3-body systems in strong-gravity

Marta Orselli

University of Perugia (Italy)
&
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THE CENTER OF GRAVITY

INFN Istituto Nazionale di Fisica Nucleare

Danish Quantum Field Theory Meeting 2025
University of Southern Denmark

Based on:

- "Strong-gravity precession resonances for binary systems orbiting a Schwarzschild black hole", M. Cocco, G. Grignani, T. Harmark, MO and D. Pica, arXiv:2505.15901, Phys. Rev. D 112, 044010
- + work in collaboration with F. Camilloni, E. Grilli, D. Panella, D. Pereñiguez Rodriguez, M. van de Meent

August 13-14, 2025

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Plan of the talk

- Introduction
- System setup: binary system+external body $\rightarrow$ 3-body system
- Tidal effects from Supermassive Black Holes (SMBHs)
- Precession resonances: Newtonian derivation
- Precession resonances in strong-gravity regime: perturbative analysis
- Numerical analysis
- Conclusions and future directions

Introduction

Gravitational waves provide invaluable information about the Universe

First direct proof of existence of black holes and of binary systems of compact objects that merge

Gravitational waves provide a unique framework to study General Relativity

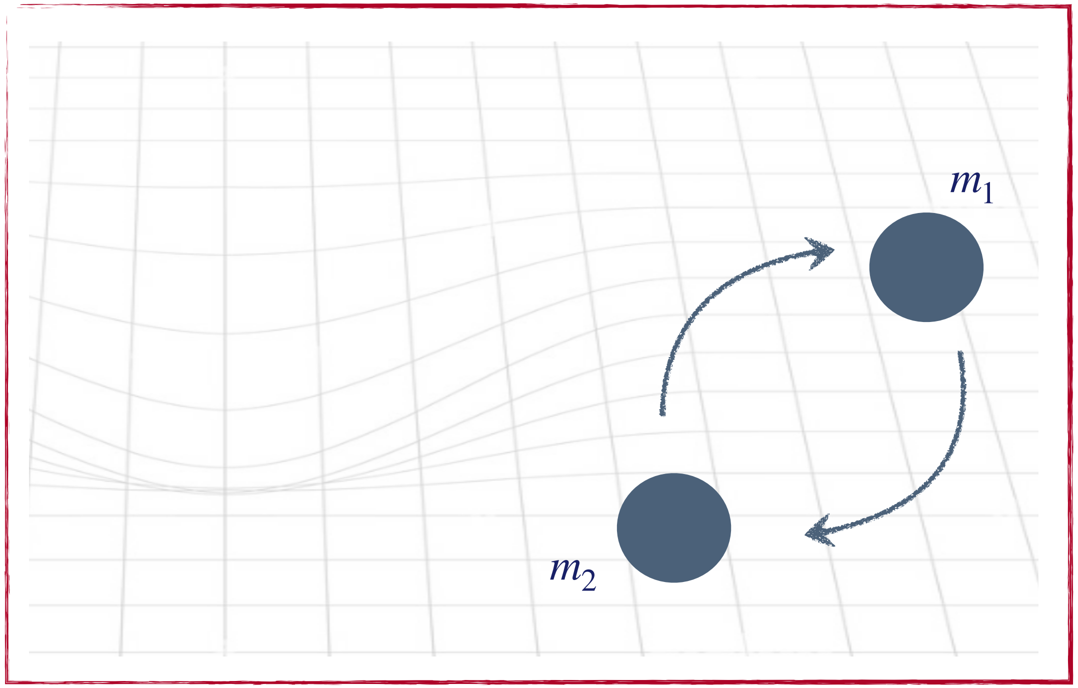

Image credit: M.Cocco

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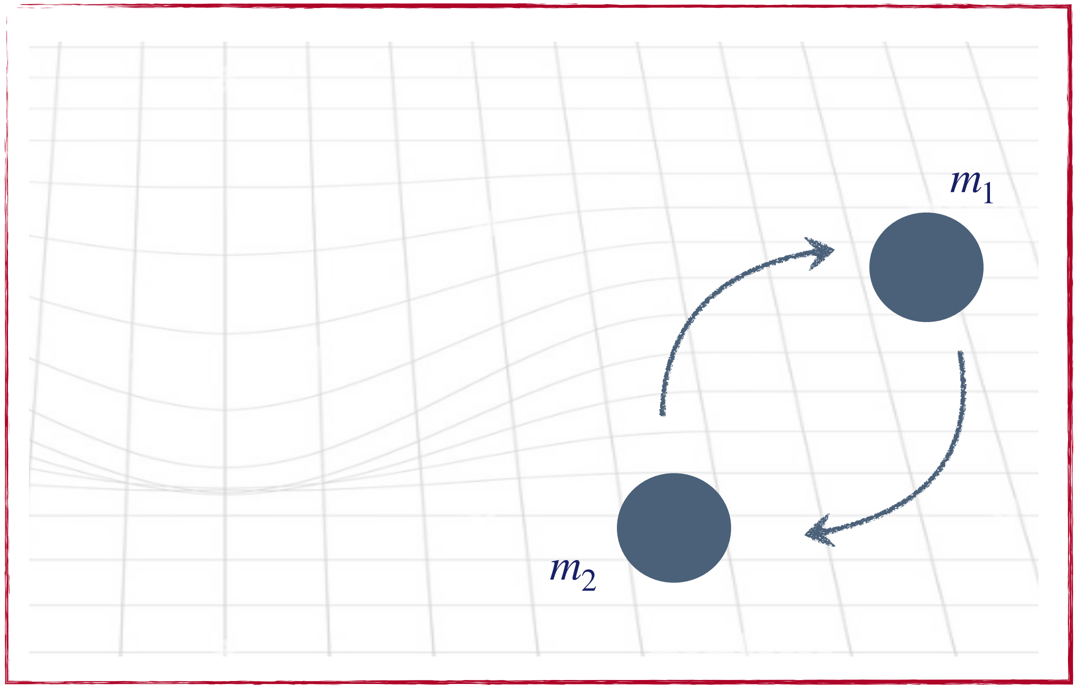

Image credit: M.Cocco

Binary systems are **not isolated**: how does the **environment** influence their dynamics?

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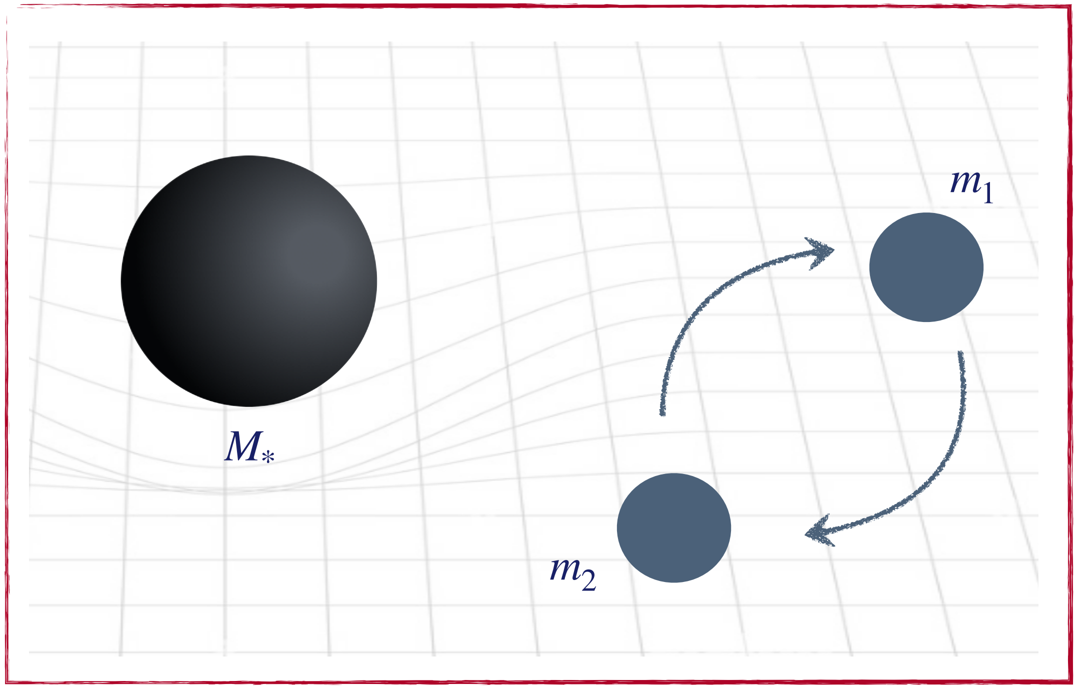

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Supermassive black hole close to binary system that affects the dynamics of the binary through **tidal forces**

SMBH + binary system form a **3-body system**

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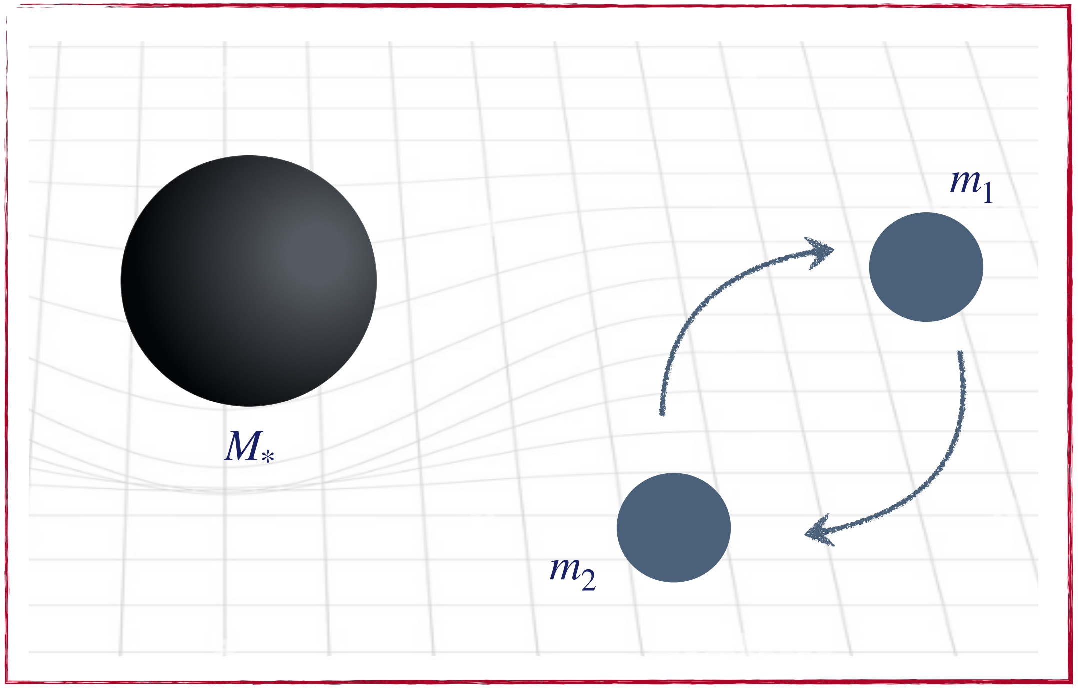

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SMBH + binary system form a **3-body system**

One needs to use **General Relativity**

Introduction: tidal effects

My motivation: understand how to describe the tidal influence of a SMBH on the dynamics of a binary system while being in a strong-gravity regime to learn about fundamental physics and General Relativity

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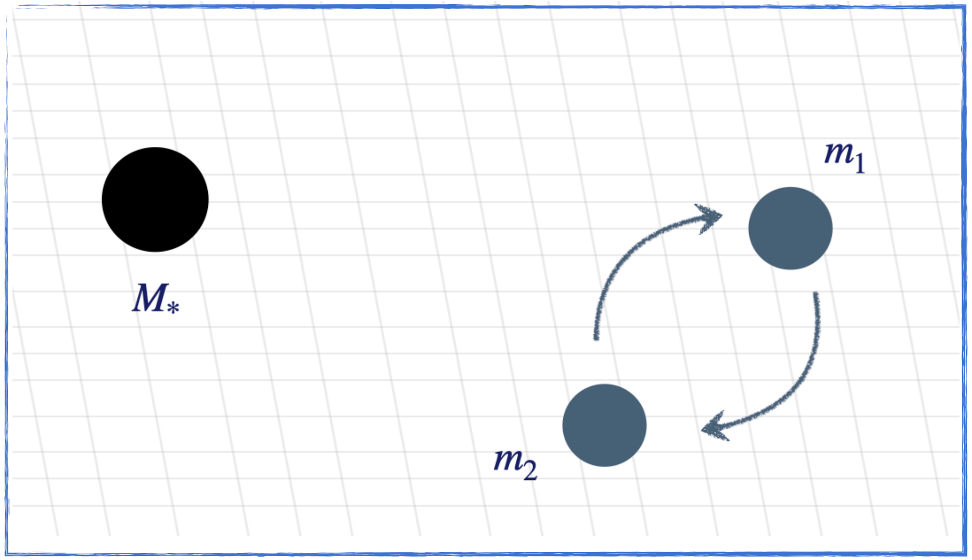
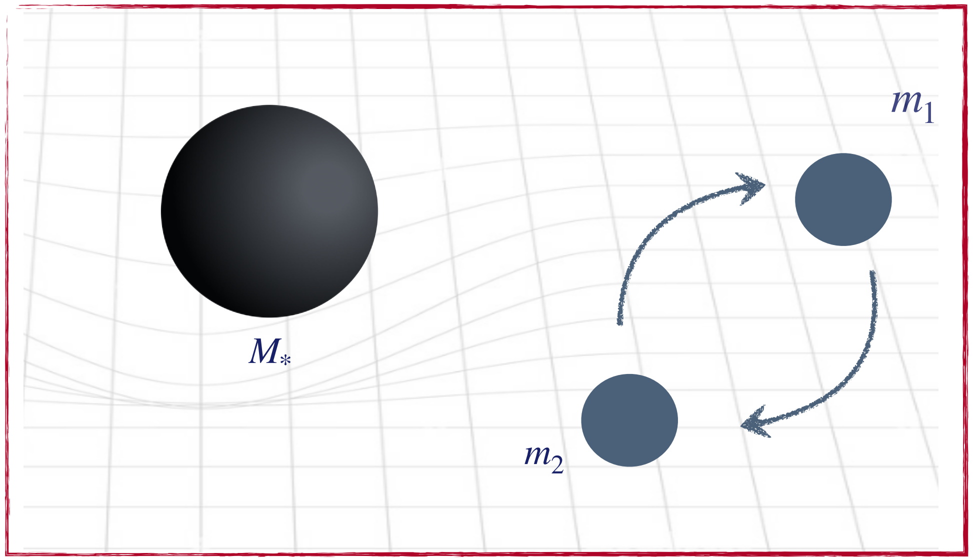
Newtonian regime

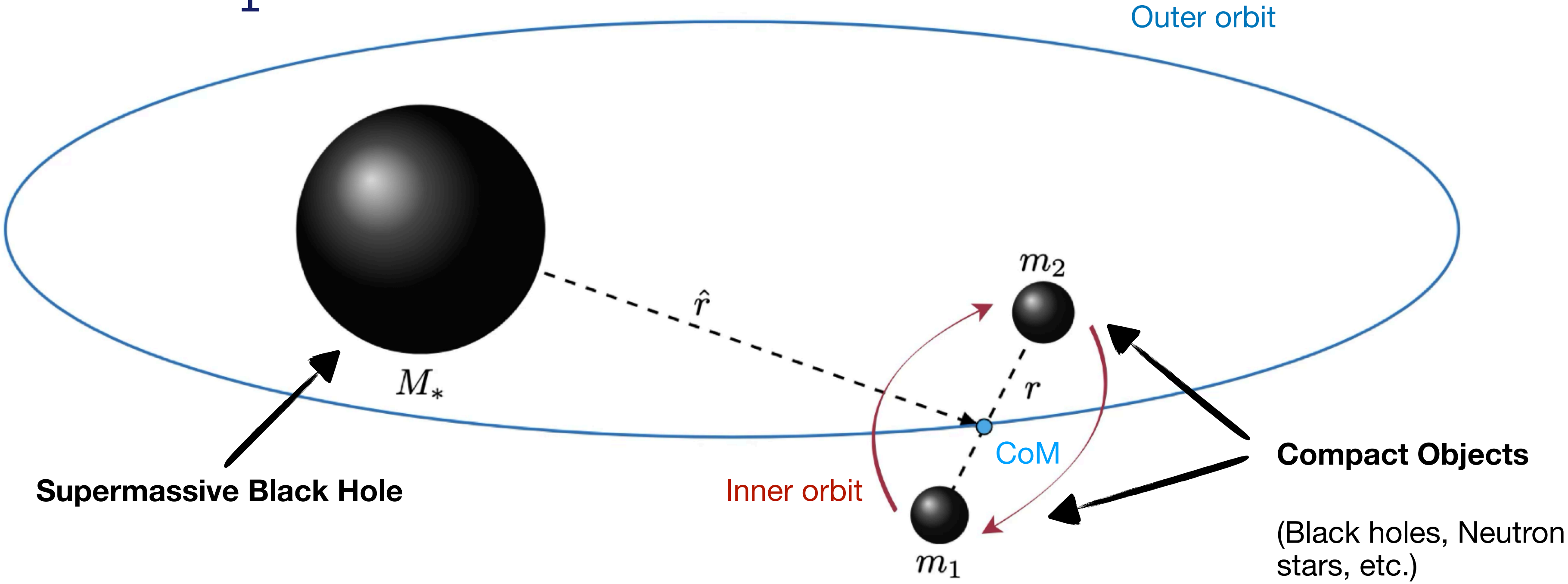


Strong-gravity regime



Images credit: M.Cocco

System setup



$$M = m_1 + m_2, \quad \mu = \frac{m_1 m_2}{M}$$

Schwarzschild black hole metric in $(\hat{t}, \hat{r}, \hat{\theta}, \hat{\phi})$ coordinates

$$ds^2 = -\left(1 - \frac{2GM_*}{c^2\hat{r}}\right)c^2 d\hat{t}^2 + \frac{d\hat{r}^2}{1 - \frac{2GM_*}{c^2\hat{r}}} + \hat{r}^2(d\hat{\theta}^2 + \sin^2\hat{\theta}\, d\hat{\phi}^2)$$

If m_1 and m_2 are black holes, we require → $\quad r \gg \frac{2Gm_1}{c^2}, \frac{2Gm_2}{c^2}$

Tidal effects from a supermassive BH

Small-tide approximation — When the characteristic scale of the binary system (m_1, m_2) is much smaller than the radius of the curvature generated by M_* → $\boxed{\frac{r}{\mathcal{R}} \ll 1}$

[Poisson & Vlasov (2009)]

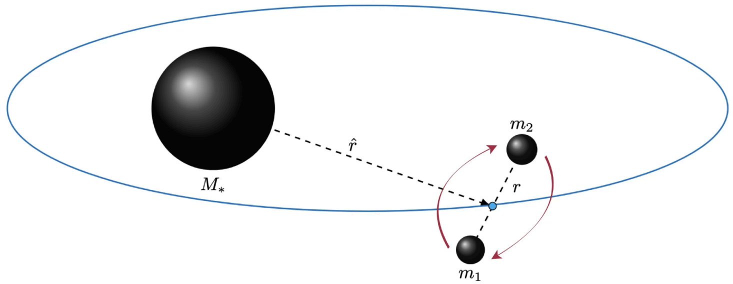
We write the metric near a geodesic of a background space-time using the Thorne-Hartle version of the Fermi-normal coordinates

$$g_{00} = -1 - \mathcal{E}_{ij} x^i x^j + \mathcal{O}(x^3/\mathcal{R}^3)$$

$$g_{0i} = -\frac{2}{3}\epsilon_{ijk}\mathcal{B}^j{}_l x^k x^l + \mathcal{O}(x^3/\mathcal{R}^3)$$

$$g_{ij} = \delta_{ij}(1 - \mathcal{E}_{kl} x^k x^l) + \mathcal{O}(x^3/\mathcal{R}^3)$$

where $i, j, k = 1,2,3$ and $x = \sqrt{x^i x^i} = r$ is the distance to the geodesic



the terms of order $x^2/\mathcal{R}^2$ capture the **quadrupole approximation** of the tidal forces

$$\mathcal{E}_{ij} = R_{0i0j}\big|_{x=0}\,, \quad \mathcal{B}_{ij} = \frac{1}{2}\epsilon_{pq(i}R^{pq}\big|_{j)0}\big|_{x=0}$$

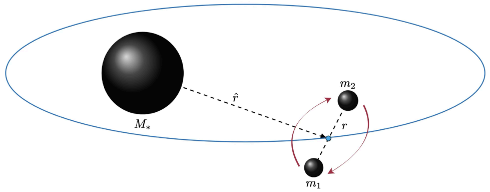
electric and magnetic quadruple tidal moments

N.B.: For now we only include electric quadrupole tidal moments

Tidal effects from a supermassive BH

Small-tide approximation — When the characteristic scale of the binary system (m_1, m_2) is much smaller than the radius of the curvature generated by M_*

$$\frac{r}{\mathscr{R}} \ll 1 \longrightarrow r \ll \sqrt{\frac{c^2 \hat{r}^3}{GM_*}}$$



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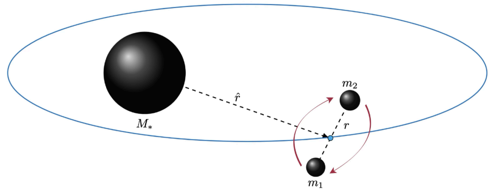

Weak-field limit Triple system hierarchical in the **distances** $\hat{r} \to \infty \quad r \ll \hat{r}$

→ ***Newtonian description*** → we lose info about strong-gravity effects

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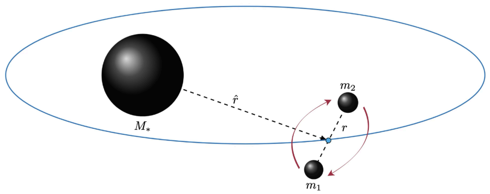

Weak-field limit

Triple system hierarchical in the **distances** $\hat{r} \to \infty \quad r \ll \hat{r}$

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Small-hole limit

$\hat{r} \sim 2GM_*/c^2 \quad \rightarrow \quad r \ll \sqrt{c^2\hat{r}^3/GM_*}$ becomes $r \ll 2GM_*/c^2$

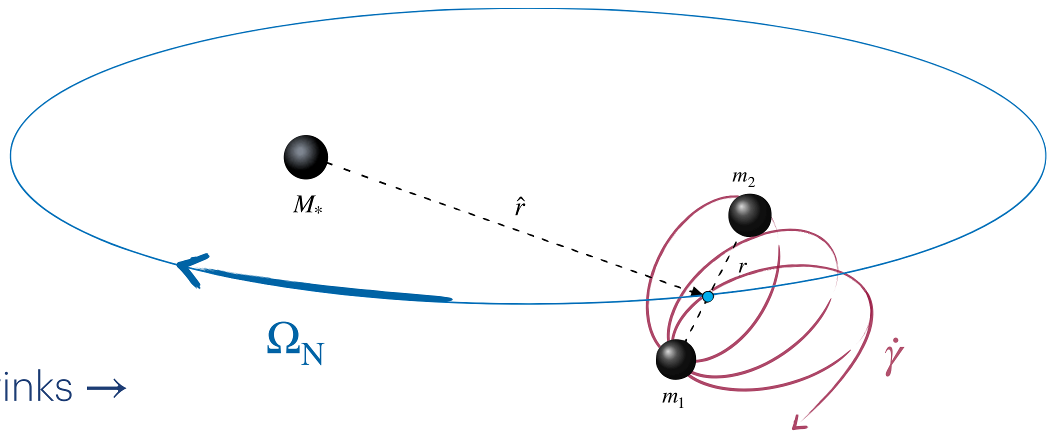
Triple system hierarchical in the **masses** $m_1, m_2 \ll M_*$

→ ***Relativistic description*** ★ we can probe the **strong-field regime**

Precession Resonance in Newtonian regime

[A. Kuntz, *Phys.Rev.D* 105 (2022) 2, 024017]

Resonance condition $2\,\dot{\gamma} = p\,\Omega_{\mathrm{N}}$



$\Omega_{\mathrm{N}} = \sqrt{GM_*/\hat{a}}$ → Newtonian frequency associated with the Keplerian motion of the binary around M_*

Inner binary emits gw during inspiral motion → its orbit shrinks → precession timescale $\sim 1/\dot{\gamma}$ shortens.

During its evolution up to merger, the inner binary passes through all the resonance conditions

Resonances amplify the eccentricity of the binary system thus leaving a clear imprint on the emitted gw

We want to see what happens when being in a strong gravity regime

The main difference is that in a relativistic spacetime there are multipole fundamental frequencies associated to bound motion

Fundamental frequencies in strong-gravity

In Newtonian description → only 1 frequency

In general relativistic description → multipole fundamental frequencies [Schmidt (2002), Flanagan & Hinderer (2008), Fujita & Hikida (2009)]

In strong-gravity regime → **Richer resonance spectrum**

In Schwarzschild spacetime $(\hat{t}, \hat{r}, \hat{\theta}, \hat{\phi})$ the fundamental frequencies, for bounded motion, wrt proper time are:

$$\omega_{\hat{t}}, \quad \omega_{\hat{r}}, \quad \omega_{\hat{\theta}}, \quad \omega_{\hat{\phi}}$$

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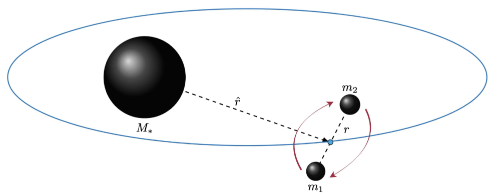
Important: there is **another fundamental frequency**, independent of the others:

→ $\omega_{\hat{\Psi}}$ frequency associated to **Marck's angle** $\hat{\Psi}$ [M. van de Meent, *Class.Quant.Grav.* 37 (2020) 14, 145007]

The **Marck's angle** $\hat{\Psi}$ ensures that the local inertial system associated with the inner binary is parallel-transported along the geodesic



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The frequency $\omega_{\hat{t}}$ accounts for gravitational time dilation between the proper time $\hat{\tau}$, which is the local time of the inner binary, and $\hat{t}$, the time measured by an asymptotic observer.

We use this to write frequencies wrt asymptotic time $\hat{t}$ → $\Omega_{\hat{r}} = \dfrac{\omega_{\hat{r}}}{\omega_{\hat{t}}}, \quad \Omega_{\hat{\theta}} = \dfrac{\omega_{\hat{\theta}}}{\omega_{\hat{t}}}, \quad \Omega_{\hat{\phi}} = \dfrac{\omega_{\hat{\phi}}}{\omega_{\hat{t}}}, \quad \Omega_{\hat{\Psi}} = \dfrac{\omega_{\hat{\Psi}}}{\omega_{\hat{t}}}$

with $(\Omega_{\hat{r}}, \quad \Omega_{\hat{\theta}}, \quad \Omega_{\hat{\phi}}, \quad \Omega_{\hat{\Psi}}) \to \Omega_{\rm N}$ in the Newtonian limit

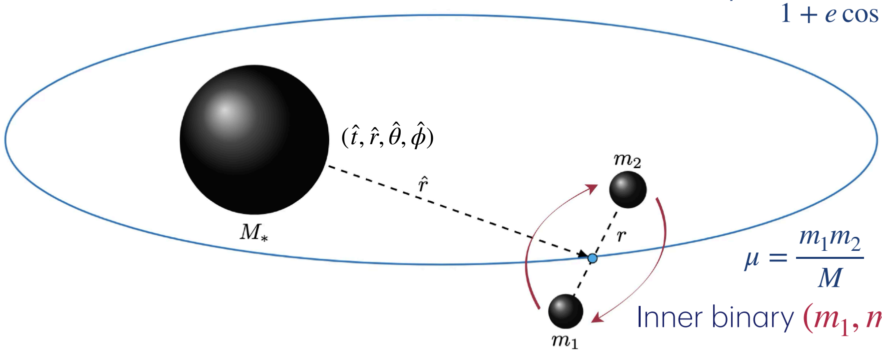
Due to spherical symmetry, we set $\hat{\theta} = \pi/2$ and therefore $\Omega_{\hat{\theta}}$ does not appear in our results.

Tidal Hamiltonian

The hamiltonian that accounts for the tidal interaction at the quadrupole order is given by

$$\mathcal{H}^{tidal}_{\text{quad}} = \frac{\mu}{2} c^2 r^2 \ \mathcal{E}^q$$

$\mathcal{E}^{\mathrm{q}}$ → scalar electric quadrupole tidal moment induced by M_*



After averaging over inner orbit and writing explicitly the electric tidal moments, it reads

$$\langle \mathcal{H}^{tidal}_{\text{quad}} \rangle = \frac{GM_*}{\hat{r}^3} \frac{\mu a^2}{2} \Bigg[\frac{2+3e^2}{2} + 3\frac{\hat{L}^2}{c^2 \hat{r}^2} \frac{2+3e^2-5e^2\cos 2\gamma}{4} \sin^2 I$$
$$-3\left(1+\frac{\hat{L}^2}{c^2\hat{r}^2}\right)\left(\frac{2+3e^2+5e^2\cos 2\gamma}{4}\cos^2(\hat{\Psi}-\vartheta) + \frac{2+3e^2-5e^2\cos 2\gamma}{4}\sin^2(\hat{\Psi}-\vartheta)\cos^2 I\right.$$
$$\left. + \frac{5e^2}{2}\sin 2\gamma \cos(\hat{\Psi}-\vartheta)\sin(\hat{\Psi}-\vartheta)\cos I\right)\Bigg]$$

[M. Cocco, G. Grignani, T. Harmark, MO and D. Pica, arXiv:2505.15901]

Action-angle variables

By employing the action-angle formalism, the geodesic motion of the inner binary around the SMBH can be solved by expressing $(\hat{r}, \hat{\Psi})$ in terms of angle variables $(q_{\hat{r}}, q_{\hat{\Psi}})$ satisfying

$$q_{\hat{r}} = \Omega_{\hat{r}}\hat{t}, \quad q_{\hat{\Psi}} = \Omega_{\hat{\Psi}}\hat{t}$$

The procedure is also commonly used for the more general Kerr spacetime [Bini & Geralico (2016), van de Meent (2019)]

Comment: only two fundamental frequencies, $\Omega_{\hat{r}}$ and $\Omega_{\hat{\Psi}}$, will enter the resonance condition.

This is because only the generalized coordinates $q_{\hat{r}}$ and $q_{\hat{\Psi}}$ appear in the Hamiltonian.

Generalizing to a Kerr black hole, an additional fundamental frequency would arise, associated with the polar motion $\rightarrow$ $\Omega_{\hat{\theta}}$.

Precession resonance in strong-gravity: perturbative analysis

[M. Cocco, G. Grignani, T. Harmark, MO and D. Pica, arXiv:2505.15901]

To derive analytically the precession resonance we perform an **expansion for small eccentricity** $\hat{e}$ of the outer orbit.

This is done by expanding $\hat{r}$ and $\hat{\Psi}$ in powers of $\hat{e}$, treating them as functions of the generalized angles q_μ with $\mu = \hat{r}, \hat{\Psi}$

$$\hat{r} = \hat{a}\left(1 - \hat{e}\cos q_{\hat{r}}\right) + \mathcal{O}(\hat{e}^2) \qquad \hat{\Psi} = q_{\hat{\Psi}} + 2\hat{e}\,\frac{\hat{\sigma}-4}{\hat{\sigma}-2}\sqrt{\frac{\hat{\sigma}-3}{\hat{\sigma}-6}}\sin q_{\hat{r}} + \mathcal{O}(\hat{e}^2) \qquad \hat{\sigma} = \hat{a}\frac{c^2}{GM_*}$$

The fundamental frequencies are given in terms of Elliptic Integrals and for small eccentricity $\hat{e}$ they read

$$\Omega_{\hat{r}} = \Omega_{\rm N}\sqrt{\frac{\hat{\sigma}-6}{\hat{\sigma}}} + \mathcal{O}(\hat{e}^2), \quad \Omega_{\hat{\Psi}} = \Omega_{\rm N}\sqrt{\frac{\hat{\sigma}-3}{\hat{\sigma}}} + \mathcal{O}(\hat{e}^2)$$

$\Omega_{\rm N} = \sqrt{GM_*/\hat{a}^3}$ → Newtonian frequency associated with the Keplerian motion of the outer orbit

It's easy to see that $(\Omega_{\hat{r}}, \Omega_{\hat{\Psi}}) \to \Omega_{\rm N}$ in the Newtonian limit $\hat{\sigma} \to \infty$

Precession resonance in strong-gravity: perturbative analysis

[M. Cocco, G. Grignani, T. Harmark, MO and D. Pica, arXiv:2505.15901]

Implementing this expansion in the hamiltonian of the system gives the following resonance conditions to first order in the outer eccentricity

$$2\dot{\gamma} = \Omega_{\hat{r}} \qquad 2\dot{\gamma} = -\Omega_{\hat{r}} + 2\,\Omega_{\hat{\Psi}} \qquad 2\dot{\gamma} = 2\,\Omega_{\hat{\Psi}} \qquad 2\dot{\gamma} = \Omega_{\hat{r}} + 2\,\Omega_{\hat{\Psi}}$$

This implies the general **resonance condition** of the form

$$q\,\dot{\gamma} = k\,\Omega_{\hat{r}} + l\,\Omega_{\hat{\Psi}}$$

$q = 2$ reflects the structure of the quadrupole moment of the inner binary.

The expression of the electric tidal moments $\mathscr{E}_{ij}$ selects the allowed values of k, l

Comparing to the Newtonian resonance condition

$$q\,\dot{\gamma} = p\,\Omega_{\mathrm{N}}$$

In strong-gravity regime → reacher spectrum of resonances

$\Omega_{\hat{r}},\ \ \Omega_{\hat{\Psi}} \rightarrow \Omega_{\mathrm{N}}$ in the Newtonian limit → $k + l \rightarrow p$ in the Newtonian limit

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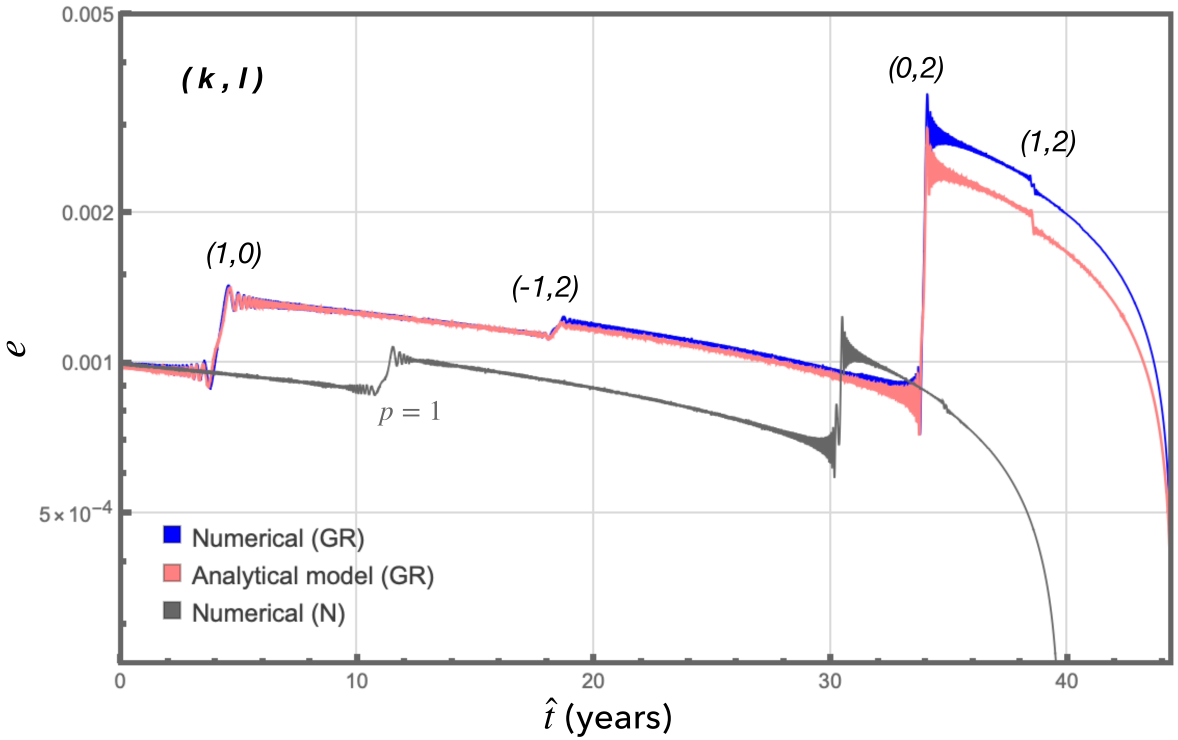

Parameters	Initial conditions
$M_* = 5\times 10^7\,M_\odot$, $M = 50\,M_\odot$, $\mu = 12.5\,M_\odot$ $\hat{a} \sim 7$ AU, $\hat{e} = 0.05$	$a_0 = 0.0014$ AU, $I_0 = 60^\circ$, $e_0 = 0.001$, $\gamma_0 = \theta_0 = 0^\circ$

Comments on parameters: $1AU \sim 1.5\times 10^8$ km

$7AU \sim 7\dfrac{2GM_*}{c^2} = 7\hat{R}_S$ → **We are deep in a strong-gravity regime**

N.B: We make sure that the inner binary is stable against tidal disruption from SMBH → $(1-\hat{e})\hat{a} \gtrsim a\left(\dfrac{3M_*}{M}\right)^{1/3}$

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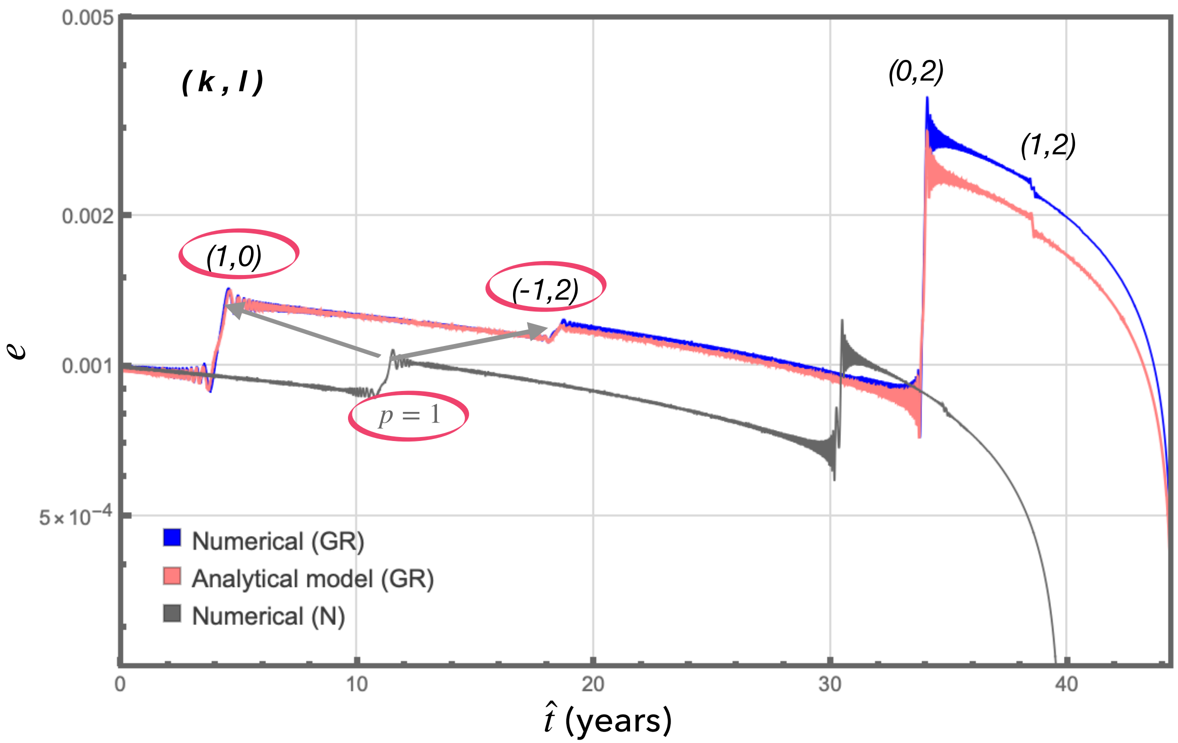

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GR

$$2\,\dot{\gamma} = k\,\Omega_{\hat{r}} + l\,\Omega_{\hat{\Psi}}$$

N

$$2\,\dot{\gamma} = p\,\Omega_{\mathrm{N}}$$

Precession resonance in strong-gravity: numerical analysis

[M. Cocco, G. Grignani, T. Harmark, MO and D. Pica, arXiv:2505.15901]

The numerical analysis is done by solving (numerically) the evolution equations for the orbital parameters of both the inner and outer orbits, without resorting to a perturbative expansion of the Hamiltonian and without the use of action-angle variables.

For the **inner orbit** the evolution equations are the **Lagrange Planetary Equations**...

$$\frac{da}{d\hat{t}} = -\frac{1}{u^{\hat{t}}}\sqrt{\frac{4a}{GM}}\frac{\partial\tilde{\mathcal{H}}}{\partial\beta}, \quad \frac{dI}{d\hat{t}} = \frac{1}{u^{\hat{t}}}\frac{1}{\sqrt{GMa(1-e^2)}\sin I}\left(\frac{\partial\tilde{\mathcal{H}}}{\partial\vartheta} - \cos I\frac{\partial\tilde{\mathcal{H}}}{\partial\gamma}\right), \quad \frac{d\beta}{d\hat{t}} = \frac{1}{u^{\hat{t}}}\left(\sqrt{\frac{4a}{GM}}\frac{\partial\tilde{\mathcal{H}}}{\partial a} + \frac{1-e^2}{\sqrt{GMae}}\frac{\partial\tilde{\mathcal{H}}}{\partial e}\right),$$

$$\frac{de}{d\hat{t}} = \frac{1}{u^{\hat{t}}}\left(\sqrt{\frac{1-e^2}{GMae^2}}\frac{\partial\tilde{\mathcal{H}}}{\partial\gamma} - \frac{1-e^2}{\sqrt{GMae}}\frac{\partial\tilde{\mathcal{H}}}{\partial\beta}\right), \quad \frac{d\gamma}{d\hat{t}} = \frac{1}{u^{\hat{t}}}\left(-\sqrt{\frac{1-e^2}{GMae^2}}\frac{\partial\tilde{\mathcal{H}}}{\partial e} + \frac{\cot I}{\sqrt{GMa(1-e^2)}}\frac{\partial\tilde{\mathcal{H}}}{\partial I}\right), \quad \frac{d\vartheta}{d\hat{t}} = -\frac{1}{u^{\hat{t}}}\frac{1}{\sqrt{GMa(1-e^2)}\sin I}\frac{\partial\tilde{\mathcal{H}}}{\partial I}.$$

....**+ radiation-reaction effects**

$$\left(\frac{da}{d\hat{t}}\right)_{\rm RR} = -\frac{1}{u^{\hat{t}}}\frac{64}{5}\frac{G^3\mu M^2}{c^5a^3}\frac{\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)}{\left(1-e^2\right)^{7/2}},$$

$$\left(\frac{de}{d\hat{t}}\right)_{\rm RR} = -\frac{1}{u^{\hat{t}}}\frac{304}{15}\frac{G^3\mu M^2}{c^5a^4}\frac{e\left(1+\frac{121}{304}e^2\right)}{\left(1-e^2\right)^{5/2}}$$

where $u^{\hat{t}} = d\hat{t}/d\hat{\tau}$ is the redshift factor

Precession resonance in strong-gravity: numerical analysis

[M. Cocco, G. Grignani, T. Harmark, MO and D. Pica, arXiv:2505.15901]

For the **outer orbit** we use

$$\hat{r} = \frac{\hat{a}(1-\hat{e}^2)}{1+\hat{e}\cos\hat{\psi}}$$

[Chandrasekar (1985)]

$\hat{\psi}$ → *relativistic anomaly*

We keep $\hat{a}$ and $\hat{e}$ fixed and evolve the relativistic anomaly $\hat{\psi}$ and Marck's angle $\hat{\Psi}$

$$\frac{d\hat{\psi}}{d\hat{t}} = \sqrt{\frac{GM_*}{\hat{a}^3(1-\hat{e}^2)^3}} \frac{\left(1+\hat{e}\cos\hat{\psi}\right)^2}{\sqrt{(2\delta-1)^2-4\delta^2\hat{e}^2}} \times \sqrt{1-2\delta(3+\hat{e}\cos\hat{\psi})}\left(1-2\delta(1+\hat{e}\cos\hat{\psi})\right),$$

where $\delta = GM_*/(c^2\hat{a}(1-\hat{e}^2))$

$$\frac{d\hat{\Psi}}{d\hat{t}} = \frac{1}{u^{\hat{t}}}\frac{\hat{E}\hat{L}}{c^2\hat{r}^2+\hat{L}^2}$$

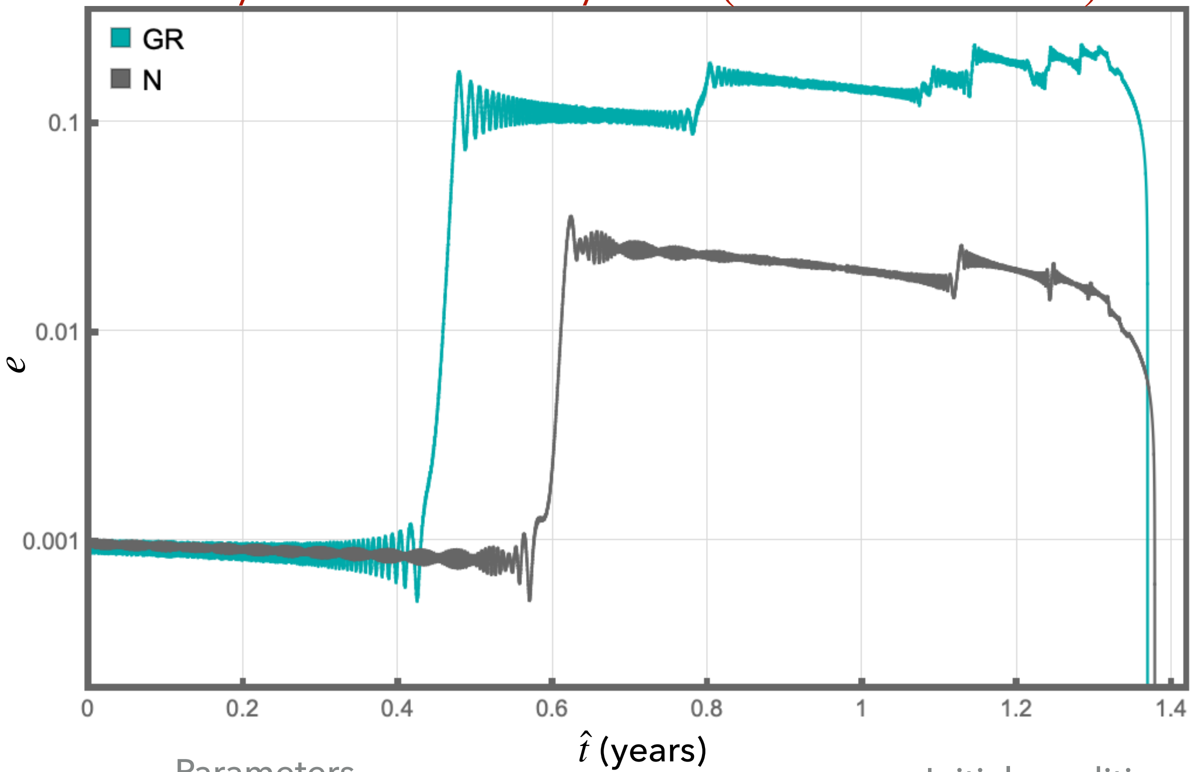
where $u^{\hat{t}} = d\hat{t}/d\hat{\tau}$ is the redshift factor

$\hat{E}, \hat{L}$ are the conserved energy per unit mass and conserved angular momentum per unit mass

Precession resonance in strong-gravity: numerical analysis

[M. Cocco, G. Grignani, T. Harmark, MO and D. Pica, arXiv:2505.15901]

Entirely in LISA sensitivity band $(0.001\text{Hz} - 0.1\text{Hz})$



Parameters

$$M_* = 4 \times 10^6\, M_\odot, \quad M = 50\, M_\odot, \mu = 12.5\, M_\odot$$

$$\hat{a} = 9\, R_S \sim 0.7\,\text{AU}, \quad \hat{e} = 0.4$$

Initial conditions

$$a_0 = 0.0006\,\text{AU}, \quad I_0 = 55^\circ,$$

$$e_0 = 0.001, \quad \gamma_0 = \theta_0 = 0^\circ$$

The numerical analysis confirms the validity of the resonance condition $2\,\dot{\gamma} = k\,\Omega_{\hat{r}} + l\,\Omega_{\hat{\Psi}}$ also for finite eccentricity $\hat{e}$

In the strong-gravity regime, the inner binary encounters more resonances compared to the Newtonian description

Having multiple resonance peaks within the LISA band provides an opportunity to discriminate precession resonance effects from other type of resonances

Conclusions and Future Directions

Strong-gravity effects significantly affect the precession resonances in hierarchical triple systems consisting of a compact binary orbiting a SMBH

In strong-gravity → richer and more intricate resonance spectrum

GR: $2\,\dot{\gamma} = k\,\Omega_{\hat{r}} + l\,\Omega_{\hat{\Psi}}$ ⟷ N: $2\,\dot{\gamma} = p\,\Omega_{\rm N}$

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Future directions

- Extend the analysis by modelling the central SMBH as a Kerr black hole:
  - additional frequencies
  - influence of the spin
- Include emission of gravitational waves associated with the outer binary's motion
- Include higher-order terms in $\hat{e}$ in the analytical model:
  - might reveal additional features of the resonance structure

Conclusions and Future Directions

Beyond electric tides $\mathcal{H}^{tidal}_{\rm quad} = \mathcal{H}^{tidal}_{\rm quad}\left(\mathcal{E}^q, \mathcal{B}^q_i\right)$

Magnetic tidal moments

- No Newtonian analog
- Typically vanish when considering secular effects
- But **not** in a **non-secular** scenario

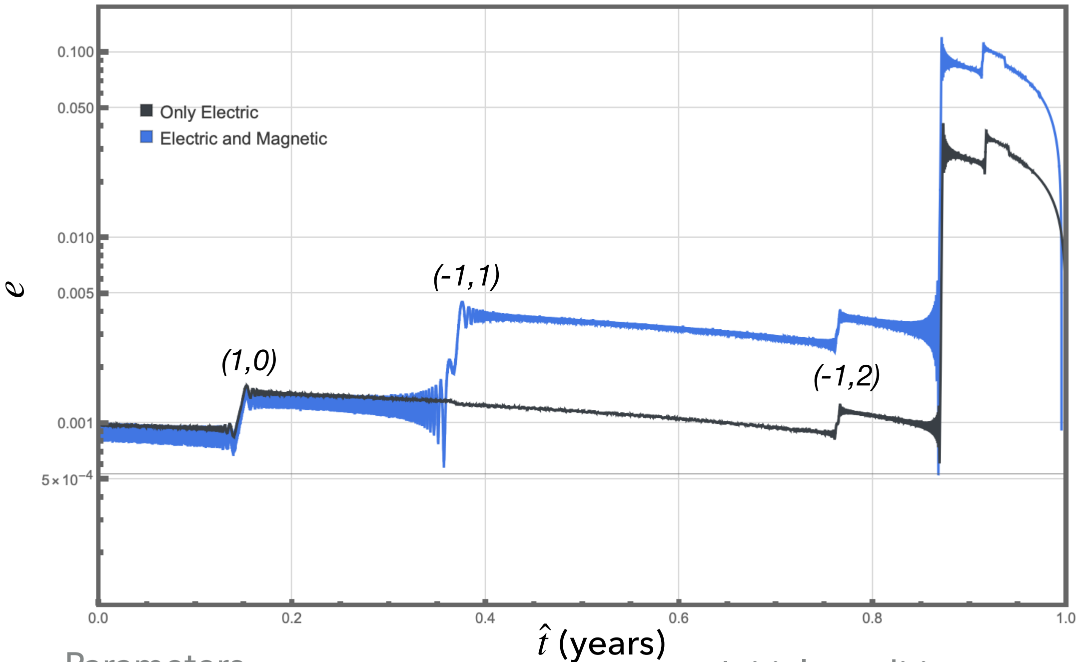
We already have preliminary results including magnetic tidal moments to study precession resonances

→ First example of a detectable effect due to magnetic tidal moments

📡 A new signature in gravitational waves?

Work in progress — stay tuned!

"Observable signature of magnetic tidal coupling in hierarchical triple systems"
M. Cocco, G. Grignani, T. Harmark, MO, D. Panella and D. Pica,, in preparation



Parameters

$M_* = 10^8\, M_\odot, \quad M = 50\, M_\odot, \mu = 12.5\, M_\odot$

$\hat{a} = 8\ GM_*/c^2 = 4R_S, \quad \hat{e} = 0.08$

Initial conditions

$a_0 = 0.0014\,\text{AU}, \quad I_0 = 25^\circ,$

$e_0 = 0.001, \quad \gamma_0 = \theta_0 = 0^\circ$

THE CENTER OF GRAVITY

Thank you for your attention!

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