A BPS Road to Holography: Decoupling Limits and Non-Lorentzian Geometries

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Vetenskapsrådet

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based on work:

2410.03591 (JHEP), 2311.10564 (PRL), 2502.20310 (JHEP) (Blair,Lahnsteiner,NO,Yan)

2506.19720 (Blair,NO,Yan)

2501.10178 (JHEP) (Harmark,Lahnsteiner,NO)

& earlier papers with Harmark,Hartong, and Bidussi/Menculini/Oling/Yan

Introduction/Motivation

- D-branes & their BPS nature underly major advances in ST
  - AdS/CFT correspondence (dual descriptions of D-branes)
  - matrix theory proposal (large N limit of D-brane wv. theory)

→ decoupling limits probe rich physics
  - simplification by removing part of spectrum
  - access to non-perturbative regimes

revisit decoupling limits in light of recent advances

- intruiging relations to non-Lorentzian corners of string theory
  e.g. NRST as a simpler corner of (relativistic) ST

Gomis,Ooguri(2000),Danielsson,Guijosa,Kruczenski(2000)
Harmark,Hartong,NO(2017)/Bergshoeff,Gomis,Yan(2018) /& many papers since

- new boostrap techniques/study of amplitudes in BFSS

Han,Hartnoll,Kruthoff(2020)/Dorey,Mouland,Zhao(2022)/Mouland(2023),….
Tropper,Wang(2023)/ Herderschee, Maldacena (2023)/Komatsu et al (2024)…

Questions

1. What are the guiding principles for mapping out self-consistent decoupling limits in ST ?

   Guided by a BPS road..

2. In the context of holography, what is the role of the ten-dimensional non-Lorentzian (NL) geometry coupled to matrix theory on the D-branes (or NRST for F-strings) ?

   To Holography …

3. How is the ten-dimensional bulk geometry generated intrinsically ?

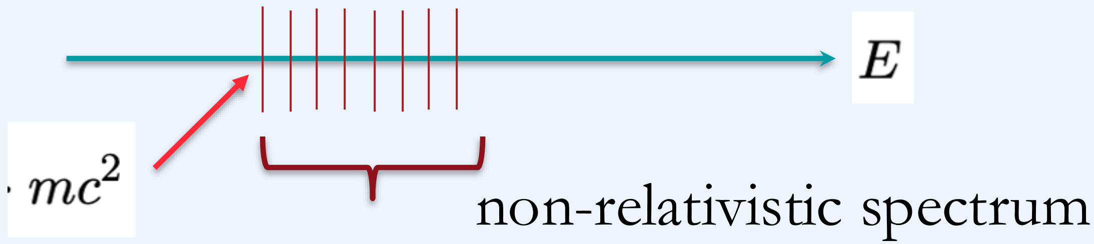
   And back via TTbar

related work on holography and NL geometries:

Avila,Guijosa,Olmedo(2023)
Lambert,Smith (2024 (3))
Fontanella,Nieto Garcia (2024 (3))
Guijosa (2025)
Bergshoeff,Lambert,Smith (2025)
Bergshoeff,Grosvenor,Romano,Yan (2025)

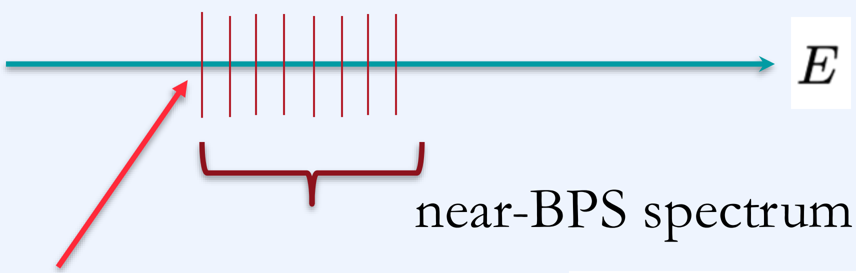
Part 1: BPS decoupling limits

Anatomy of BPS decoupling limits



$$E = \sqrt{(mc^2)^2 + (pc)^2} - \Phi q = \sqrt{(mc^2)^2 + (pc)^2} - 1 \cdot mc^2 \simeq \frac{p^2}{2m}$$

- same for near-BPS limit



BPS state (charge = mass)

$$E = \sqrt{M_1^2 + M^2} - A_1 M_1 \simeq \frac{M^2}{2M_1}$$

$$M_1 \to \omega^2, M \to \omega, \omega \to \infty \quad , \quad A_1 \to 1$$

- scale to supersymmetric background, cancel divergence with critical electric field

Matrix Theory: A BPS perspective

Matrix theory arises from BPS decoupling limit of D0-brane wv.

→ relies on fact that tension=charge & fine tuning RR gauge field

$$S_{\mathrm{D0}} = -\frac{1}{g_{\mathrm{s}}\sqrt{\alpha'}} \int \mathrm{d}\tau \sqrt{-\dot{X}^{\mu}\,\dot{X}_{\mu}} + \frac{1}{\sqrt{\alpha'}} \int C^{(1)}\,, \qquad \mu = 0\,, \cdots, 9\,.$$

take limit:

$$g_{\mathrm{s}} \to \omega^{-3/2}\, g_{\mathrm{s}}\,, \qquad X^0 \to \sqrt{\omega}\, X^0\,, \qquad X^i \to \frac{X^i}{\sqrt{\omega}}\,, \qquad C^{(1)} \to \omega^2\, g_{\mathrm{s}}^{-1}\,\mathrm{d}X^0\,,$$

$$\omega \to \infty.$$

gives NR action: $$S = \frac{1}{g_{\mathrm{s}}\sqrt{\alpha'}} \int \mathrm{d}\tau \left(\tfrac{1}{2}\,\dot{X}^i\,\dot{X}^i + 2\,\psi^{\intercal}\,\dot{\psi}\right),$$

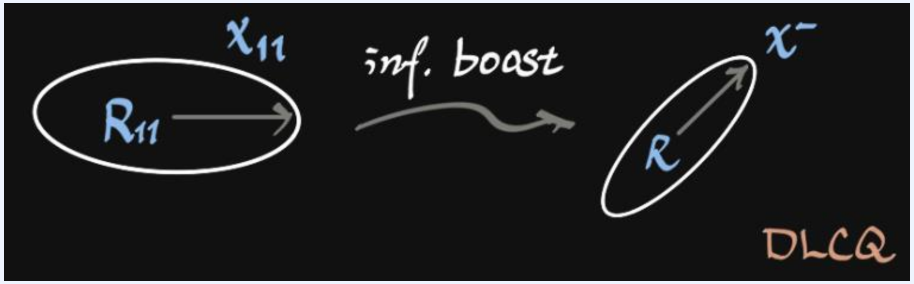
BFSS

non-abelian (& SUSY) generalization is BFSS:

$$S_{\mathrm{BFSS}} = \frac{1}{R}\int \mathrm{d}\tau\, \mathrm{tr}\left[\tfrac{1}{2}\dot{X}^i\dot{X}^i + \tfrac{1}{4}[X^i\,,\,X^j]\,[X_i\,,\,X_j] + 2\Big(\psi^\intercal\,\dot{\psi} - \psi^\intercal\,\gamma^i\,[\psi\,,\,X^i]\Big)\right],$$

- BFSS (1996) conjecture: N,R to infinity = M-theory on flat spacetime
- Susskind (1997) fixed N: DLCQ of M-theory

-Seiberg/Sen (1997):
light like circle as
infinite boost of spacelike circle



Prelude: Non-Lorentzian geometry

- start with flat relativistic metric

$$ds^2 = \omega\, dx^A\, dx^B\, \eta_{AB} + \omega^{-1}\, dx^{A'}\, dx^{A'},$$

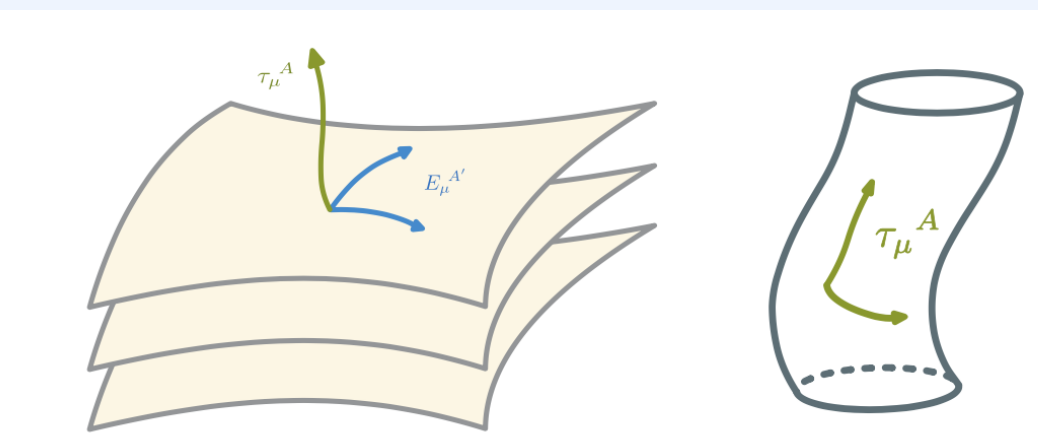
A =0..p longitudinal directions
A'= p+1,.. 9-p transverse directions

$\omega \to \infty$. geometry → non-Lorentzian: p-brane Newton-Cartan (NC) geometry

→ local SO(1,p) and SO(9-p) geometry
& p-brane boost symmetry

$$\delta_G x^A = 0\,, \qquad \delta_G x^{A'} = \Lambda^{A'}{}_A\, x^A$$

- generalize to curved bgrs. $dx^A \to \tau_\mu{}^A\, dx^\mu$ $\quad dx^{A'} \to E_\mu{}^{A'}\, dx^\mu$



$$\tau_{\mu\nu} = \tau_\mu{}^A\, \tau_\nu{}^B\, \eta_{AB}$$

$$E_{\mu\nu} = E_\mu{}^{A'}\, E_\nu{}^{A'}$$

Matrix 0-brane theory (M0T)

DLCQ M-Theory $\xrightarrow[\textit{compactification}]{\textit{lightlike}}$ **M0T: BFSS Matrix theory**

can go beyond the limit in flat space:

curved space of M0T

$$G_{\mu\nu} = \omega\,\tau_{\mu\nu} + \omega^{-1}\,E_{\mu\nu}\,, \qquad \Phi = \varphi - \tfrac{3}{2}\,\ln\omega\,, \qquad B^{(2)} = b^{(2)}\,,$$

$$C^{(1)} = \omega^2\,e^{-\varphi}\,\tau^0 + c^{(1)}\,, \qquad C^{(q)} = c^{(q)} \quad \text{if } q \neq 1\,.$$

$$\omega \to \infty.$$

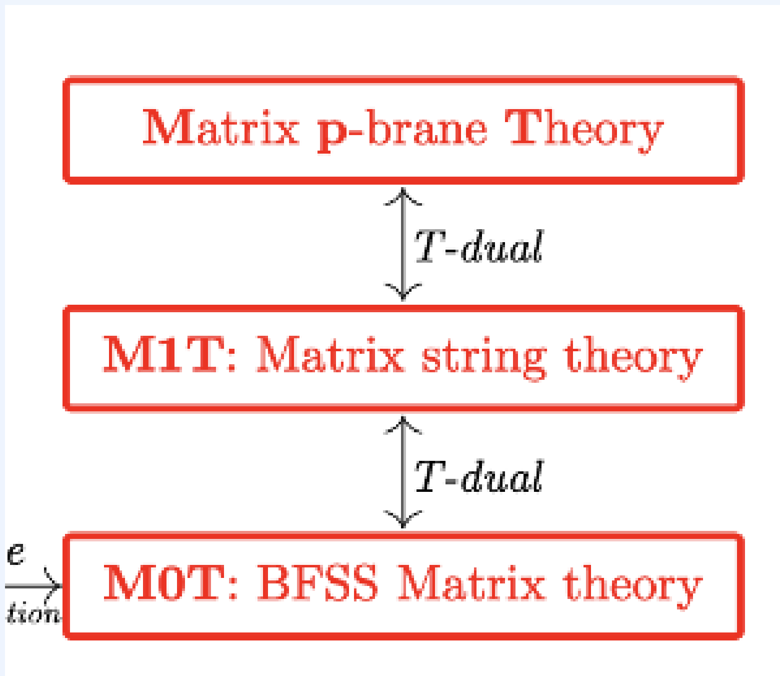
Matrix theory of D0-branes (M0T) couples to non-Lorentzian 10D geometry:

M0T target space = Newton-Cartan geometry + other string fields

$$S_{\text{D0}}^{\text{M0T}} = \frac{1}{2\sqrt{\alpha'}}\int d\tau\,e^{-\varphi}\,\frac{\dot{X}^\mu\,\dot{X}^\nu\,E_{\mu\nu}}{\dot{X}^\mu\,\tau_\mu{}^0} + \frac{1}{\sqrt{\alpha'}}\int c^{(1)}\,.$$

- as opposed to earlier work: now applied to the full type II string theory containing all possible extended objects and in general backgrounds

MpT in curved spacetime



via T-duality → MpT = near-BPS decoupling limit of Dp-branes

MpT target space = p-brane NC geometry + other string fields

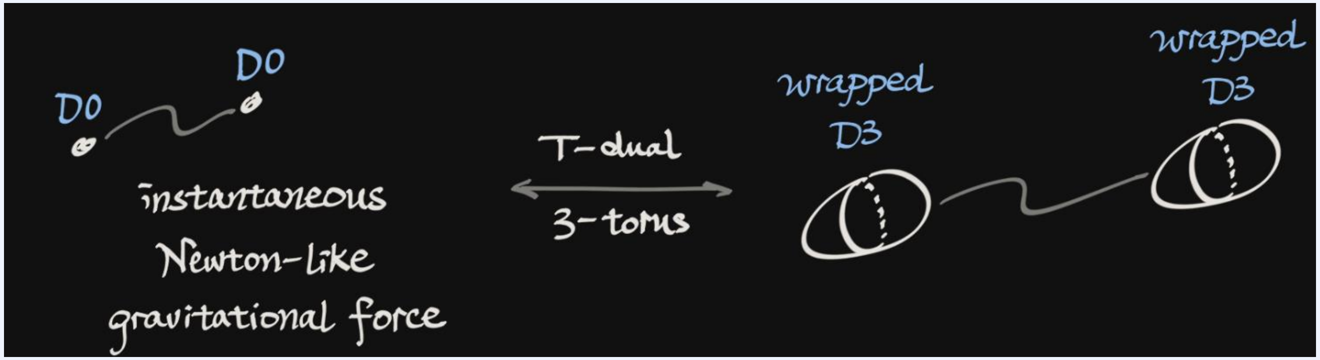
Dp-branes in MpT

can study wv. actions of generic Dq-branes and F-string in MpT
- form of actions related to BPS structure

e.g. Dp-branes fundamental dof in MpT:

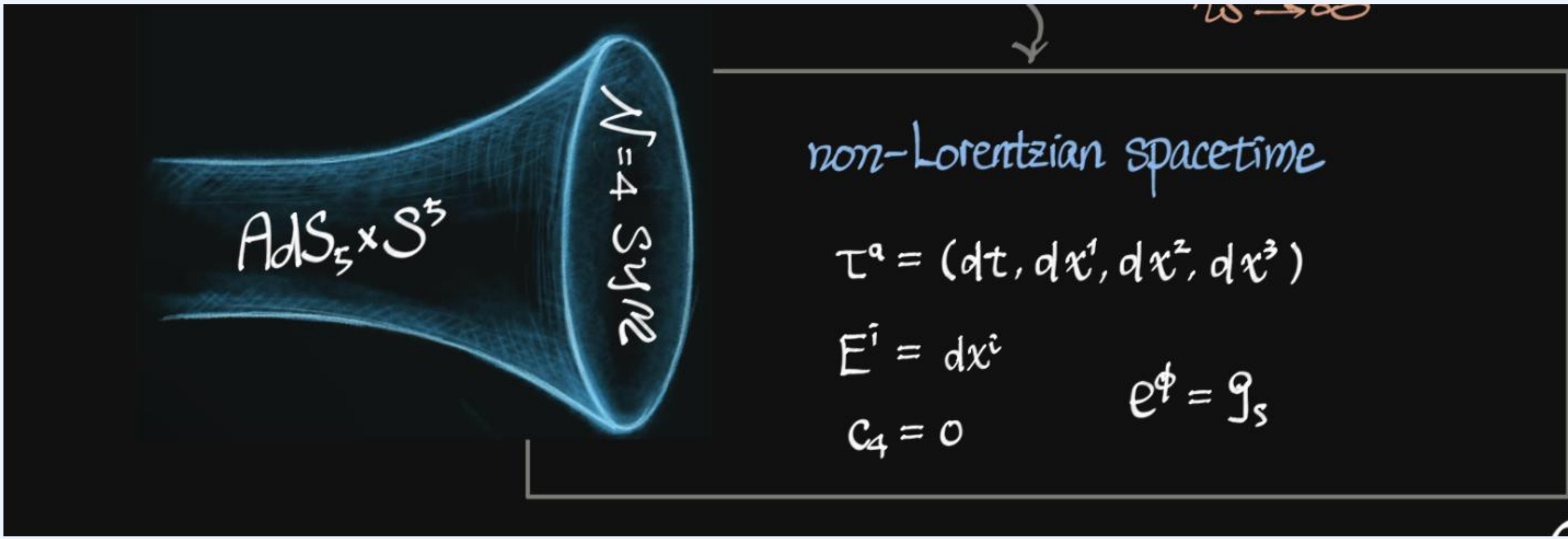
$$S_{\mathrm{D}p}^{\mathrm{MpT}} = -\frac{T_p}{2}\int \mathrm{d}^{p+1}\sigma\, e^{-\varphi}\sqrt{-\tau}\,\tau^{\alpha\beta}\,E_{\alpha\beta} + T_p\int c^{(p+1)}\,.$$

- wrapped Dp-branes behave like NR particles, experiencing Newton-like forces



- especially interesting is p=3:
  M3T → N=4 SYM sees 10D non-Lorentzian spacetime

Part 1I: Application to Holography in string theory



BPS decoupling = near-horizon in bulk

- BPS decoupling limit at asymptotic infinity
  → N=4 SYM coupled to non-Lorentzian M3T geometry

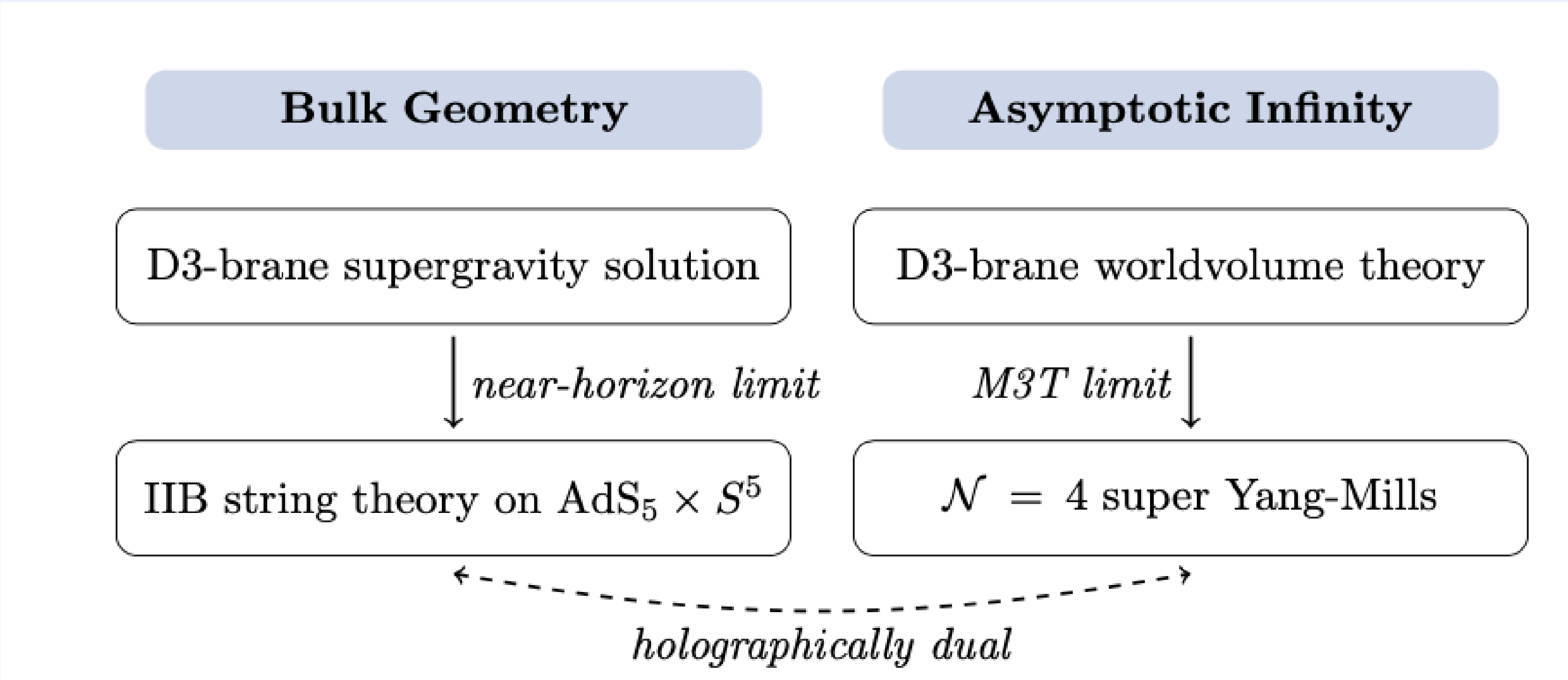
$$(X^0, \cdots, X^p) = \omega^{\frac{1}{2}} \, (t\,, x^1, \cdots, x^p)\,,$$
$$(X^{p+1}, \cdots, X^9) = \omega^{-\frac{1}{2}} \, (x^{p+1}, \cdots, x^9)\,,$$
$$G_s = \omega^{\frac{p-3}{2}} \, g_s$$

$$C^{(p+1)} = \omega^2 \, g_s^{-1} \, \mathrm{d}t \wedge \cdots \wedge \mathrm{d}x^p\,.$$

- **same** asymptotic BPS decoupling limit, applied to D3-brane geometry generates the near-horizon limit !

$$\mathrm{d}s^2 = \Omega(r) \, \mathrm{d}x^A \, \mathrm{d}x^B \, \eta_{AB} + \frac{\mathrm{d}r^2 + r^2 \, \mathrm{d}\Omega_{8-p}^2}{\Omega(r)}\,, \qquad \Omega(r) = \left(\frac{r}{\ell}\right)^{\frac{7-p}{2}},$$

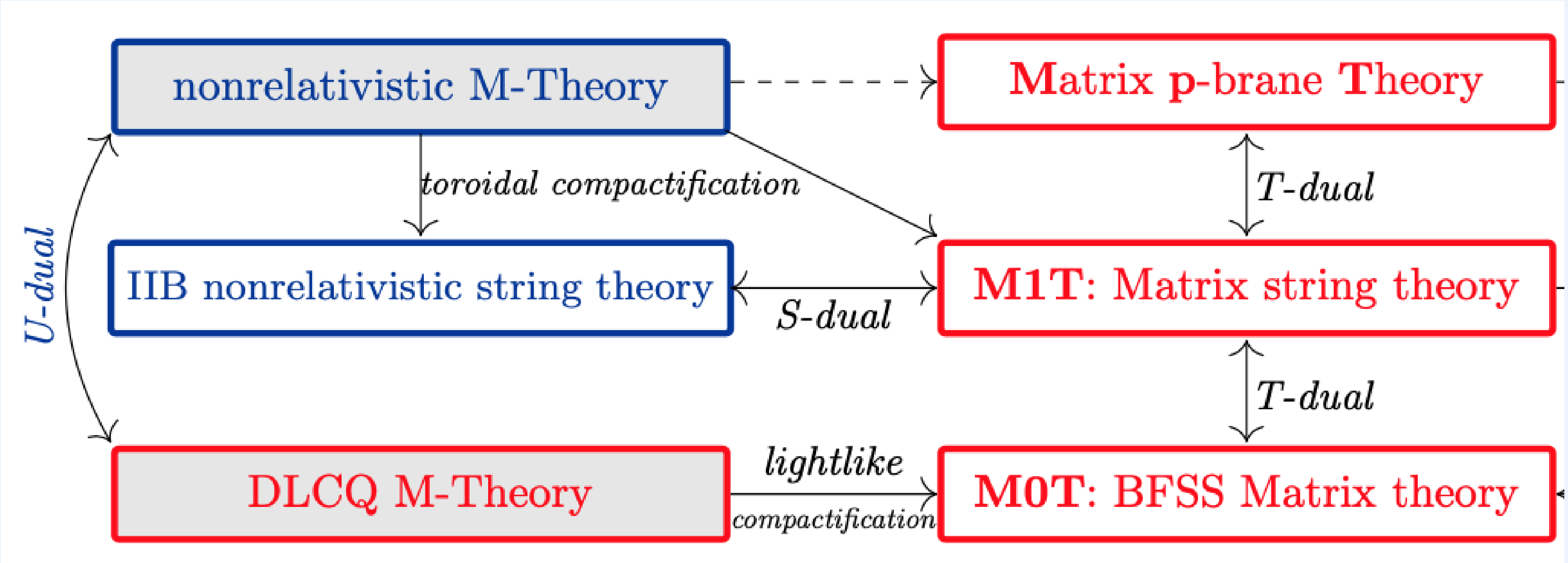
AdS/CFT



- applies to general Dp-brane solutions as well as F-string (IMSY)

- near-horizon geometries themselves asymptotically approach an MpT limit
  - yields back NL geometry seen by SYM at asymptotic infinity
- geometrically explains:
  relationship between matrix theory & AdS/CFT decoupling

Connections to NR string theory/M-theory



- near crtical D1-string dual to near critical F1-string (=NRST)

- M-theory uplift (NR M-theory): near critical M2-brane

  - wrapped membranes WM2
  - open M2-branes on M5 (OM)

    (studied in 2000s)

see also:
Blair,Gallegos,Zinnato(2021)
Ebert,Sun,Yan(2021)
Ebert,Yan(2023)
Lambert,Smith(2024)

Example of NRST in holography

Harmark,Lahnsteiner,NO (2025)

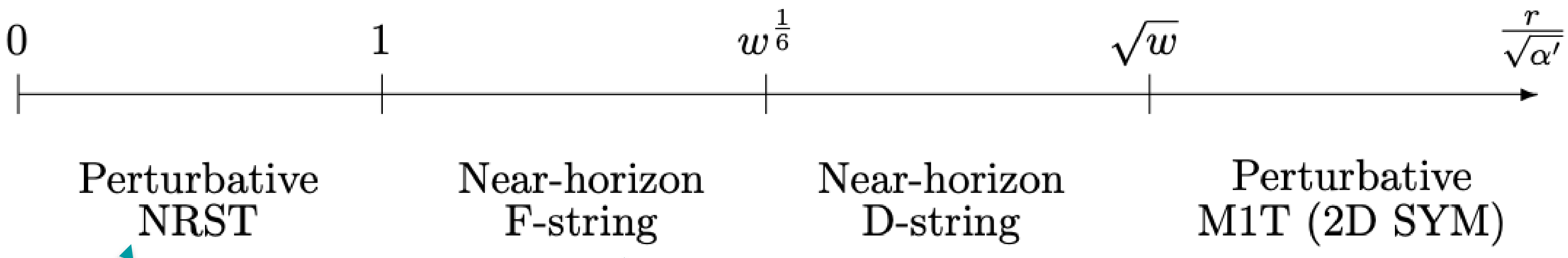


Figure 2: Phase diagram for the NRST limit of type IIB string theory.

NRST on torsional string Newton-Cartan

NRST limit of the F-string solution = near-horizon limit

Avila,Guijosa,Olmedo

matrix string theory: 2nd quantization of NRST
describes multi string states, joining/splitting via interaction term

Dijkgraaf,Verlinde,Verlinde/Motl

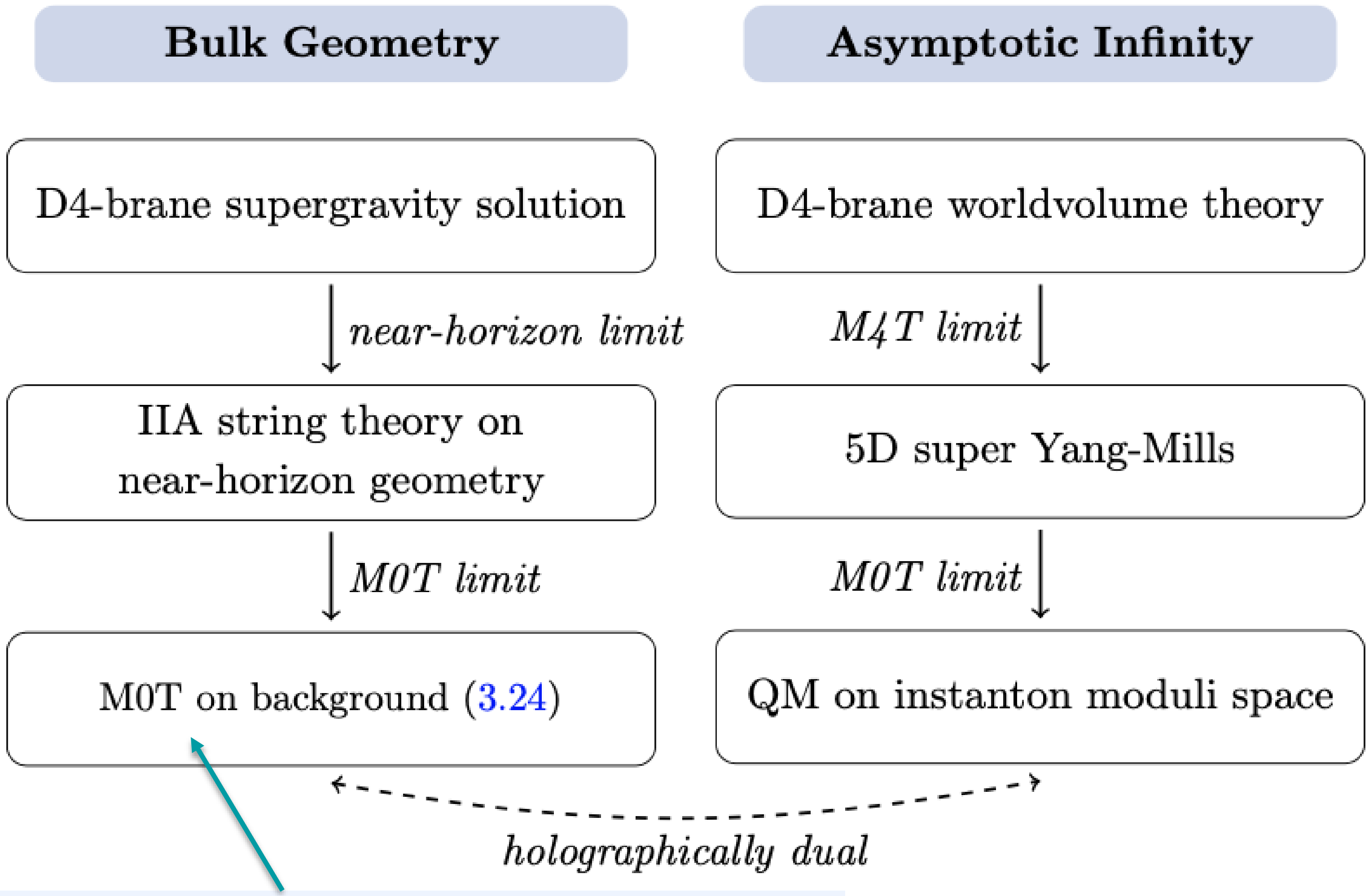
Landscape of holographic duals

- can use this perspective to generate
  new examples of holographic bulk geometries

→ can do asymptotic MpT limit and apply to bulk Dq-brane with (p not q)

- involves double BPS decoupling limit zooming in on intersecting background bound states

Example of multi-critical BPS limit



Newton-Cartan bulk background = M0T limit of $\mathrm{AdS}_6 \times S^4$.

many more duals can be generated: involving NL geometries in the bulk !

see also: Lambert,Smith(2024)

M-theory uplift and U-duality orbits

can unify the BPS deocupling limit using M-theory and U-dualities

- M0T BPS decoupling limit = DLCQ in M-theory

  - single BPS decoupling limits lie in U-duality orbit for ½ BPS (one DLCQ)
  - double BPS decoupling limits lie in U-duality orbit for ¼ BPS (two DLCQ's)

Blair,Lahnsteiner,NO,Yan (2023) Gomis,Yan (2023)
Dijkgraaf,de Boer,Harmark,NO (unpublished)

Holography as DLCQ^n/DLCQ^m correspondence

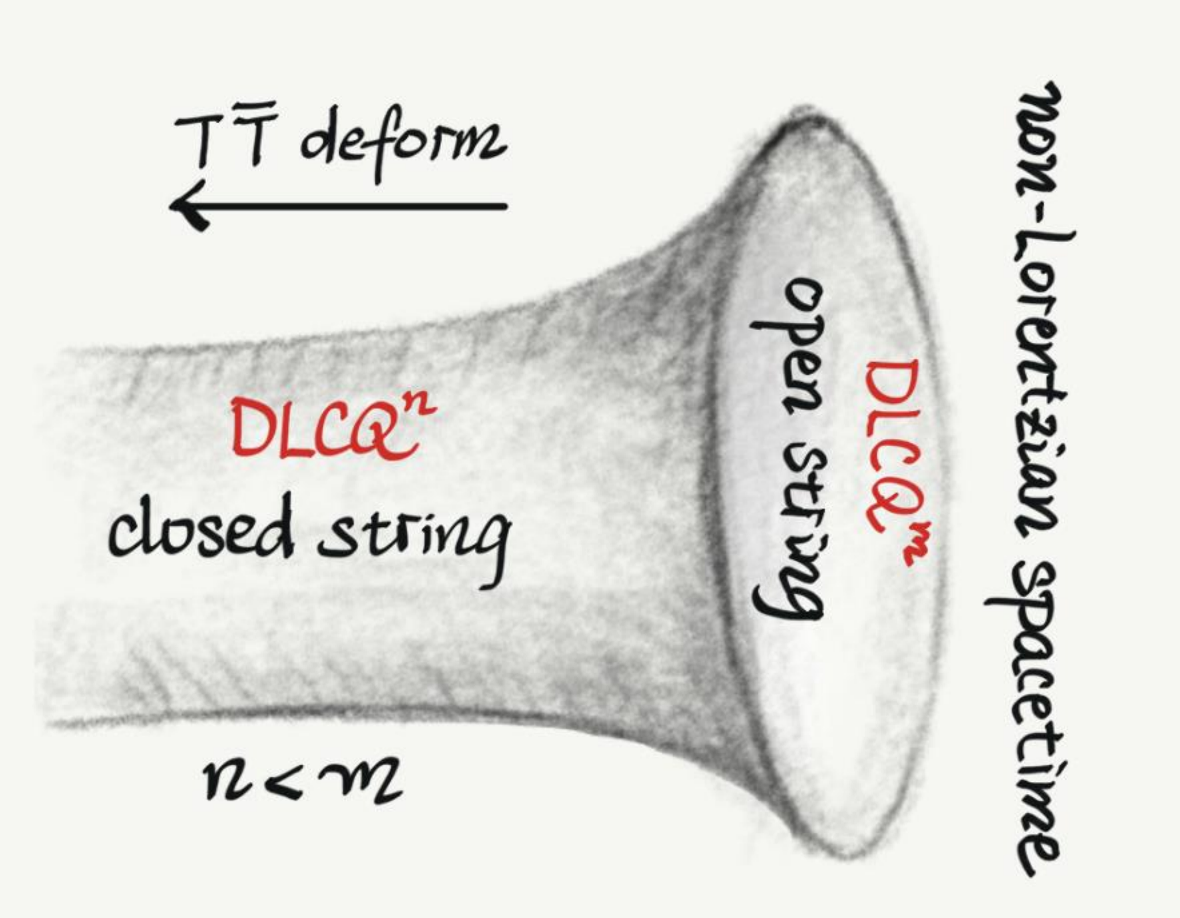
→ holographic dulaities unified as DLCQ^n/DLCQ^m with m > n

- AdS_5/CFT_4 = DLCQ^0/DLCQ^1 :
  DLCQ at asympt. inf = near-horizon
  similarly for the other Dp-branes and NRST

- AdS_3/CFT_2 = DLCQ^0/DLCQ^2

- novel holographic dualities with NL geometry in bulk: DLCQ^1/DLCQ^2

Part III: Undoing the BPS limit → TTbar like deformation



Generating near-horizon bulk geometry

- intrinsic perspective on relation between asymptotic infinity and bulk geometry
- can we``invert'' the BPS decoupling limit ?

Yes: related to TTbar deformation

$$\frac{\partial \mathcal{L}(\boldsymbol{t})}{\partial \boldsymbol{t}} \sim \det T_{\alpha\beta}(\boldsymbol{t}) \,,$$

$$T_{\alpha\beta}(\boldsymbol{t}) = -\frac{2}{\sqrt{-\det\tau}} \frac{\partial \mathcal{L}(\boldsymbol{t})}{\partial \tau^{\alpha\beta}}$$

NRST can be TTbar deformed back to relativistic string theory

Blair (2020)

New Dp-brane TTbar

→ via duality web: new Dp-brane TTbar deformations

- deformations that induce a flow from SYM to DBI action (so far: abelian)

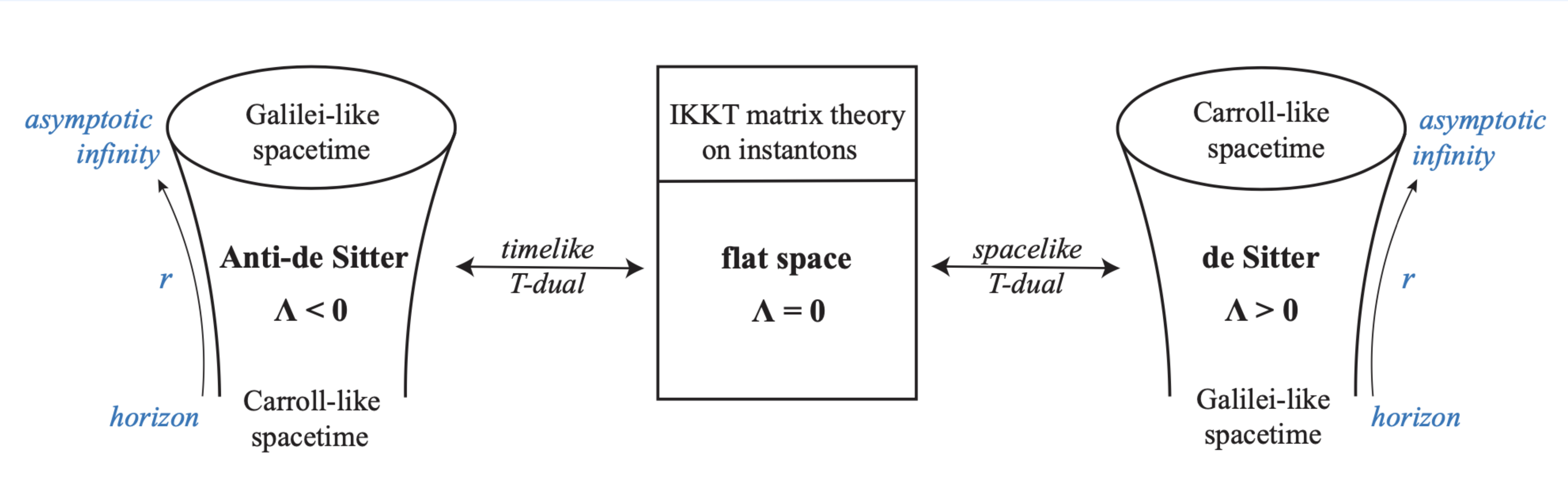
- universal result for (p+1)-dimensional free scalar field theory to DNG

$$\frac{\partial \mathcal{L}}{\partial t} = \frac{\sqrt{-\det \tau}}{2\,t^2}\left\{\mathrm{tr}\left(\mathbb{1} - t\,\mathcal{T}\right) - (p-1)\left[\det\left(\mathbb{1} - t\,\mathcal{T}\right)\right]^{\frac{1}{p-1}} - 2\right\}.$$

- also expressions including worldvolume gauge fields
- checks with various results from other methods

Carroll geometry meets dS via holography

Blair,NO,Yan (2025)

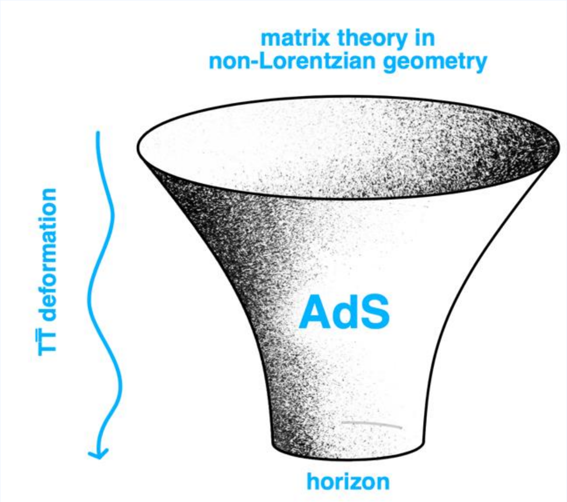


- extends AdS case to (hypotethical) dS holography in string theory
- elaborates on work of Hull (1998): E-branes in type IIB*
- flat space corresp. of IKKT = T-dual to geometry with (conformal) dS
- geometry at asymptotic infinity is Carroll-like

Outlook

- correspondence between SUGRA and matrix QM: revisit

- elements of the AdS/CFT correspondence: deeper insight into relationships between brane configs and possible decoupling limits



-non-Lorentzian holography: solutions, EOMs, torsions constraints

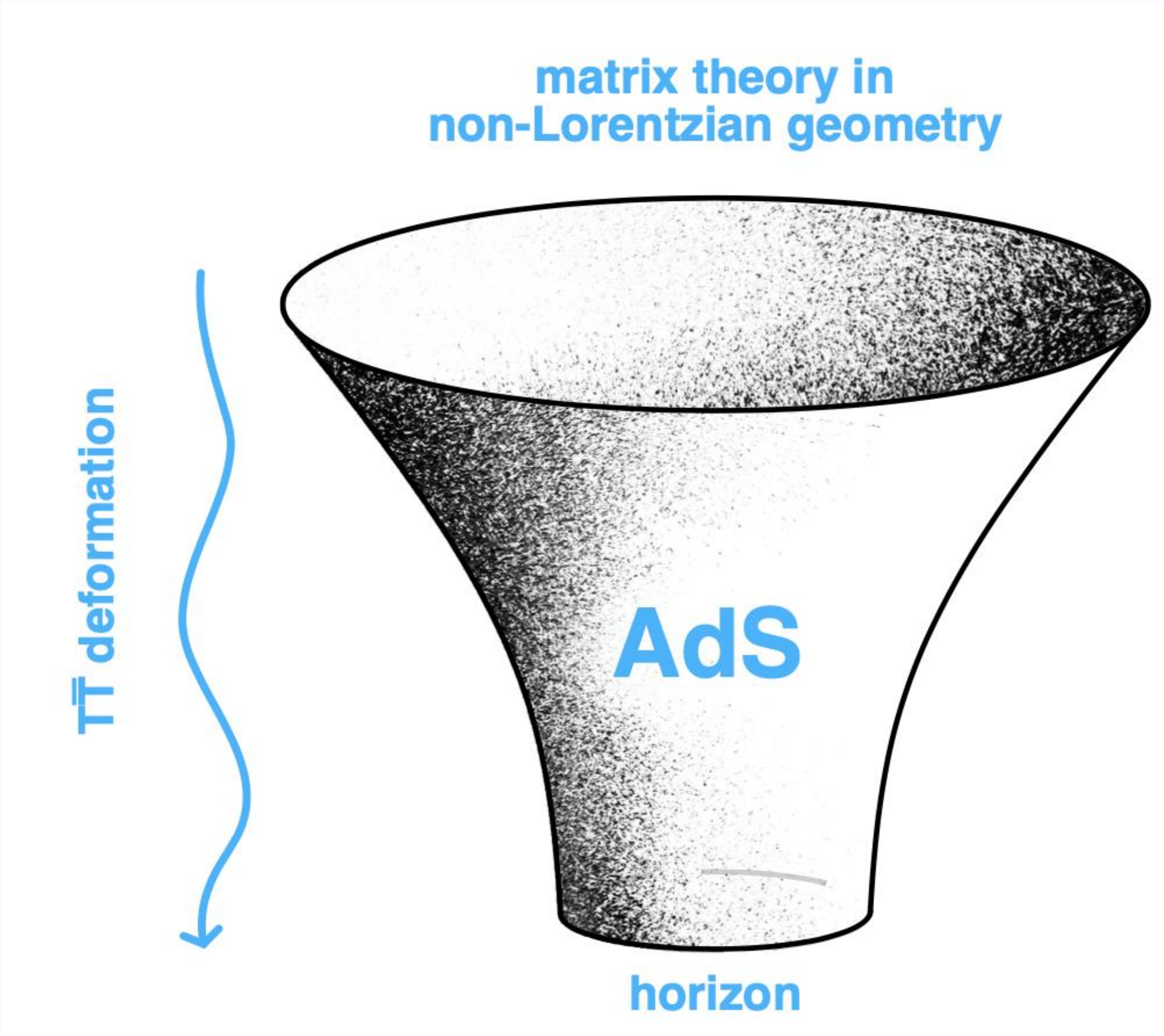
- p-brane generalisations of TTbar deformation

- algebraic aspects of decoupling (novel IW contactions of superalgebra)

- matrix valued TSNC geometry ? (in 2nd quantized NRST)

- extensions: tensionless, Carrollian/dS, heterotic

The end


matrix theory in
non-Lorentzian geometry
$T\bar{T}$ deformation
AdS
horizon

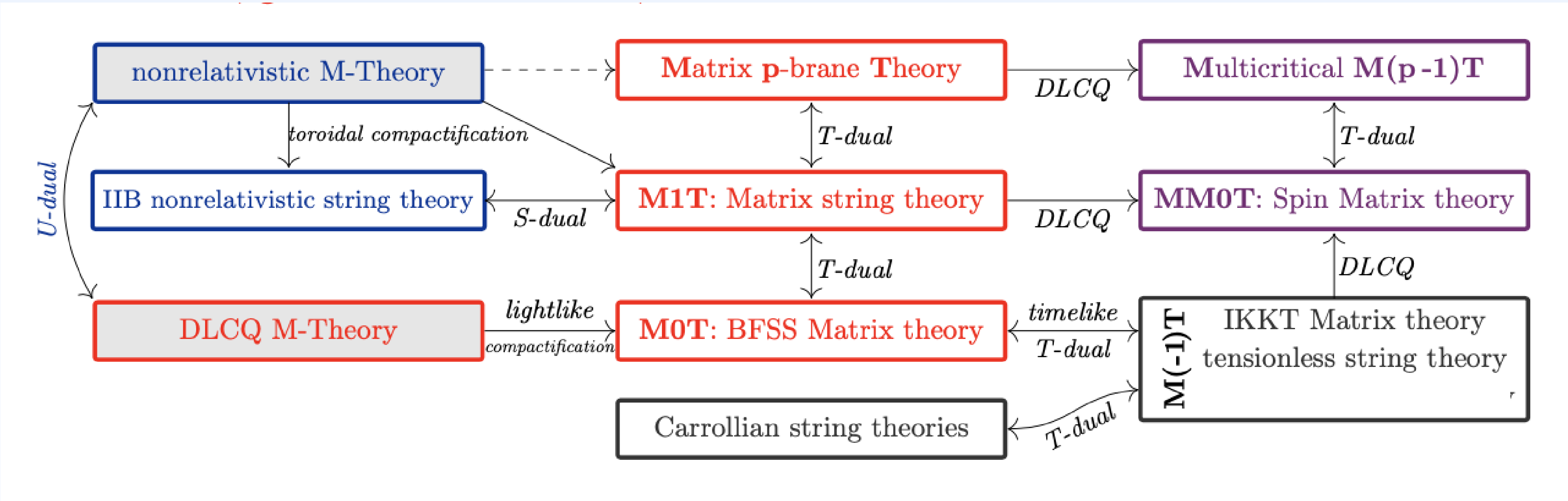
Extra slides after this

Unified web of non-Lorentzian string theories

Blair,Lahnsteiner,NO,Yan (2023)
Gomis,Yan (2023)

obtained from insights using
- non-relativistic string theory on curved spactimes
- applying string theory U-dualities
- anatomy of BPS mass formulae and decoupling limits

de Boer,Dijkgraaf,Harmark,NO (2000s, unpublished)



each node describes decoupled theory including actions for:
- fundamental (light) degrees of freedom
  coupling to an appropriate (non-Lorentzian) target spacetime
- other (heavy) probe objects in the theory

Lessons from the web

- near-BPS limits in ST can be viewed as natural generalizations of non-relativistic point particle limit

- novel perspective on Matrix theory/NRST via non-Lorentzian backgrounds → new insights into holography

- new corners of the duality maze (via solution generating techniques): IKKT, Carrollian string theory,..

- multi-critical limits & relation to spin-Matrix theory

- limits and geometries can also be understood from novel IW contractions of the M-theory superalgebra

-

Remarks on M0T

objects in M0T

- D0-branes: BFSS theory (on curved Newton-Cartan bgr)
- D2-branes: NCYSM
- F1-string: novel action (target space Galilean/world-sheet Carrollian boost)

important/relevant later: emergent dilation symmetry
for (sufficiently) well-behaved function Delta

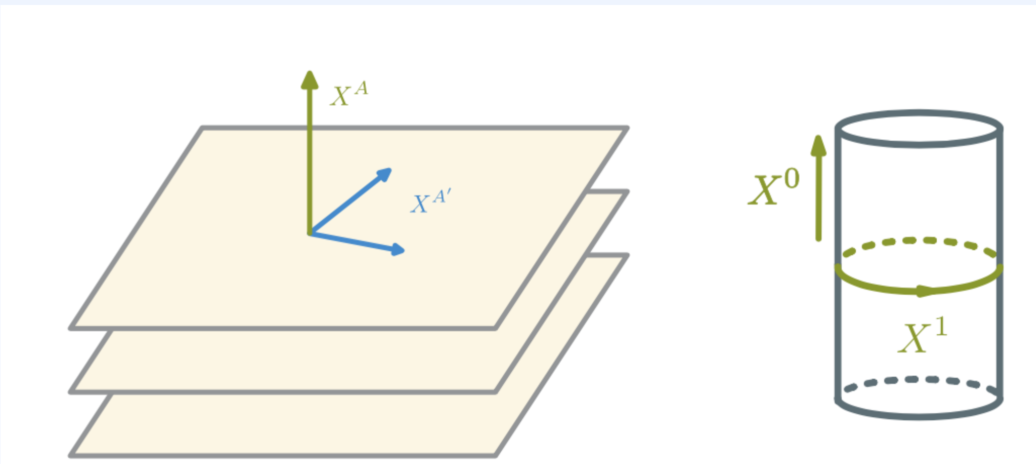
$$\omega \to \omega\, \Delta^{-1}$$

$$\tau^0 \to \Delta^{\frac{1}{2}}\, \tau^0\,, \qquad E^i \to \Delta^{-\frac{1}{2}}\, E^i, \qquad e^{\varphi} \to \Delta^{-\frac{3}{2}}\, e^{\varphi}\,.$$

underlying symmetry from (two) algebraic perspectives:
extension of super-Bargmann altebra with extra bosonic charges
can also see as D0-brane adapted IW contraction of relativistic IIA superalgebra

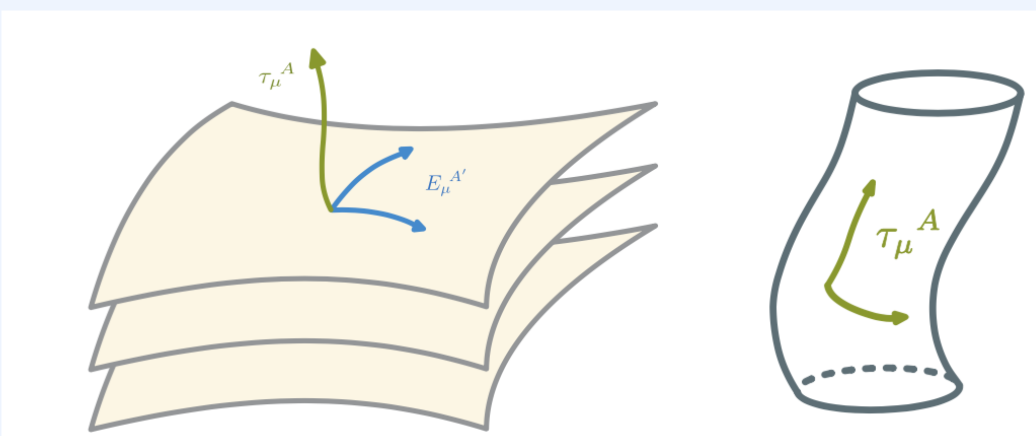
How does MpT relate to NR string theory (analogous decoupling limit) ?

Gomis/Ooguri NR string lives in flat space



figures from
review on NRST
Gerben Oling & Ziqi Yan
(2202.12698)

General target space probed by NR strings



multi-critical examples - NL geometry in bulk

- multicritical limit adapted to D0-D4 → bulk M0T geometry

$$\tau_{\mu\nu}\, dx^\mu\, dx^\nu \equiv \tau^A\, \tau^B\, \eta_{AB} = -\left(\frac{r}{\tilde{\ell}}\right)^{\frac{3}{2}} dx^0\, dx^0,$$

$$E_{\mu\nu}\, dx^\mu\, dx^\nu \equiv E^{A'} E^{A'} = \left(\frac{\tilde{\ell}}{r}\right)^{\frac{3}{2}} \left[\left(\frac{r}{\tilde{\ell}}\right)^3 dx^I dx^I + dr^2 + r^2\, d\Omega_4^2\right]$$

M0T limit of $\mathrm{AdS}_6 \times S^4$.

multi-critical examples - NL geometry in bulk

- multicritical limit adapted to D1-Dp → bulk torsional string Newton-Cartan

$$\tau_{\mu\nu} dx^\mu dx^\nu = -\left(\frac{r}{\ell}\right)^{(6-p)/2} dt^2 + \left(\frac{r}{\ell}\right)^{-(6-p)/2} dv^2 \ ,$$

$$E_{\mu\nu} dx^\mu dx^\nu = \left(\frac{r}{\ell}\right)^{(6-p)/2} \sum_{a=1}^{p} (dy^a)^2 + \left(\frac{r}{\ell}\right)^{-(6-p)/2} \sum_{i=1}^{8-p} dr^i dr^i \ ,$$

$$a_{(p+1)} = \left(\frac{r}{\ell}\right)^{6-p} dt \wedge dy^1 \wedge \cdots \wedge dy^p \ , \quad g_s e^{\varphi} = g_s \left(\frac{r}{\ell}\right)^{-(6-p)(3-p)/4}$$

$$\ell^{6-p} = \frac{(2\pi\sqrt{\alpha'})^{7-p} g_s N}{(6-p)\Omega_{7-p} R_v}$$

- torsion constraints from SUSY?

- can also consider NS5-F1 → reproduces solition that is earlier found

Bergshoeff et al (2021)

Duality asymmetry of NL geometries

Blair,Lahnsteiner,NO,Yan (2025)

- rich structure:

T- and S-duality transformations exhibit novel asymmetric properties:

depending on choice of transformation & value of the background fields, codimension of foliation structure of NL geometry can change/stay same)

- duality asymmetry between NL geometries for Matrix theory underlying Morita equivalence in matrix theory and non-commutative YM

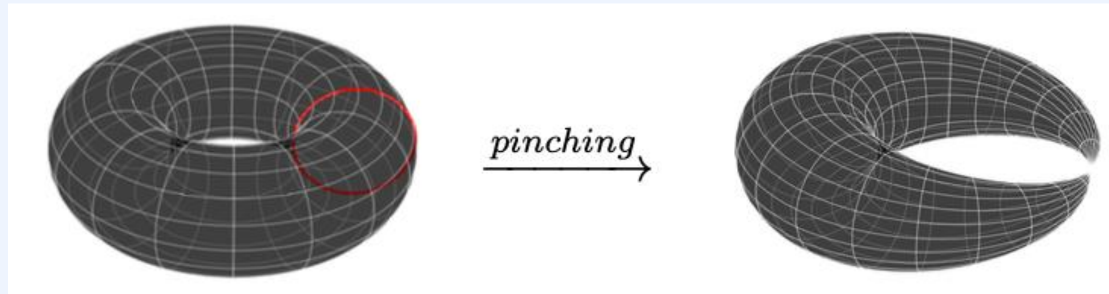
- Hashimoto-Itzhaki/Maldacena-Russo holography for noncommutative YM with B-field & generalizations

Nodal Riemann spheres

interesting degenerate topologies of 2D surfaces show up both on world-sheetand target space

Gomis,Yan (2023)

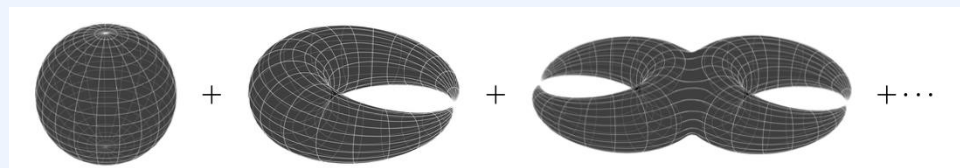
- F-string in M0T:



modular parameter

$$\tau \to i\infty\,.$$

genus expansion
(cf. ambitwistor)



Geyer,Mason,Monteiro,Tourkine(2015)

- pinched torus
in target space: