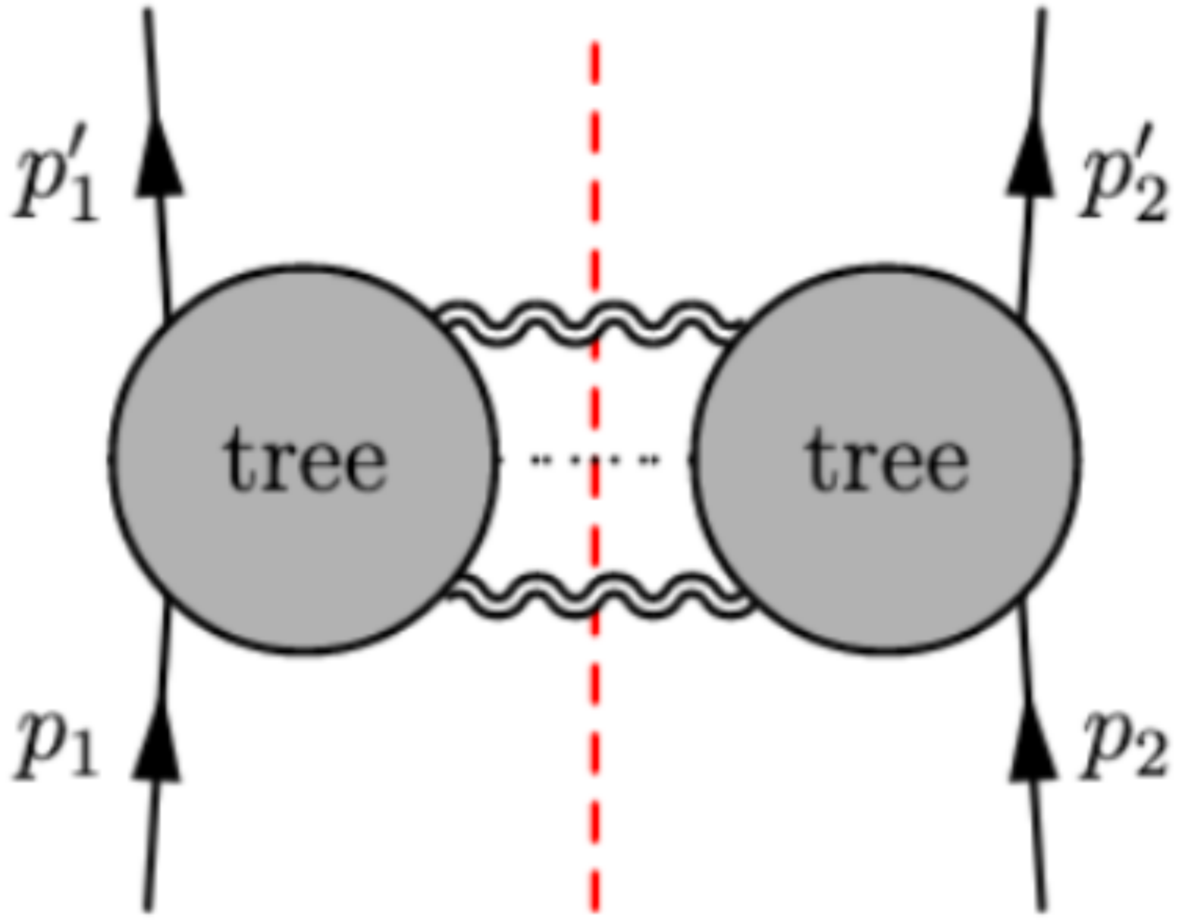

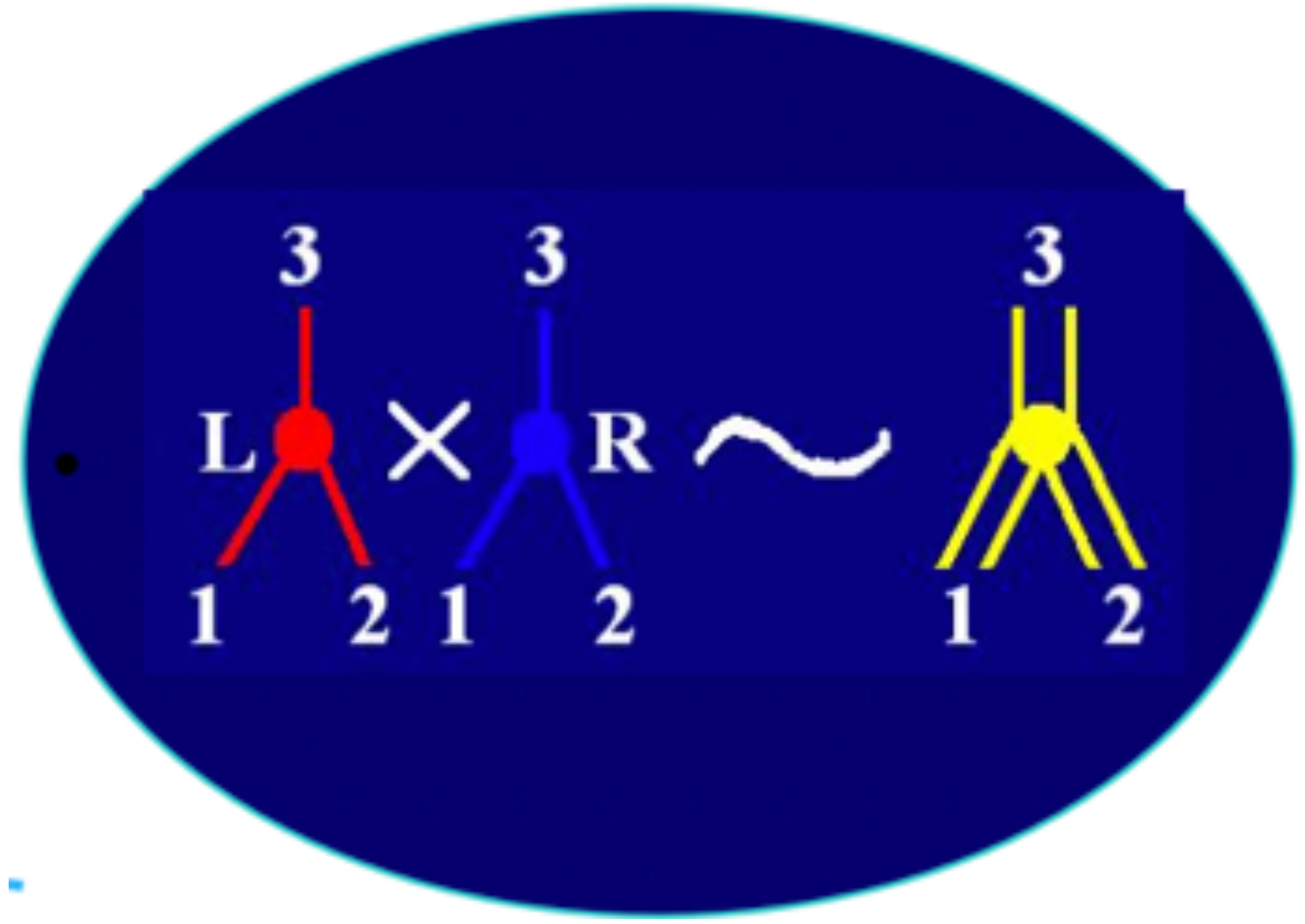

Using on-shell amplitude techniques
(Neill, Rothstein; NEJBB, Donoghue, Vanhove)

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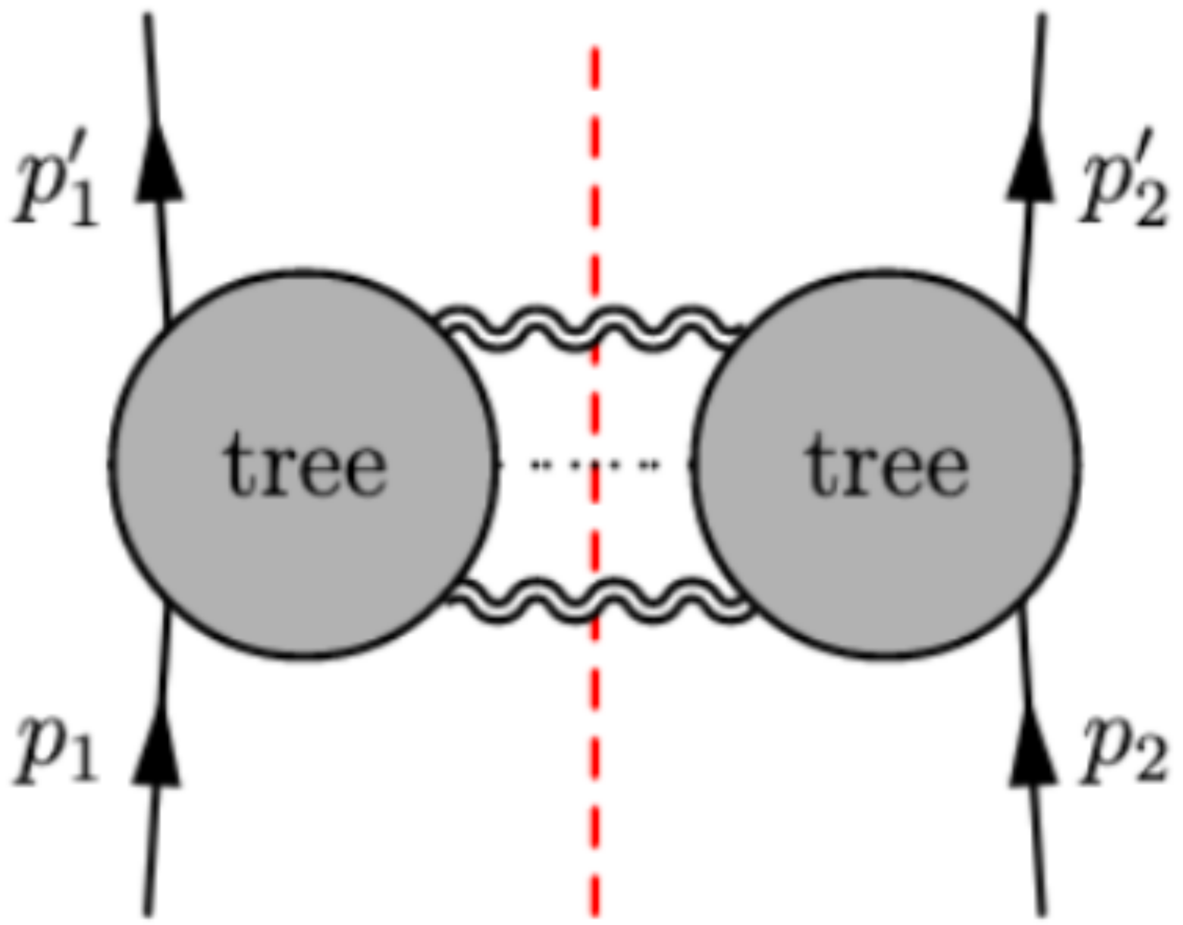

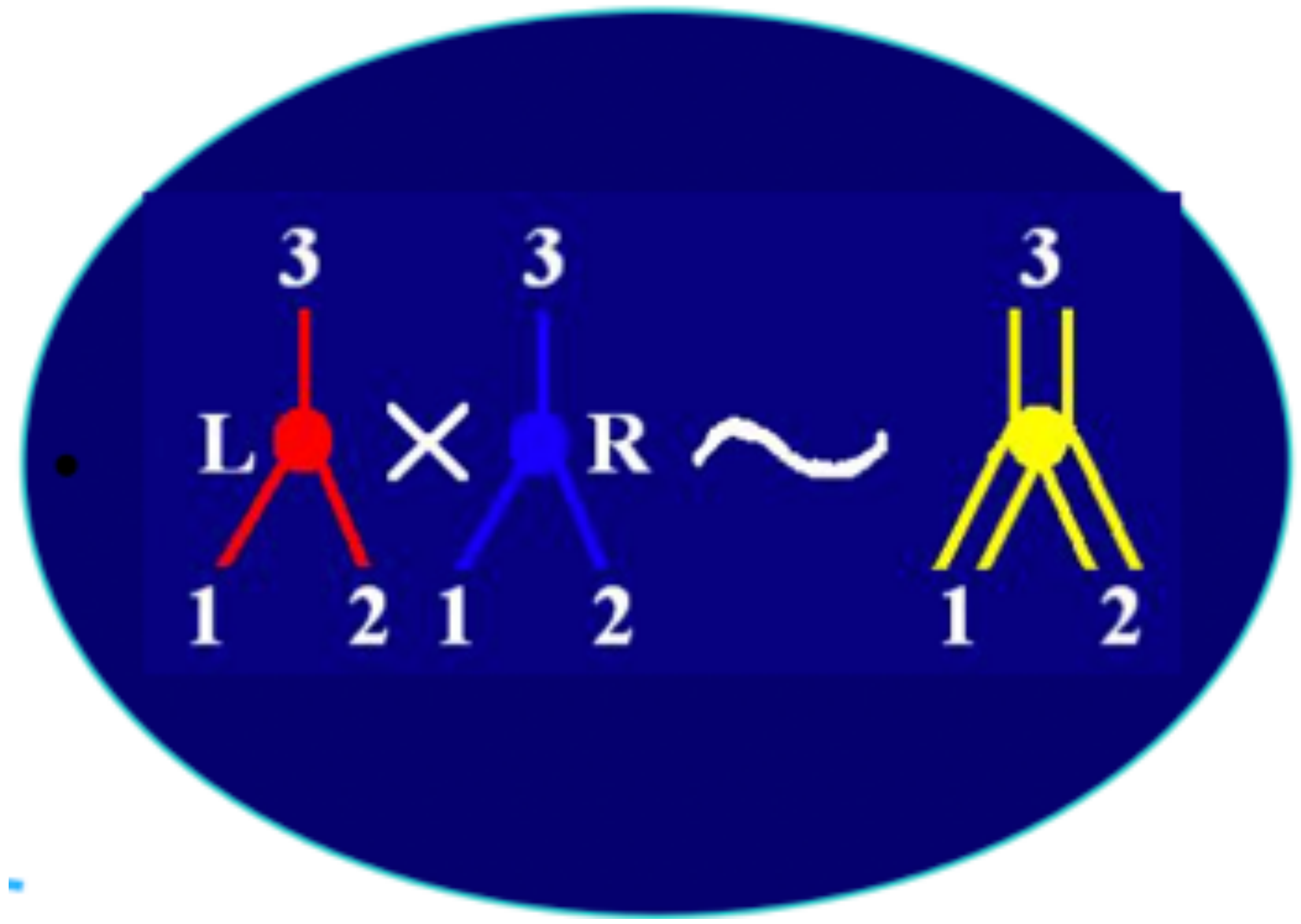

KLT+on-shell input trees (e.g. Badger et al., Forde, Kosower) recycled from Yang-Mills -> gravity

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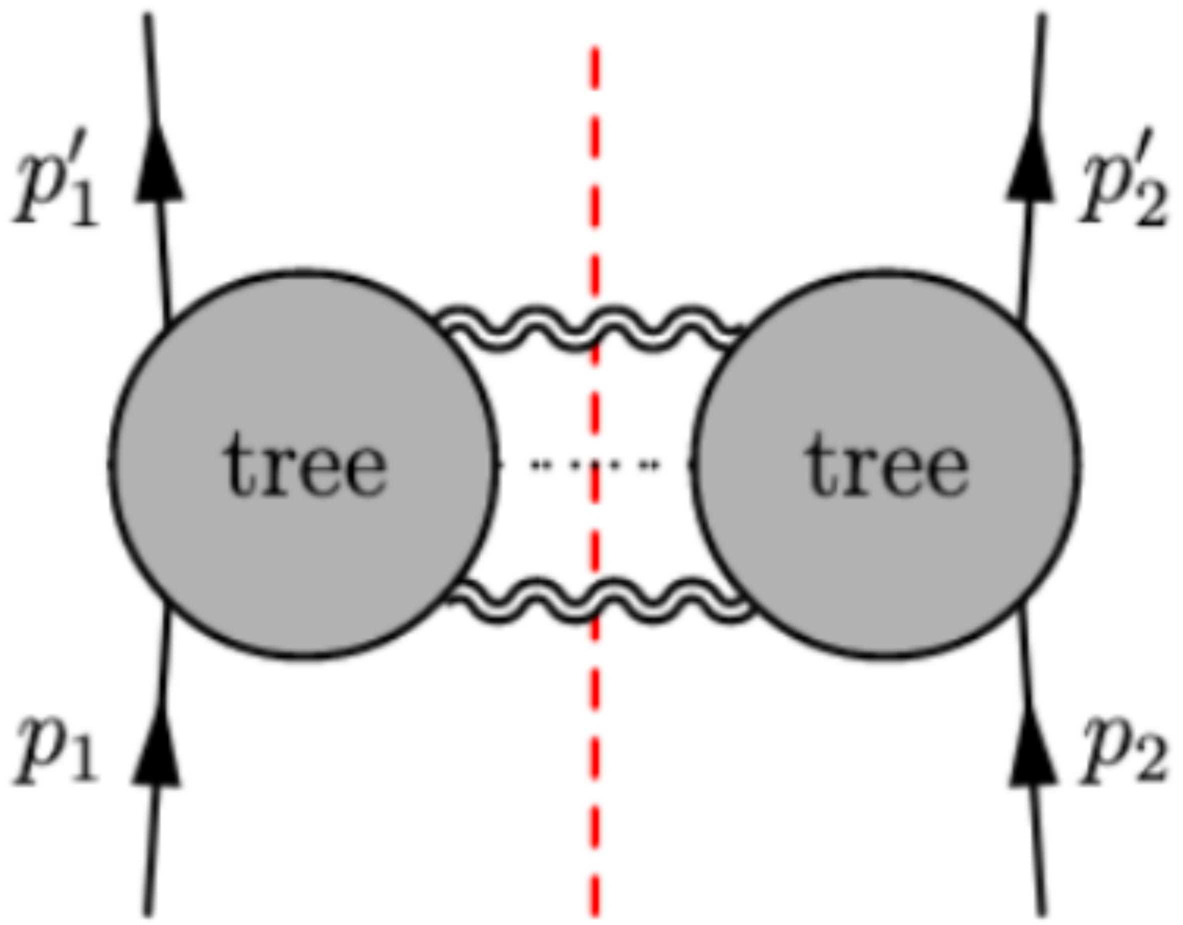

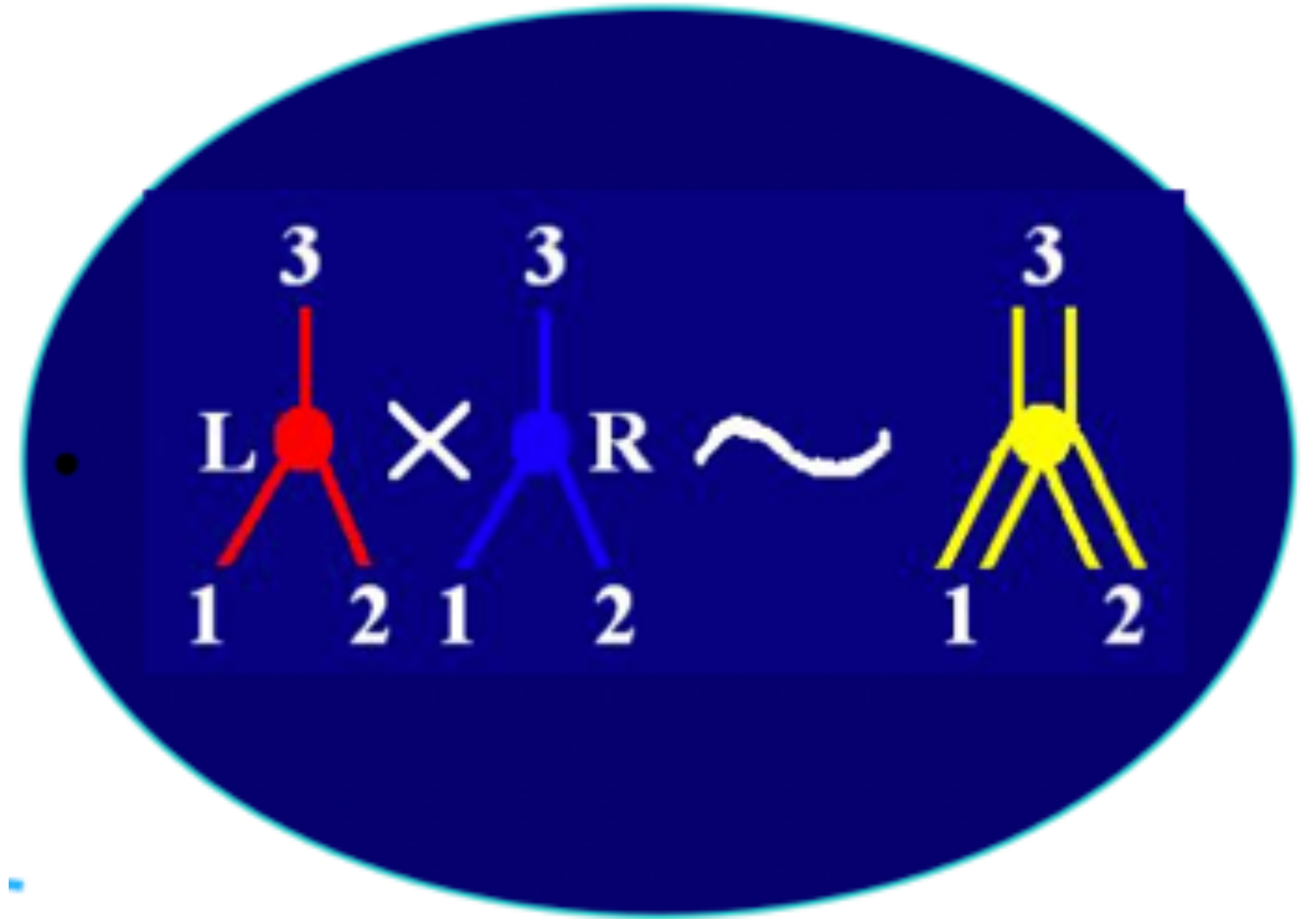

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In D-dimensions from CHY (NEJBB, Cristofoli, Damgaard, Gomez; NEJBB, Plante, Vanhove)

Using on-shell amplitude techniques (Neill, Rothstein; NEJBB, Donoghue, Vanhove)

Long range behaviour

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Four point amplitude take the form

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$$\mathcal{M} \sim \left(A + Bq^2 + \ldots + \alpha\kappa^4 \frac{1}{q^2} + \beta_1\kappa^4 \ln(-q^2) + \beta_2\kappa^4 \frac{m}{\sqrt{-q^2}} + \ldots \right)$$

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(no leading higher derivative contributions)

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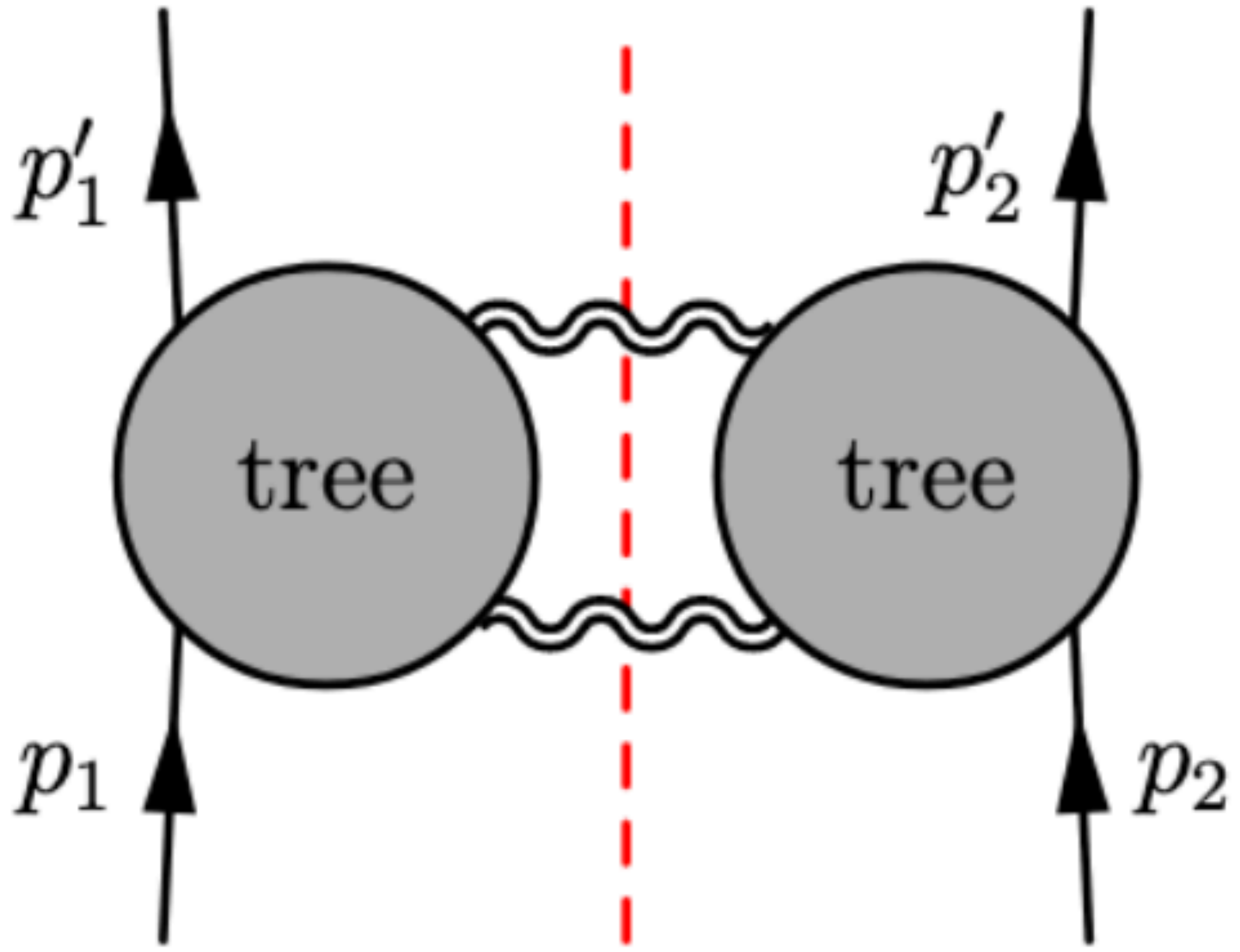
(NEJB, Donoghue, Holstein)

Example: One-loop amplitude potential



$$\mathcal{M}_1(\gamma, \underline{q}^2, \hbar) = \mathcal{M}_1^{\square} + \mathcal{M}_1^{\triangleright} + \mathcal{M}_1^{\triangleleft} + \mathcal{M}_1^{\circ}$$

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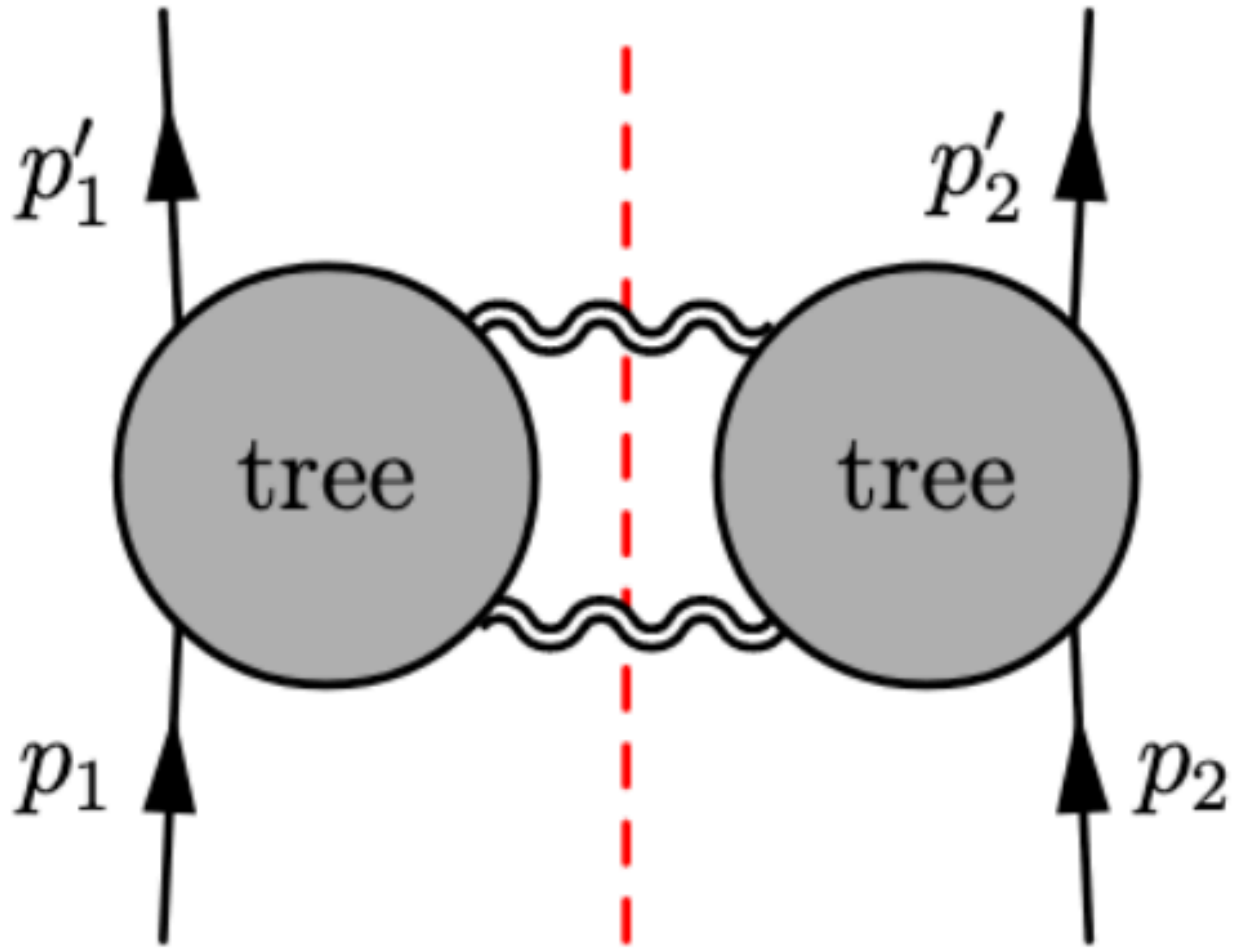

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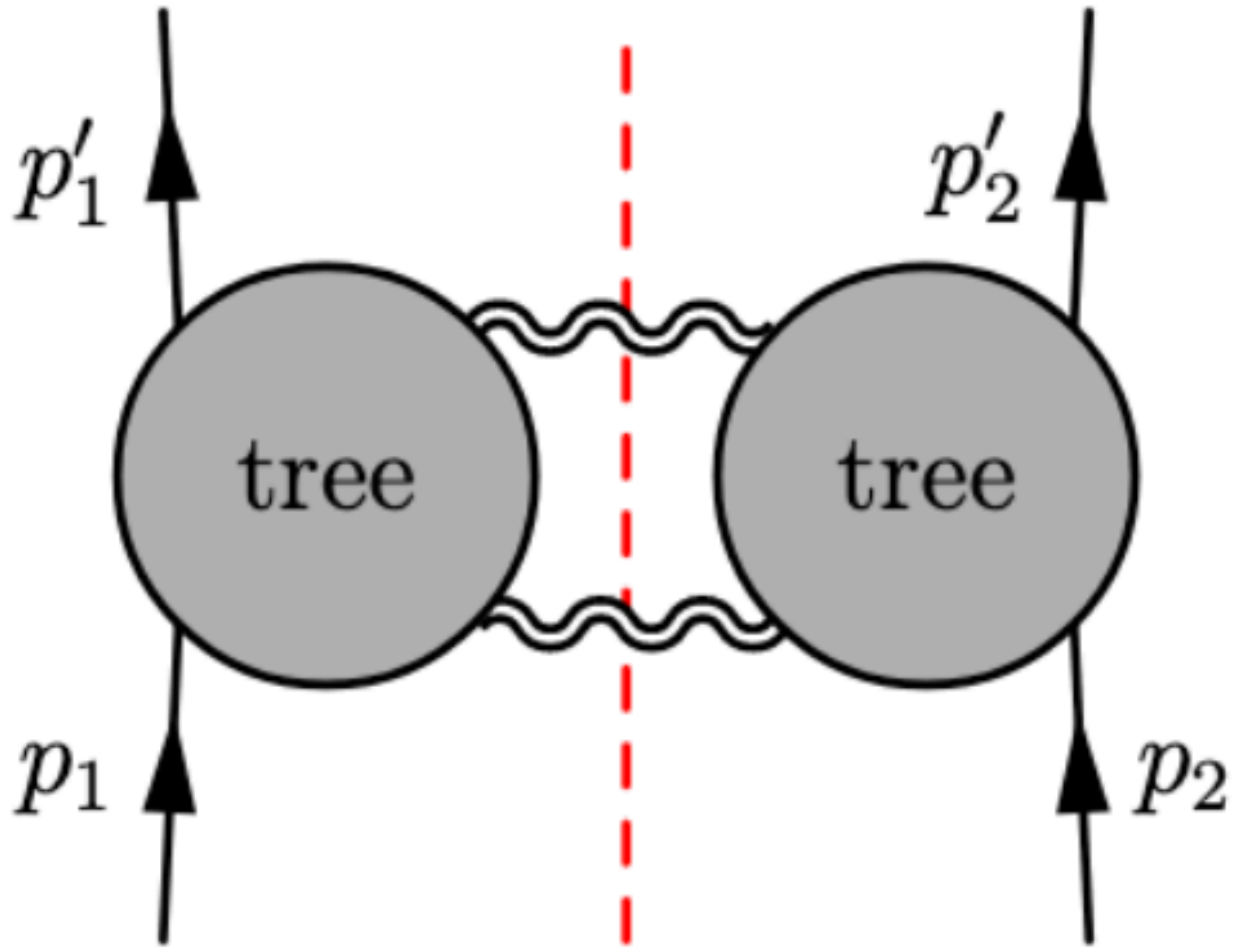

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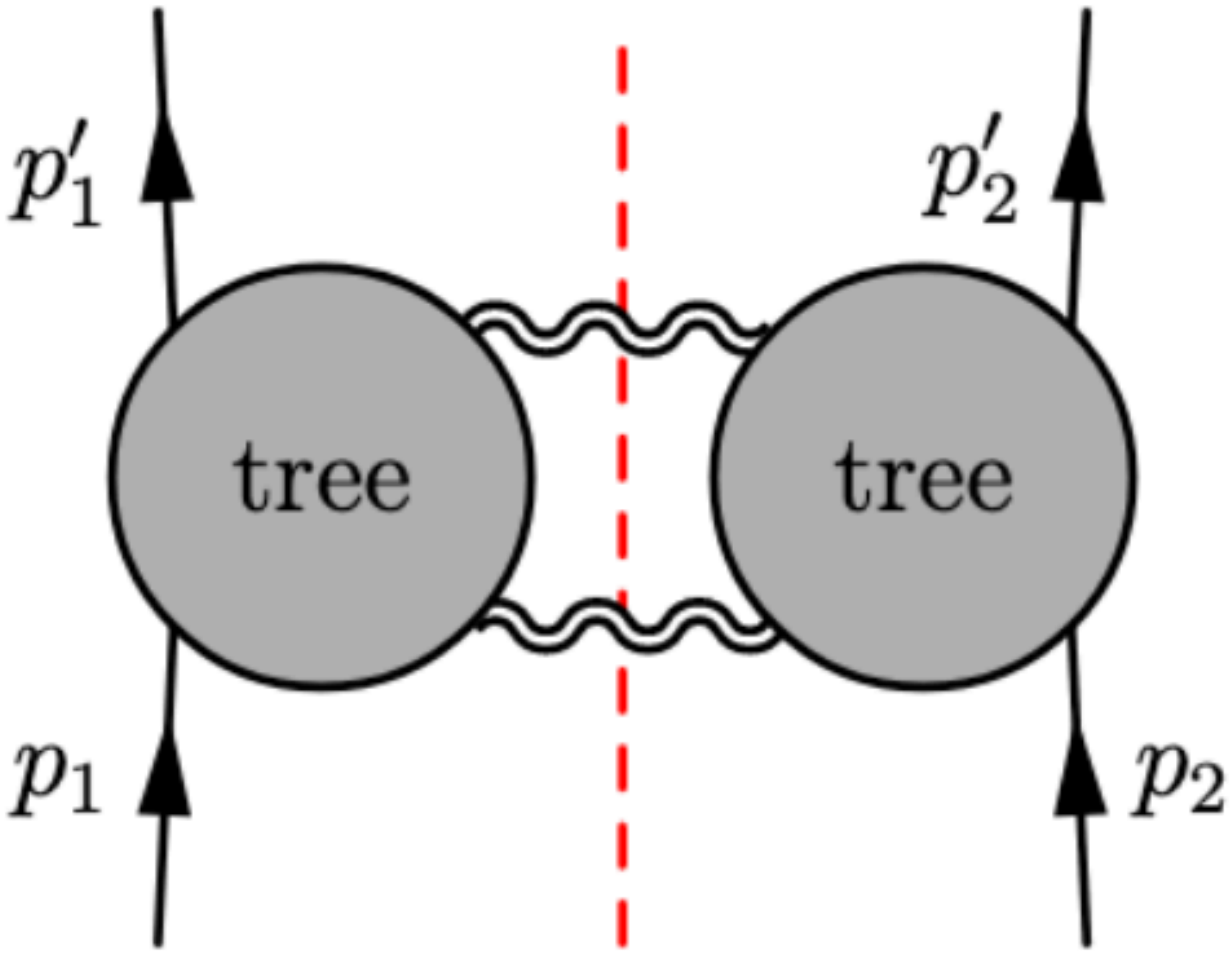

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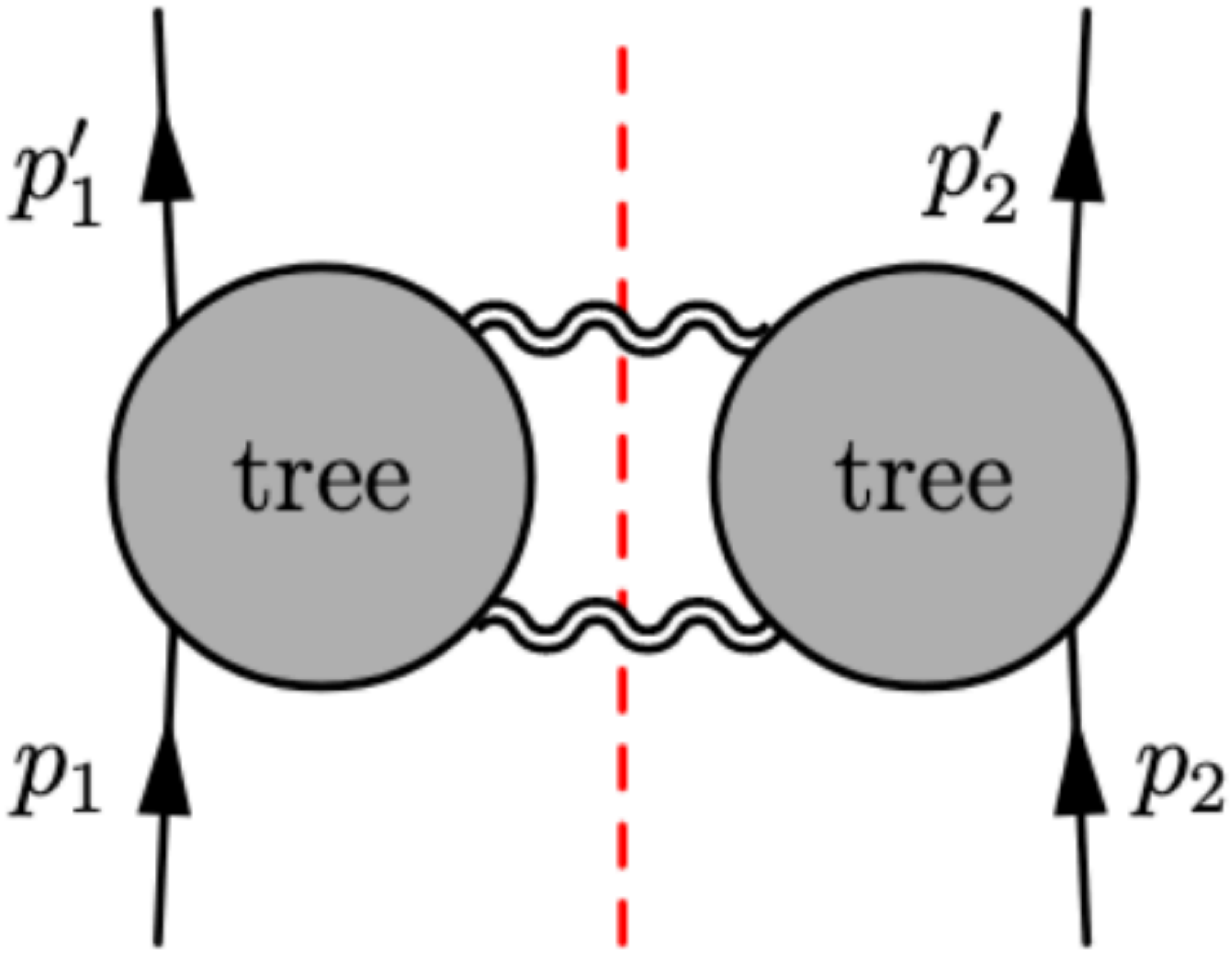

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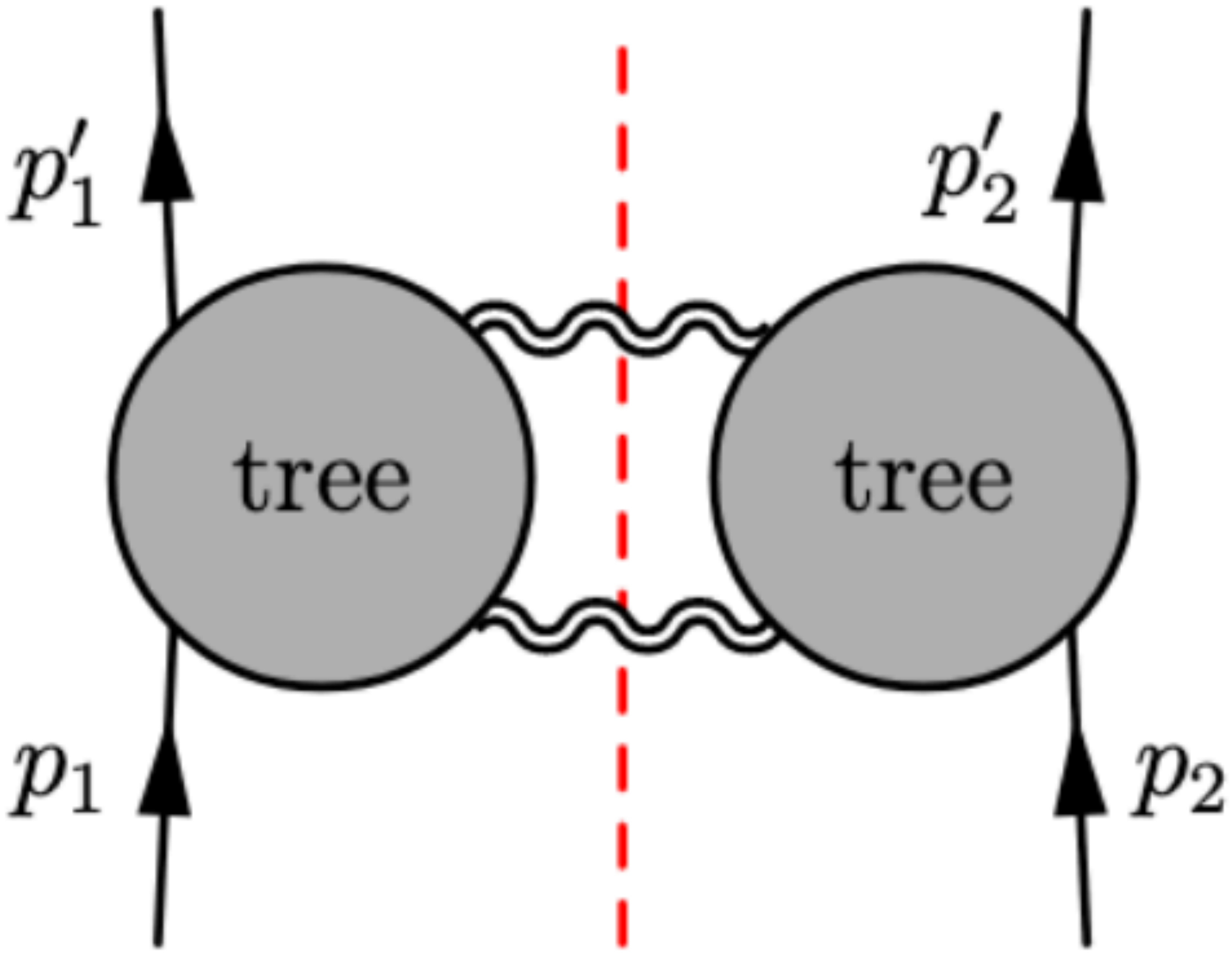

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Provide long-range behaviour

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$$\mathcal{M}_1(\gamma, \underline{q}^2, \hbar) = \frac{1}{|\underline{q}|^{4-D}} \left(\frac{\mathcal{M}_1^{(-2)}(\gamma, \underline{q}^2)}{\hbar^2} + \frac{\mathcal{M}_1^{(-1)}(\gamma, \underline{q}^2)}{\hbar} + \mathcal{M}_1^{(0)}(\gamma, \underline{q}^2) + \mathcal{O}(\hbar) \right)$$

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Organise order by order in Planck’s constant

PM potential one-loop amplitude

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$$c_\Box = c_{\bowtie} = 16 m_1^4 m_2^4 \frac{(1-(D-2)\sigma^2)^2}{(D-2)^2},$$

$$c_\triangleright = \frac{4m_1^4 m_2^2\,(D - 7 + (D(4D-17)+19)\sigma^2)}{(D-2)^2}$$

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Link to Einstein's theory

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- Problem in scattering theory to relate a scattering loop amplitude M to an interaction potential V.
- In post-Newtonian computations, we consider non-relativistic quantum mechanics, and this can be generalized to the relativistic case.

Classical potential from a Lippmann-Schwinger equation

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- Extension is given by Lippmann Schwinger eq. (involves iterations/ subtractions)

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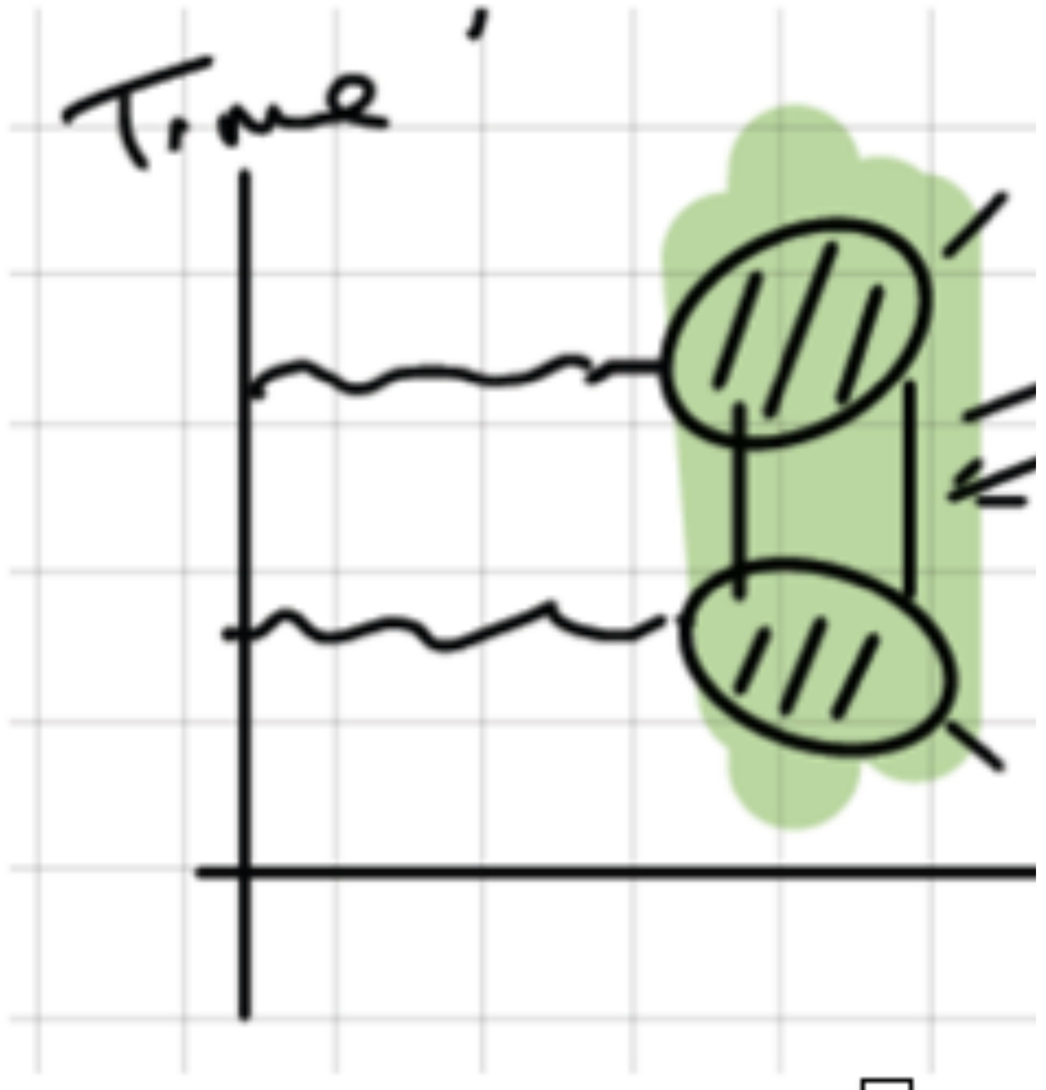
- Non-relativistic limit, the tree classical potential is simply equal to the amplitude after a Fourier transform:

$$\mathcal{V}(r,p) = \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot r} \mathcal{V}(p,q) = \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot r} \tilde{\mathcal{M}}(p,q)$$



- Extension is given by Lippmann Schwinger eq. (involves iterations/ subtractions)

$$\tilde{\mathcal{M}}(p,p') = \mathcal{V}(p,p') + \int \frac{d^3k}{(2\pi)^3} \frac{\mathcal{V}(p,k)\mathcal{M}(k,p')}{E_p - E_k + i\varepsilon}$$

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We will also assume (classical) long-distance scattering (this has the consequence that we can focus on non-analytic contributions -> ideal for unitarity)

(NEJBB, Donoghue, Holstein; Cristofoli, NEJBB, Damgaard, Vanhove)

One-loop

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Subtraction important to make contact with classical physics potential

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$$\mathcal{M}^{\text{Iterated}} = \frac{i\pi G_N^2}{E_p^3\xi}\frac{4c_1^2}{|\vec{p}|}\frac{(\log|\vec{q}|^2 - \frac{2}{3-d})}{|\vec{q}|^2} + \frac{2\pi^2 G_N^2}{E_p^3\xi^2|\vec{q}|}\left(\frac{c_1^2(\xi-1)}{2E_p^2\xi} - 4c_1 p_1\cdot p_3\right)$$

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Follows from the Lippmann-Schwinger subtraction. Again same result as from matching (Bern et al), the effect is that singular terms are gone!

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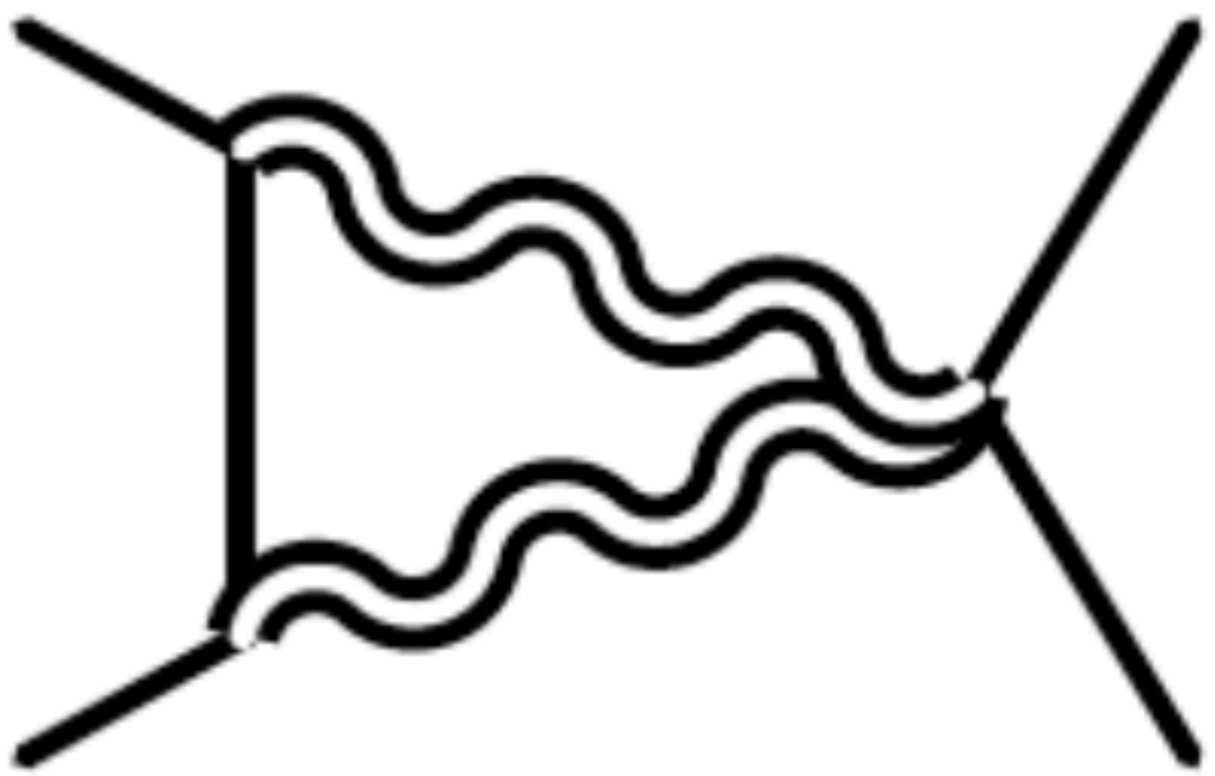
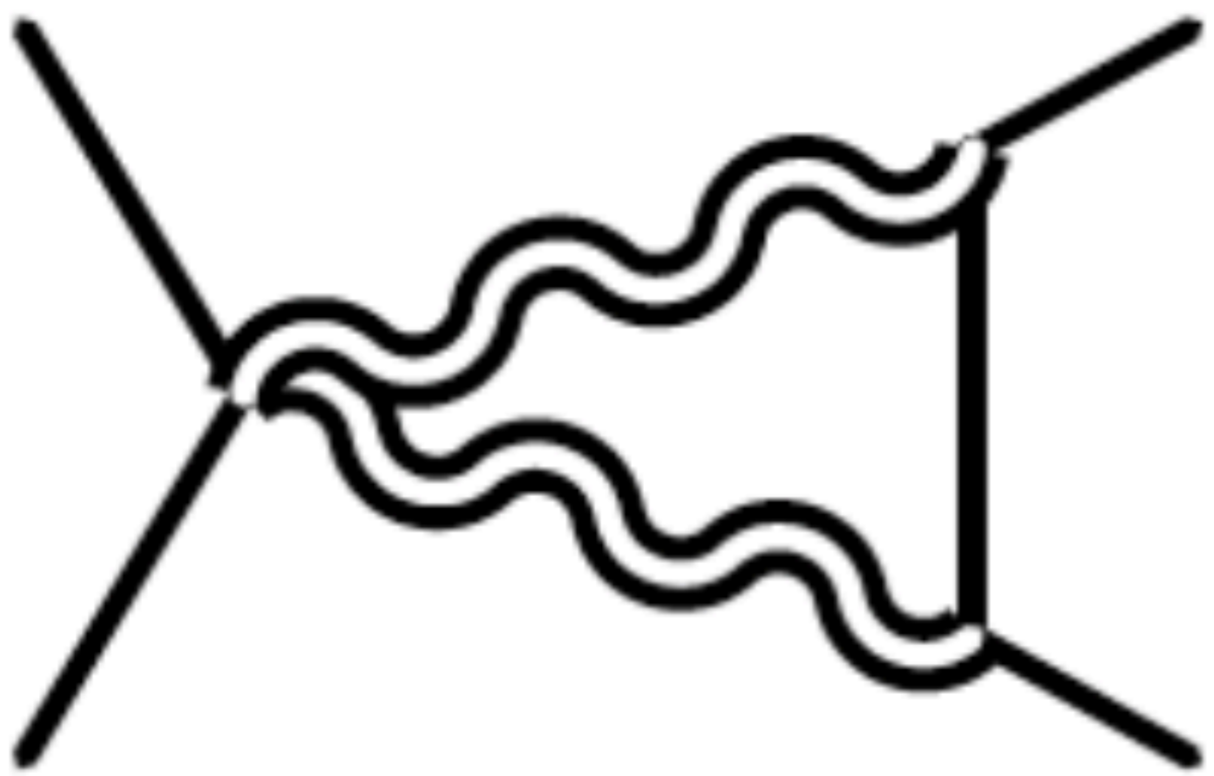
One-loop level

$\mathcal{M}_2 =$  $+$ 

$$= -i(8\pi G)^2\left(\frac{c(m_1,m_2)I_{\triangleright}(p_1,q)}{(q^2-4m_1^2)^2}+\frac{c(m_2,m_1)I_{\triangleright}(p_4,-q)}{(q^2-4m_2^2)^2}\right)$$

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We will now consider what happens at two-loops

Classical gravitational scattering:
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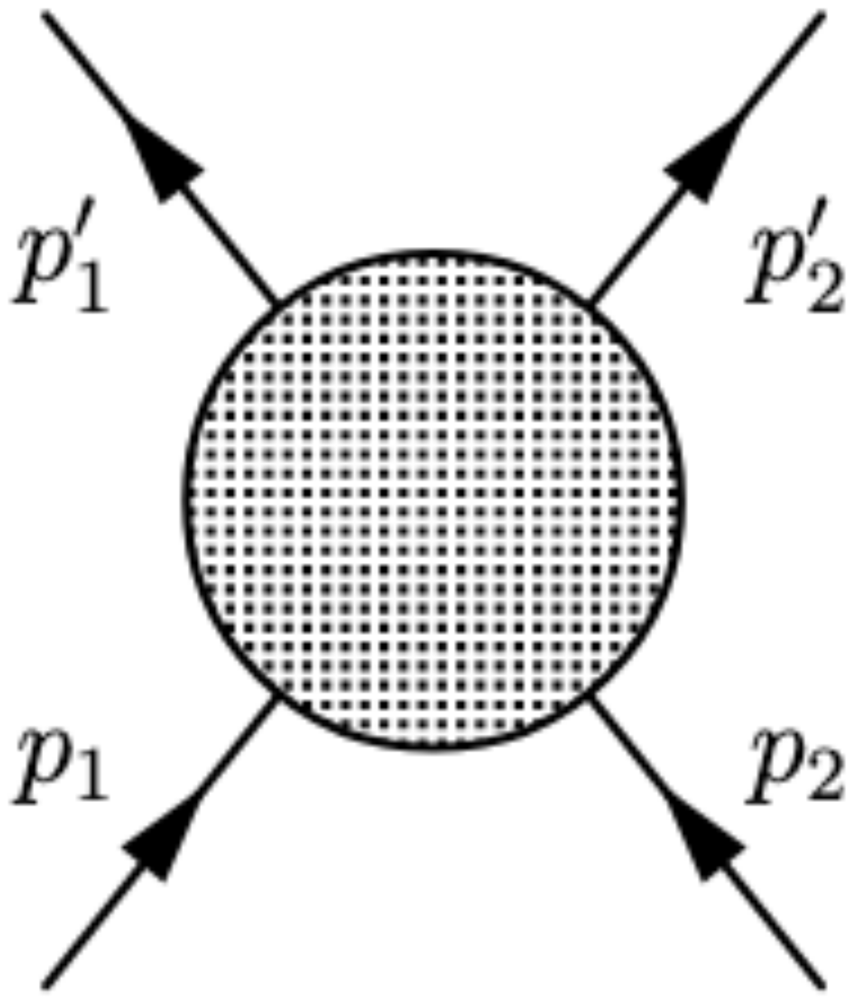

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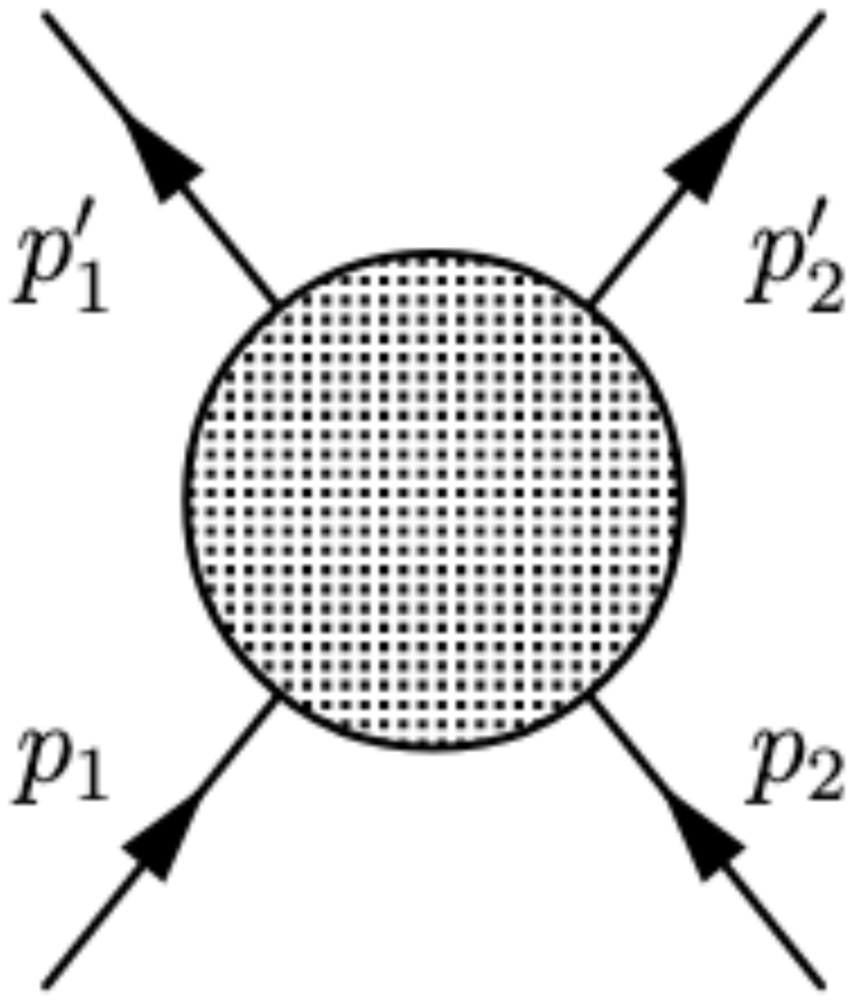

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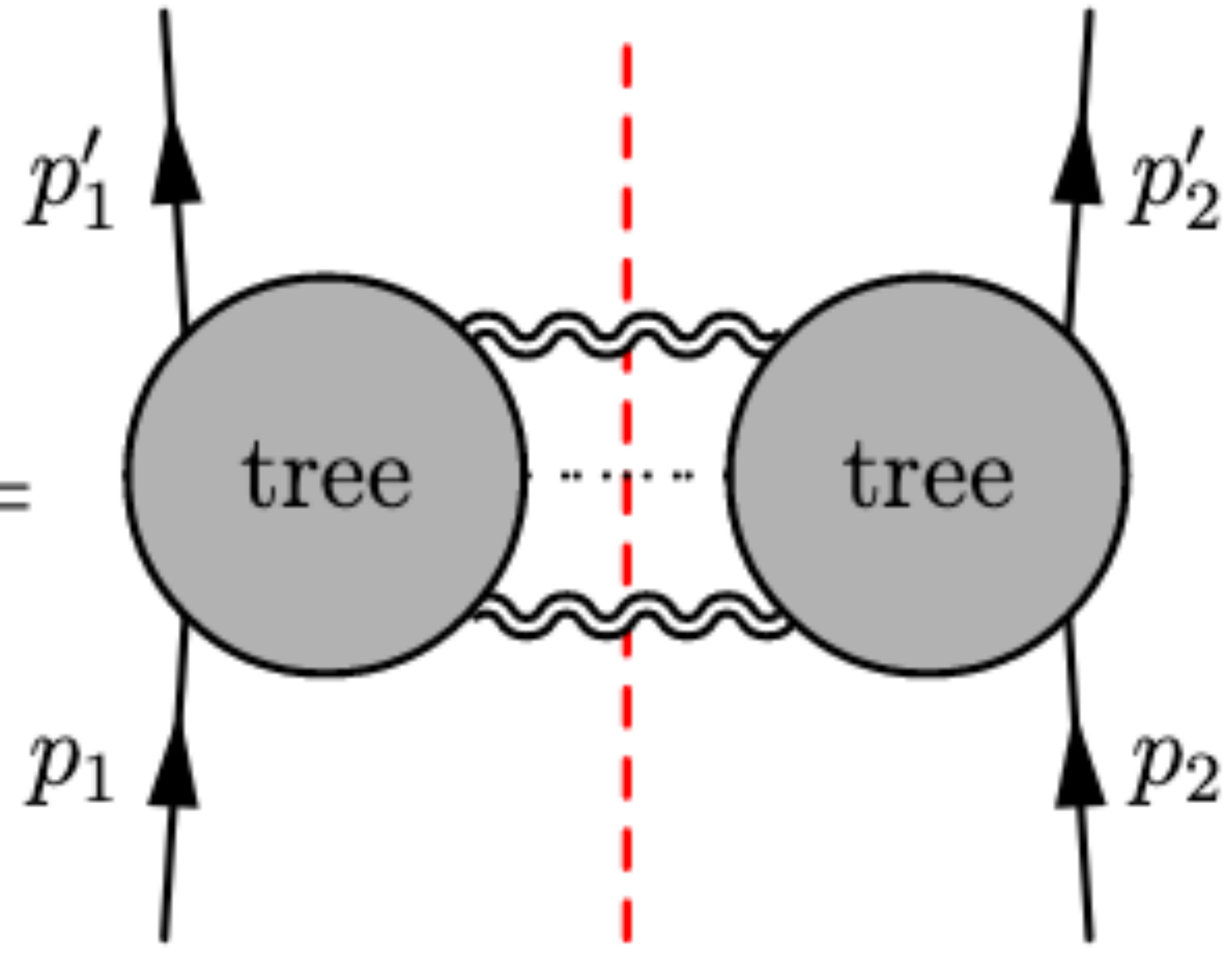

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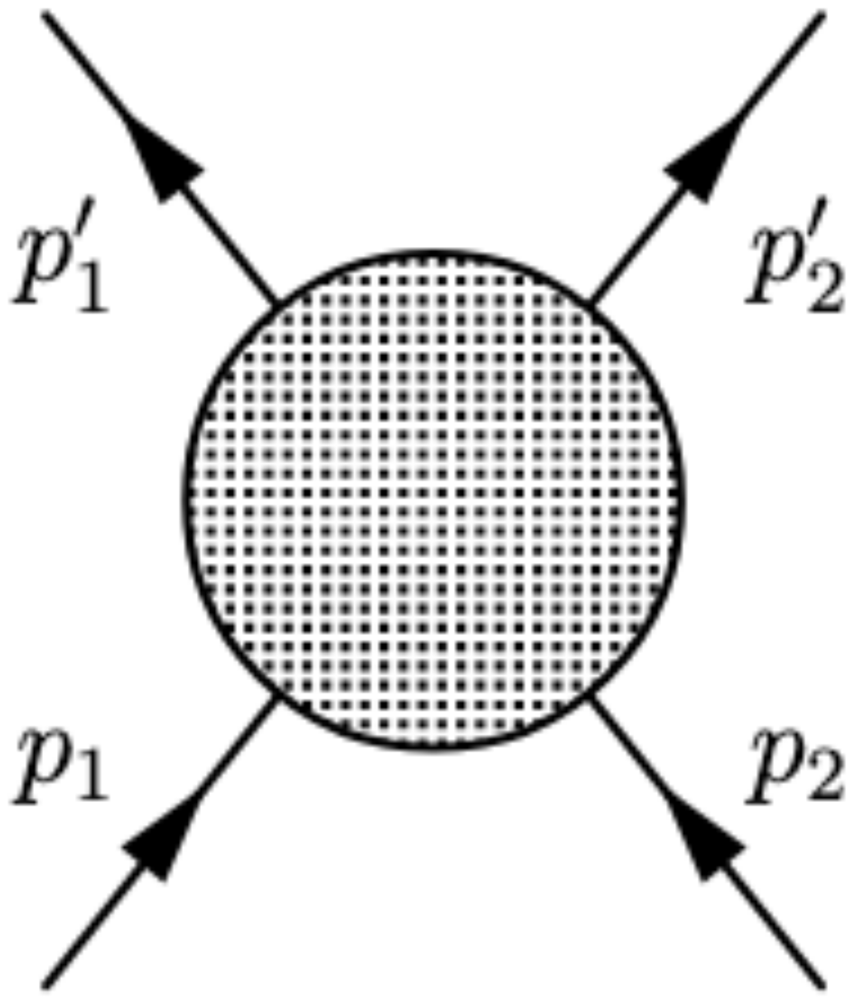
Extraction of integrand similar to QCD
Spinor-helicity and D-dimension
covariant tree
amplitudes can be used in cuts



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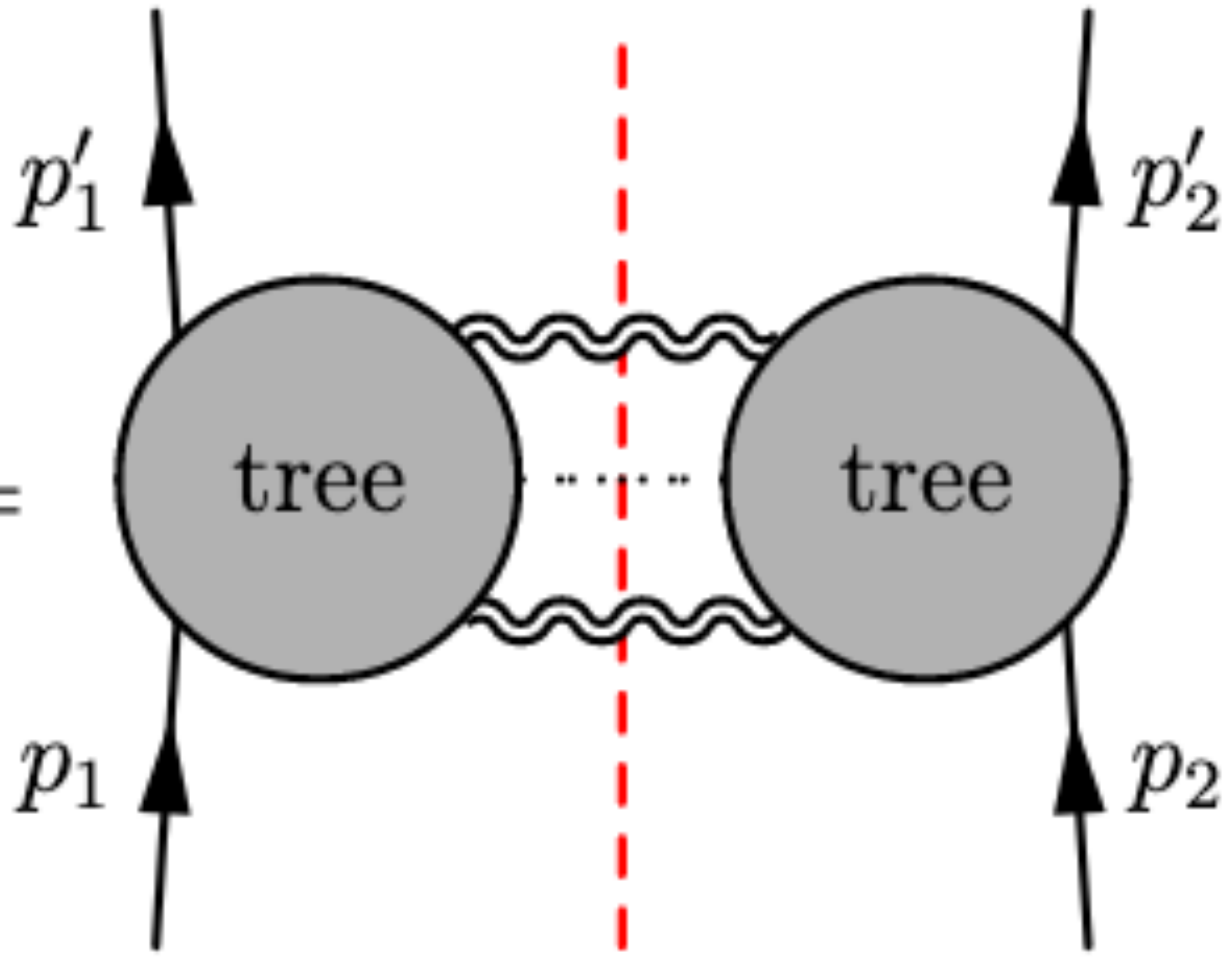

Example: Einstein gravity at two-loop order

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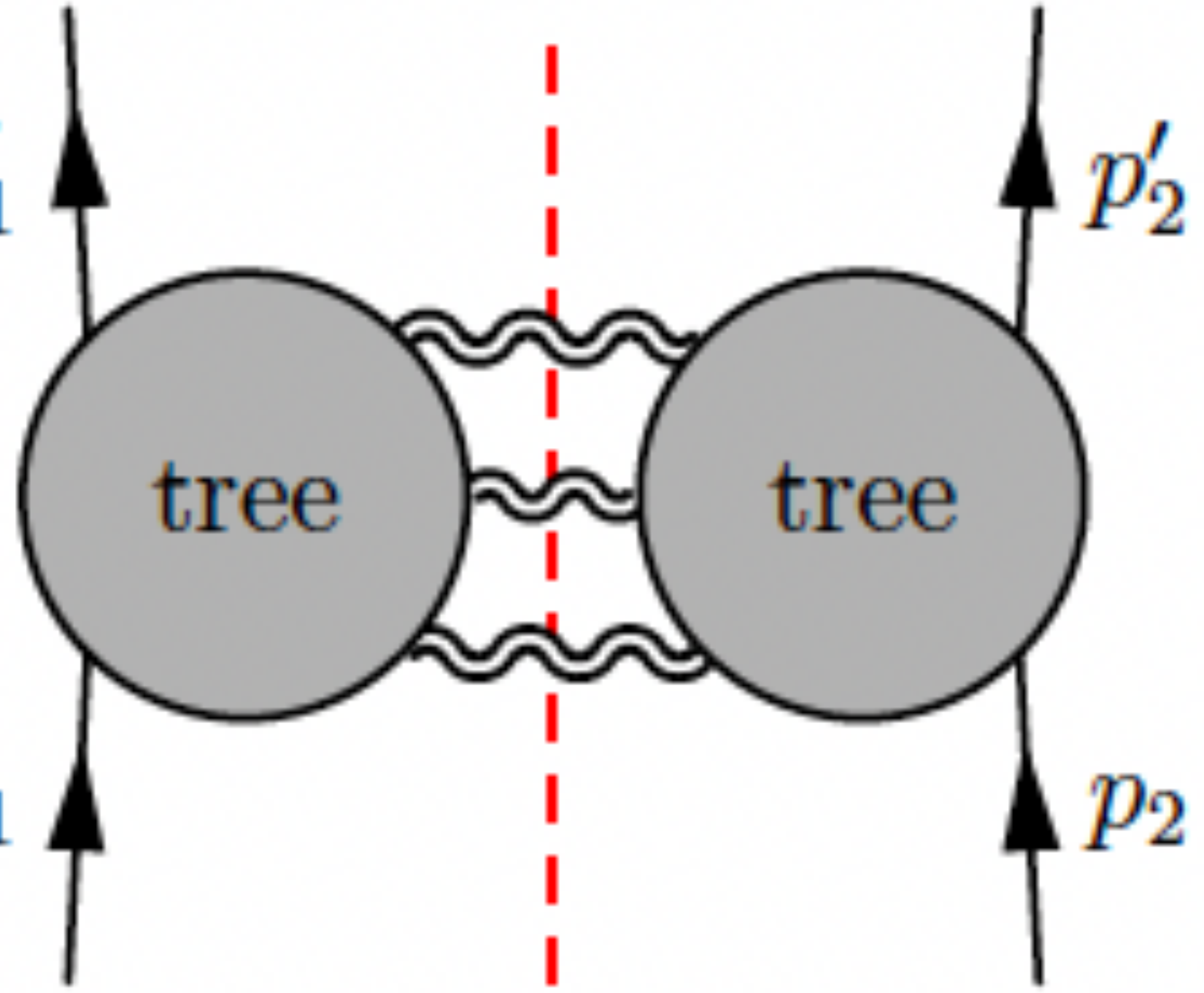
$$\mathcal{M}^2_{3-\text{cut}}(\sigma, q^2) =$$



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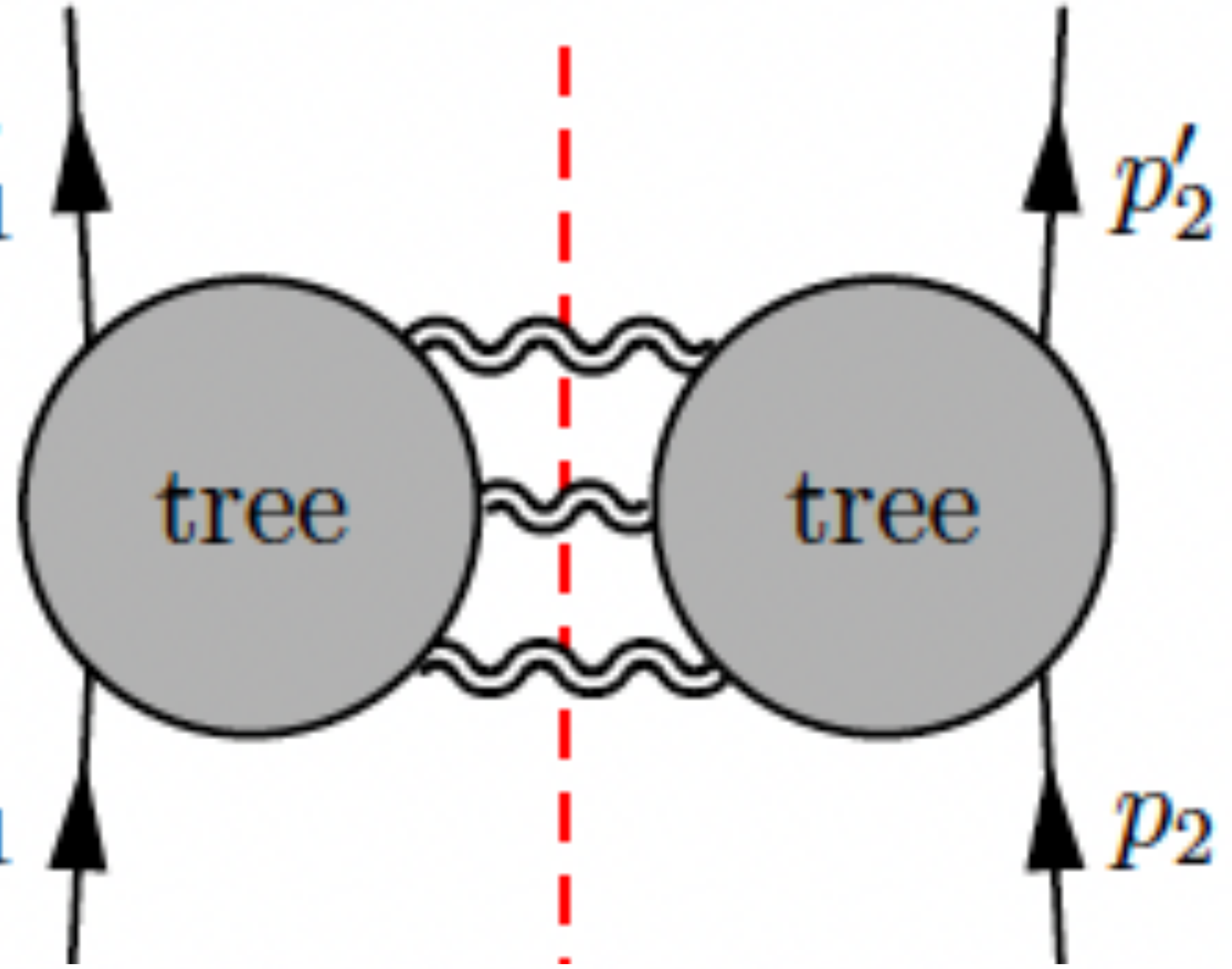
$\mathcal{M}^2_{3-\text{cut}}(\sigma, q^2) =$ 

$$\mathcal{M}_2^{3-\text{cut}}(\sigma, q^2) = \int \frac{d^D l_1 d^D l_2 d^D l_3}{(2\pi)^{3D}} (2\pi)^D \delta^{(D)}(l_1 + l_2 + l_3 + q) \frac{i^3}{l_1^2 l_2^2 l_3^2}$$
$$\times \frac{1}{3!} \sum_{\substack{\text{Perm}(l_1,l_2,l_3) \\ \lambda_1=\pm,\lambda_2=\pm,\lambda_3=\pm}} \mathcal{M}_0(p_1, p'_1, l_1^{\lambda_1}, l_2^{\lambda_2}, l_3^{\lambda_3})(\mathcal{M}_0(p_2, p'_2, -l_1^{\lambda_1}, -l_2^{\lambda_2}, -l_3^{\lambda_3}))^*$$

Back: Next integral basis

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New integrals

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$$\mathcal{M}_2(\gamma, q^2) = \mathcal{M}_2^{3-\text{cut}}(\gamma, q^2) + \mathcal{M}_2^{\text{SE}}(\gamma, q^2)$$

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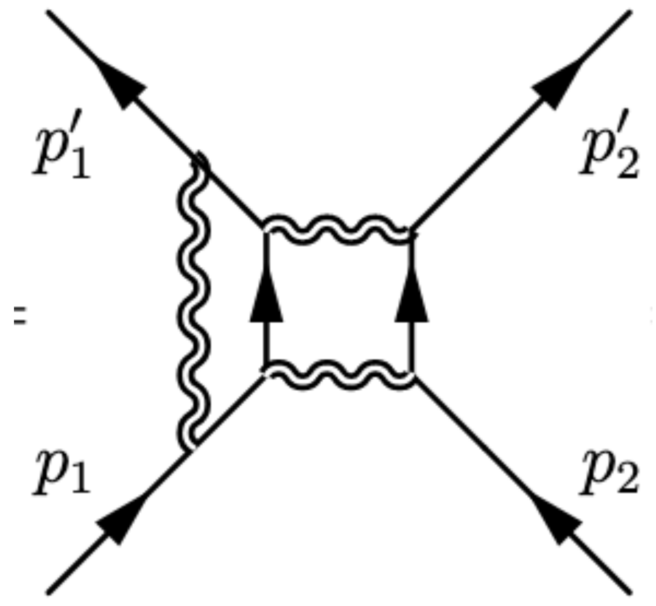

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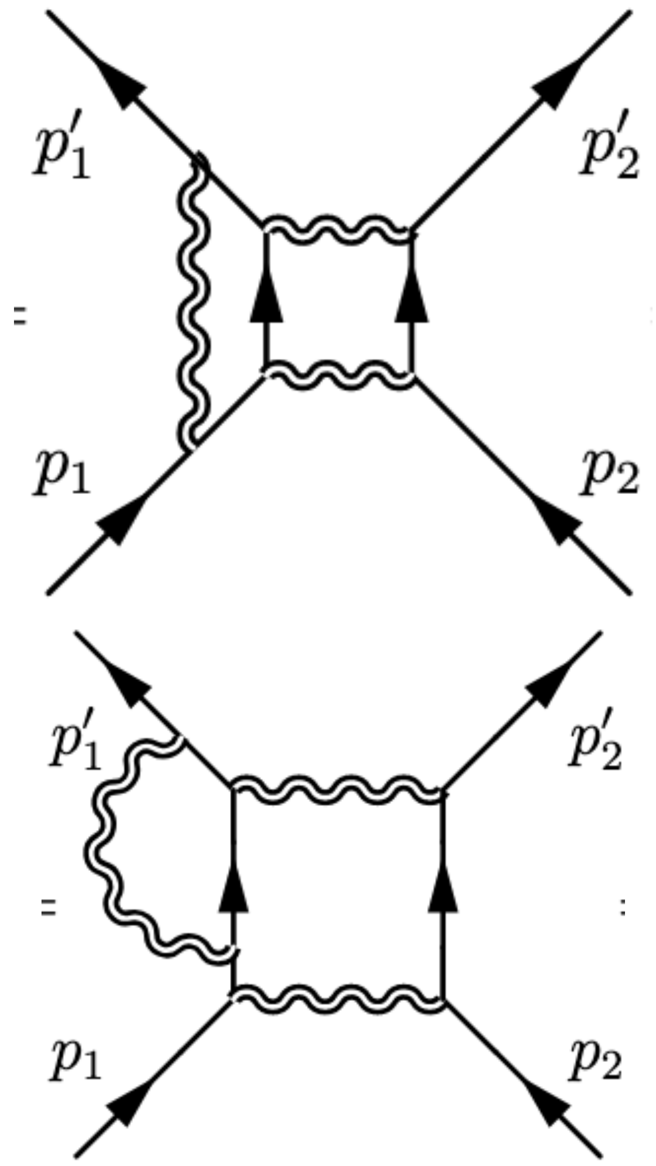

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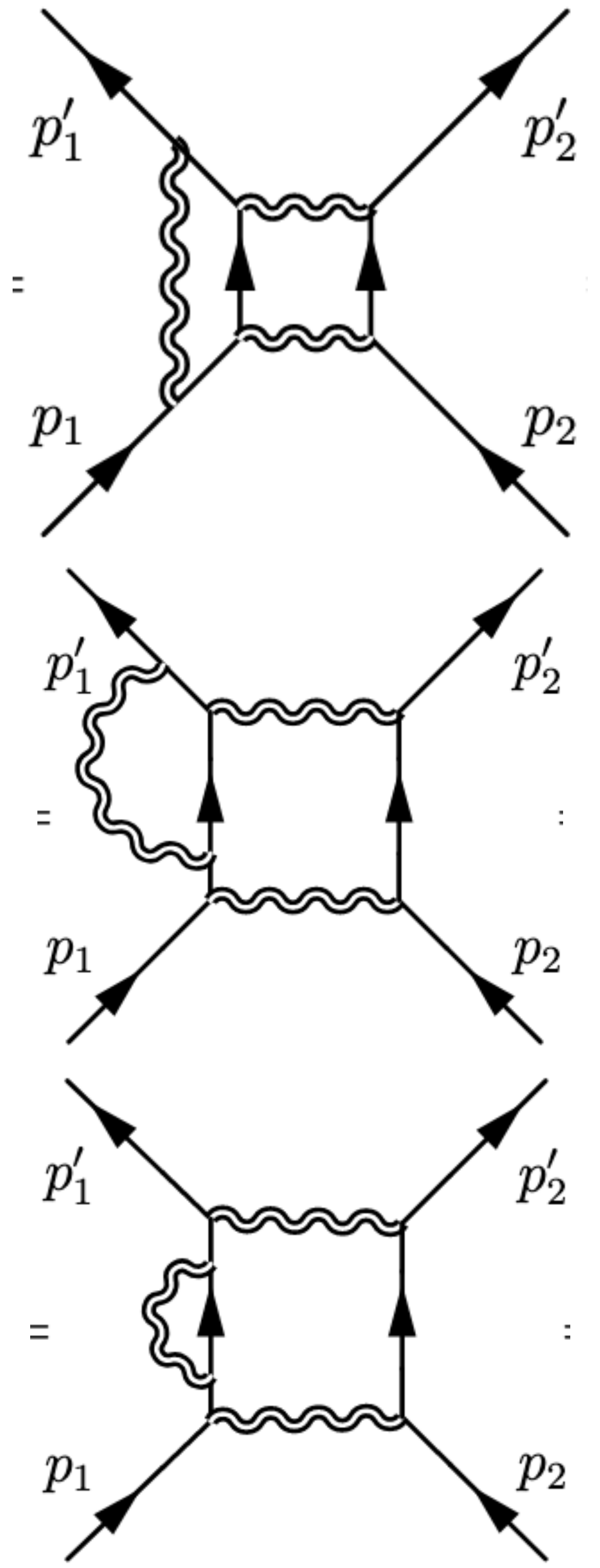

Einstein gravity at two-loop order

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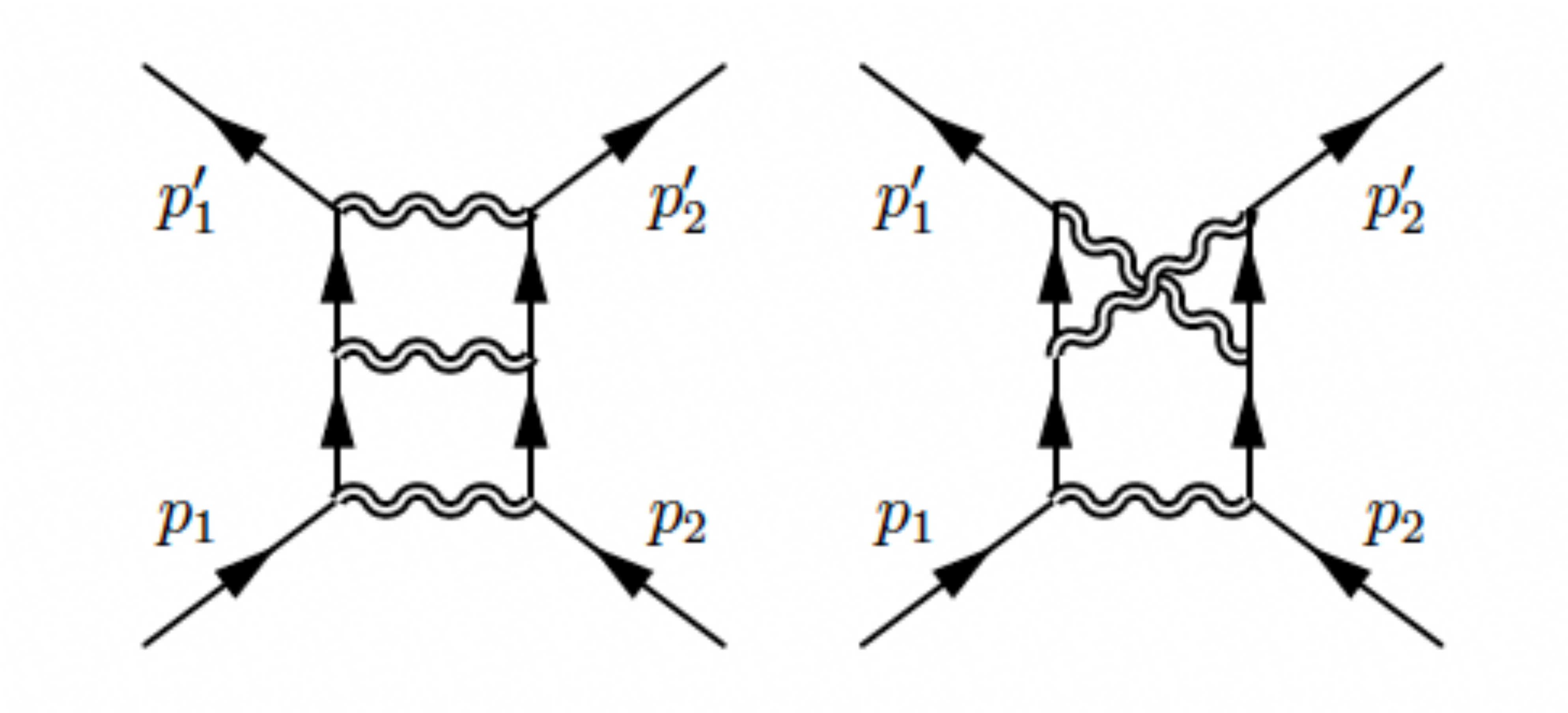

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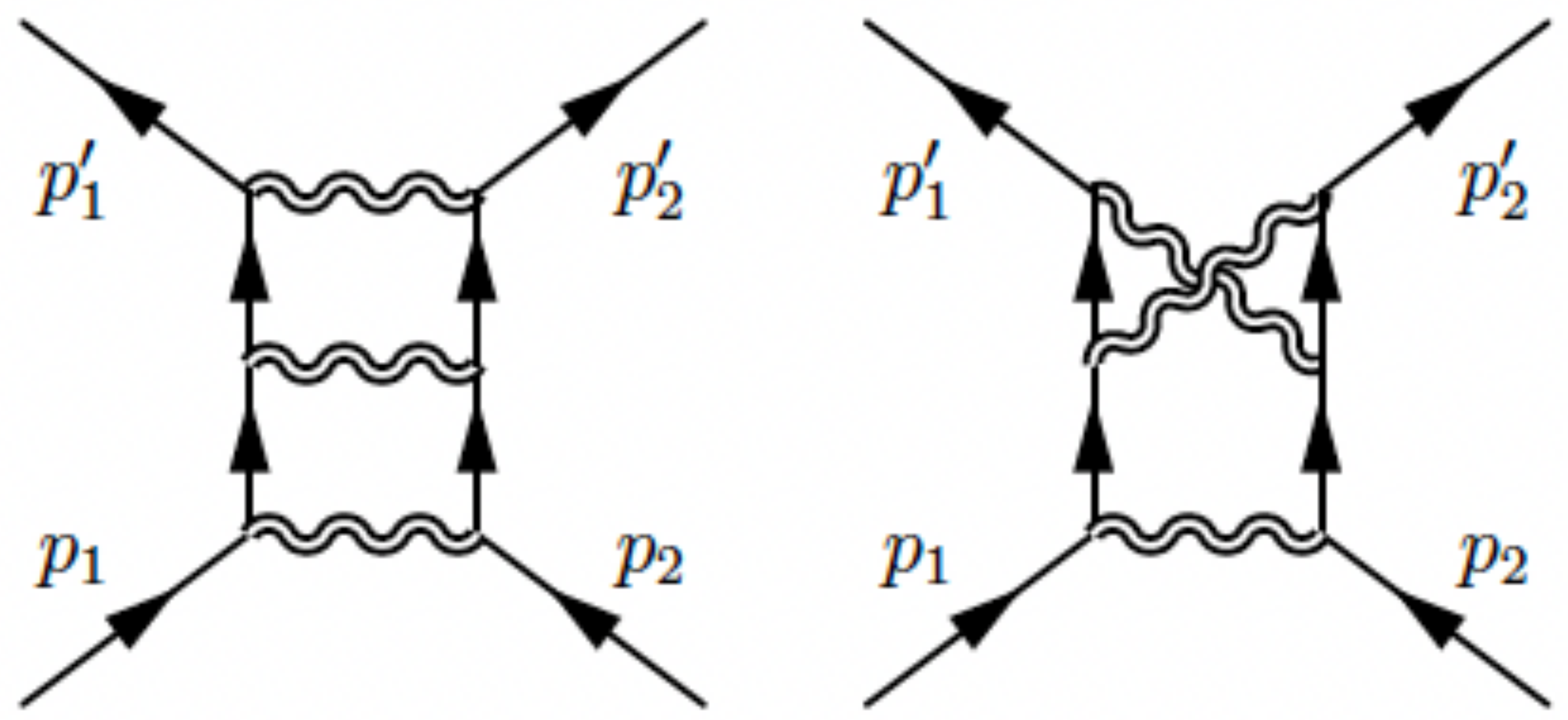

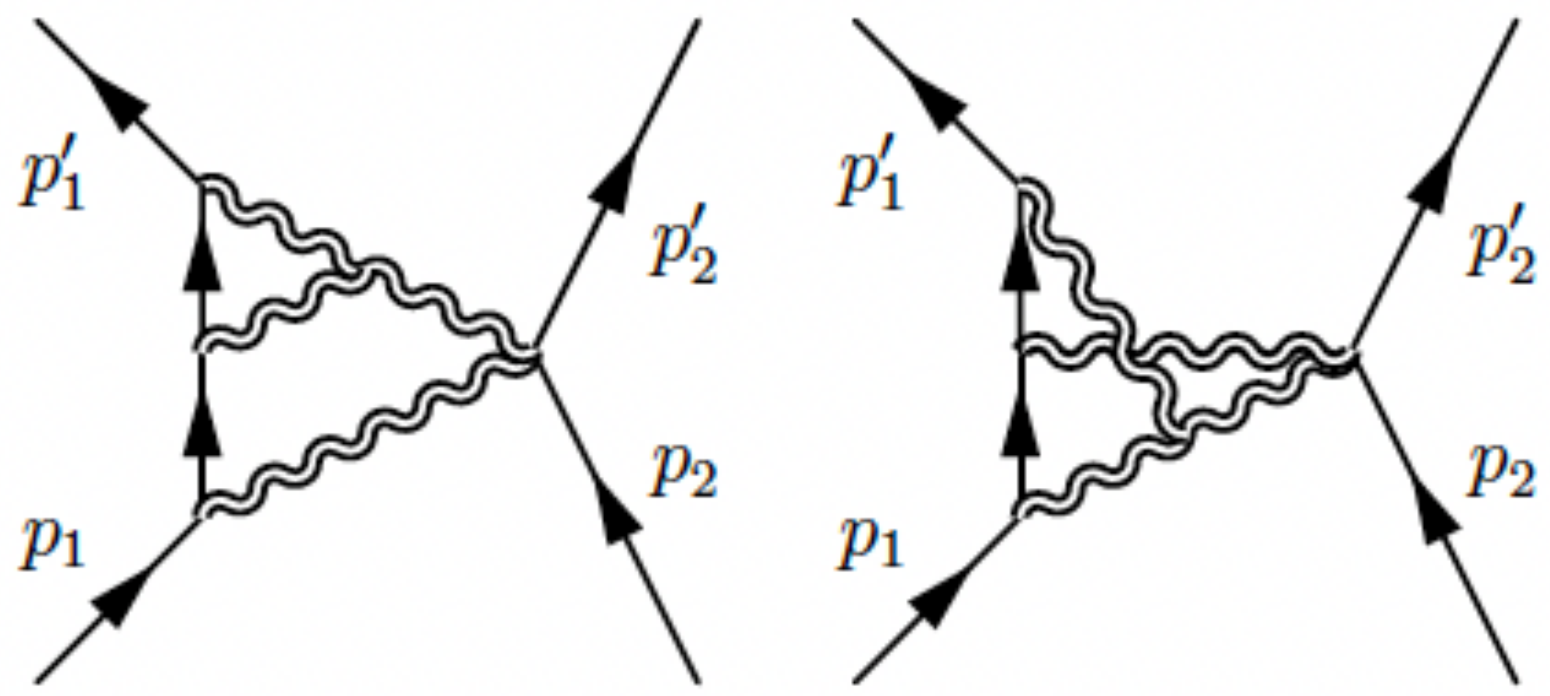

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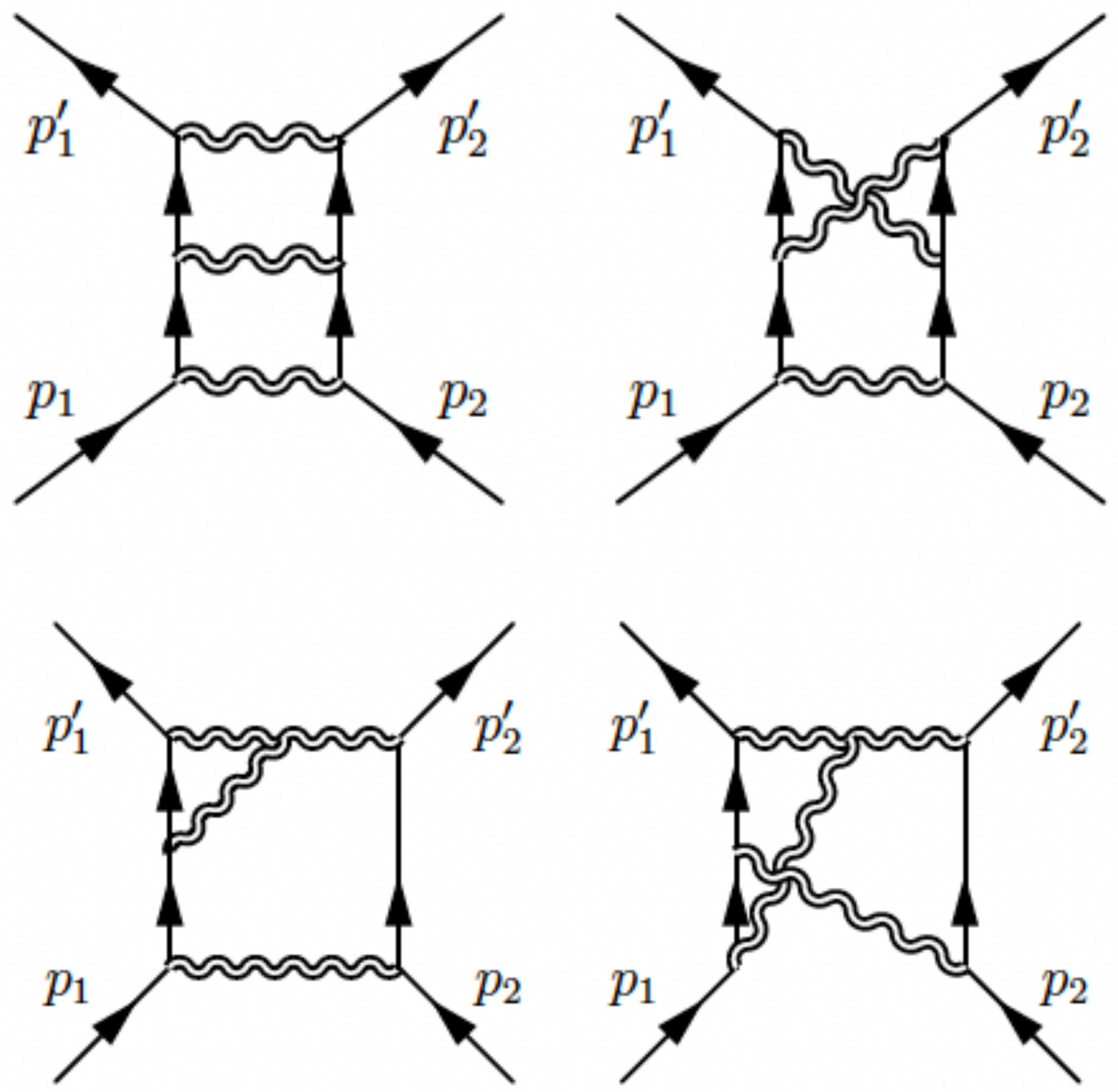

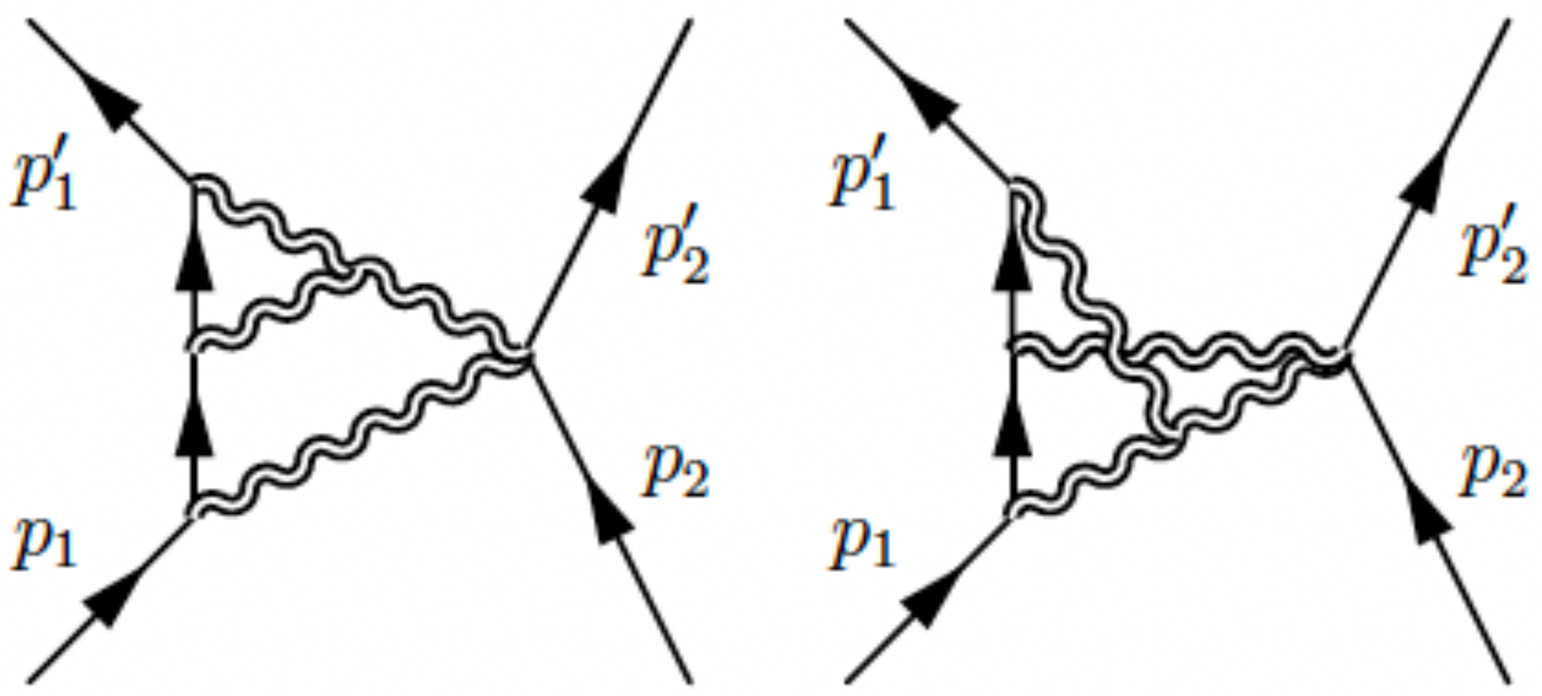

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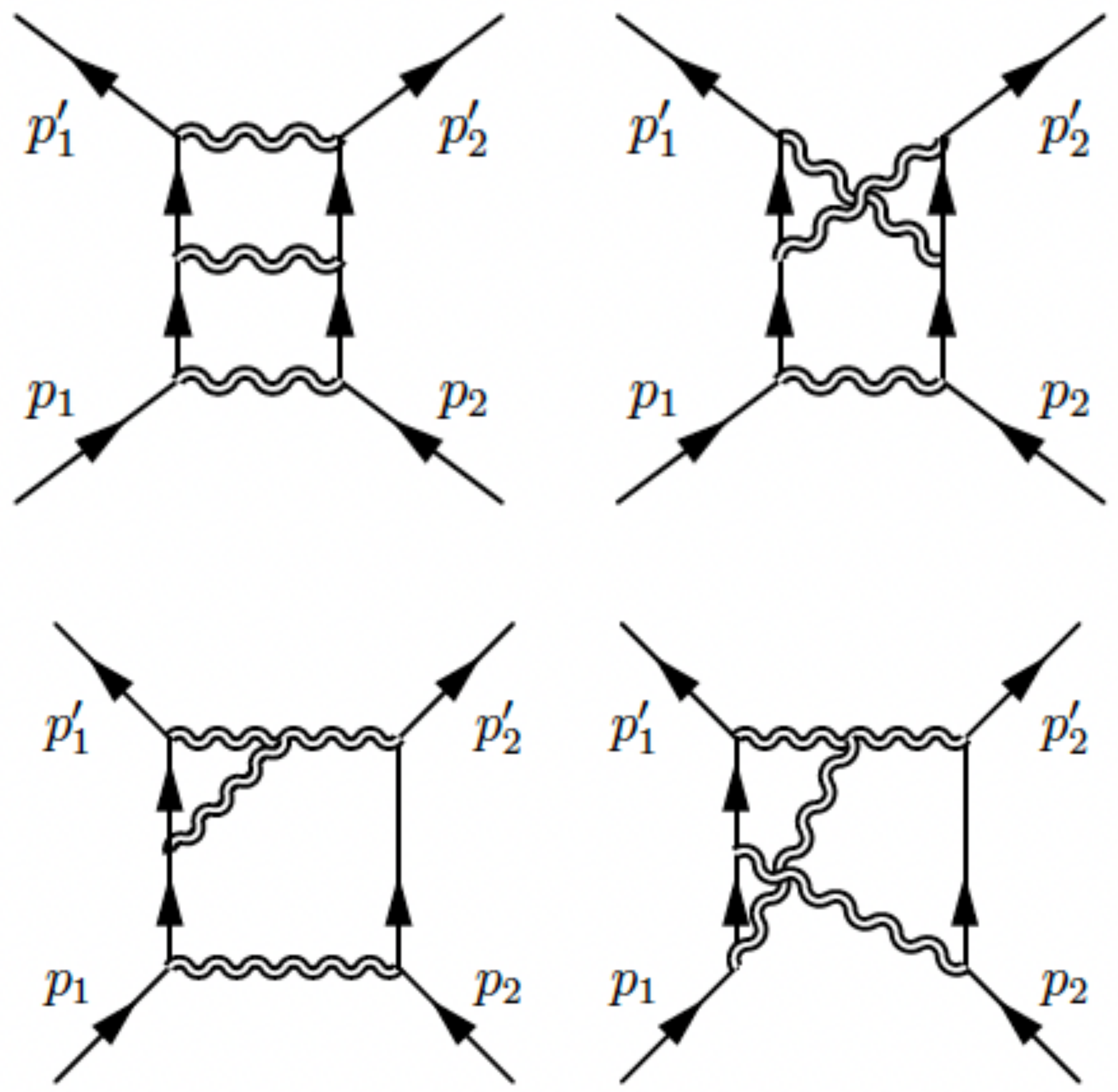

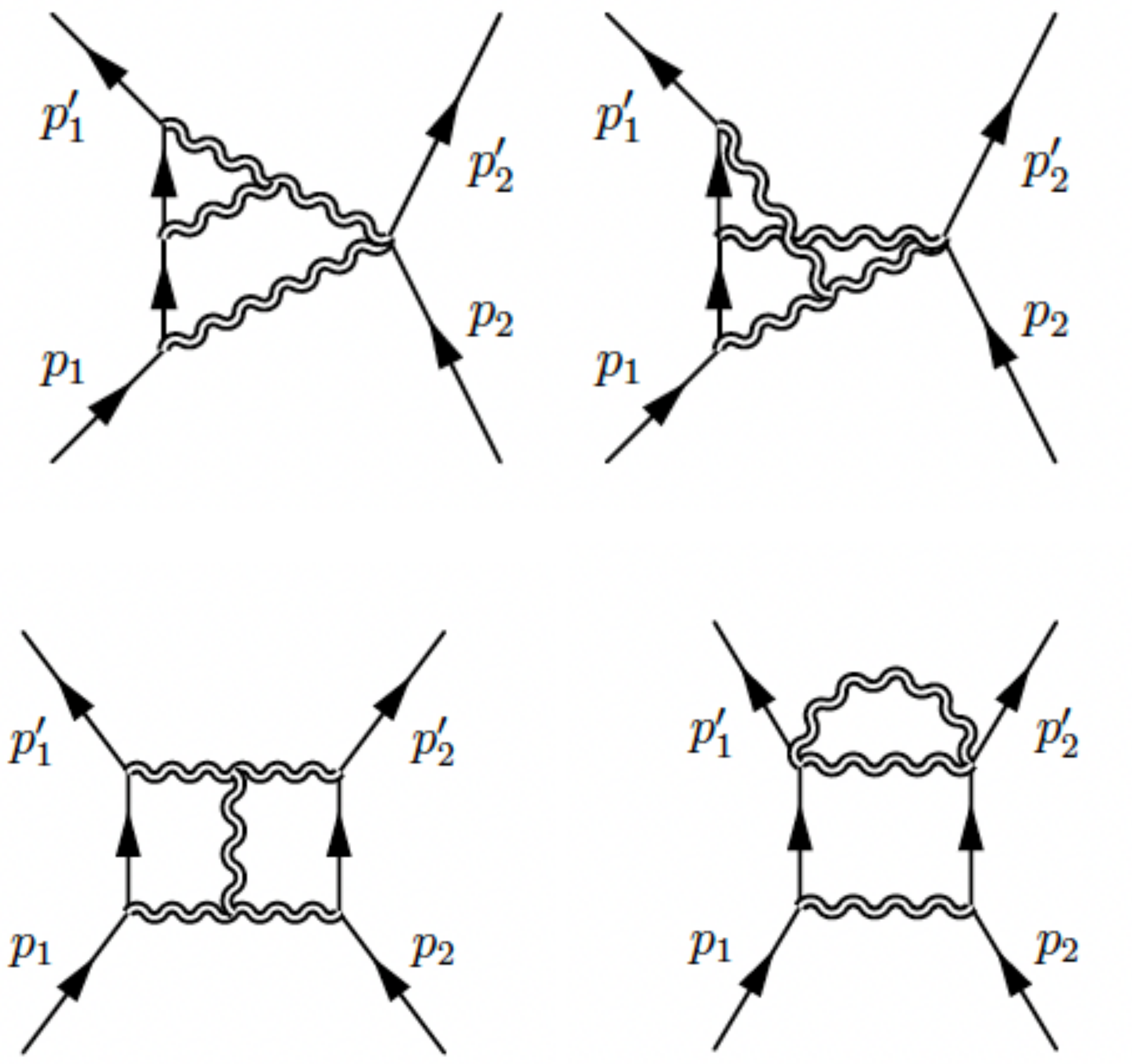

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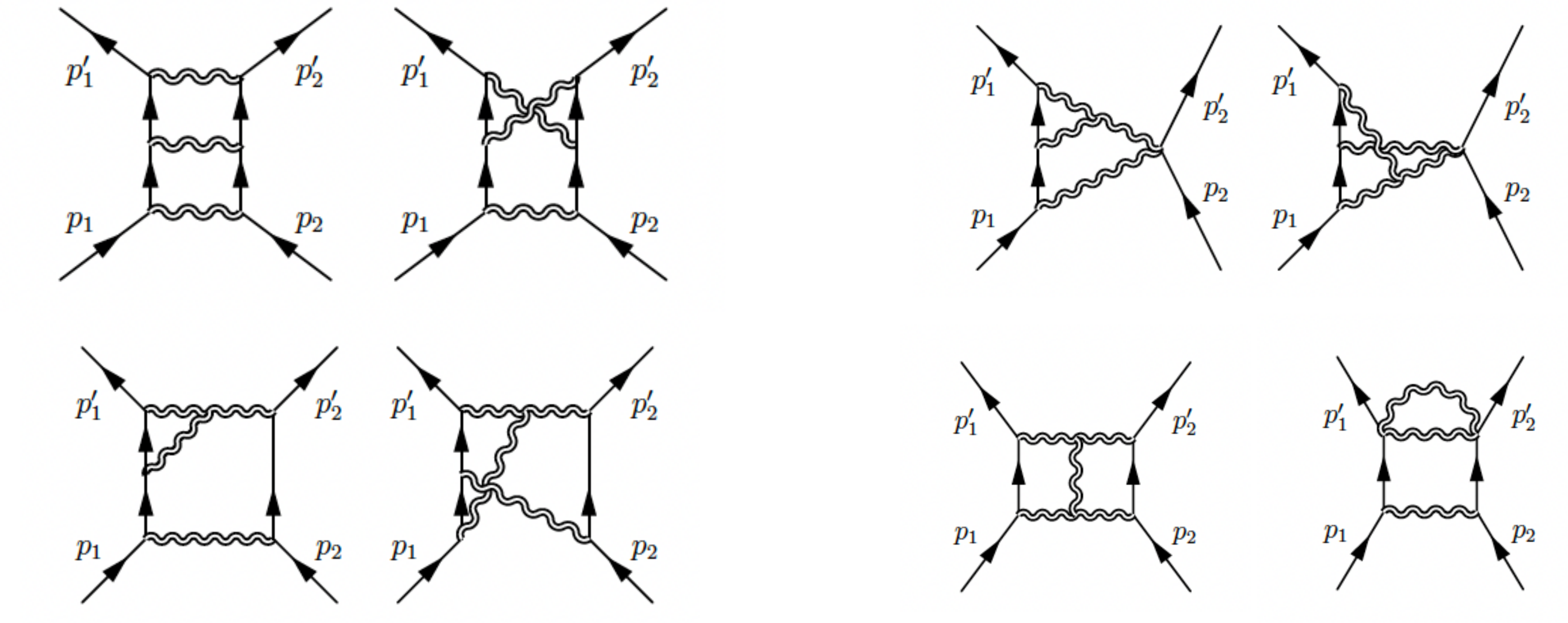

Needed master integrals at two-loops for the conservative part of the amplitude - determined by LiteRed/FIRE6/KIRA etc.

Einstein gravity at two-loop order

$$\mathcal{M}_2^{3-\mathrm{cut}(-1)}(\sigma,q^2) = \frac{2(4\pi e^{-\gamma_E})^{2\epsilon}\pi G_N^3 m_1^2 m_2^2}{3\epsilon|\underline{q}|^{4\epsilon}\hbar}\left(\frac{3s(2\sigma^2-1)^3}{(\sigma^2-1)^2}\right.$$
$$+\frac{im_1m_2(2\sigma^2-1)}{\pi\epsilon(\sigma^2-1)^{\frac{3}{2}}}\left(\frac{1-49\sigma^2+18\sigma^4}{5}-\frac{6\sigma(2\sigma^2-1)(6\sigma^2-7)\,\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)$$
$$-\frac{9(2\sigma^2-1)(1-5\sigma^2)s}{2(\sigma^2-1)}+\frac{3}{2}(m_1^2+m_2^2)(-1+18\sigma^2)-m_1m_2\sigma(103+2\sigma^2)$$
$$+\frac{12m_1m_2(3+12\sigma^2-4\sigma^4)\,\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}$$
$$\left.-\frac{6im_1m_2(2\sigma^2-1)^2}{\pi\epsilon\sqrt{\sigma^2-1}}\left(\frac{-1}{4(\sigma^2-1)}\right)^{\epsilon}\frac{d}{d\sigma}\left(\frac{(2\sigma^2-1)\,\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)\right).$$

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$$
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&+ \frac{im_1m_2(2\sigma^2-1)}{\pi\epsilon(\sigma^2-1)^{\frac{3}{2}}}\left(\frac{1-49\sigma^2+18\sigma^4}{5} - \frac{6\sigma(2\sigma^2-1)(6\sigma^2-7)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right) \\
&- \frac{9(2\sigma^2-1)(1-5\sigma^2)s}{2(\sigma^2-1)} + \frac{3}{2}(m_1^2+m_2^2)(-1+18\sigma^2) - m_1m_2\sigma(103+2\sigma^2) \\
&+ \frac{12m_1m_2(3+12\sigma^2-4\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} \\
&- \frac{6im_1m_2(2\sigma^2-1)^2}{\pi\epsilon\sqrt{\sigma^2-1}}\left(\frac{-1}{4(\sigma^2-1)}\right)^{\epsilon}\frac{d}{d\sigma}\left(\frac{(2\sigma^2-1)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)\Bigg).
\end{aligned}
$$

Einstein gravity at two-loop order

$$\mathcal{M}_2^{3-\mathrm{cut}(-1)}(\sigma,q^2) = \frac{2(4\pi e^{-\gamma_E})^{2\epsilon}\pi G_N^3 m_1^2 m_2^2}{3\epsilon|\underline{q}|^{4\epsilon}\hbar}\left(\frac{3s(2\sigma^2-1)^3}{(\sigma^2-1)^2}\right.$$
$$+\frac{i m_1 m_2(2\sigma^2-1)}{\pi\epsilon(\sigma^2-1)^{\frac{3}{2}}}\left(\frac{1-49\sigma^2+18\sigma^4}{5}-\frac{6\sigma(2\sigma^2-1)(6\sigma^2-7)\,\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)$$
$$-\frac{9(2\sigma^2-1)(1-5\sigma^2)s}{2(\sigma^2-1)}+\frac{3}{2}(m_1^2+m_2^2)(-1+18\sigma^2)-m_1m_2\sigma(103+2\sigma^2)$$
$$+\frac{12m_1m_2(3+12\sigma^2-4\sigma^4)\,\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}$$
$$\left.-\frac{6im_1m_2(2\sigma^2-1)^2}{\pi\epsilon\sqrt{\sigma^2-1}}\left(\frac{-1}{4(\sigma^2-1)}\right)^{\epsilon}\frac{d}{d\sigma}\left(\frac{(2\sigma^2-1)\,\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)\right).$$

Imaginary

Gravity amplitude in powers of hbar

Gravity amplitude in powers of hbar

$$\mathcal{M}_2(\sigma, |\underline{q}|) = \frac{1}{|\underline{q}|^{4\epsilon}} \left(\mathcal{M}_2^{(-3)}(\sigma, |\underline{q}|) + \mathcal{M}_2^{(-2)}(\sigma, |\underline{q}|) + \mathcal{M}_2^{(-1)}(\sigma, |\underline{q}|) + \mathcal{O}(\hbar^0) \right)$$

Gravity amplitude in powers of hbar

$$\mathcal{M}_2(\sigma, |\underline{q}|) = \frac{1}{|\underline{q}|^{4\epsilon}} \left(\mathcal{M}_2^{(-3)}(\sigma, |\underline{q}|) + \mathcal{M}_2^{(-2)}(\sigma, |\underline{q}|) + \mathcal{M}_2^{(-1)}(\sigma, |\underline{q}|) + \mathcal{O}(\hbar^0) \right)$$

$$\mathcal{M}_2^{(-3)}(\sigma, |\underline{q}|) = -\frac{8\pi G_N^3 m_1^4 m_2^4 (2\sigma^2 - 1)^3 \Gamma(-\epsilon)^3 \Gamma(1 + 2\epsilon)}{3\hbar^3 |\underline{q}|^2 (\sigma^2 - 1)(4\pi)^{-2\epsilon} \Gamma(-3\epsilon)}$$

Gravity amplitude in powers of hbar

$$\mathcal{M}_2(\sigma, |\underline{q}|) = \frac{1}{|\underline{q}|^{4\epsilon}} \left(\mathcal{M}_2^{(-3)}(\sigma, |\underline{q}|) + \mathcal{M}_2^{(-2)}(\sigma, |\underline{q}|) + \mathcal{M}_2^{(-1)}(\sigma, |\underline{q}|) + \mathcal{O}(\hbar^0) \right)$$

$$\mathcal{M}_2^{(-3)}(\sigma, |\underline{q}|) = -\frac{8\pi G_N^3 m_1^4 m_2^4 (2\sigma^2 - 1)^3 \Gamma(-\epsilon)^3 \Gamma(1 + 2\epsilon)}{3\hbar^3 |\underline{q}|^2 (\sigma^2 - 1)(4\pi)^{-2\epsilon} \Gamma(-3\epsilon)}$$

$$\mathcal{M}_2^{(-2)}(\sigma, |\underline{q}|) = \frac{6i\pi^2 G_N^3 (m_1 + m_2) m_1^3 m_2^3 (2\sigma^2 - 1)(1 - 5\sigma^2)(4\pi e^{-\gamma_E})^{2\epsilon}}{\epsilon\sqrt{\sigma^2 - 1}\hbar^2 |\underline{q}|} + \mathcal{O}(\epsilon^0)$$

Gravity amplitude in powers of hbar

$$\mathcal{M}_2(\sigma,|\underline{q}|) = \frac{1}{|\underline{q}|^{4\epsilon}}\left(\mathcal{M}_2^{(-3)}(\sigma,|\underline{q}|) + \mathcal{M}_2^{(-2)}(\sigma,|\underline{q}|) + \mathcal{M}_2^{(-1)}(\sigma,|\underline{q}|) + \mathcal{O}(\hbar^0)\right)$$

$$\mathcal{M}_2^{(-3)}(\sigma,|\underline{q}|) = -\frac{8\pi G_N^3 m_1^4 m_2^4 (2\sigma^2-1)^3 \Gamma(-\epsilon)^3 \Gamma(1+2\epsilon)}{3\hbar^3 |\underline{q}|^2 (\sigma^2-1)(4\pi)^{-2\epsilon}\Gamma(-3\epsilon)}$$

$$\mathcal{M}_2^{(-2)}(\sigma,|\underline{q}|) = \frac{6i\pi^2 G_N^3 (m_1+m_2) m_1^3 m_2^3 (2\sigma^2-1)(1-5\sigma^2)(4\pi e^{-\gamma_E})^{2\epsilon}}{\epsilon\sqrt{\sigma^2-1}\,\hbar^2|\underline{q}|} + \mathcal{O}(\epsilon^0)$$

$$\mathcal{M}_2^{(-1)}(\sigma,|\underline{q}|) = \frac{2\pi G_N^3 (4\pi e^{-\gamma_E})^{2\epsilon} m_1^2 m_2^2}{\hbar\epsilon}\Bigg(\frac{s(2\sigma^2-1)^3}{(\sigma^2-1)^2}$$

$$+\frac{i m_1 m_2 (2\sigma^2-1)}{\pi\epsilon(\sigma^2-1)^{\frac{3}{2}}}\left(\frac{1-49\sigma^2+18\sigma^4}{15} - \frac{2\sigma(7-20\sigma^2+12\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)$$

$$-\frac{3(2\sigma^2-1)(1-5\sigma^2)s}{2(\sigma^2-1)} + \frac{1}{2}(m_1^2+m_2^2)(18\sigma^2-1) - \frac{1}{3}m_1 m_2 \sigma(103+2\sigma^2)$$

$$+\frac{4m_1m_2(3+12\sigma^2-4\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}$$

$$-\frac{2i m_1 m_2 (2\sigma^2-1)^2}{\pi\epsilon\sqrt{\sigma^2-1}}\left(\frac{-1}{4(\sigma^2-1)}\right)^{\epsilon}\left(-\frac{11}{3} + \frac{d}{d\sigma}\left(\frac{(2\sigma^2-1)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)\right)\Bigg).$$

Gravity amplitude in powers of hbar

$$\mathcal{M}_2(\sigma,|\underline{q}|)=\frac{1}{|\underline{q}|^{4\epsilon}}\left(\mathcal{M}_2^{(-3)}(\sigma,|\underline{q}|)+\mathcal{M}_2^{(-2)}(\sigma,|\underline{q}|)+\mathcal{M}_2^{(-1)}(\sigma,|\underline{q}|)+\mathcal{O}(\hbar^0)\right)$$

$$\mathcal{M}_2^{(-3)}(\sigma,|\underline{q}|)=-\frac{8\pi G_N^3 m_1^4 m_2^4(2\sigma^2-1)^3\Gamma(-\epsilon)^3\Gamma(1+2\epsilon)}{3\hbar^3|\underline{q}|^2(\sigma^2-1)(4\pi)^{-2\epsilon}\Gamma(-3\epsilon)}$$

$$\mathcal{M}_2^{(-2)}(\sigma,|\underline{q}|)=\frac{6i\pi^2 G_N^3(m_1+m_2)m_1^3m_2^3(2\sigma^2-1)(1-5\sigma^2)(4\pi e^{-\gamma_E})^{2\epsilon}}{\epsilon\sqrt{\sigma^2-1}\hbar^2|\underline{q}|}+\mathcal{O}(\epsilon^0)$$

$$\begin{aligned}\mathcal{M}_2^{(-1)}(\sigma,|\underline{q}|)=\frac{2\pi G_N^3(4\pi e^{-\gamma_E})^{2\epsilon}m_1^2m_2^2}{\hbar\epsilon}\Bigg(&\frac{s(2\sigma^2-1)^3}{(\sigma^2-1)^2}\\&+\frac{im_1m_2(2\sigma^2-1)}{\pi\epsilon(\sigma^2-1)^{\frac{3}{2}}}\left(\frac{1-49\sigma^2+18\sigma^4}{15}-\frac{2\sigma(7-20\sigma^2+12\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)\\&-\frac{3(2\sigma^2-1)(1-5\sigma^2)s}{2(\sigma^2-1)}+\frac{1}{2}(m_1^2+m_2^2)(18\sigma^2-1)-\frac{1}{3}m_1m_2\sigma(103+2\sigma^2)\\&+\frac{4m_1m_2(3+12\sigma^2-4\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\\&-\frac{2im_1m_2(2\sigma^2-1)^2}{\pi\epsilon\sqrt{\sigma^2-1}}\left(\frac{-1}{4(\sigma^2-1)}\right)^{\epsilon}\left(-\frac{11}{3}+\frac{d}{d\sigma}\left(\frac{(2\sigma^2-1)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)\right)\Bigg).\end{aligned}$$

Laurant expansion in Planck's constant

- imaginary contribution cancelled by radiative contributions

Gravity amplitude in powers of hbar

$$\mathcal{M}_2(\sigma,|\underline{q}|) = \frac{1}{|\underline{q}|^{4\epsilon}}\left(\mathcal{M}_2^{(-3)}(\sigma,|\underline{q}|) + \mathcal{M}_2^{(-2)}(\sigma,|\underline{q}|) + \mathcal{M}_2^{(-1)}(\sigma,|\underline{q}|) + \mathcal{O}(\hbar^0)\right)$$

$$\mathcal{M}_2^{(-3)}(\sigma,|\underline{q}|) = -\frac{8\pi G_N^3 m_1^4 m_2^4 (2\sigma^2-1)^3 \Gamma(-\epsilon)^3 \Gamma(1+2\epsilon)}{3\hbar^3 |\underline{q}|^2 (\sigma^2-1)(4\pi)^{-2\epsilon}\Gamma(-3\epsilon)}$$

$$\mathcal{M}_2^{(-2)}(\sigma,|\underline{q}|) = \frac{6i\pi^2 G_N^3 (m_1+m_2) m_1^3 m_2^3 (2\sigma^2-1)(1-5\sigma^2)(4\pi e^{-\gamma_E})^{2\epsilon}}{\epsilon\sqrt{\sigma^2-1}\,\hbar^2 |\underline{q}|} + \mathcal{O}(\epsilon^0)$$

$$\mathcal{M}_2^{(-1)}(\sigma,|\underline{q}|) = \frac{2\pi G_N^3 (4\pi e^{-\gamma_E})^{2\epsilon} m_1^2 m_2^2}{\hbar\epsilon}\Bigg(\frac{s(2\sigma^2-1)^3}{(\sigma^2-1)^2} + \frac{i m_1 m_2 (2\sigma^2-1)}{\pi\epsilon(\sigma^2-1)^{\frac{3}{2}}}\left(\frac{1-49\sigma^2+18\sigma^4}{15} - \frac{2\sigma(7-20\sigma^2+12\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)$$
$$- \frac{3(2\sigma^2-1)(1-5\sigma^2)s}{2(\sigma^2-1)} + \frac{1}{2}(m_1^2+m_2^2)(18\sigma^2-1) - \frac{1}{3}m_1 m_2 \sigma(103+2\sigma^2) + \frac{4m_1 m_2(3+12\sigma^2-4\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}$$
$$- \frac{2i m_1 m_2 (2\sigma^2-1)^2}{\pi\epsilon\sqrt{\sigma^2-1}}\left(\frac{-1}{4(\sigma^2-1)}\right)^{\epsilon}\left(-\frac{11}{3} + \frac{d}{d\sigma}\left(\frac{(2\sigma^2-1)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)\right)\Bigg).$$

Laurant expansion in Planck's constant
- imaginary contribution cancelled by radiative contributions

(Di Vecchia, Heissenberg, Russo, Veneziano)

Gravity amplitude in powers of hbar

$$\mathcal{M}_2(\sigma,|\underline{q}|) = \frac{1}{|\underline{q}|^{4\epsilon}}\left(\mathcal{M}_2^{(-3)}(\sigma,|\underline{q}|) + \mathcal{M}_2^{(-2)}(\sigma,|\underline{q}|) + \mathcal{M}_2^{(-1)}(\sigma,|\underline{q}|) + \mathcal{O}(\hbar^0)\right)$$

$$\mathcal{M}_2^{(-3)}(\sigma,|\underline{q}|) = -\frac{8\pi G_N^3 m_1^4 m_2^4 (2\sigma^2-1)^3 \Gamma(-\epsilon)^3 \Gamma(1+2\epsilon)}{3\hbar^3 |\underline{q}|^2 (\sigma^2-1)(4\pi)^{-2\epsilon}\Gamma(-3\epsilon)}$$

(Bern et al, Parra-Martinez et al)

$$\mathcal{M}_2^{(-2)}(\sigma,|\underline{q}|) = \frac{6i\pi^2 G_N^3 (m_1+m_2) m_1^3 m_2^3 (2\sigma^2-1)(1-5\sigma^2)(4\pi e^{-\gamma_E})^{2\epsilon}}{\epsilon\sqrt{\sigma^2-1}\,\hbar^2 |\underline{q}|} + \mathcal{O}(\epsilon^0)$$

$$\begin{aligned}\mathcal{M}_2^{(-1)}(\sigma,|\underline{q}|) = \frac{2\pi G_N^3 (4\pi e^{-\gamma_E})^{2\epsilon} m_1^2 m_2^2}{\hbar\epsilon}\Bigg(&\frac{s(2\sigma^2-1)^3}{(\sigma^2-1)^2} \\ &+ \frac{i m_1 m_2 (2\sigma^2-1)}{\pi\epsilon(\sigma^2-1)^{\frac{3}{2}}}\left(\frac{1-49\sigma^2+18\sigma^4}{15} - \frac{2\sigma(7-20\sigma^2+12\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right) \\ &- \frac{3(2\sigma^2-1)(1-5\sigma^2)s}{2(\sigma^2-1)} + \frac{1}{2}(m_1^2+m_2^2)(18\sigma^2-1) - \frac{1}{3}m_1 m_2 \sigma(103+2\sigma^2) \\ &+ \frac{4m_1 m_2 (3+12\sigma^2-4\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} \\ &- \frac{2i m_1 m_2 (2\sigma^2-1)^2}{\pi\epsilon\sqrt{\sigma^2-1}}\left(\frac{-1}{4(\sigma^2-1)}\right)^{\epsilon}\left(-\frac{11}{3} + \frac{d}{d\sigma}\left(\frac{(2\sigma^2-1)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\right)\right)\Bigg).\end{aligned}$$

Laurant expansion in Planck's constant

- imaginary contribution cancelled by radiative contributions

(Di Vecchia, Heissenberg, Russo, Veneziano)

Gravity amplitude in b-space have iterative structure

Gravity amplitude in b-space have iterative structure

$$\widetilde{\mathcal{M}}_2(\sigma, b) = \frac{1}{4E_{\rm c.m.}P}\int_{\mathbb{R}^{D-2}} \frac{d^{D-2}\underline{\vec{q}}}{(2\pi)^{D-2}}\mathcal{M}_2(p_1, p_2, p_1', p_2')e^{i\underline{\vec{q}}\cdot\vec{b}}$$

Gravity amplitude in b-space have iterative structure

$$\widetilde{\mathcal{M}}_2(\sigma, b) = \frac{1}{4E_{\text{c.m.}}P}\int_{\mathbb{R}^{D-2}} \frac{d^{D-2}\vec{\underline{q}}}{(2\pi)^{D-2}} \mathcal{M}_2(p_1, p_2, p_1', p_2') e^{i\vec{\underline{q}}\cdot\vec{b}}$$

$$\widetilde{\mathcal{M}}_2(\sigma, b) = -\frac{1}{6}\left(\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\right)^3 + i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\left(\widetilde{\mathcal{M}}_1^{\text{Cl.}}(\sigma, b) + \widetilde{\mathcal{M}}_1^{\text{Qt.}}(\sigma, b)\right) + \widetilde{\mathcal{M}}_2^{\text{Cl.}}(\sigma, b) + \mathcal{O}(\hbar^0).$$

Gravity amplitude in b-space have iterative structure

$$\widetilde{\mathcal{M}}_2(\sigma, b) = \frac{1}{4E_{\text{c.m.}}P}\int_{\mathbb{R}^{D-2}} \frac{d^{D-2}\vec{\underline{q}}}{(2\pi)^{D-2}} \mathcal{M}_2(p_1, p_2, p_1', p_2') e^{i\vec{\underline{q}}\cdot\vec{b}}$$

$$\widetilde{\mathcal{M}}_2(\sigma, b) = -\frac{1}{6}\left(\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\right)^3 + i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\left(\widetilde{\mathcal{M}}_1^{\text{Cl.}}(\sigma, b) + \widetilde{\mathcal{M}}_1^{\text{Qt.}}(\sigma, b)\right) + \widetilde{\mathcal{M}}_2^{\text{Cl.}}(\sigma, b) + \mathcal{O}(\hbar^0).$$

$$\widetilde{\mathcal{M}}_2^{\Box\Box(-3)}(\sigma, b) = -\frac{1}{6}\left(\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\right)^3,$$

$$\widetilde{\mathcal{M}}_2^{\Box\Box(-2)}(\sigma, b) = i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\widetilde{\mathcal{M}}_1^{\Box(-1)}(\sigma, b),$$

$$\widetilde{\mathcal{M}}_2^{\lhd\Box(-2)}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\Box\rhd(-2)}(\sigma, b) = i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\left(\widetilde{\mathcal{M}}_1^{\lhd(-1)}(\sigma, b) + \widetilde{\mathcal{M}}_1^{\rhd(-1)}(\sigma, b)\right)$$

$$\widetilde{\mathcal{M}}_2^{\Box\Box(-1)}(\sigma, b) = i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\widetilde{\mathcal{M}}_1^{\Box(0)}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\Box\Box\ \text{Cl.}}(\sigma, b),$$

$$\widetilde{\mathcal{M}}_2^{\lhd\Box(-1)}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\Box\rhd(-1)}(\sigma, b) = i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\left(\widetilde{\mathcal{M}}_1^{\lhd(0)}(\sigma, b) + \widetilde{\mathcal{M}}_1^{\rhd(0)}(\sigma, b)\right) + \widetilde{\mathcal{M}}_2^{\lhd\Box\ \text{Cl.}}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\Box\rhd\ \text{Cl.}}(\sigma, b),$$

$$\widetilde{\mathcal{M}}_2^{\Box\circ(-1)}(\sigma, b) = i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma, b)\widetilde{\mathcal{M}}_1^{\circ(0)}(\sigma, b) + \widetilde{\mathcal{M}}_2^{\Box\circ\ \text{Cl.}}(\sigma, b),$$

Gravity amplitude in b-space have iterative structure

$$\widetilde{\mathcal{M}}_2(\sigma,b)=\frac{1}{4E_{\text{c.m.}}P}\int_{\mathbb{R}^{D-2}}\frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}}\mathcal{M}_2(p_1,p_2,p_1',p_2')e^{i\vec{q}\cdot\vec{b}}$$

$$\widetilde{\mathcal{M}}_2(\sigma,b)=-\frac{1}{6}\left(\widetilde{\mathcal{M}}_0^{(-1)}(\sigma,b)\right)^3+i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma,b)\left(\widetilde{\mathcal{M}}_1^{\text{Cl.}}(\sigma,b)+\widetilde{\mathcal{M}}_1^{\text{Qt.}}(\sigma,b)\right)+\widetilde{\mathcal{M}}_2^{\text{Cl.}}(\sigma,b)+\mathcal{O}(\hbar^0).$$

$$\widetilde{\mathcal{M}}_2^{\square\square(-3)}(\sigma,b)=-\frac{1}{6}\left(\widetilde{\mathcal{M}}_0^{(-1)}(\sigma,b)\right)^3,$$

$$\widetilde{\mathcal{M}}_2^{\square\square(-2)}(\sigma,b)=i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma,b)\widetilde{\mathcal{M}}_1^{\square(-1)}(\sigma,b),$$

$$\widetilde{\mathcal{M}}_2^{\triangleleft\square(-2)}(\sigma,b)+\widetilde{\mathcal{M}}_2^{\square\triangleright(-2)}(\sigma,b)=i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma,b)\left(\widetilde{\mathcal{M}}_1^{\triangleleft(-1)}(\sigma,b)+\widetilde{\mathcal{M}}_1^{\triangleright(-1)}(\sigma,b)\right)$$

$$\widetilde{\mathcal{M}}_2^{\square\square(-1)}(\sigma,b)=i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma,b)\widetilde{\mathcal{M}}_1^{\square(0)}(\sigma,b)+\widetilde{\mathcal{M}}_2^{\square\square\ \text{Cl.}}(\sigma,b),$$

$$\widetilde{\mathcal{M}}_2^{\triangleleft\square(-1)}(\sigma,b)+\widetilde{\mathcal{M}}_2^{\square\triangleright(-1)}(\sigma,b)=i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma,b)\left(\widetilde{\mathcal{M}}_1^{\triangleleft(0)}(\sigma,b)+\widetilde{\mathcal{M}}_1^{\triangleright(0)}(\sigma,b)\right)+\widetilde{\mathcal{M}}_2^{\triangleleft\square\ \text{Cl.}}(\sigma,b)+\widetilde{\mathcal{M}}_2^{\square\triangleright\ \text{Cl.}}(\sigma,b),$$

$$\widetilde{\mathcal{M}}_2^{\square\circ(-1)}(\sigma,b)=i\widetilde{\mathcal{M}}_0^{(-1)}(\sigma,b)\widetilde{\mathcal{M}}_1^{\circ(0)}(\sigma,b)+\widetilde{\mathcal{M}}_2^{\square\circ\ \text{Cl.}}(\sigma,b),$$

Again iterative structure like one-loop, part of a bigger scheme..Seen after Fourier transform to b space

Scattering angle from amplitudes

Scattering angle from amplitudes

$$1 + i \sum_{L \geq 0} \widetilde{\mathcal{M}}_L(\sigma, b) = (1 + 2i\Delta(\sigma, b)) \exp\left(\frac{2i}{\hbar} \sum_{L \geq 0} \delta_L(\sigma, b)\right)$$

Scattering angle from amplitudes

$$1 + i\sum_{L\geq 0}\widetilde{\mathcal{M}}_L(\sigma, b) = (1 + 2i\Delta(\sigma, b))\exp\left(\frac{2i}{\hbar}\sum_{L\geq 0}\delta_L(\sigma, b)\right)$$

Gravity eikonal

Scattering angle from amplitudes

$$1 + i \sum_{L \geq 0} \widetilde{\mathcal{M}}_L(\sigma, b) = (1 + 2i\Delta(\sigma, b)) \exp\left(\frac{2i}{\hbar} \sum_{L \geq 0} \delta_L(\sigma, b) \right)$$

Gravity eikonal

$$\delta_0(\sigma, b) = -\frac{G_N m_1 m_2 (2\sigma^2 - 1)}{2\epsilon\sqrt{\sigma^2 - 1}} (\pi b^2 e^{\gamma_E})^\epsilon + \mathcal{O}(\epsilon),$$

Scattering angle from amplitudes

$$1 + i \sum_{L \geq 0} \widetilde{\mathcal{M}}_L(\sigma, b) = (1 + 2i\Delta(\sigma, b)) \exp\left(\frac{2i}{\hbar} \sum_{L \geq 0} \delta_L(\sigma, b) \right)$$

Gravity eikonal

$$\delta_0(\sigma, b) = -\frac{G_N m_1 m_2 (2\sigma^2 - 1)}{2\epsilon\sqrt{\sigma^2 - 1}} (\pi b^2 e^{\gamma_E})^{\epsilon} + \mathcal{O}(\epsilon),$$

$$\delta_1(\sigma, b) = \frac{3\pi G_N^2 (m_1 + m_2) m_1 m_2 (5\sigma^2 - 1)}{8b\sqrt{\sigma^2 - 1}} (\pi b^2 e^{\gamma_E})^{2\epsilon}$$

Scattering angle from amplitudes

$$1 + i \sum_{L \geq 0} \widetilde{\mathcal{M}}_L(\sigma, b) = (1 + 2i\Delta(\sigma, b)) \exp\left(\frac{2i}{\hbar} \sum_{L \geq 0} \delta_L(\sigma, b)\right)$$

Gravity eikonal

$$2\Delta_1 = \widetilde{\mathcal{M}}_1^{\text{Qt.}}(\sigma, b)$$

$$\begin{aligned}\delta_2(\sigma, b) = \frac{G_N^3 m_1 m_2 (\pi b^2 e^{\gamma_E})^{3\epsilon}}{2b^2\sqrt{\sigma^2 - 1}} \Bigg(& \frac{2s(12\sigma^4 - 10\sigma^2 + 1)}{\sigma^2 - 1} \\ & - \frac{4m_1 m_2 \sigma}{3}(25 + 14\sigma^2) + \frac{4m_1 m_2 (3 + 12\sigma^2 - 4\sigma^4)\,\text{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}} \\ + \frac{2m_1 m_2 (2\sigma^2 - 1)^2}{\sqrt{\sigma^2 - 1}} & \frac{1}{(4(\sigma^2 - 1))^\epsilon}\left(-\frac{11}{3} + \frac{d}{d\sigma}\left(\frac{(2\sigma^2 - 1)\,\text{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}}\right)\right)\Bigg).\end{aligned}$$

Scattering angle from amplitudes

Scattering angle from amplitudes

$$\sin\left(\frac{\chi}{2}\right)\Big|_{3PM} = -\frac{\sqrt{s}}{m_1 m_2\sqrt{\sigma^2-1}}\frac{\partial\delta_2(\sigma,b)}{\partial b}$$

Scattering angle from amplitudes

$$\sin\left(\frac{\chi}{2}\right)\Big|_{3PM} = -\frac{\sqrt{s}}{m_1 m_2\sqrt{\sigma^2-1}}\frac{\partial\delta_2(\sigma,b)}{\partial b}$$

$$J = \frac{m_1 m_2\sqrt{\sigma^2-1}}{\sqrt{s}} b\cos\left(\frac{\chi}{2}\right)$$

Scattering angle from amplitudes

$$\sin\left(\frac{\chi}{2}\right)\Big|_{3PM} = -\frac{\sqrt{s}}{m_1 m_2\sqrt{\sigma^2-1}}\frac{\partial\delta_2(\sigma,b)}{\partial b}$$

$$J = \frac{m_1 m_2\sqrt{\sigma^2-1}}{\sqrt{s}}b\cos\left(\frac{\chi}{2}\right)$$

$$\chi_{1PM} = \frac{2G_N m_1 m_2(2\sigma^2-1)}{J\sqrt{\sigma^2-1}},$$

$$\chi_{2PM} = \frac{3\pi G_N^2 m_1^2 m_2^2(m_1+m_2)(5\sigma^2-1)}{4J^2\sqrt{s}};$$

Scattering angle from amplitudes

Scattering angle from amplitudes

$$\widehat{\chi}_{3PM} = \frac{2G_N^3 m_1^3 m_2^3 \left(64\sigma^6 - 120\sigma^4 + 60\sigma^2 - 5\right)}{3J^3 \left(\sigma^2 - 1\right)^{\frac{3}{2}}}$$
$$+ \frac{8G_N^3 m_1^4 m_2^4 \sqrt{\sigma^2 - 1}}{3J^3 s} \left(\sigma(-25 - 14\sigma^2) + \frac{3(3 + 12\sigma^2 - 4\sigma^4)\,\mathrm{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}} \right)$$

Scattering angle from amplitudes

$$\widehat{\chi}_{3PM} = \frac{2G_N^3 m_1^3 m_2^3 \left(64\sigma^6 - 120\sigma^4 + 60\sigma^2 - 5\right)}{3J^3 \left(\sigma^2 - 1\right)^{\frac{3}{2}}}$$

$$+ \frac{8G_N^3 m_1^4 m_2^4 \sqrt{\sigma^2 - 1}}{3J^3 s}\left(\sigma(-25 - 14\sigma^2) + \frac{3(3 + 12\sigma^2 - 4\sigma^4)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}}\right)$$

$$\chi_{3PM}^{\text{Rad.}} = \frac{4G_N^3 m_1^4 m_2^4 (2\sigma^2 - 1)^2}{J^3 s}\frac{1}{(4(\sigma^2 - 1))^{\epsilon}}\left(-\frac{11}{3} + \frac{d}{d\sigma}\left(\frac{(2\sigma^2 - 1)\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}}\right)\right)$$

Scattering angle from amplitudes

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Match with expectations

(Bern at al, Damour; Di Vecchia et al; Hermann et al)

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What is nice to see is the fact that everything matches up!

- the cancellation of terms that is demonstrated explicitly gives important consistency of computations. Quantum terms are important for getting to get the correct eikonal exponentiation

(NEJB, Damgaard, Plante, Vanhove)

Simpler integrand - velocity cuts tree topologies!

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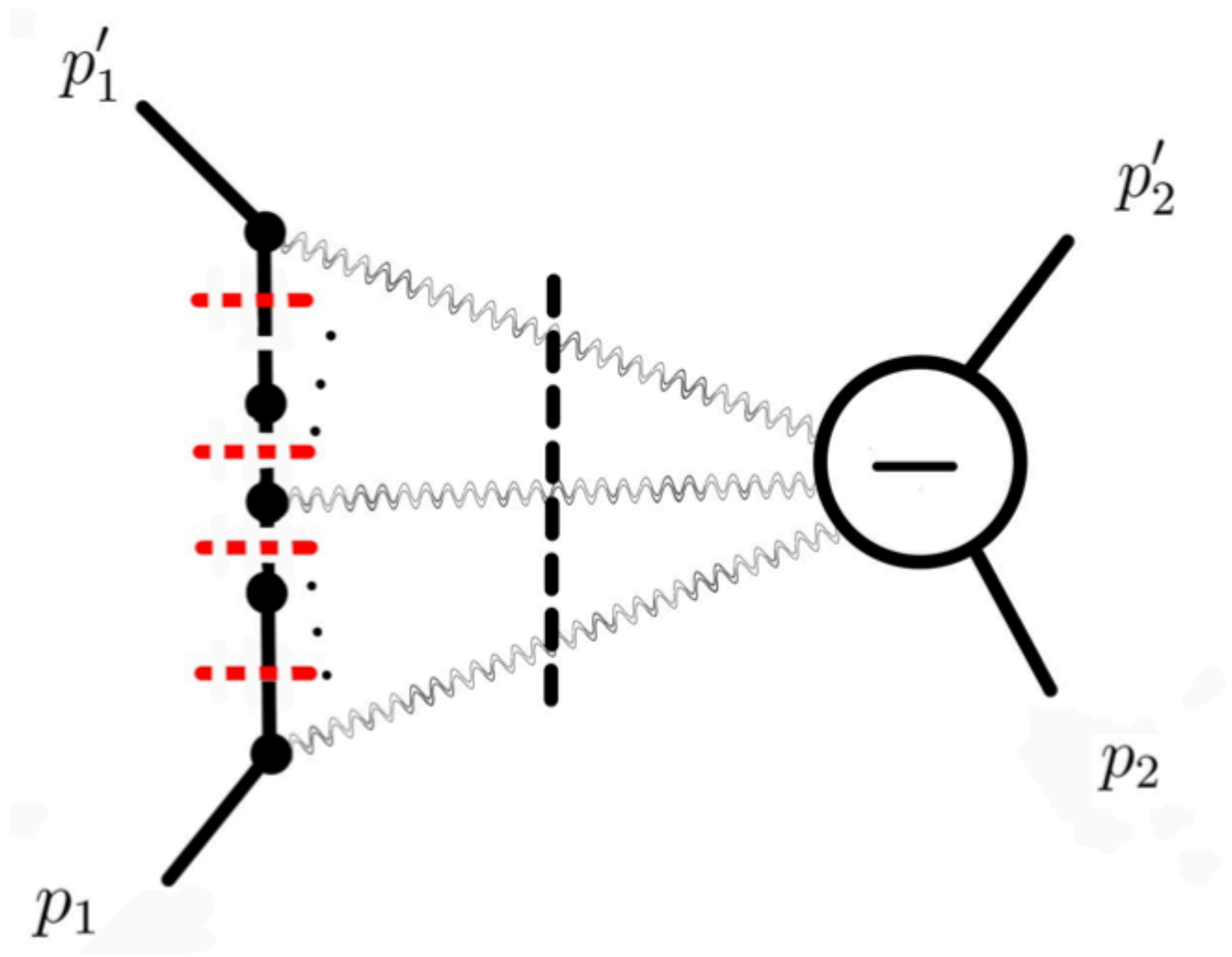

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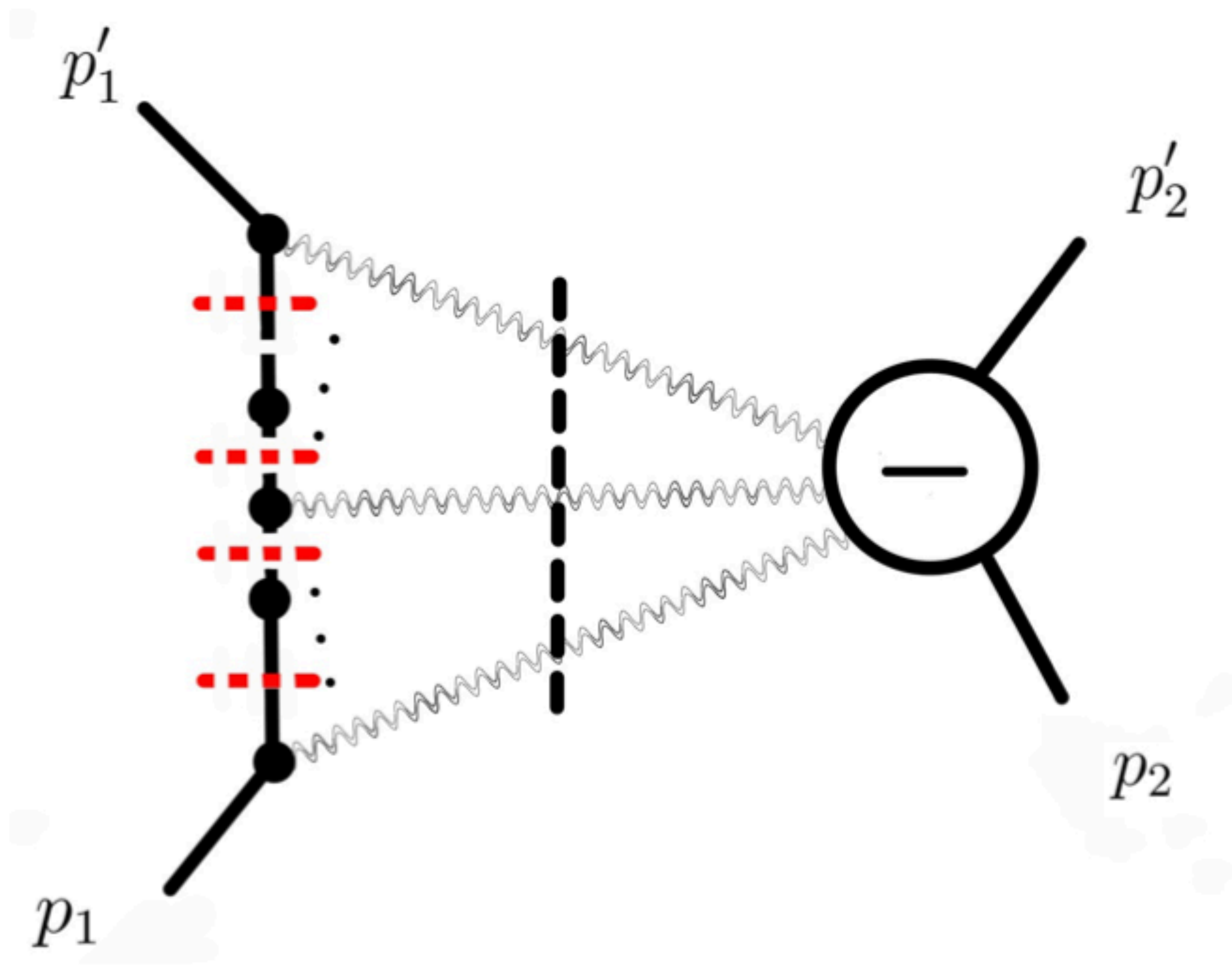

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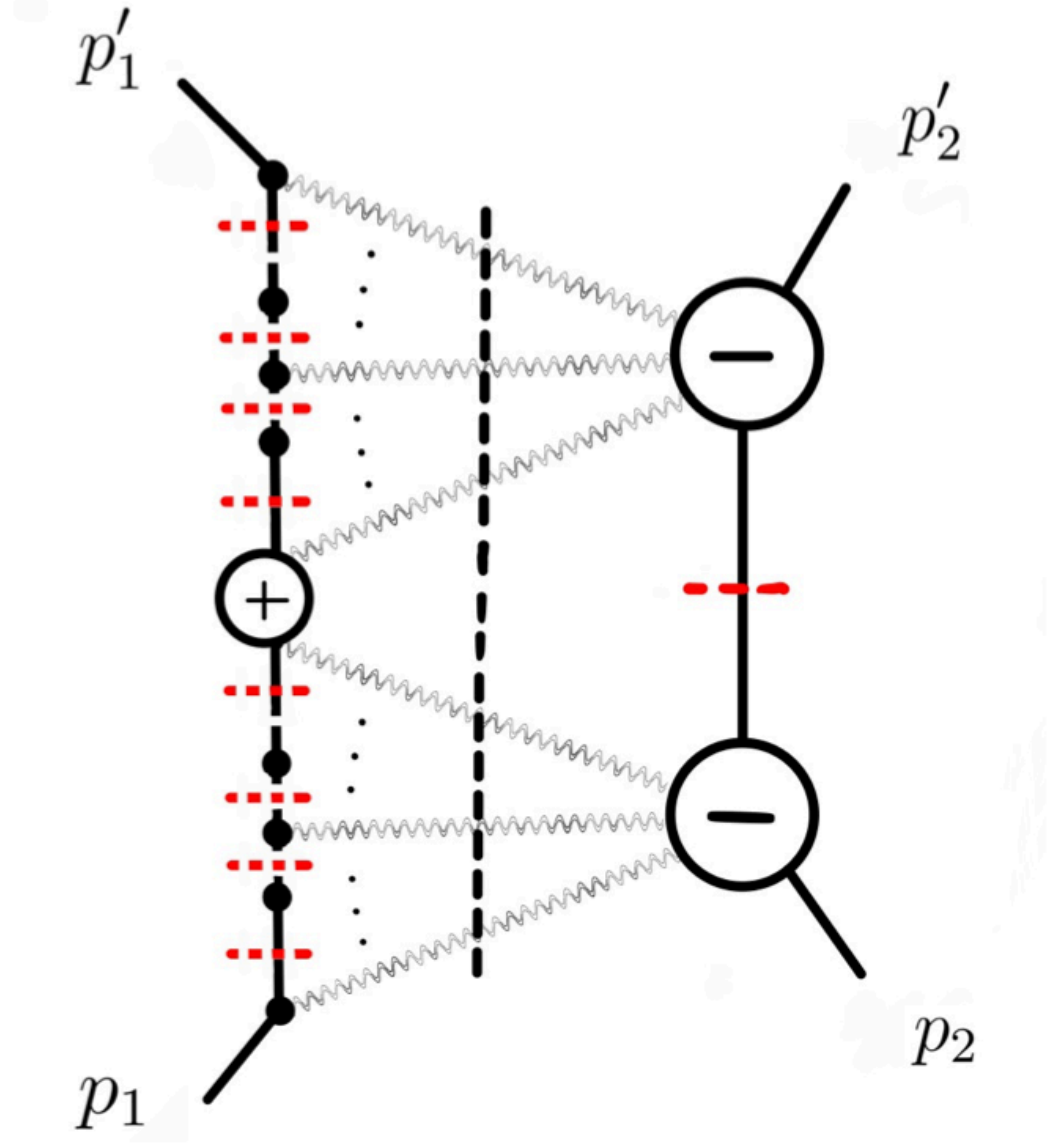

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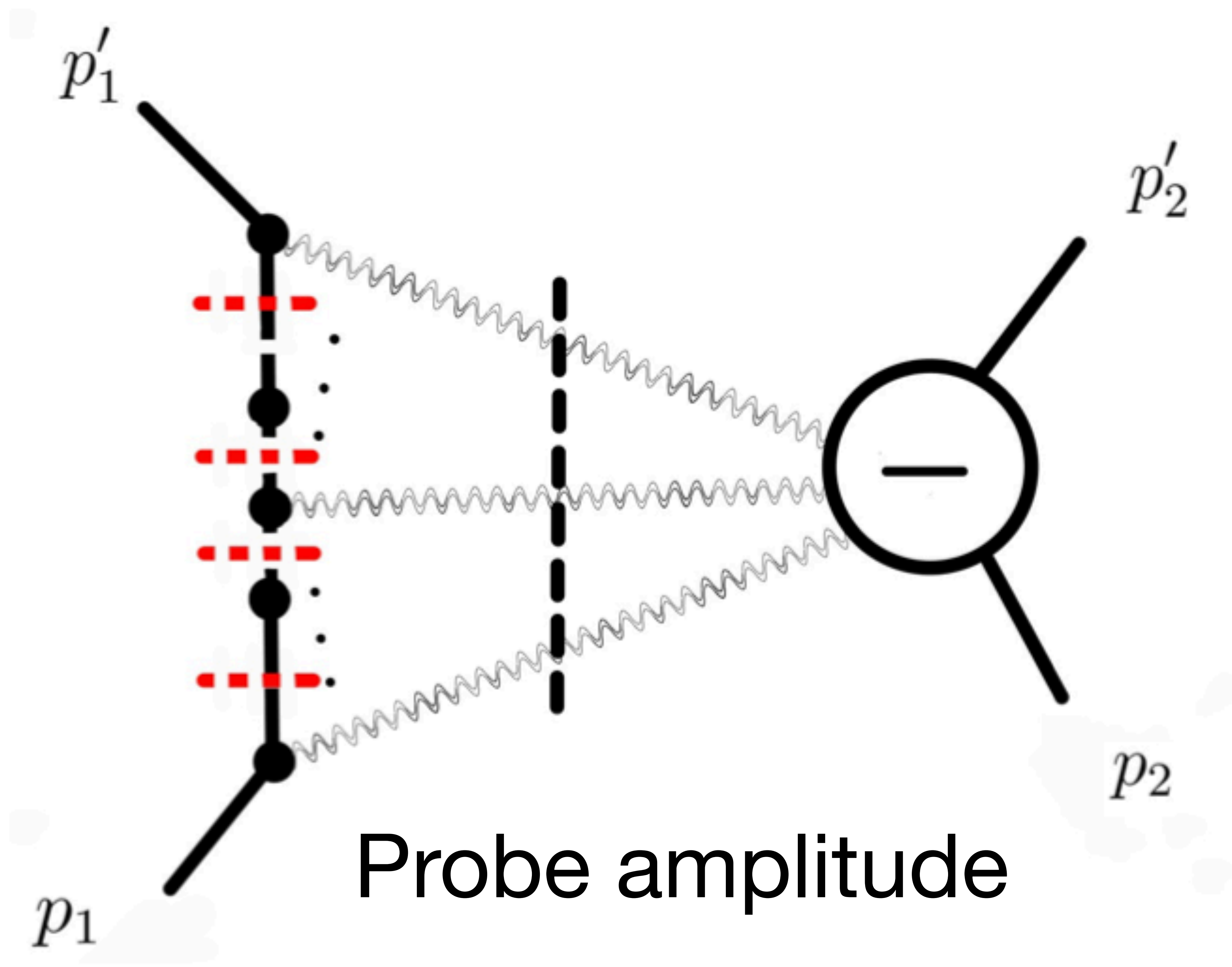

Probe amplitude

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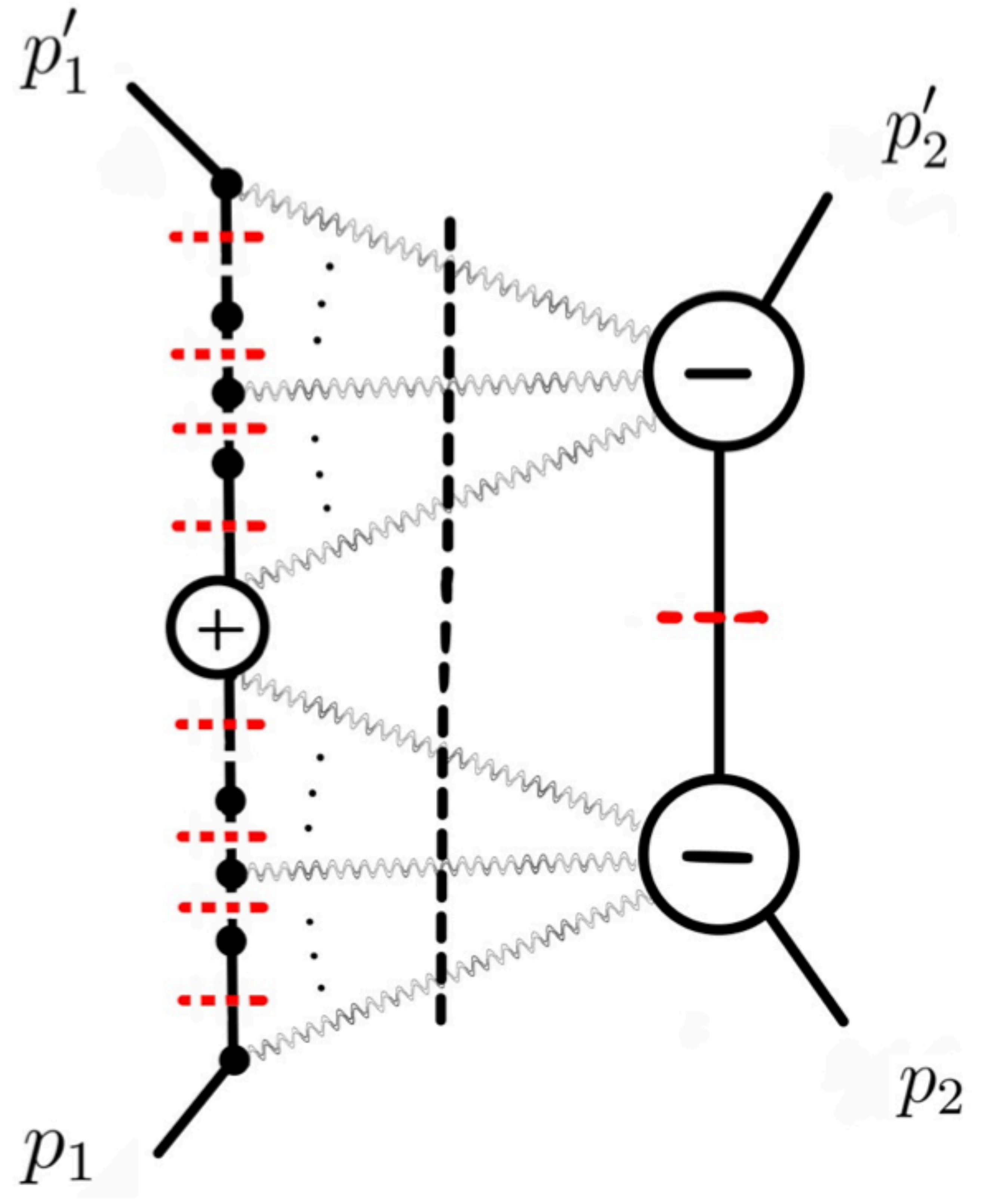

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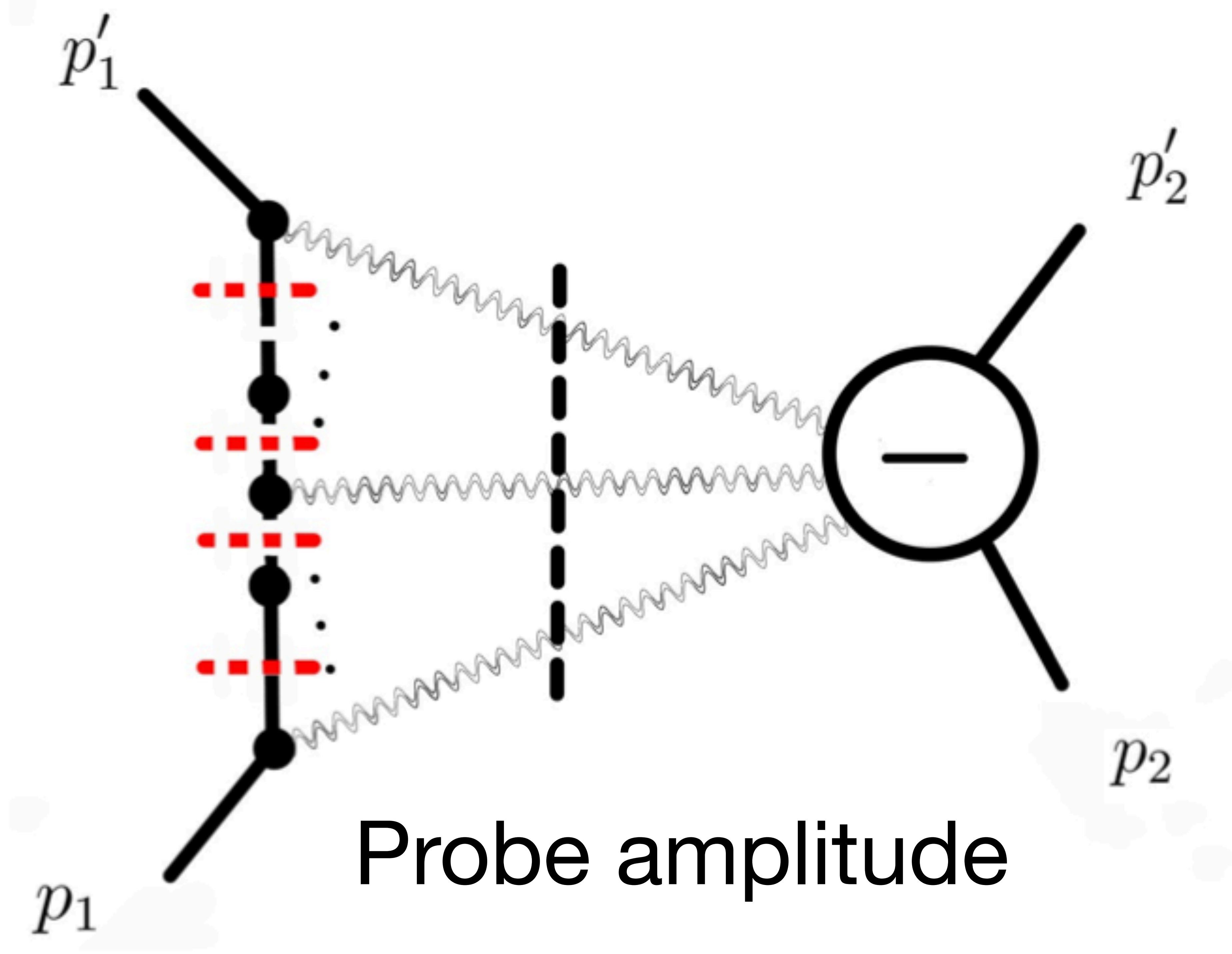

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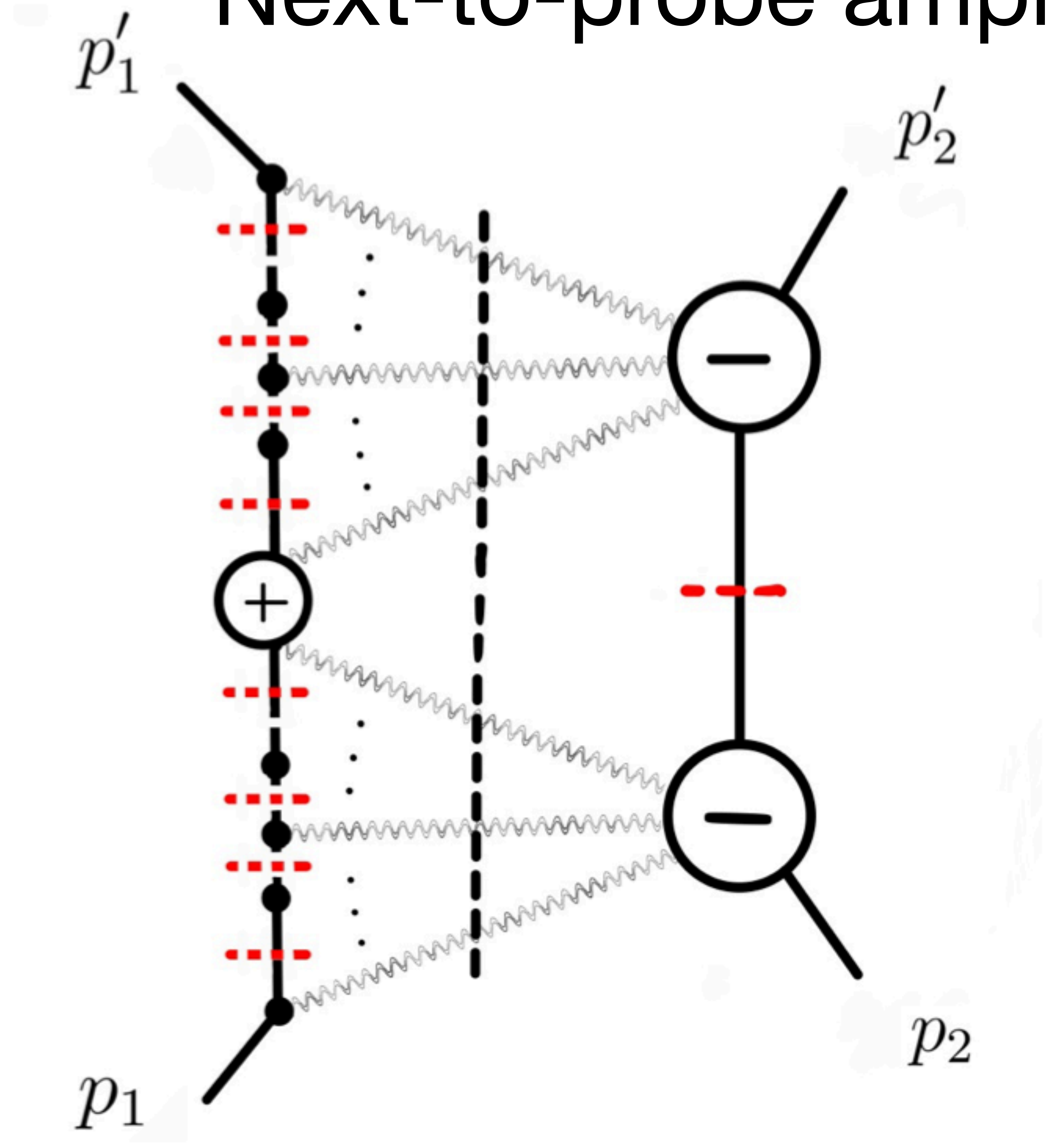

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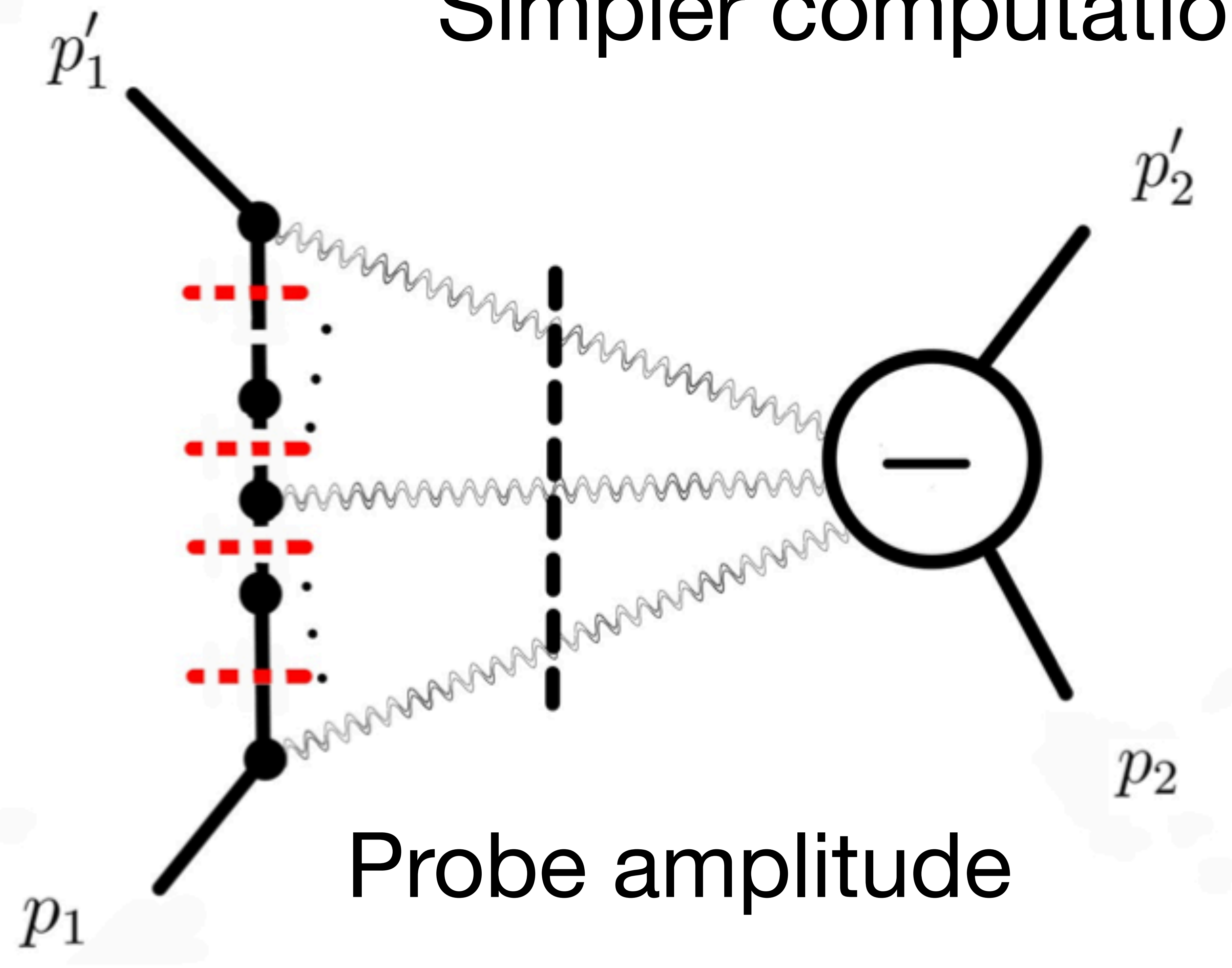

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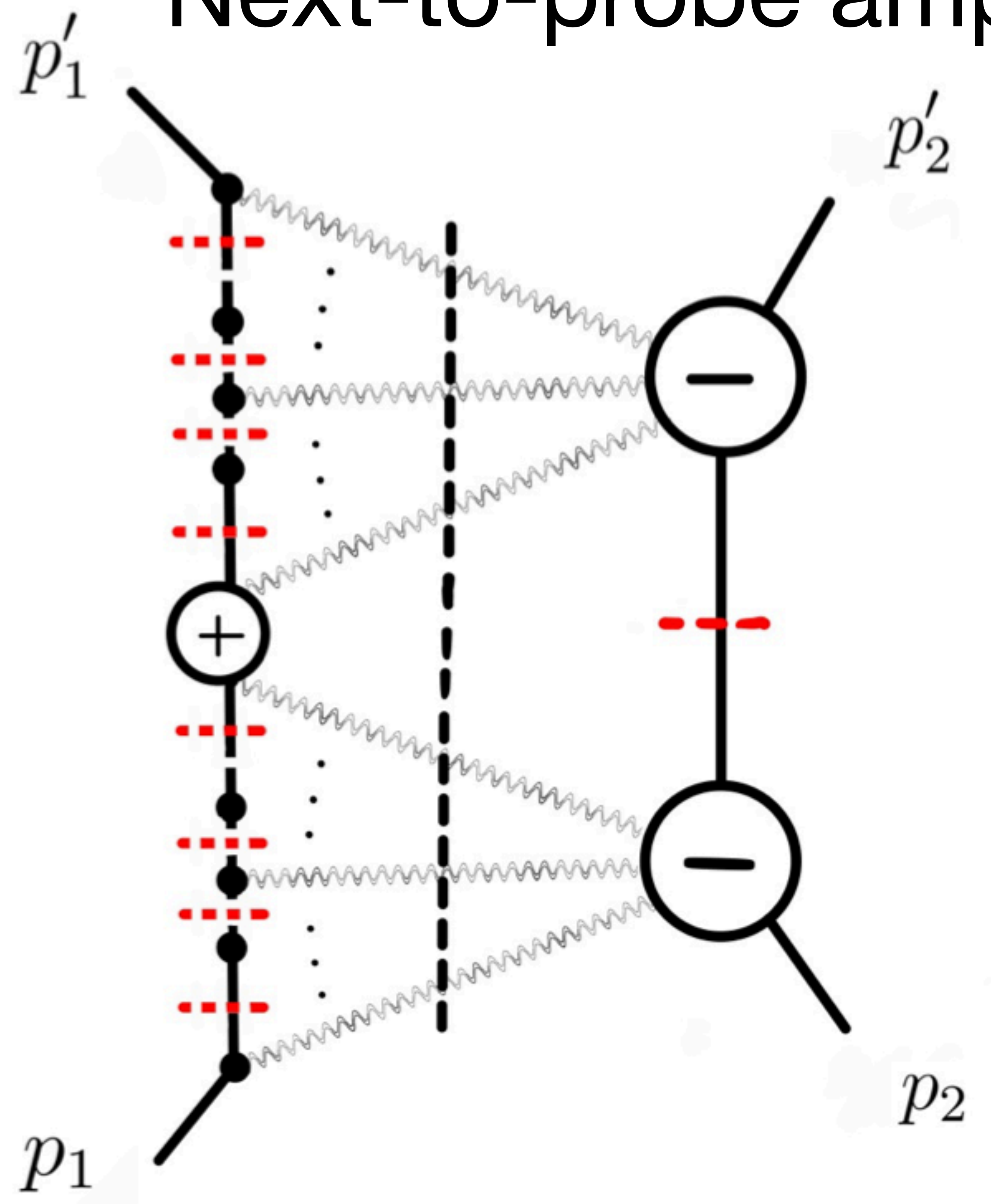

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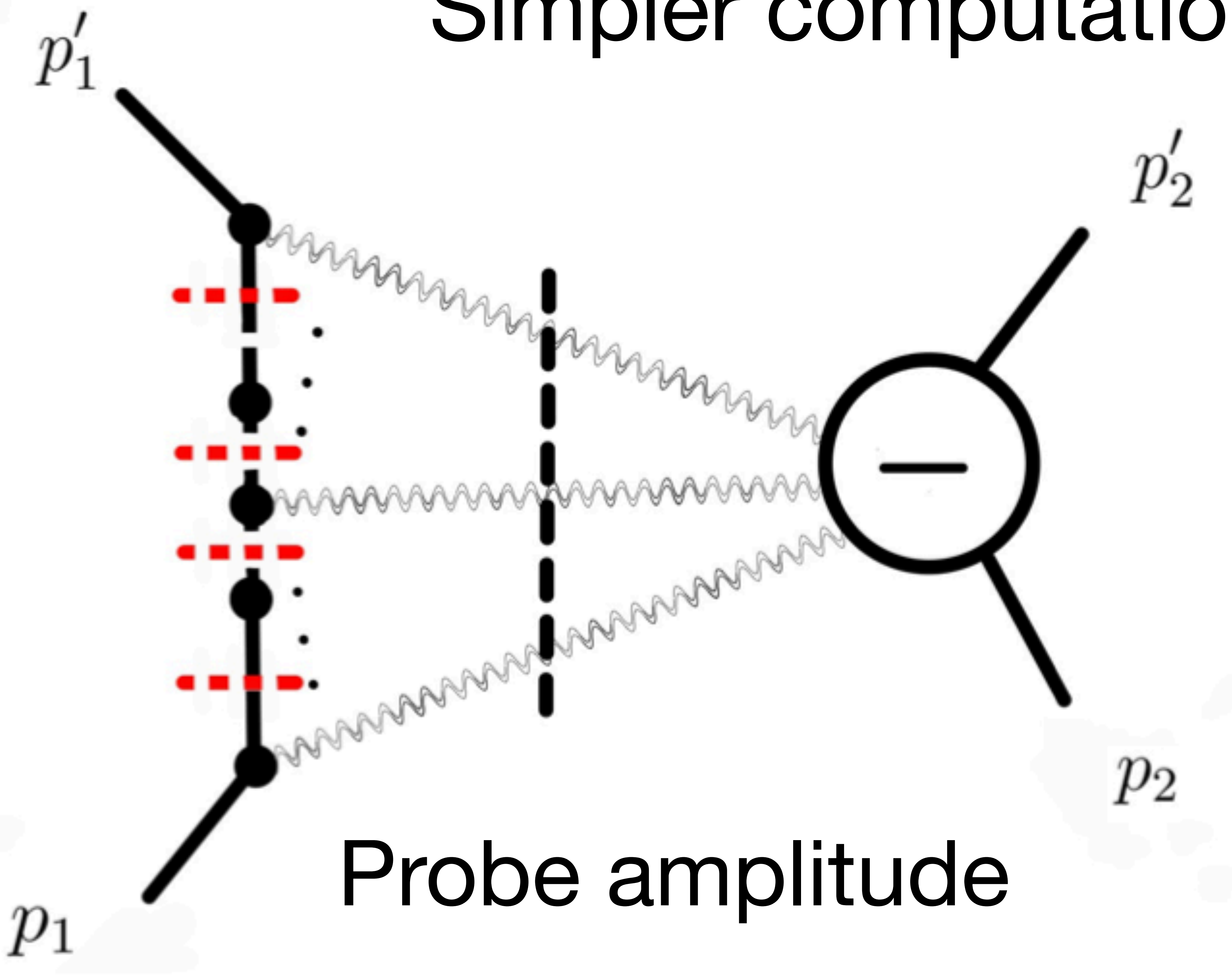

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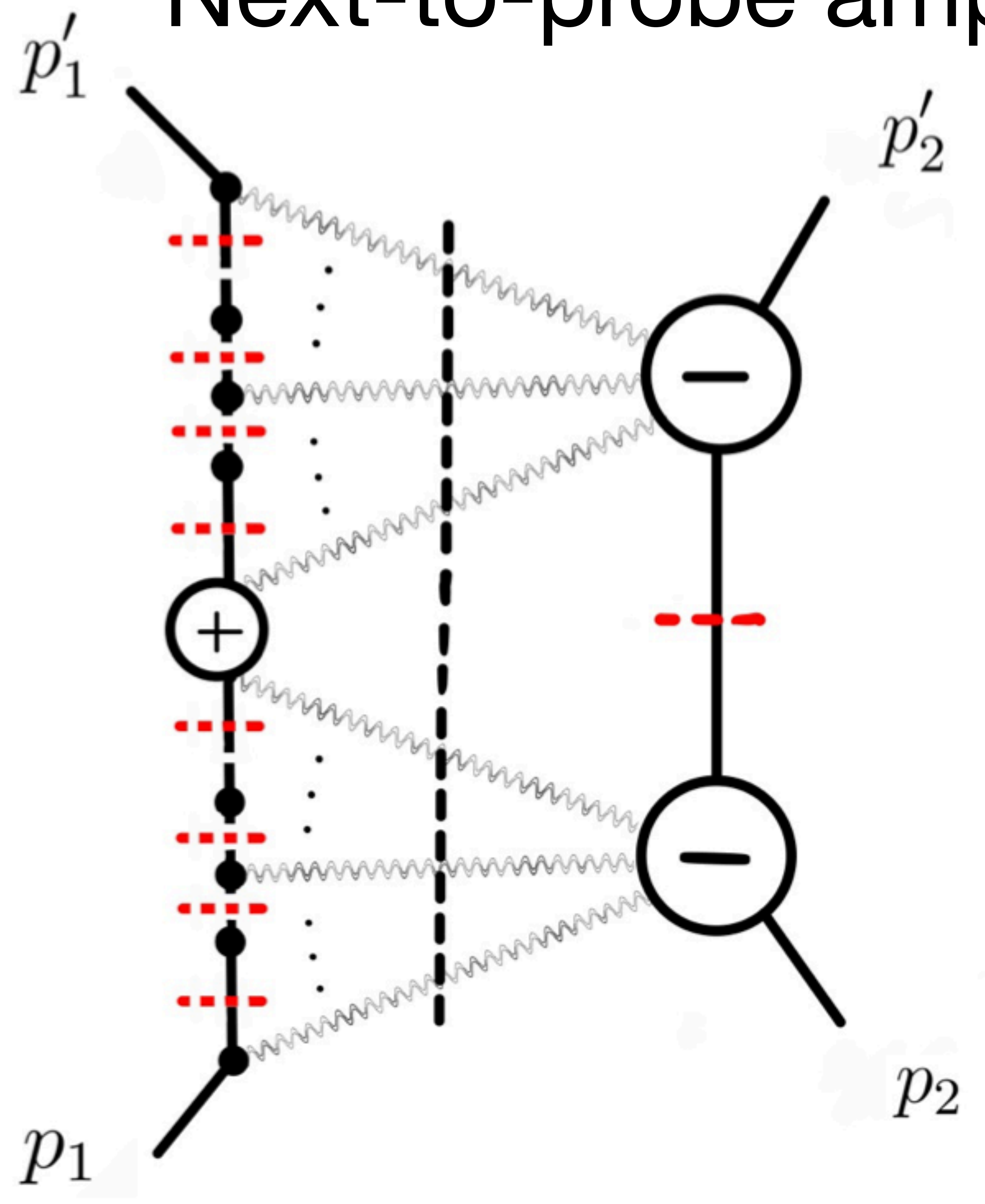

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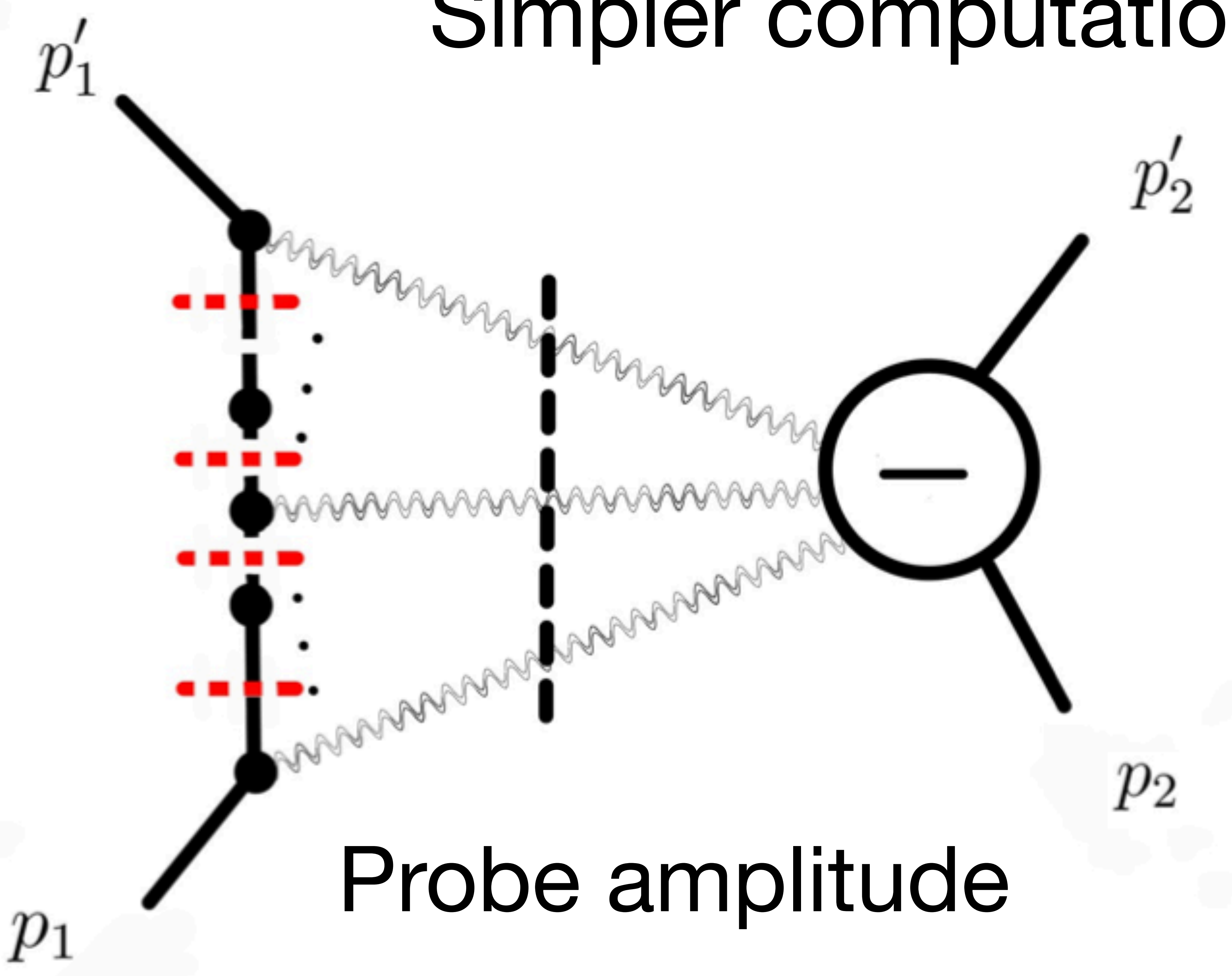

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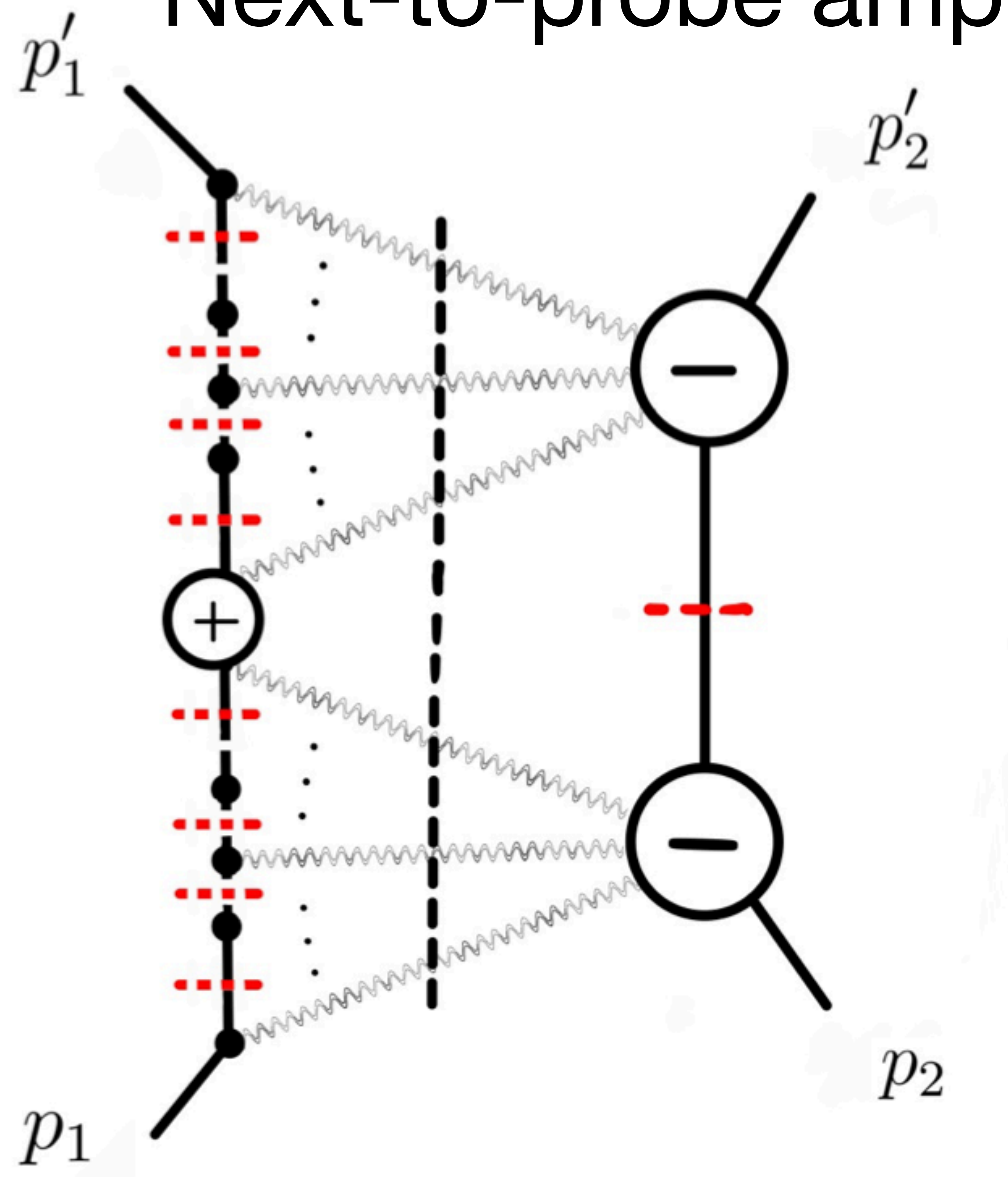

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- some similarities / some differences

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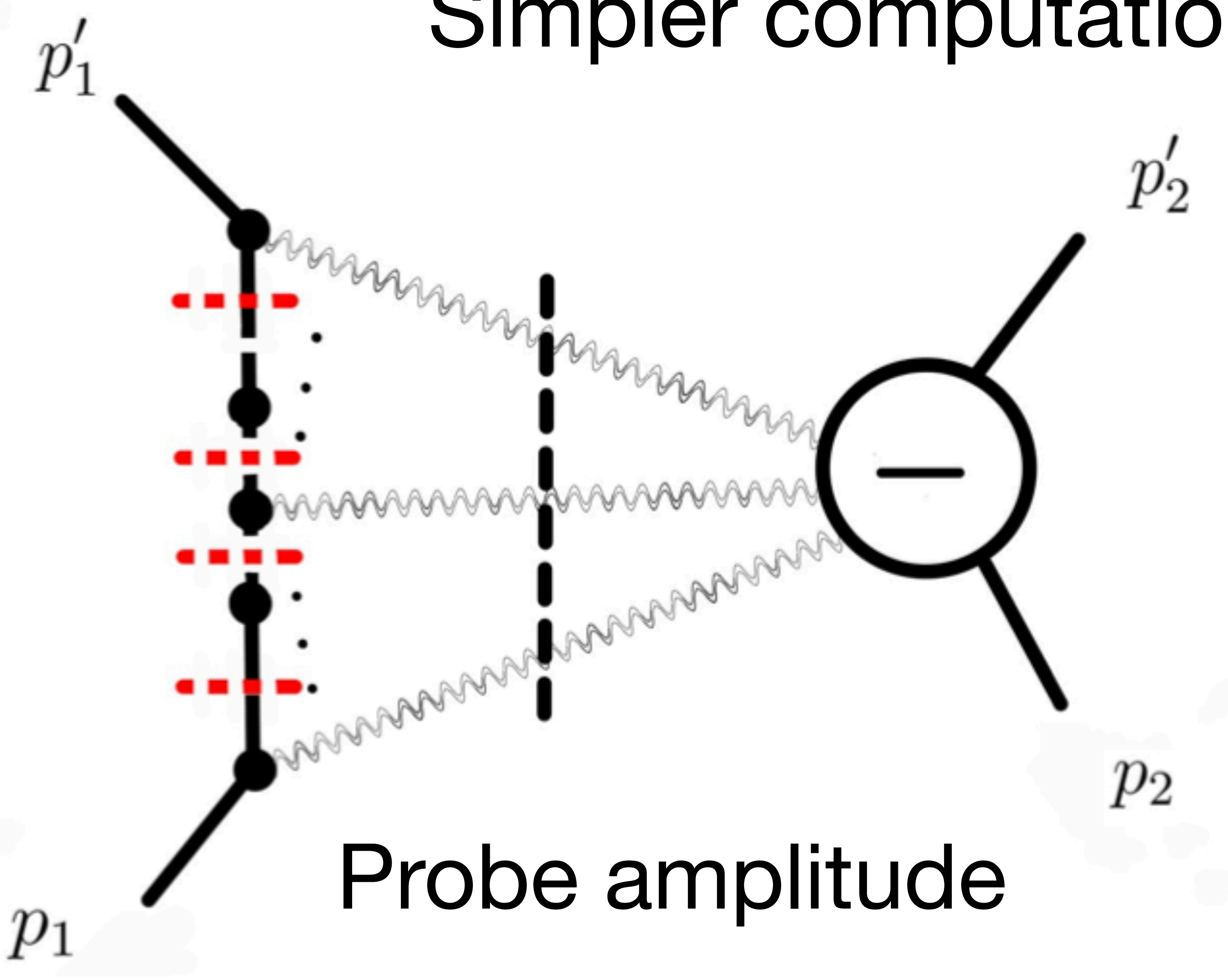

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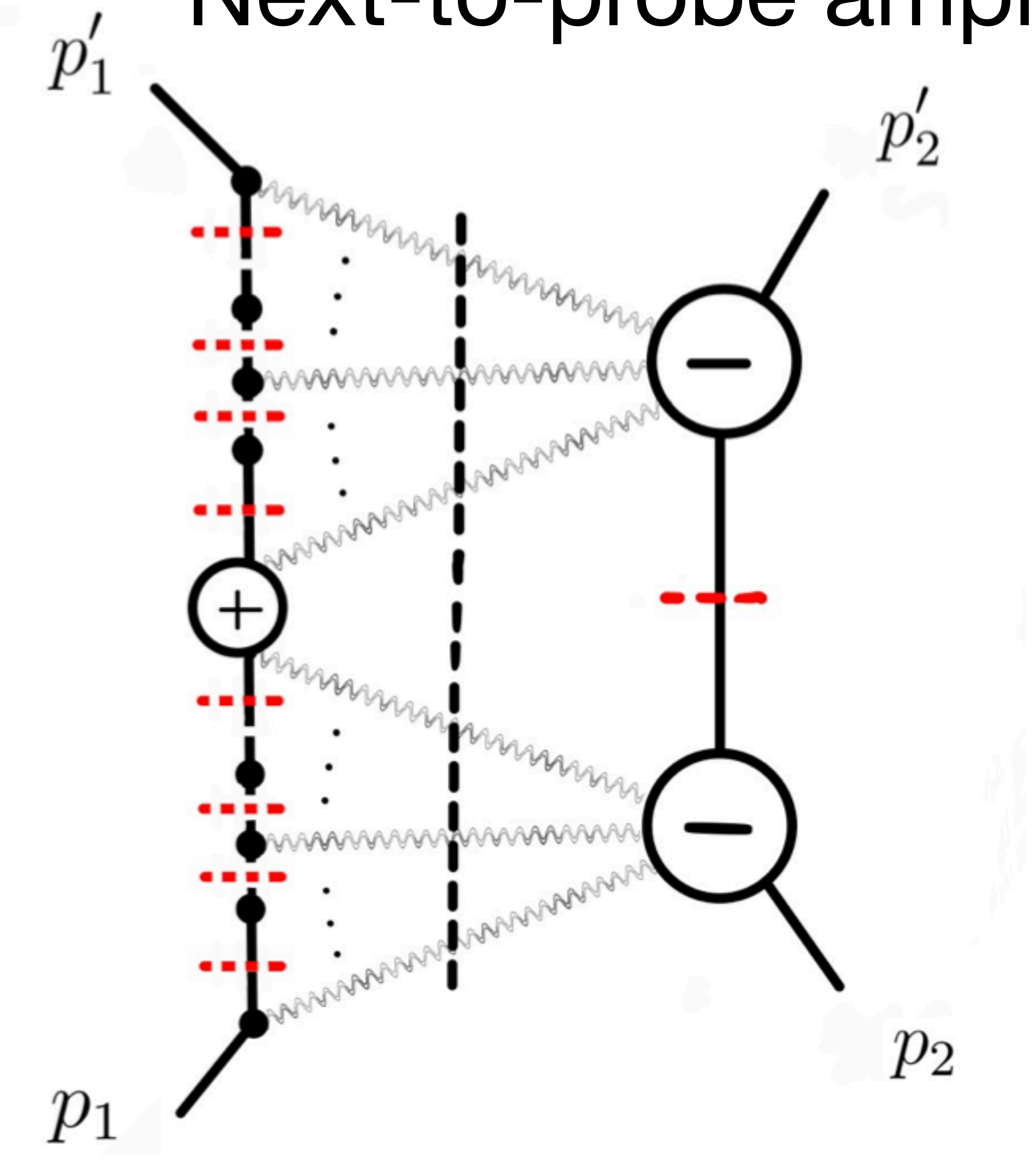

Interesting stuff to investigate

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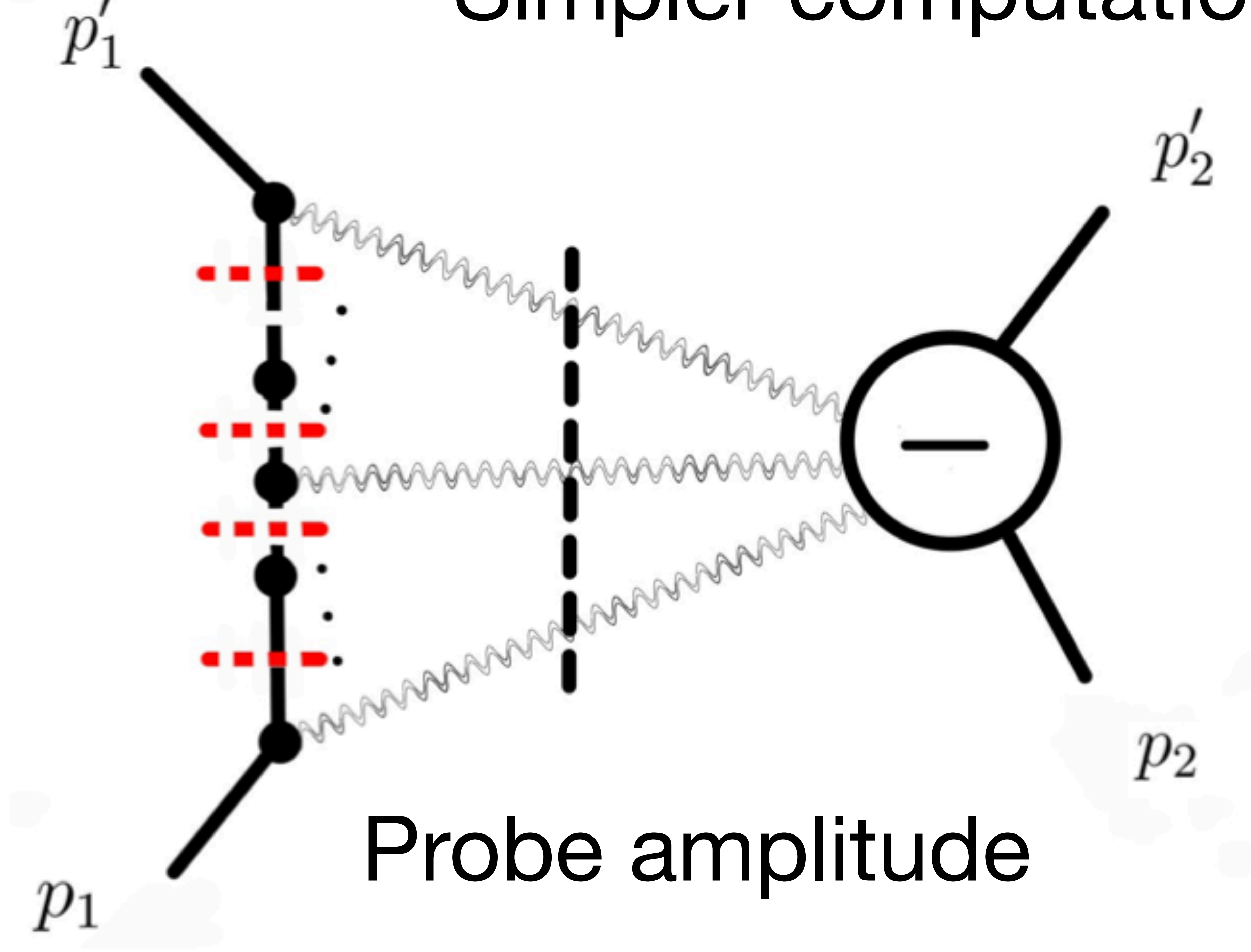

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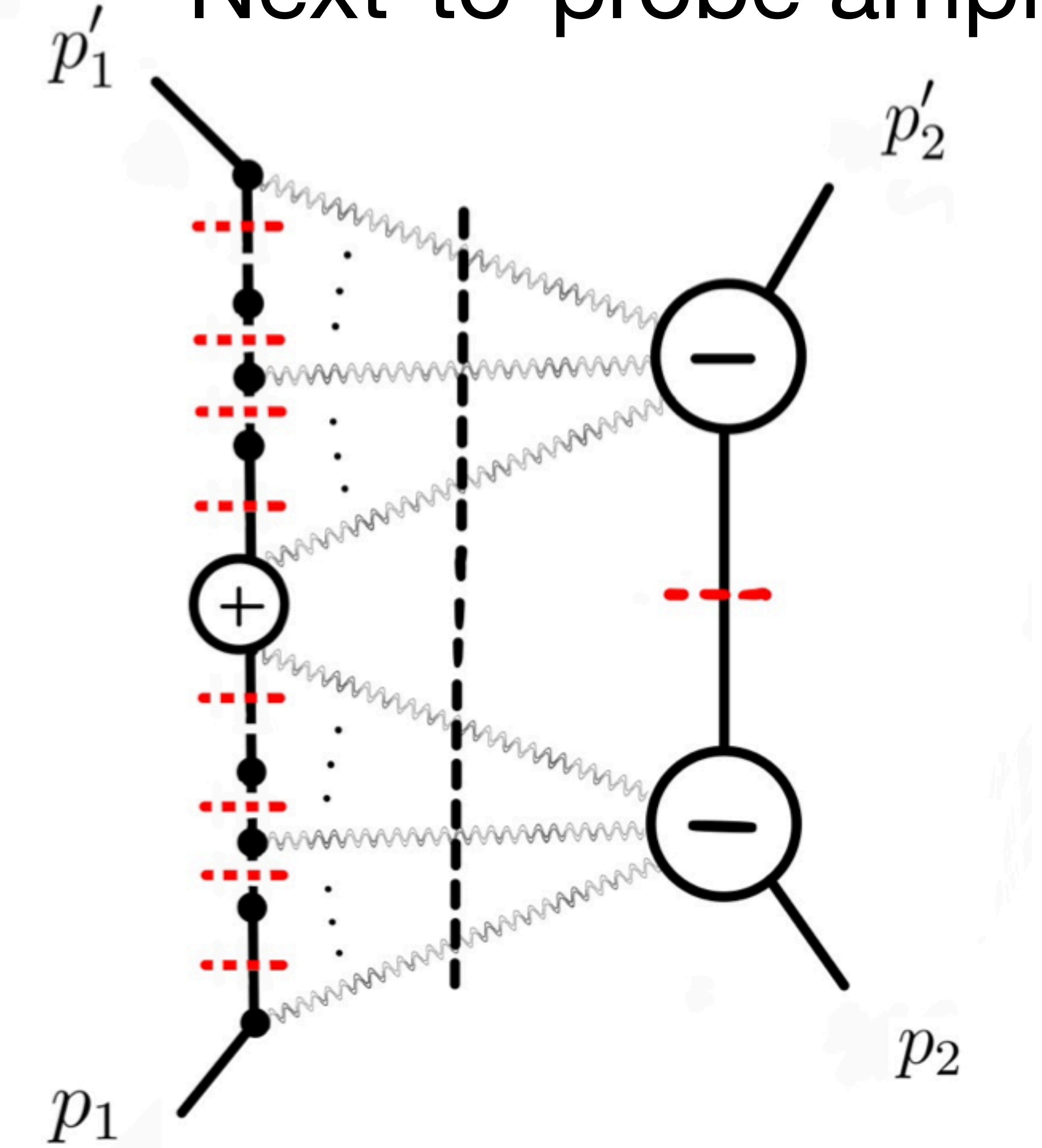

Interesting stuff to investigate

Heavy-quark—EFT inspiration:
(Damgaard, Haddad, Helset)

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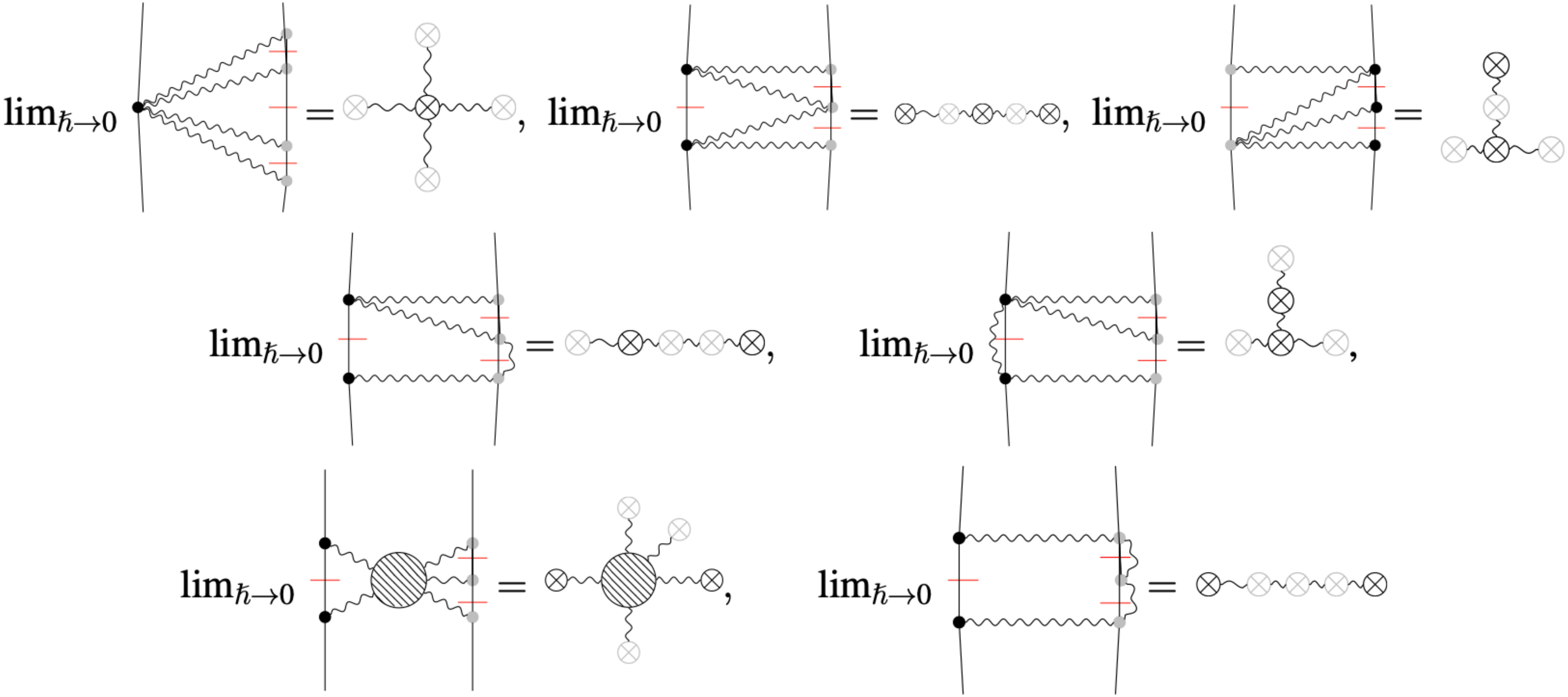
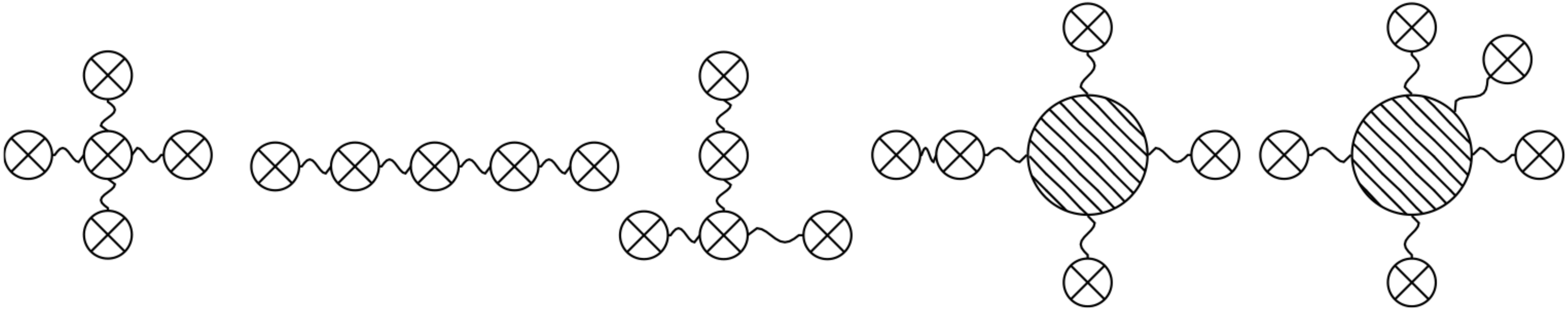
Enables a simple extension to fourth order in Newton's constant



Extension to fourth order in Newton's constant

Only five integrand topologies have to be considered



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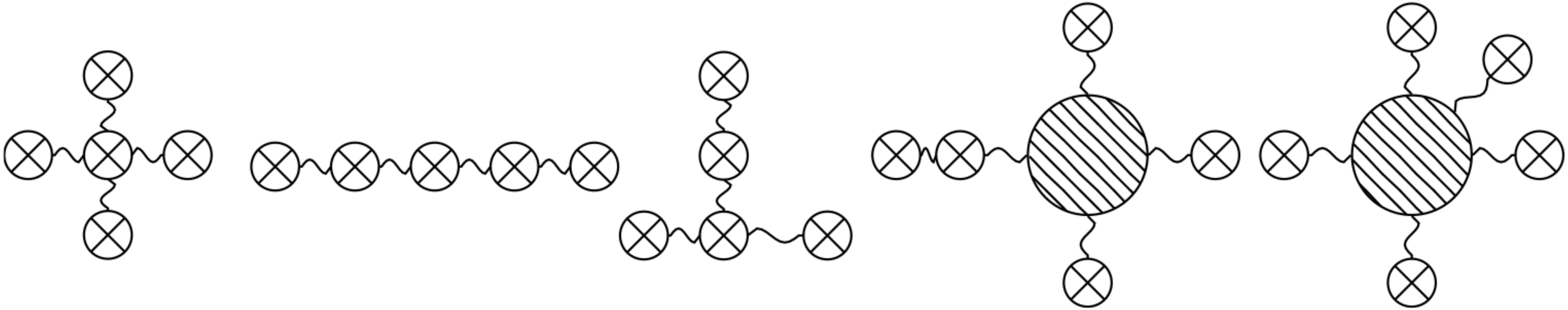

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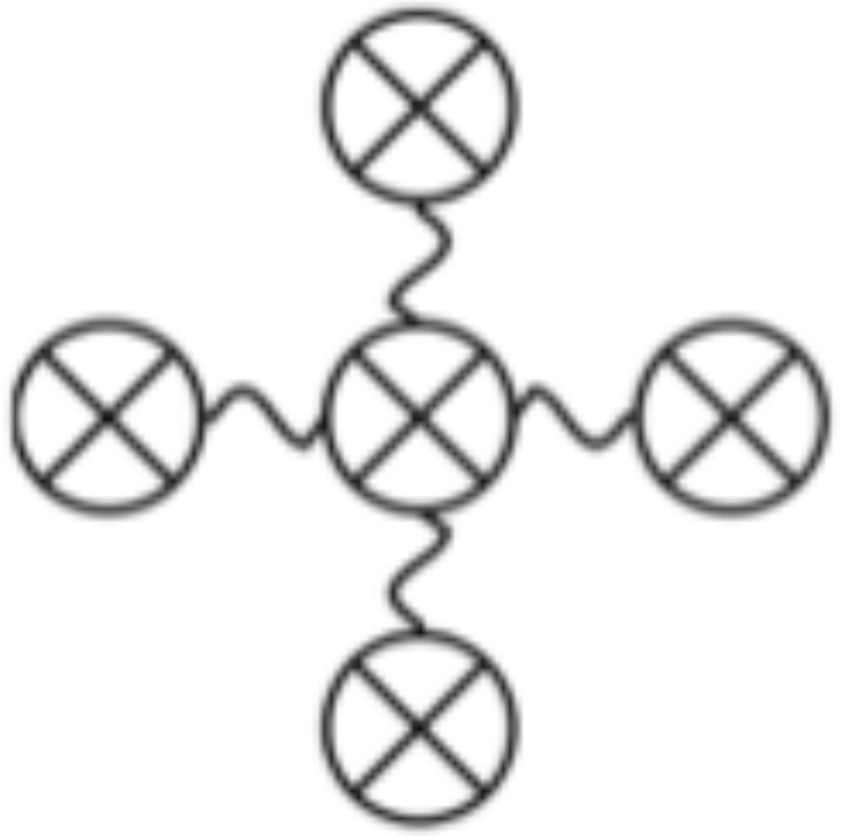
Extension to fourth order in Newton's constant

$$\mathscr{M}_{4\mathrm{PM}}(\gamma,\underline{q}^2) = \lim_{\varepsilon\to 0}\sum_{i=1}^{40} c\left(\{n_j\};\gamma,\underline{q}^2\right)\mathscr{I}\left(\{n_j\};\gamma,\varepsilon\right)$$

For instance the probe result is



$$\mathscr{M}_{4\mathrm{PM}}^{\mathrm{probe}}(\gamma,\underline{q}^2) = \lim_{\varepsilon\to 0}\frac{(8\pi G_N)^4}{|\underline{q}|^{-1+3\varepsilon}} m_1^2 m_2^2 (m_1^3+m_2^3)\frac{(1-2\varepsilon)^3}{(2-2\varepsilon)^4}\frac{c_3(\gamma,\varepsilon)}{(\gamma^2-1)^3} I_{\mathrm{PP}}^1(1,\varepsilon)$$

$$= G_N^4 (m_1^3+m_2^3) m_1^2 m_2^2 |\underline{q}|\pi^3 \frac{35i\left(33\gamma^4-18\gamma^2+1\right)}{8\left(\gamma^2-1\right)},$$

Post-Minkowskian framework and amplitudes

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- Focus so far has been on precision classical physics: But all these techniques are readily available for quantum terms as well (however tiny effects — no possible observation)

- Challenge in making quantum interpretation: Classical physics can be understood from taking the classical limit and comparing to general relativity — lacking a good framework for quantum effects…

Outlook

Amplitude toolbox for computations already provided many new efficient methods for computation

- Amplitude tools very useful
  - Double-copy/KLT
  - Unitarity
  - Spinor-helicity
  - CHY formalism
  - Low energy limits of string theory

- Identifying IBP-relations solving DE equations/integral
- Recycling tools from QCD computations
- Numerical programs for amplitude computation

Conclusion

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Endless tasks ahead

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Interesting to focus on quantum effects from a theoretical perspective/consistency

Conclusion

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Interesting perspectives in the analysis of gravity in the context of general relativity vs a quantum field theory framework

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THANKS!!!