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Four-Fermion Deformations on Line Defects in QED_4 with Yukawas

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Introduction

- Defects in CFTs
- Preliminaries: scalar and fermionic QED_4
- What's new: Yukawa interaction and change in fixed point dynamics

Why CFTs?

- Conformal Field Theories play an important role in many branches of physics.
- Ex. #1. **Statistical Physics**: description of 2nd order phase transitions as we dial the temperature.

Ising Model

Microscopic model for ferromagnetism.

$$H = \frac{1}{T} \sum_{<ij>} (1 - s_i s_j)$$

we find: $< s(r)s(0) > \sim e^{-r/\xi(T)}$

correlation lenght: $\xi(T) \sim (T - T_c)^{-\nu}$

- We only need to know one number: the critical exponent ν!
- Good news: ν can be understood in terms of "CFT data"!

$$\nu = \frac{1}{d - \Delta}$$

- At the critical point the system is described as an EFT by the Landau-Ginzburg theory

$$S = \int d^d x (\nabla \phi)^2 + \lambda \phi^4$$

- Ex. #2. **QFT**

Every well-defined QFT has a fixed point in the UV

↓

CFTs are lampposts to study QFTs!

↓

Many systems become easier to study.

Ex.: QCD in the conformal window

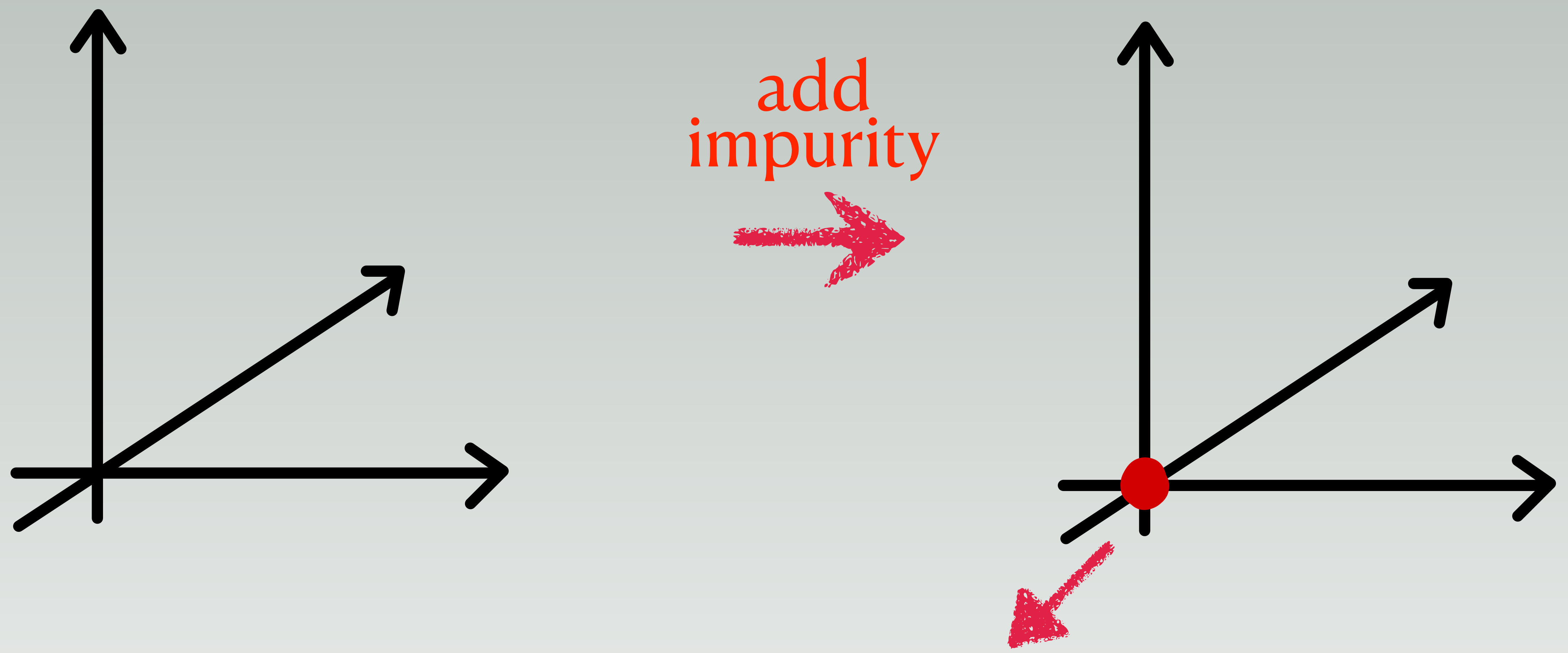
Line Operators in Conformal Theories

- Disclaimer: by "line operator" here we will mean a Wilson Line localised at $\vec{x} = \vec{0}$

CFT | **DCFT**

Invariant under:

$\mathrm{SO}(d+1,1)$



$W_q(\gamma) = e^{iq\int_\gamma A}$

A "conformal Wilson Line" preserves the maximal allowed subgroup that leaves $\vec{x} = \vec{0}$ invariant.

$$t' = \frac{at+b}{ct+d}, \quad ad - bc = 1$$

$\mathrm{SL}(2,\mathbb{R})$

- In a DCFT we distinguish between bulk operators $O_i(\vec{x}, t)$ with scaling dimensions Δ_i and defect operators $\hat{U}_\alpha(t)$ with defect scaling dimensions $\hat{\Delta}_\alpha$
- Powerful tool: **bulk-to-defect OPE**

$$O_i(\vec{x}, t) = \sum_\alpha \frac{a_{i\alpha}}{r^{\Delta_i - \hat{\Delta}_\alpha}} \hat{U}_\alpha(t), \qquad r \sim 0$$

- **GOAL:** understand the long distance effect of these impurities (RG flows)

Examples:

1. Scalar QED_4
2. Fermionic QED_4
3. Fermionic QED_4 with Yukawa interaction

O. Aharony, G. Cuomo, Z. Komargodski, M. Mezei, A. Raviv-Moshe, 2310.00045

→ A. D'Alise, G.M., F. Sannino, 2504.19686

Scalar QED$_4$

Action of the theory in presence of the defect (after rescaling $\Phi = \phi/e$):

$$S = \frac{1}{e^2}\int d^dx\Big(-\frac{1}{4}F^2 + |D\phi|^2 + \frac{\lambda}{2e^2}|\phi|^4 - e^2 q\delta^3(\vec{x})\delta_0^\mu A_\mu\Big)$$

We work in the limit:

$$e \to 0, \quad \frac{\lambda}{e} = \text{fxed}, \quad e^2 q = \text{fxed},$$

$$\lambda \to 0, \quad q \to \infty$$

This allows:

1. Working in semiclassical approximation
2. Treating the theory as a CFT

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$$ds^2 = dt^2 - dr^2 - r^2 d\Omega^2 = r^2 ds^2_{AdS_2 \times S^2}$$

We exploit Weyl invariance and move the theory to $AdS_2 \times S^2$ $(\phi \to r\phi)$

EOMs => $A_0 = \frac{e^2 q}{4\pi r} = \frac{g}{r}$

quantum fluctuations of scalar field:

$$\phi \sim \alpha r^{1/2-\nu} + \beta r^{1/2+\nu}, \qquad \nu = \sqrt{\frac{1}{4} - g^2}$$

=> $\hat{\Delta}(\alpha) = \frac{1}{2} - \nu, \quad \hat{\Delta}(\beta) = \frac{1}{2} + \nu$

→ relevant operator in the theory: $\hat{\Delta}(\alpha^\dagger \alpha) = 1 - 2\nu < 1$!!

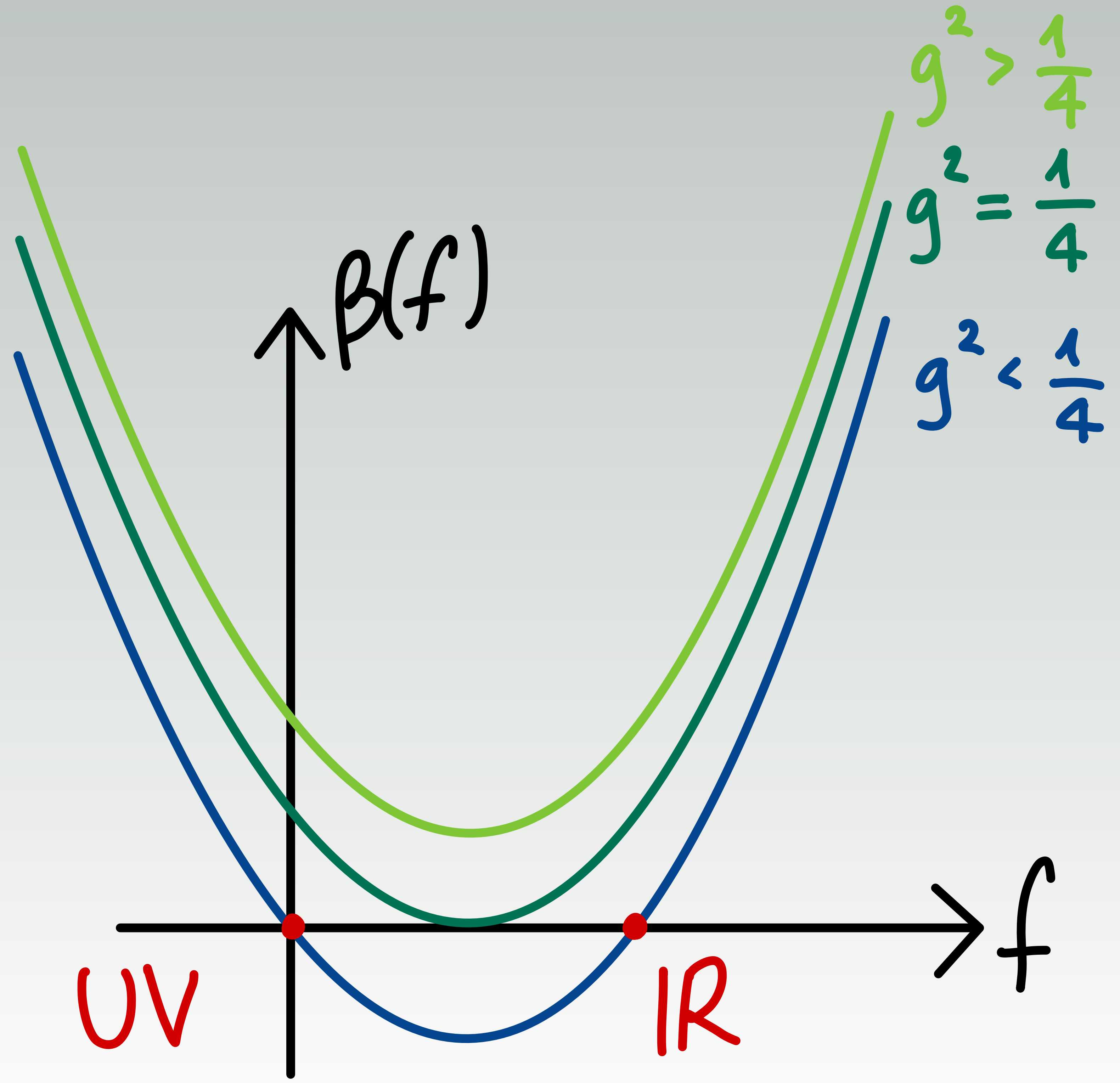
inconsistent to ignore it in the theory!

The “naive” Wilson Line must be modified:

$$W_q(\gamma) \quad \to \quad W_q'(\gamma) = \exp\left(iq \int A_\mu dx^\mu - f \int dt \phi^\dagger \phi \right)$$

f is a relevant coupling. We find:

$$\beta(f) = f(f - 2\nu)$$



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Fermionic QED$_4$

Action of the theory in presence of the defect (after rescaling $\Psi = \psi/e$):

$$S = \frac{1}{e^2}\int d^4x\Big(-\frac{1}{4}F^2 + i\bar{\psi}\not{D}\psi - e^2 q\delta^3(\vec{x})\delta_0^\mu A_\mu\Big)$$

We work in the limit:

$$e \to 0, \quad q \to \infty, \quad e^2 q = \text{fixed}$$

so we can employ a semiclassical approximation

and a CFT treatment

Every fermion has 4 components => many more possible relevant operators!

Counting only the ones preserving Lorentz and chiral symmetries, we are left with 4.

Their beta functions turn out to be:

$$\beta_f = -2\nu f + 2(1+\nu)[f^2 + k^2 + h^2 + q^2]$$

$$\beta_k = -2\nu k + 4(1+\nu)kf$$

$$\beta_h = -2\nu h + 4(1+\nu)hf$$

$$\beta_q = -2\nu q + 4(1+\nu)qf$$

Much richer fixed point structure!

1. $(f, k, h, q) = (0,0,0,0)$ unstable fixed point
2. $(f, k, h, q) = \left(\frac{\nu}{m+\nu}, 0, 0, 0\right)$ stable fixed point
3. $f = \frac{\nu}{2(m+\nu)}$ while $k^2 + h^2 + q^2 = \frac{\nu^2}{4(m+\nu)^2}$ manifold of unstable fixed points

Combining fermions and scalars: Fermionic QED_4 with Yukawa interaction

- **Why?**

  Adding a Yukawa interaction brings us closer to a case of physical interest: the Standard Model!

- Action of the theory in presence of the defect (after rescaling $\Phi = \phi/e$, $\Psi = \psi/e$):

$$S = \frac{1}{e^2}\int d^4x \left\{ -\frac{1}{4}F^2 + i\bar{\psi}\not{D}\psi + \frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4e^2}\phi^4 - \frac{y}{e}\bar{\psi}\phi\psi - e^2 q\delta^3(\vec{x})\delta_0^{\mu}A_{\mu}(x) \right\}$$

we work in the limit:

$$e \to 0, \quad \frac{\lambda}{e^2} = \text{fixed}, \quad \frac{y}{e} = \text{fixed}, \quad e^2 q = \text{fixed}$$

$$\lambda \to 0, \quad y \to 0, \quad q \to \infty$$

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EOMs:

$$\partial_\mu F^{\mu\nu} = -j^\nu + Q\delta^3(\vec{x})\delta^\nu_0$$

$$(i\not{D} - y\phi)\psi = 0$$

$$(\Box - \lambda\phi^2)\phi = -y\bar{\psi}\psi$$

→ The solution for ψ becomes a source term for ϕ!

=> ϕ will also contribute to the β functions!

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- In the regime $e^2 q < \pi\sqrt{15}$ not only quadratic operators are relevant, but also four-fermion operators!
- Fixed point structure of the theory changes drastically.
- In the analysis the "ψ^4" operators turn out to be the dominating contribution to the running of the theory.
- New fixed points:

1. $k = h = q = 0, \quad f[144\pi\nu^2(1+2\nu)f - 5y^2] \neq 0;$
2. $(f, h, k, q) = \left(\frac{5y^2}{192\nu^2(1+2\nu)\pi}, 0, 0, 0\right);$
3. $(f, h, k, q) = \left(\frac{5y^2}{144\nu^2(1+2\nu)\pi}, -\frac{5y^2}{144\nu^2(1+2\nu)\pi}, 0, 0\right).$

All unstable fixed points!

Take-home messages

- Inserting a line operator (infinitely massive impurity) in a CFT can compromise the conformality of the theory → new phases of the theory uncovered;
- In a scenario with Yukawa interaction the regime where "ψ^4" operators dominate turns out to be more interesting → completely new fixed point structure;
- Further studies are possible: defect finite mass effects, asymptotically safe theories...

Thank You!

Backup

Boundary conditions and beta function

$$\phi \sim \alpha r^{1/2-\nu} + \beta r^{1/2+\nu}$$

Boundary conditions on α and β found imposing $\delta S = 0$

We add:

$$S^{(1)}_{bdy} = -\frac{1-2\nu}{2} \int_{r=r_0} dt \sqrt{-\hat{g}}\ \Phi^\dagger \Phi, \quad S^{(2)}_{bdy} = -f_0 \int_{r_0} dt \sqrt{-\hat{g}}\ r_0^{2\nu}\ \Phi^\dagger \Phi$$

Then:

$$\delta S_{bulk} + \delta S^{(1)}_{bdy} + \delta S^{(2)}_{bdy} = 0 \quad => \quad \frac{\beta}{\alpha} = \frac{f}{2\nu - f}\ r_0^{-2\nu}$$

$$\frac{\partial}{\partial \log \mu}\left(\frac{\beta}{\alpha}\right) = 0 \implies \beta_f = f(f - 2\nu)$$

Details on fermionic solution

Fermionic solution found via Kaluza-Klein decomposition

$$\Psi = \frac{1}{r^{\frac{d-1}{2}}} \sum_{\ell,s} \sum_{\delta=+,-} \psi^{(\delta)}_{\ell s}(t,r) \otimes \chi^{(\delta)}_{\ell s}(\hat{n})$$

$\rightarrow$ $\psi^{(\delta)}_{\ell s}(t,r)$ solution of Dirac equation on AdS_2 with gauge bkg $A_0 = g/r$

$$\left[i \left(r\gamma^1 \partial_r - \frac{1}{2}\gamma^1 - ig\gamma^0 \right) - m_\ell \right] \psi(r) = 0$$

$$\psi_\ell = \begin{pmatrix} \alpha_\ell r^{\frac{1}{2}-\nu_\ell} + \frac{g}{m_\ell + \nu_\ell} \beta_\ell r^{\frac{1}{2}+\nu_\ell} \\ \beta_\ell r^{\frac{1}{2}+\nu_\ell} + \frac{g}{m_\ell + \nu_\ell} \alpha_\ell r^{\frac{1}{2}-\nu_\ell} \end{pmatrix}$$