A hypersurface model for Kerr black hole

Gang Chen(NBI)

Based on the work 2503.20538 w/

Emil Bjerrum-Bohr *Chenliang Su* *Tianheng Wang*

@SDU

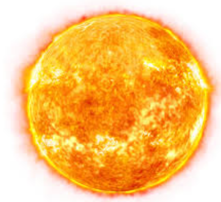
Motivation

- Gravitational waveform for Kerr black hole
- Test the no-hair hypothesis of black hole at dynamic level
- Fundamental theory for Kerr black hole?

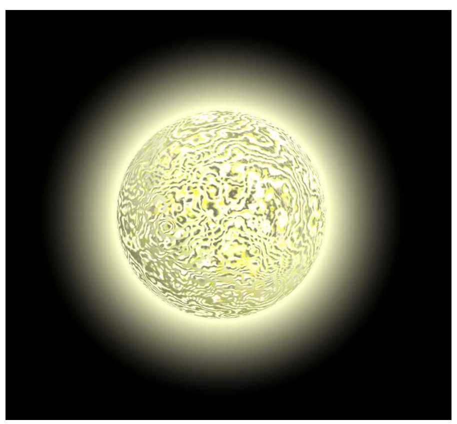
Background

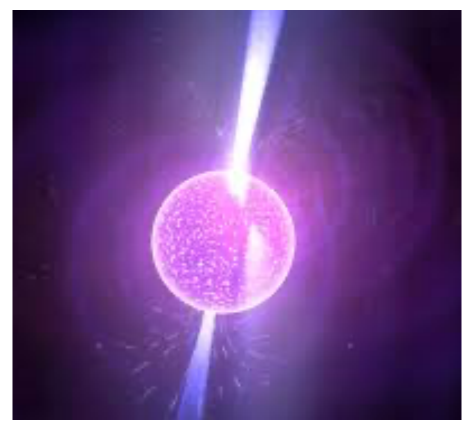
Gravitational collapse of a star

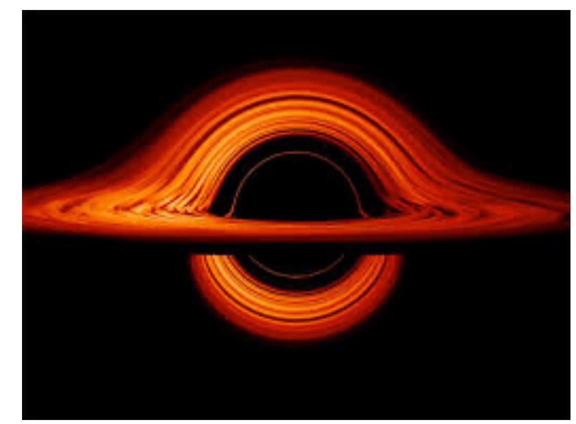
$1.4 g/cm^3$



$< 8M_\odot$ | $8M_\odot - 25M_\odot$ | $> 25M_\odot$



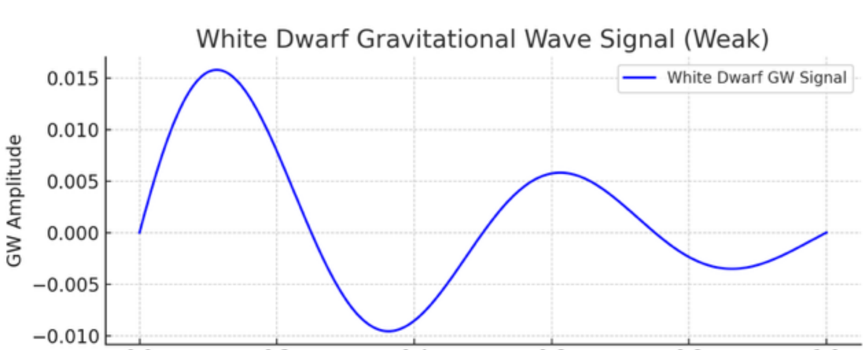


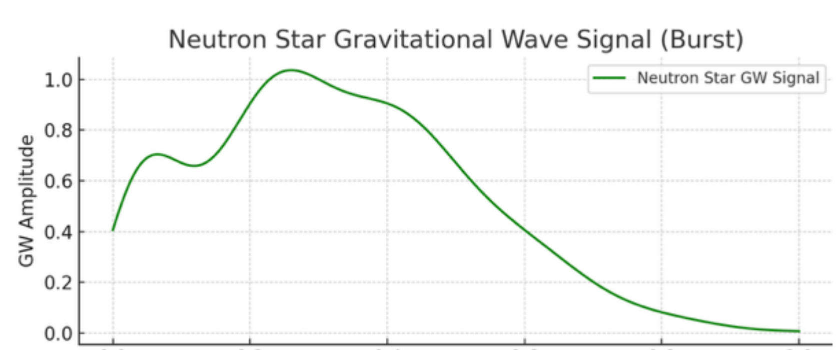


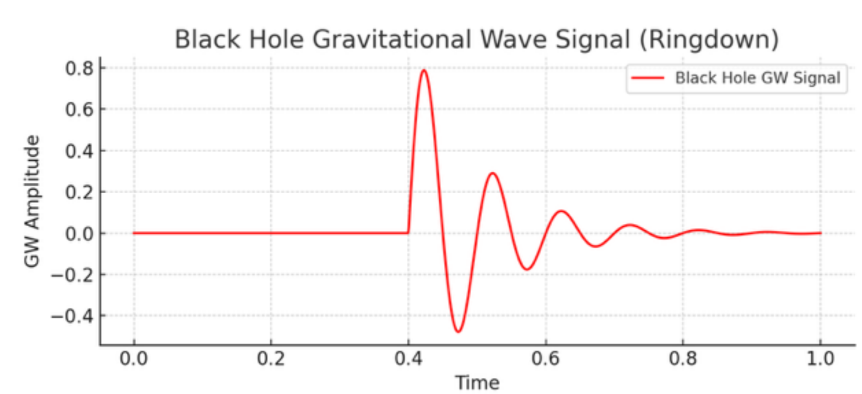
White dwarf: carbon-oxygen

Neutron star: neutron

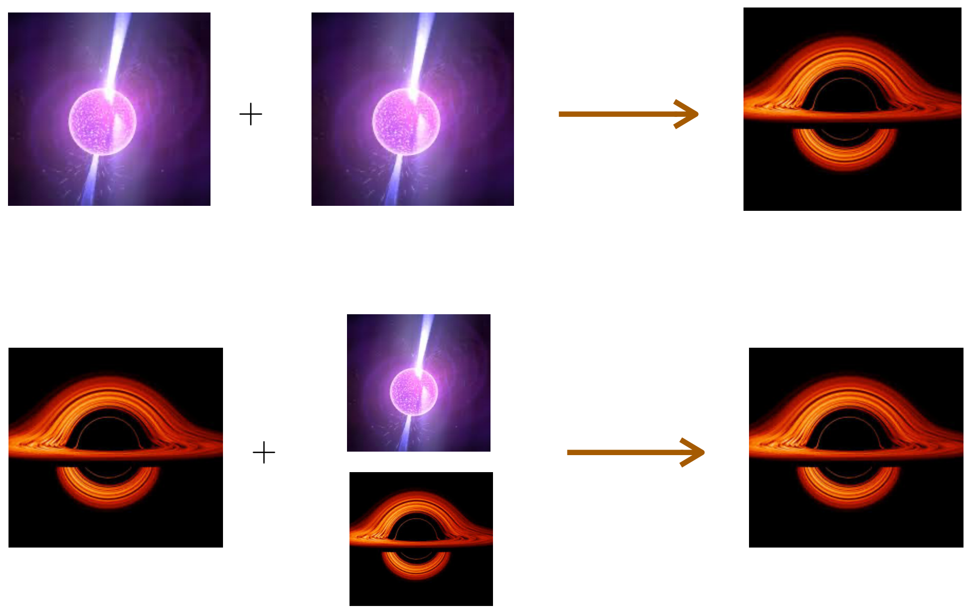
Black hole: effective geometry point

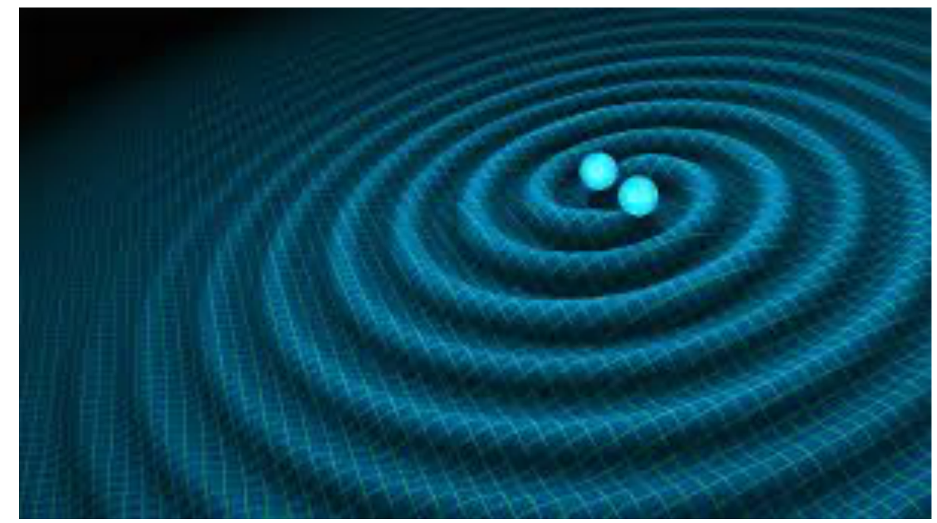


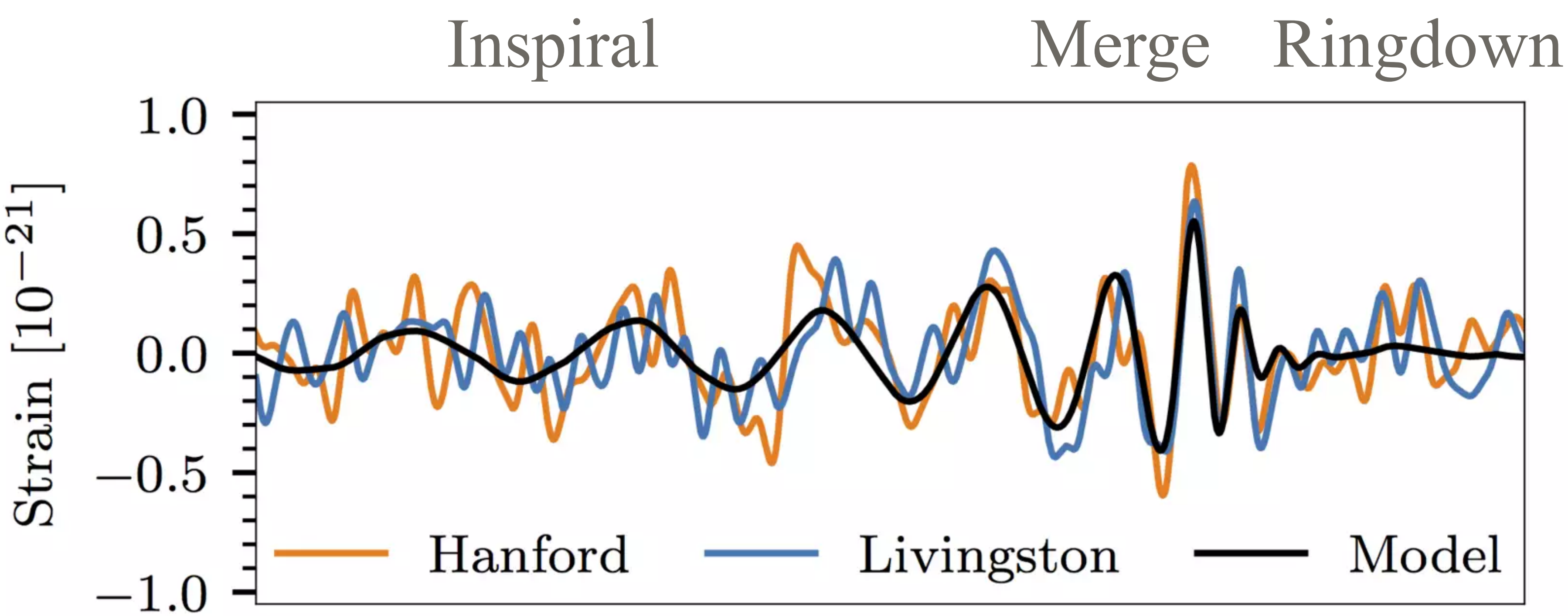




Merge of compact objects





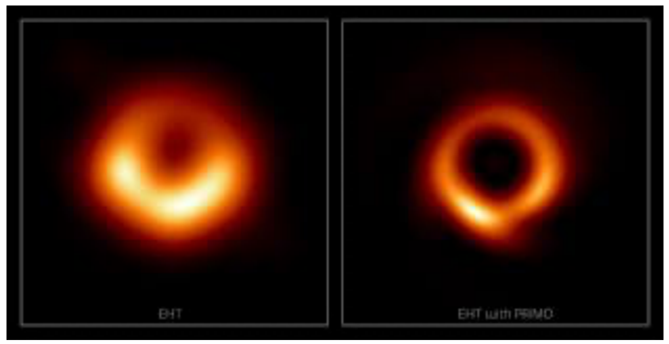


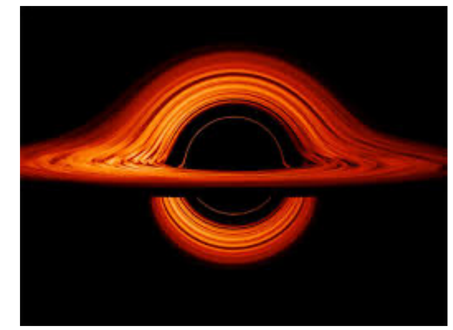
GW170104:© LIGO/Phys. Rev. Lett. 118, 221101

$m_1 = 31.2 M_\odot$ $m_2 = 19.4 M_\odot$ $M_f = 48.7 M_\odot, a_f = 0.64$

Shape of black holes

View from space





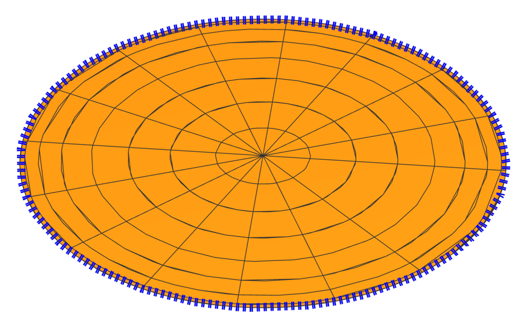
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Black hole support(geometry singularity)

Schwarzchild

Kerr

Israel,PRD(1970)



radius |a|

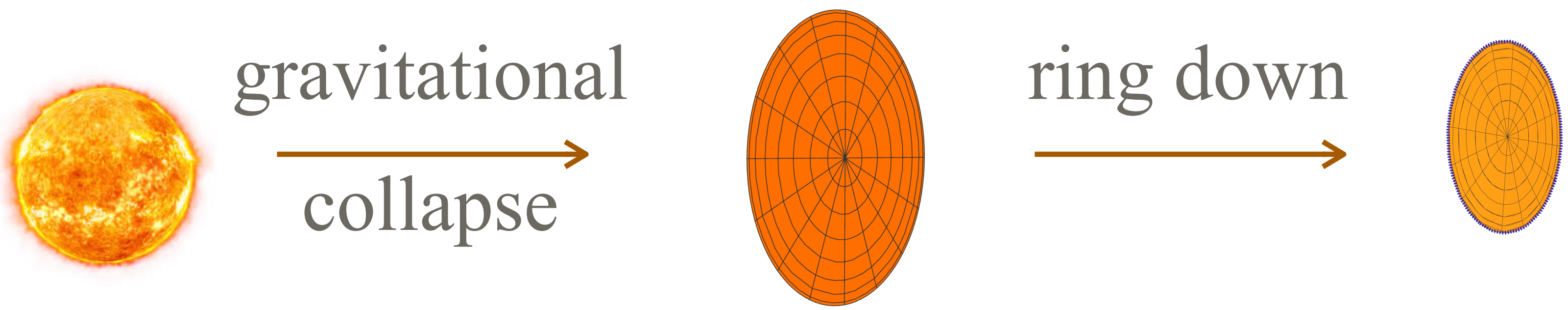
$$1km \sim 10^6 km$$

$$S = \int d\tau \dot{X}^\mu \dot{X}^\nu g_{\mu\nu}$$

$$S_{\text{Kerr}} = ???$$

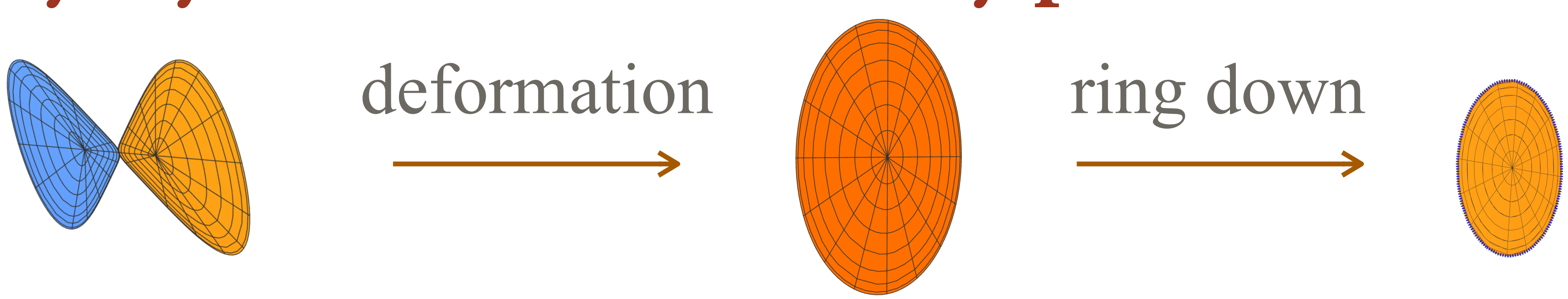
Related processes for hypersurface dynamic

Gravitational collapse



Binary Dynamics I: Two body problem

Binary Dynamics II: One-body problem



Main Task: Formulate the Kerr Action

Kerr action

Fundamental requirement for spinning extend body

- Lorentz invariant in flat background spacetime
- Naturally extend to curved background and keep differmorphism invariant
- Include the finite size effective
- Parameterise invariant of the inertial coordinate
- Free of multi time problem

Extend the Polyakov action in string theory

Spinning black holes in flat spacetime

Extra constraints

- 2+1 dimensional
- Inertial time independent
- Axial rotation symmetry

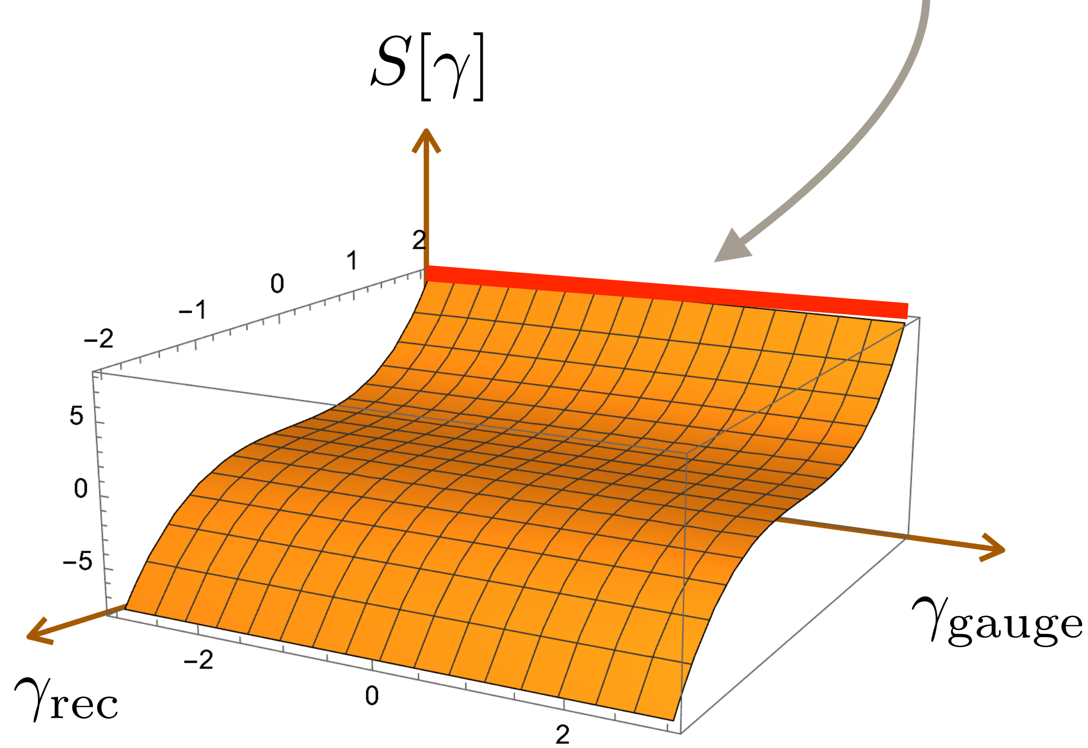
Worldvolumn metric γ: rigid reparameterize equivalent class(rec)

$$ds^2 = d\tau^2 - a_w^2\left(d\theta^2 + f(\theta)d\phi^2\right)$$

$$f(\theta) = \sin^2(\theta)$$

Action in flat spacetime

$$S_0 = -\frac{m}{8\pi a_w^2}\int_{\mathbb{S}^2\times\mathbb{R}} d^3\sigma\sqrt{\gamma}\left(\partial_a Z^\mu \partial_b Z^\nu \gamma^{ab}\eta_{\mu\nu} + 1\right),$$



Solution of EOM

State of black hole(state function)

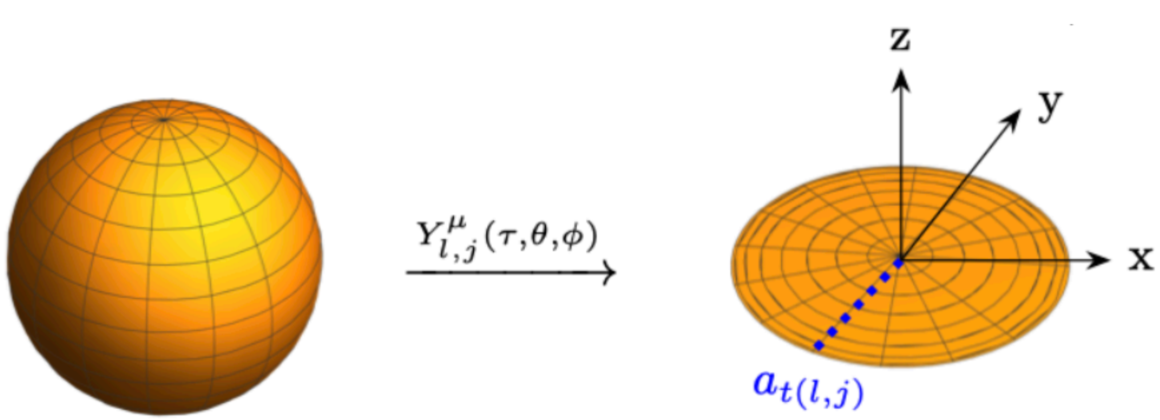
$$Z^\mu = X^\mu(\tau) + Y^\mu$$

$$X^\mu = x^\mu + v^\mu \tau$$

$$Y^\mu = \sum_{l>j\geq 0} Y^\mu_{l,j}$$

$$Y^\mu_{l,j} = a_w\Big(c_{l,j}\,\beta^\mu_{\rm x}\cos\big(\frac{\sqrt{l(l+1)}}{a_w}\tau + j\phi\big) + c'_{l,j}\,\beta^\mu_{\rm y}\sin\big(\frac{\sqrt{l(l+1)}}{a_w}\tau + j\phi\big)\Big)\mathcal{P}^j_l\big(\cos(\theta)\big)$$

rotational symmetry: $c_{l,j} = c'_{l,j}$



Spin relations

$$a_{t(l,j)} \equiv a_w c_{l,j}$$

$$S^{\mu\nu} = m|a|(\beta^\mu_{\rm x}\beta^\nu_{\rm y} - \beta^\mu_{\rm y}\beta^\nu_{\rm x})$$

$$|a| = \frac{\sqrt{l(l+1)}(l+j)!}{(2l+1)(l-j)!}a_w c^2_{l,j}$$

Action in curved background

assumption on effective geometric point

- only couple to metric or Riemann tensor $(\mathcal{D}Z)^2 \equiv \partial_a Z^\mu \partial_b Z^\nu \gamma^{ab} \mathcal{G}_{\mu\nu}(Z)$

$$S = -\frac{m}{8\pi a_w^2}\int_{\mathbb{S}^2\times\mathbb{R}} d^3\sigma\sqrt{\gamma}\Big[(\mathcal{D}Z)^2+1$$
$$+a_w^2\mathcal{R}_{\mu\nu\rho\lambda}(Z)\partial_a Z^\mu\partial_b Z^\rho\partial_c Z^\nu\partial_d Z^\lambda\Big(\gamma^{ab}\gamma^{cd}(\sum_{j=0}^{\infty}\xi_{2j+1}[(\mathcal{D}Z)^2]^j)+a_w^2\varrho^{acbd}(\sum_{j=0}^{\infty}\xi_{2j+2}[(\mathcal{D}Z)^2]^j)\Big)\Big]$$

Completeness

Fit any three point amplitude
(related to the spin vector "k.a")

Unique

Metric fix the free parameters uniquely
(consistent with no-hair hypothesis)

Checked upto spin 99 order

Integral techniques

Typical term in amplitude

$$\int_\tau \int_\theta \int_\phi F(\theta)\partial_{a_1} Y^{\mu_1} \cdots \partial_{a_n} Y^{\mu_n} Y^{\nu_1} \cdots Y^{\nu_m}\, e^{i\,\tau\, k\cdot v}$$

ϕ integral

Remove tau dependents in Y

m+n being odd : 0

m+n being even: $[\partial_\tau Y^\mu Y^\nu] \to [S^{\mu\nu}]$

$$[Y^\mu Y^\nu], \quad [\partial_b Y^\mu \partial_b Y^\nu] \to \left[v^\mu v^\mu - \eta^{\mu\nu} - \frac{a^\mu a^\nu}{|a|^2}\right]$$

θ integral

Fit hypersurface radius to Kerr disc radius

$(1,1)$ $\quad \cosh\left(\frac{a_{t(1,1)}}{|a|} k\cdot a\right), \sinh\left(\frac{a_{t(1,1)}}{|a|} k\cdot a\right) \longrightarrow \cosh\left(k\cdot a\right), \sinh\left(k\cdot a\right)$

$(l, l \text{ or } l-1), l>1$ entire hypergeometry fun.

others general entire fun.

τ integral

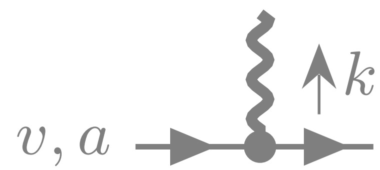
Heavy mass on-shell condition $\delta(k\cdot v)$

Kerr action in general background

By fitting Kerr amplitude

Justin Vines 2017
Arkani-Hamed, Huang, and Huang;
Guevara, Ochirov, Vines

$$A^{(1,1)}_{3,\mathrm{Kerr}}(v,a,k) =$$



$$= -i\kappa(mv\cdot\varepsilon)\left(mv\cdot\varepsilon\cosh\left(k\cdot a\right) + ik\cdot S\cdot\varepsilon\frac{\sinh\left(k\cdot a\right)}{k\cdot a}\right)$$

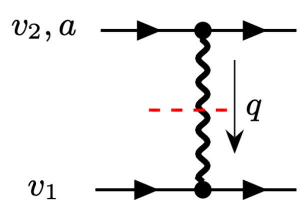
Kerr action

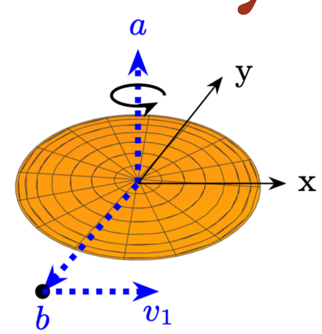
$$S = -\frac{m}{8\pi a_w^2}\int_{\mathbb{S}_2\times\mathbb{R}} d^3\sigma\sqrt{\gamma}\left[(\mathcal{D}Z)^2 + 1\right.$$
$$\left. + a_w^2\mathcal{R}_{\mu\nu\rho\lambda}(Z)\partial_a Z^\mu\partial_b Z^\rho\partial_c Z^\nu\partial_d Z^\lambda\left(\frac{1}{3}\gamma^{ab}\gamma^{cd} - \frac{1}{3^4}a_w^2\varrho^{acbd} + \frac{4}{3^4}a_w^2\varrho^{acbd}(\mathcal{D}Z)^2\right)\right]$$

Three extra terms

Induced spacetime singularity

Detect singularity from 1PM bending angle

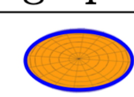
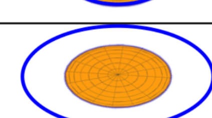
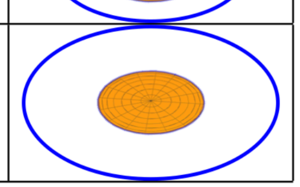




$$\chi_P^{(1,1)} = \frac{\pi\kappa^2\sqrt{s}}{16\pi^2(y^2-1)}\Bigg[(2y^2-1)\left[\frac{27|b|^2}{16|a|^3}\operatorname{arctanh}\left(\frac{|a|}{|b|}\right) - \frac{27|b|}{16|a|^2} + \frac{7}{16|a|}\operatorname{arctanh}\left(\frac{|a|}{|b|}\right)\right] + y\sqrt{y^2-1}\left[\frac{6}{|a|} - \frac{6|b|}{|a|^2}\operatorname{arctanh}\left(\frac{|a|}{|b|}\right)\right]\Bigg]$$

$$\begin{aligned}\chi_P^{(2,1)} = \frac{\kappa^2\sqrt{s}}{16\pi^2(y^2-1)}\pi\Bigg[& (2y^2-1)\Bigg({}_3F_2\left(3,\frac{7}{2},\frac{7}{2};\frac{19}{4},\frac{21}{4};\frac{9a_{t(2,1)}^2}{4|b|^2}\right)\frac{2700|a|^2a_{t(2,1)}^4}{17017|b|^7} \\ & + {}_3F_2\left(1,\frac{3}{2},\frac{3}{2};\frac{7}{4},\frac{9}{4};\frac{9a_{t(2,1)}^2}{4|b|^2}\right)\frac{3a_{t(2,1)}^2}{5|b|^3} \\ & + {}_3F_2\left(2,\frac{5}{2},\frac{5}{2};\frac{15}{4},\frac{17}{4};\frac{9a_{t(2,1)}^2}{4|b|^2}\right)\frac{1395|a|^2a_{t(2,1)}^2}{4004|b|^5} \\ & + {}_3F_2\left(1,\frac{3}{2},\frac{3}{2};\frac{11}{4},\frac{13}{4};\frac{9a_{t(2,1)}^2}{4|b|^2}\right)\frac{25|a|^2}{84|b|^3} + \frac{1}{|b|}\Bigg) \\ & - y\sqrt{y^2-1}\Bigg({}_3F_2\left(\frac{3}{2},2,\frac{5}{2};\frac{11}{4},\frac{13}{4};\frac{9a_{t(2,1)}^2}{4|b|^2}\right)\frac{12|a|a_{t(2,1)}^2}{7|b|^4} \\ & + {}_3F_2\left(\frac{1}{2},1,\frac{3}{2};\frac{7}{4},\frac{9}{4};\frac{9a_{t(2,1)}^2}{4|b|^2}\right)\frac{2|a|}{|b|^2}\Bigg)\Bigg]\end{aligned}$$

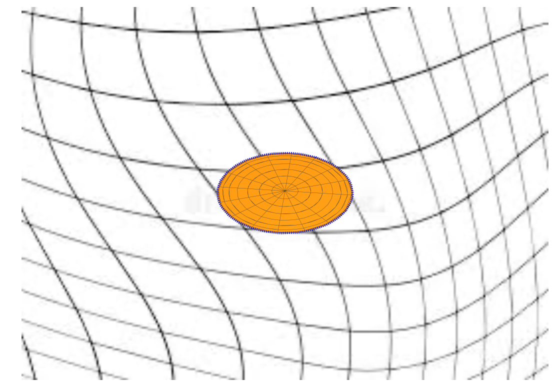
Singular rings

mode	ring radius	disc radius	graph
$(1,1)$	$\lvert a\rvert$	$\lvert a\rvert$	
$(2,1)$	$\frac{3}{2}a_{t(2,1)}$	$a_{t(2,1)}$	
$(2,2)$	$3a_{t(2,2)}$	$a_{t(2,2)}$	

Action	Singularity
Polyakov(1,1)	Log
Kerr(1,1)	Simple pole
General (1,1)	Mix
Higher Mode	Complex

Conclusion

- A Kerr action in hypersurface model
- Can apply to dynamics of Kerr in "fixed curved Background"



Outlook

- High point amplitude from Kerr action
- Test no-hair hypothesis in (1,1) mode by general action
- High mode physics and relation to unstable black hole
- Quantisation

“Thanks!”