Quantum Gravity as a QFT

Alessia Platania

Danish QFT meeting 2025
14.08.2025

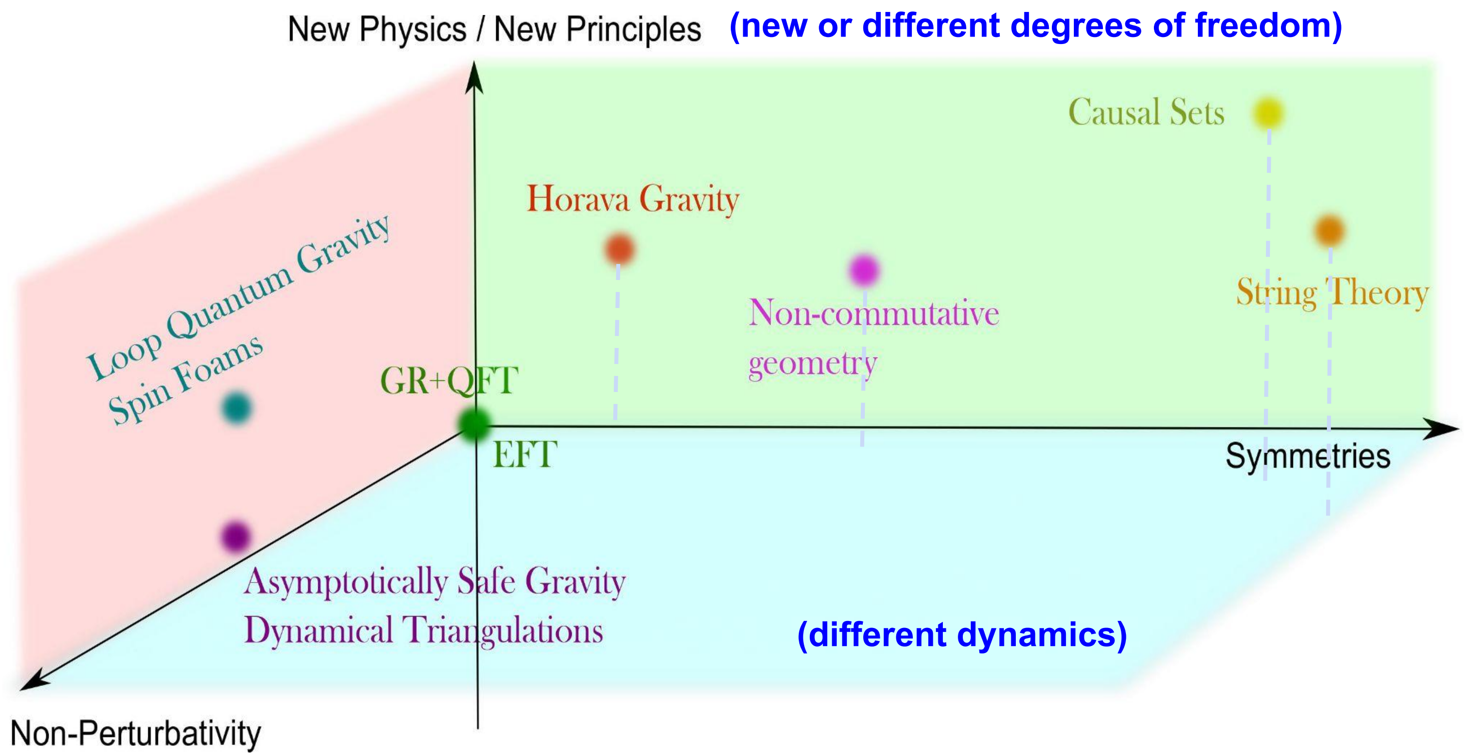
The Niels Bohr International Academy

THE CENTER OF GRAVITY

UNIVERSITY OF COPENHAGEN

VILLUM FONDEN

The Realm of Quantum Gravity



State of the art in Quantum Gravity

SciPost Physics Lecture Notes Submission

Lectures in Quantum Gravity

Ivano Basile[1⋆], Luca Buoninfante[2†], Francesco Di Filippo[3‡], Benjamin Knorr[4∘], Alessia Platania[5¶] and Anna Tokareva[6‖]

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Abstract

Formulating a quantum theory of gravity lies at the heart of fundamental theoretical physics. This collection of lecture notes encompasses a selection of topics that were covered in six mini-courses at the Nordita PhD school *"Towards Quantum Gravity"*. The scope was to provide a coherent picture, from its foundation to forefront research, emphasizing connections between different areas. The lectures begin with perturbative quantum gravity and effective field theory. Subsequently, two ultraviolet-complete approaches are presented: asymptotically safe gravity and string theory. Finally, elements of quantum effects in black hole spacetimes are discussed.

SciPost Physics Community Reports Submission

Visions in Quantum Gravity

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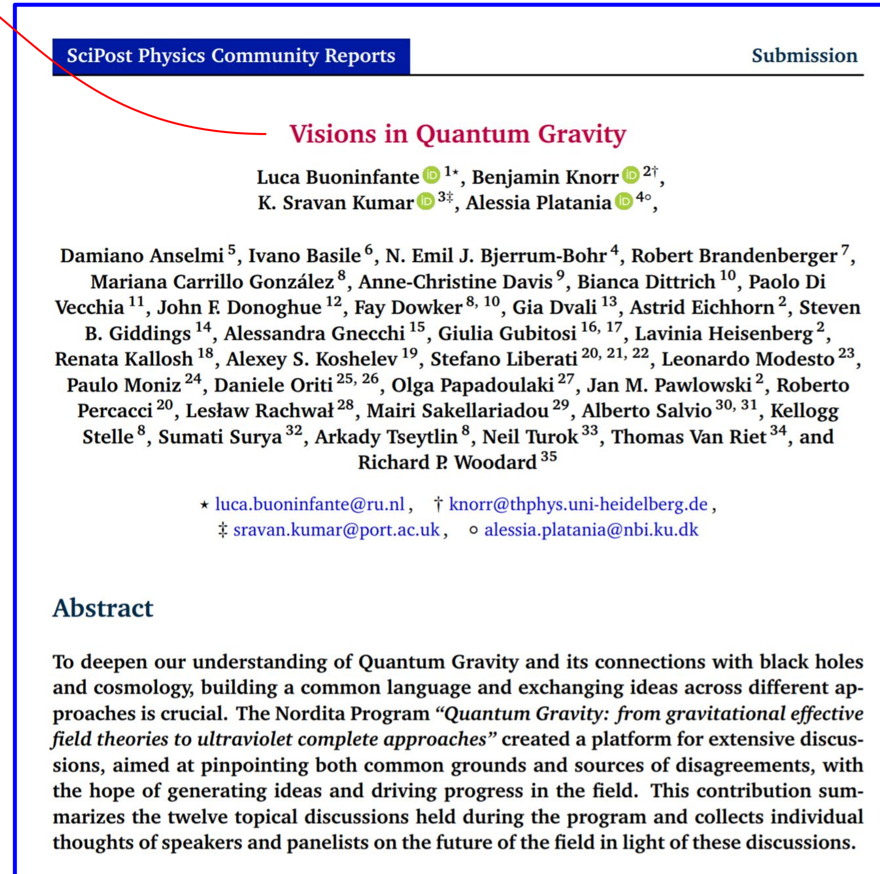
To deepen our understanding of Quantum Gravity and its connections with black holes and cosmology, building a common language and exchanging ideas across different approaches is crucial. The Nordita Program *"Quantum Gravity: from gravitational effective field theories to ultraviolet complete approaches"* created a platform for extensive discussions, aimed at pinpointing both common grounds and sources of disagreements, with the hope of generating ideas and driving progress in the field. This contribution summarizes the twelve topical discussions held during the program and collects individual thoughts of speakers and panelists on the future of the field in light of these discussions.

State of the art in Quantum Gravity

Long story short: **we even disagree on which questions are important**

Many open questions:

- ***What are the features of quantum gravity?*** Non-locality, holography, non-perturbativity, non-commutativity…
- ***What are the fundamental degrees of freedom?*** metric, strings, area metrics, torsion, holonomies…
- ***"Swampland program": universal features of quantum gravity at low energy?***
- ***Coupling with matter***: which quantum gravity theories are compatible with the Standard Model?
- ***Is quantum gravity a QFT or do we need to go beyond it?***
- …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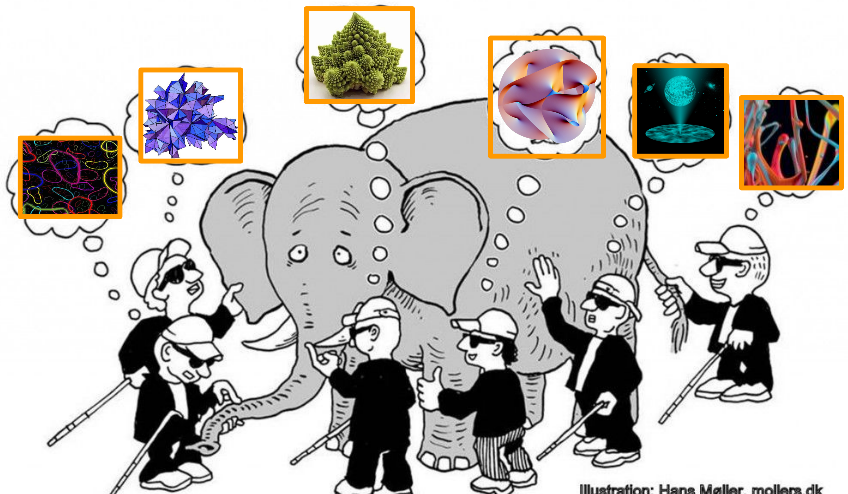

Different mathematical descriptions of the same physics?

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Key broader questions

How do we discriminate between different approaches?

How do we test the different theories (theoretically/phenomenologically)?

Which theories are the most promising?

Can different theories be different mathematical descriptions of the same physics?

Key broader questions…and problems

How do we discriminate between different approaches?
…QG lives at the Planck scale, difficult to constrain phenomenologically, difficult to make (testable) predictions → **need to translate UV details in IR predictions**

How do we test the different theories (theoretically/phenomenologically)?
…Theoretically: answer depends on threeshold criteria, pheno: QG effects may be small modulo amplification effects → **need amplification effects**

Which theories are the most promising?
…Define "promising" across approaches is difficult: agreement with SM? Can make (correct?) predictions? Unitary/causal/stable? → **need universal criteria**

Can different theories be different mathematical descriptions of the same physics?
…Yes, but hard to compare: different theories speak different languages → **need common language**

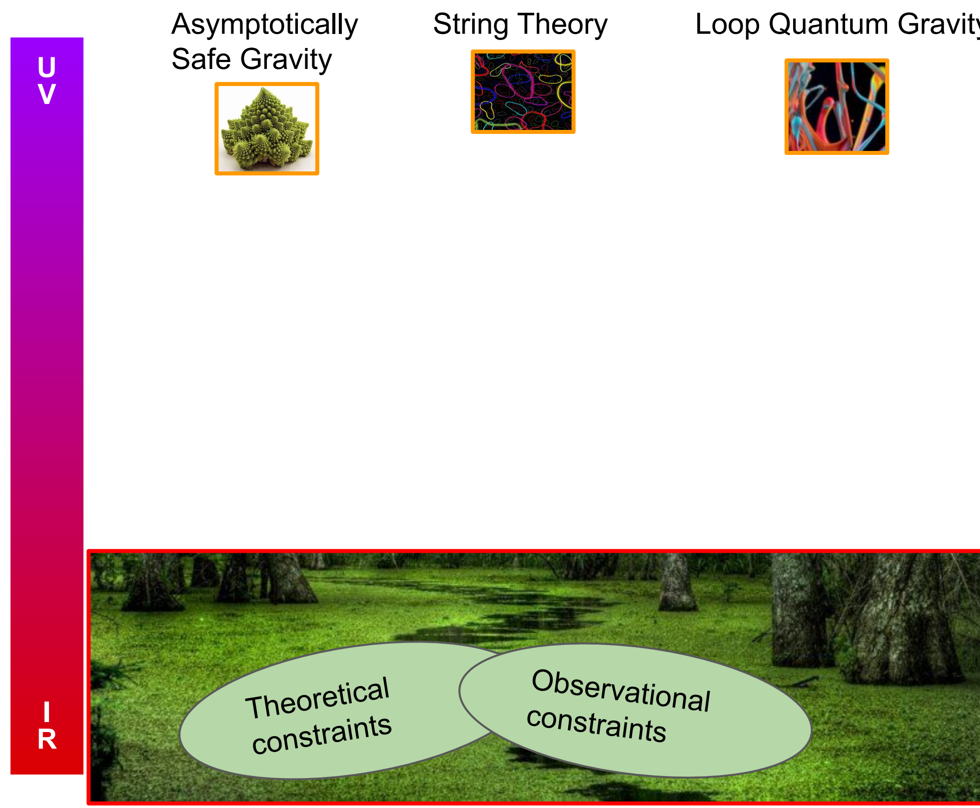
One direction for progress in QG:

Combining strategies and concepts across fields?

- **Need to translate UV details into IR predictions**
  - **⇒ e.g., RG = mathematical version of a microscope**
- **Need amplification effects**
  - **⇒ Inspiration from asymptotically safe gravity and matter**
- **Need universal criteria**
  - **⇒ Inspiration from swampland program + EFT**
- **Need common language**
  - **⇒ EFT (even if not all QG communities would agree)**

Let us put these ingredients together:

- UV: different theories
- IR: EFT-constr./observations
- Need common language
- Need to translate UV→IR



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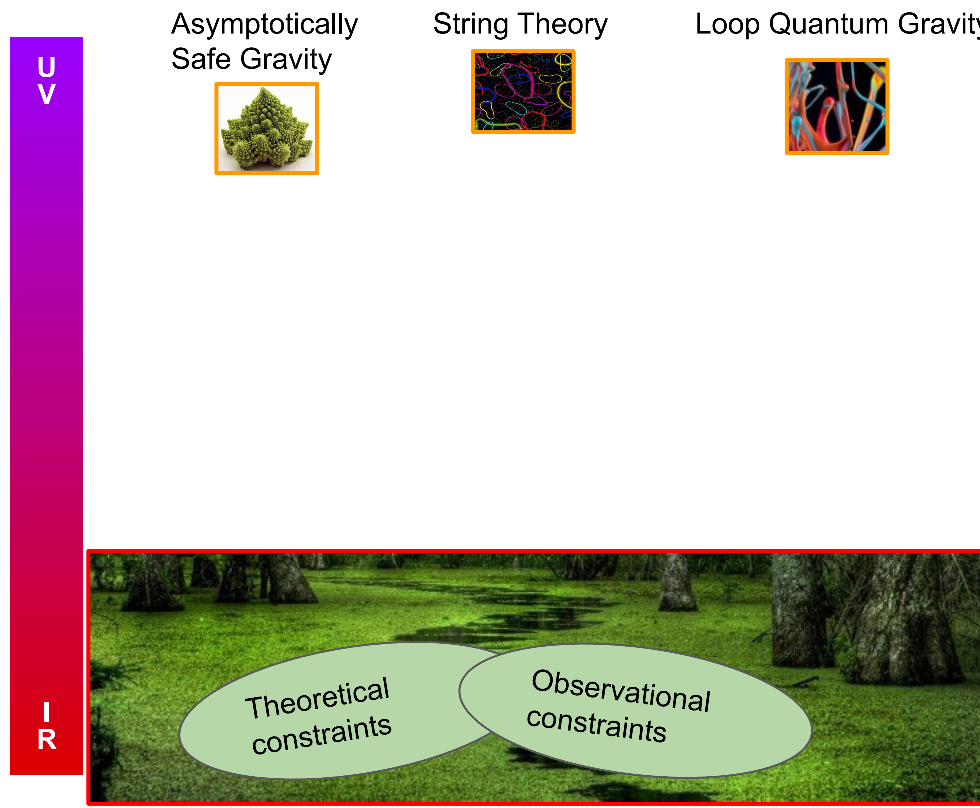

Quantum gravity through the lens of effective field theory

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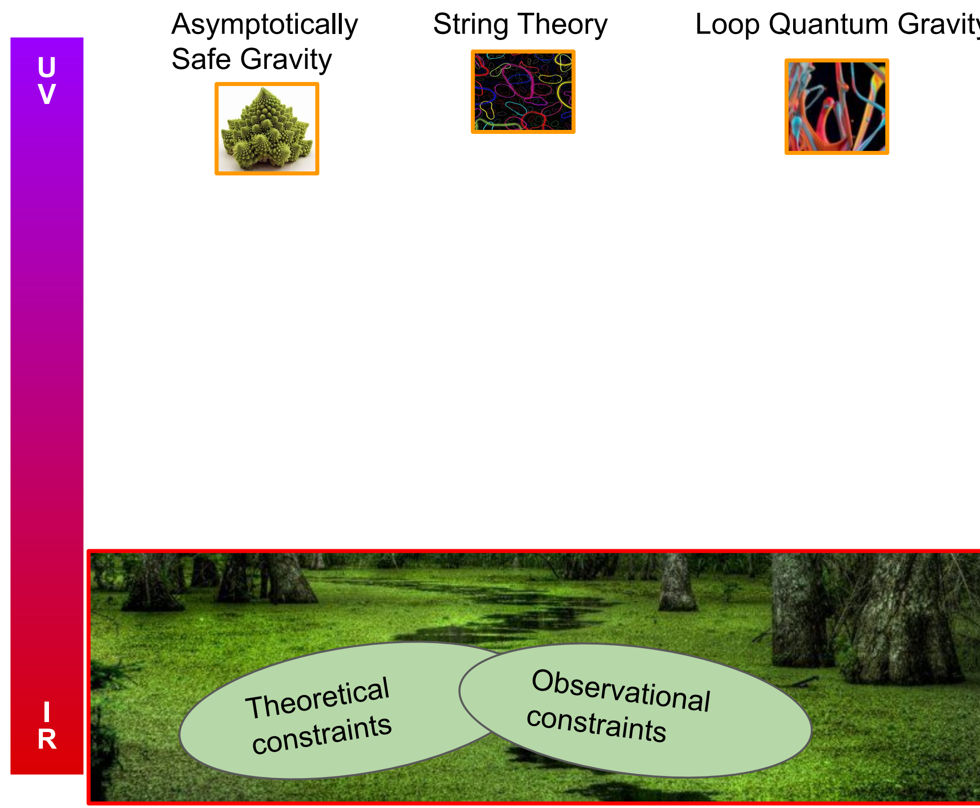

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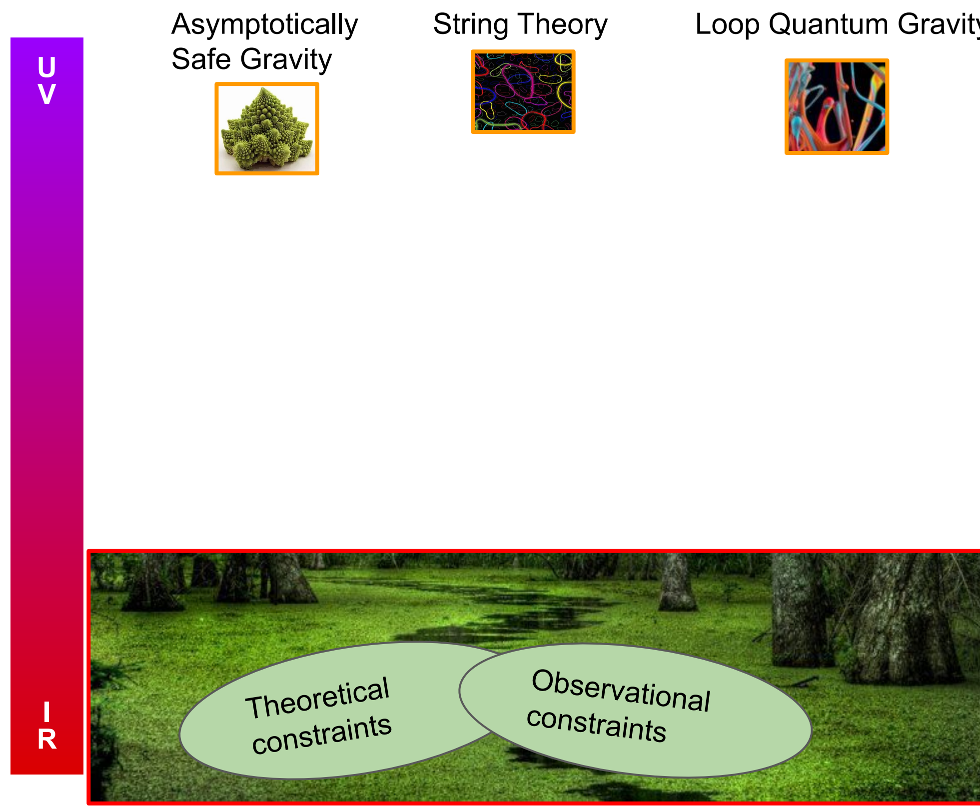
  use concept of “landscapes”



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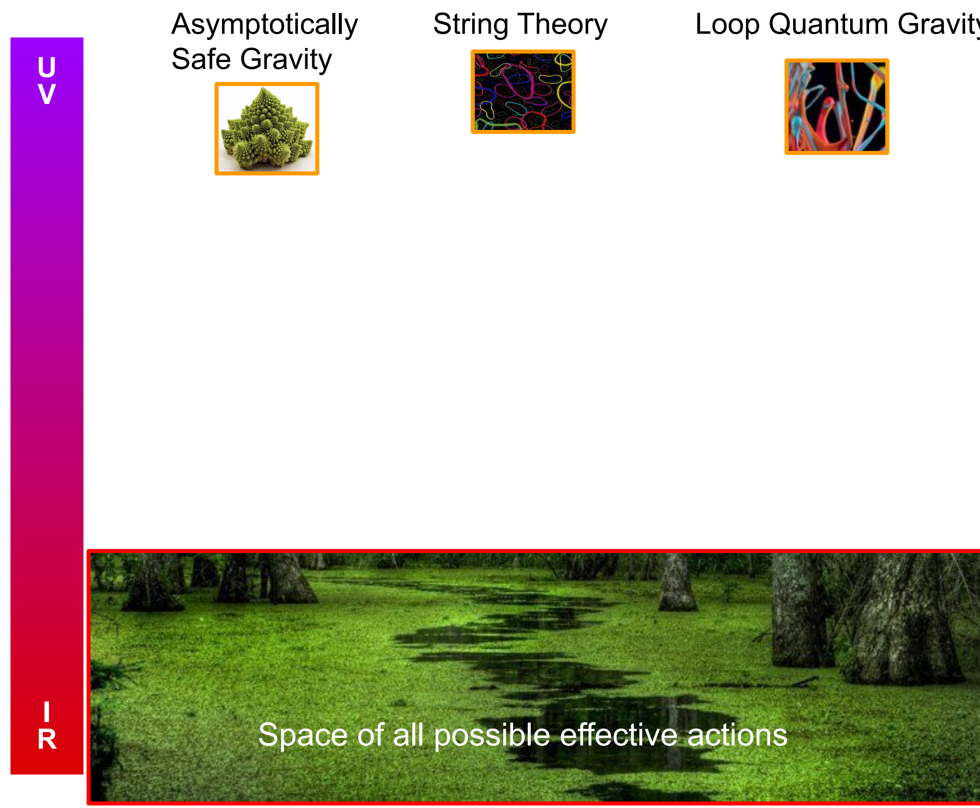

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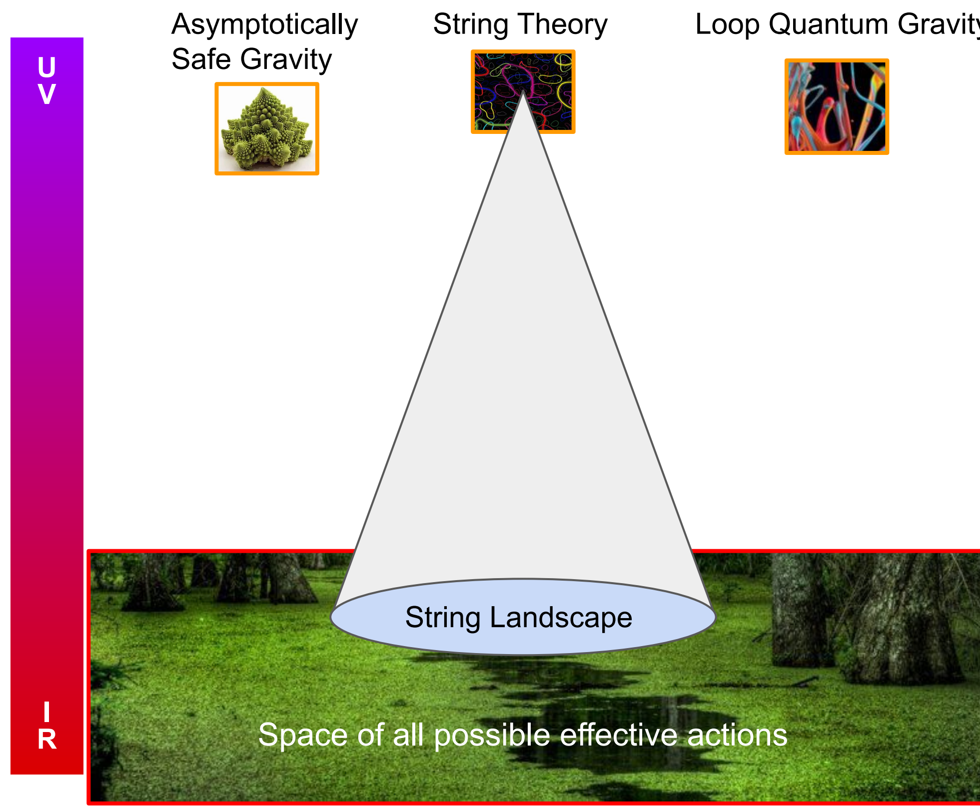

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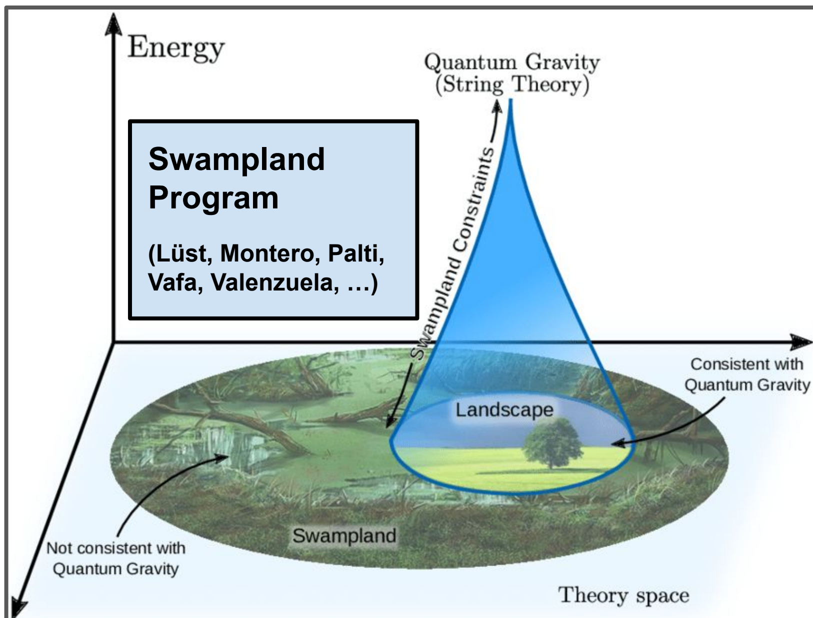
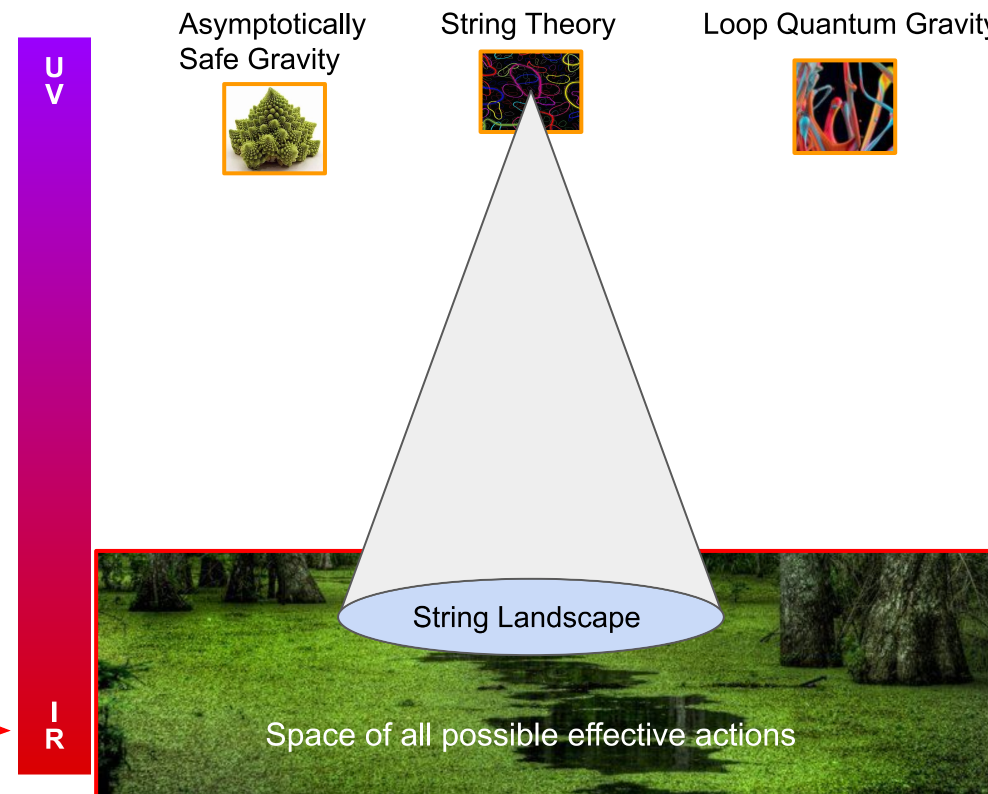
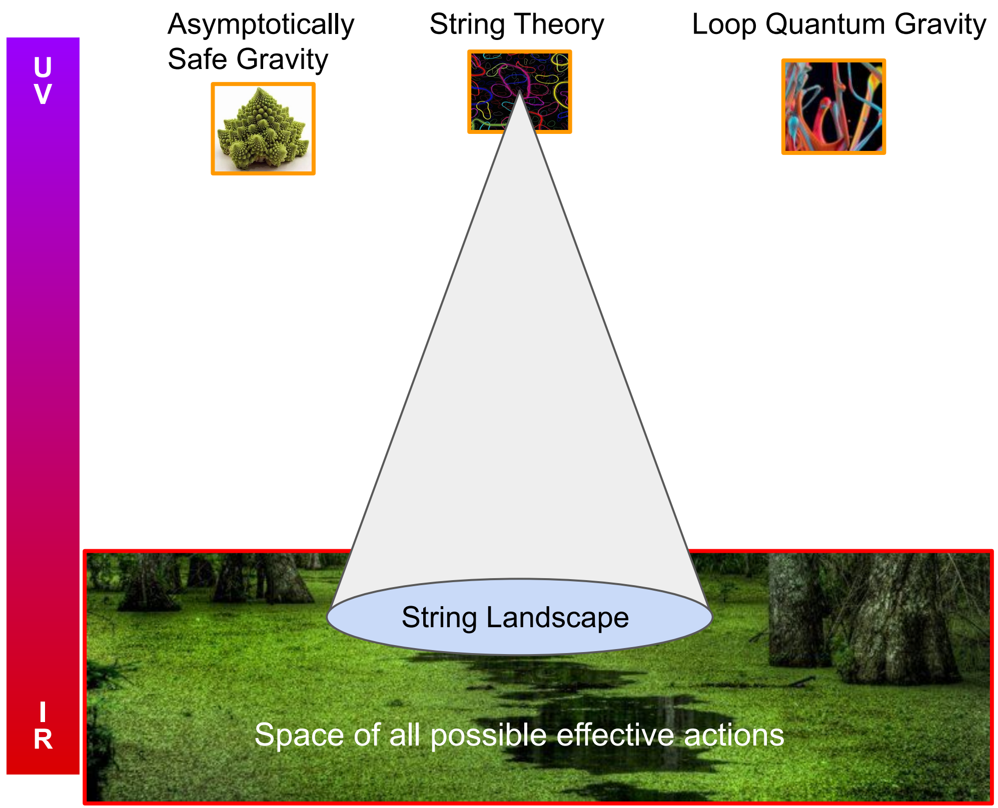




Quantum gravity through the lens of effective field theory

Can the "big picture" of the swampland program be generalized?

[Basile, AP, '21]
[Eichhorn, Hebecker, Pawlowski, Walcher, '24]



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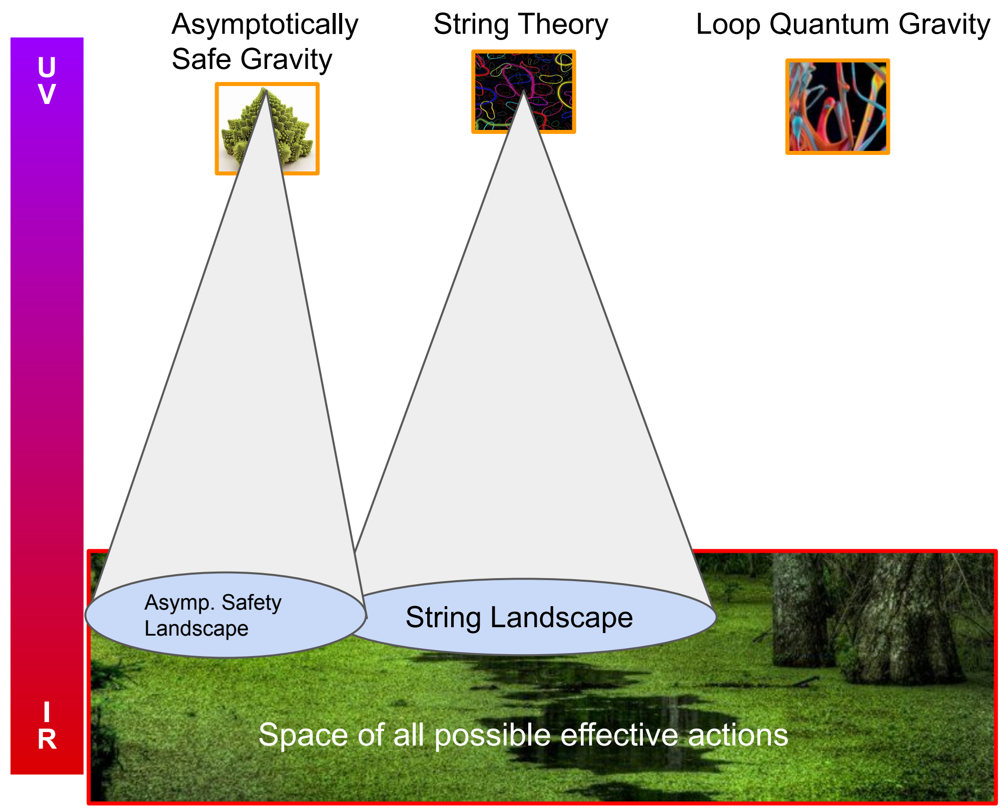

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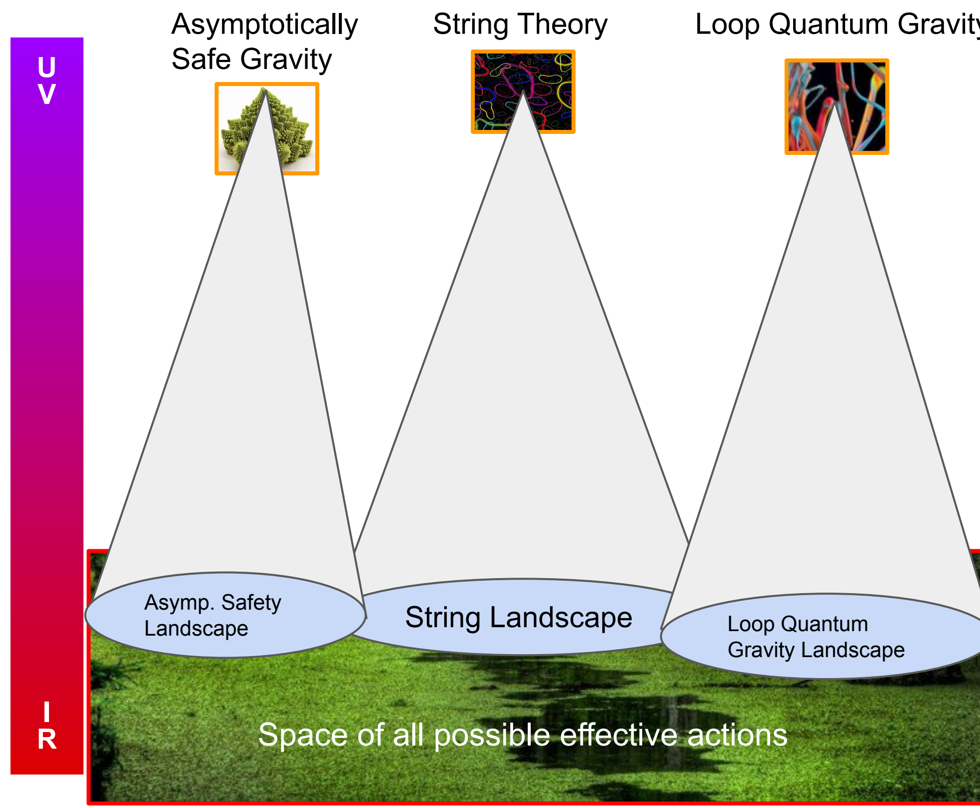

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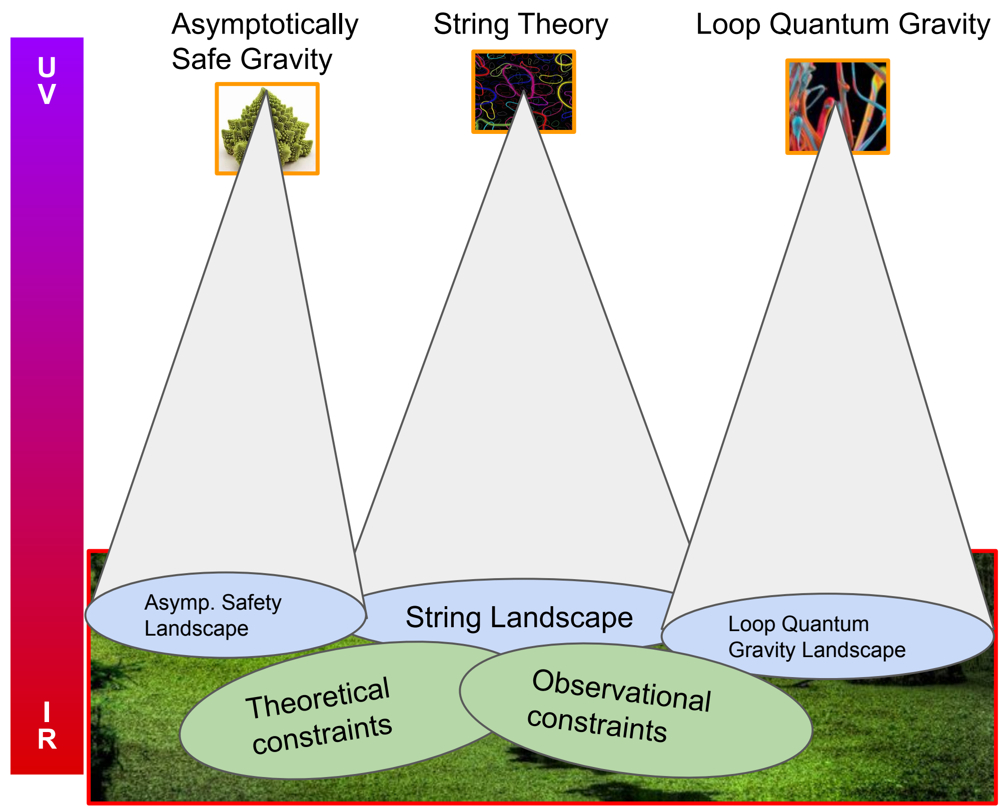


Quantum gravity through the lens of effective field theory

Several interesting questions at the intersections:

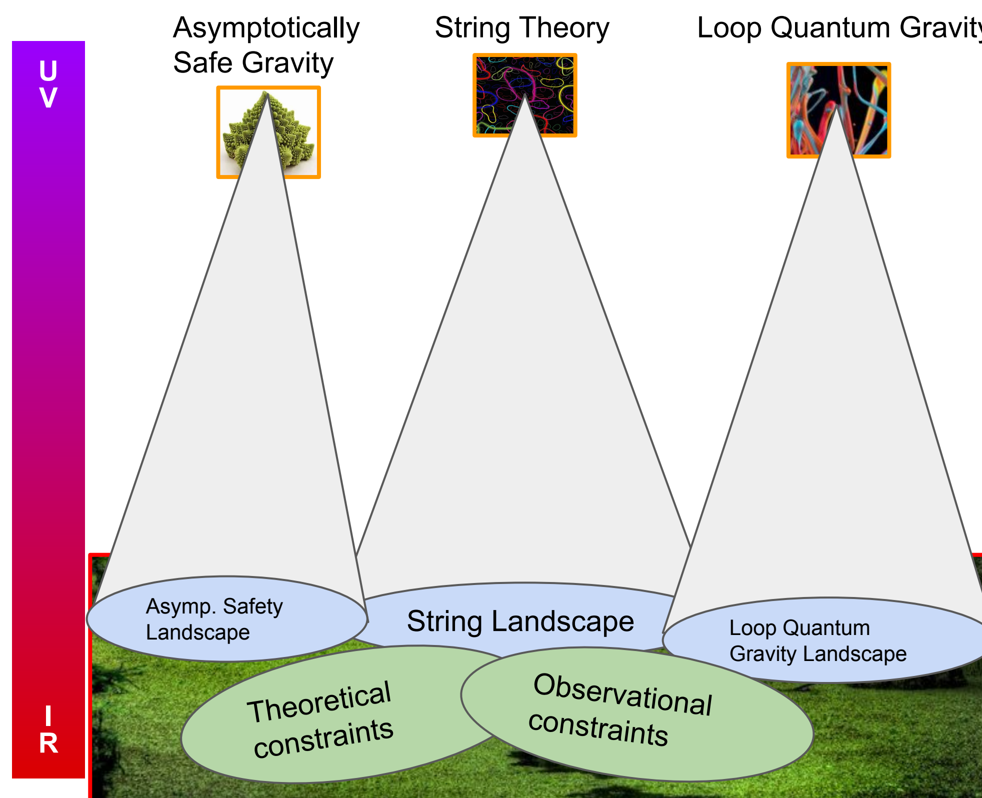
- **Consistency**, e.g., compatibility of QG predictions with positivity bounds (unitarity, causality, stability)

- **Tests of Swampland Constraints & string "universality"**: are they all general? Do they apply to all (consistent) QG or they only identify EFTs stemming from ST?

  c.f. String Lamppost Principle [Montero, Vafa, '21]: *"All consistent quantum gravity theories are part of the string landscape"*

- Comparison between **predictions of different QG approaches**? Connections between approaches?

- Comparison with bounds from **observations**?



…still many theories, where to start from?

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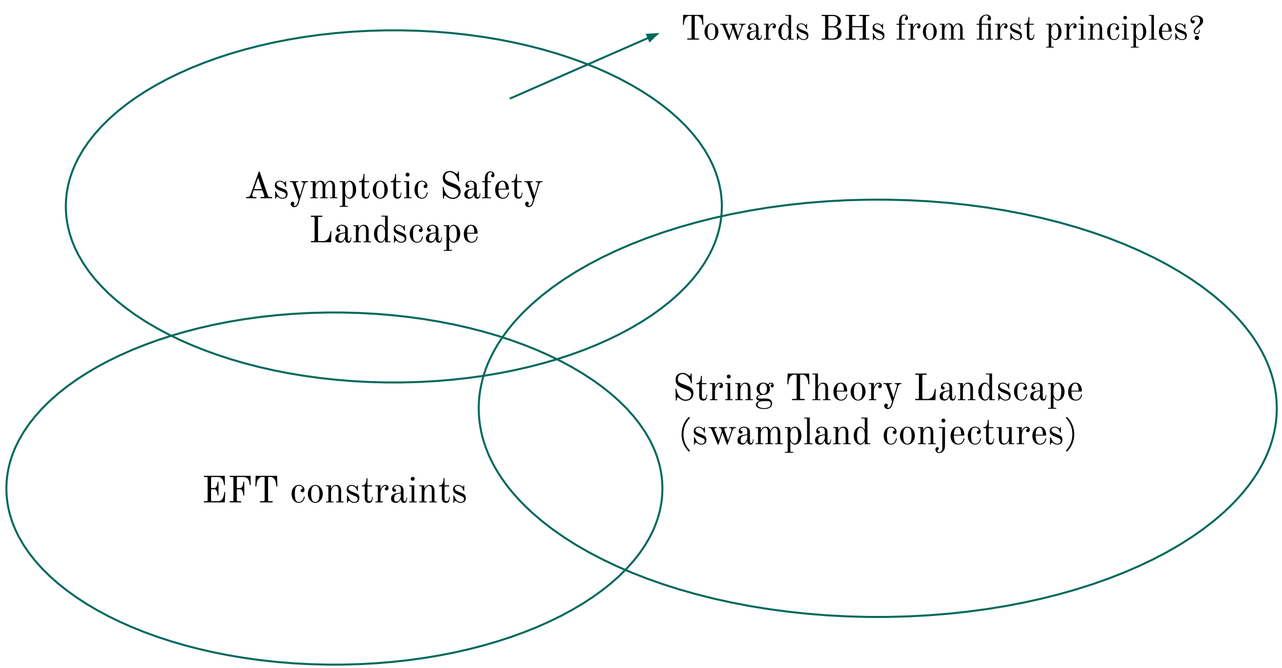
Personal criteria:

- Consistent to date (consistency: unitarity, causality, stability, renormalizability)
- Yields GR at low energies
- Connects with EFT

Based on current state-of-the-art:

- **String theory** = *beyond QFT, theory of extended objects*
- **Asymptotically safe gravity** = *gravity as a QFT*

This talk's focus

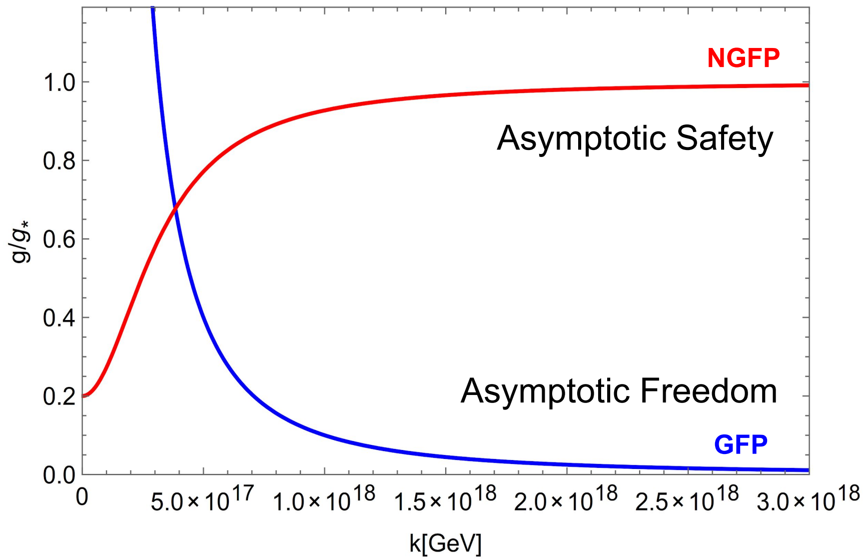


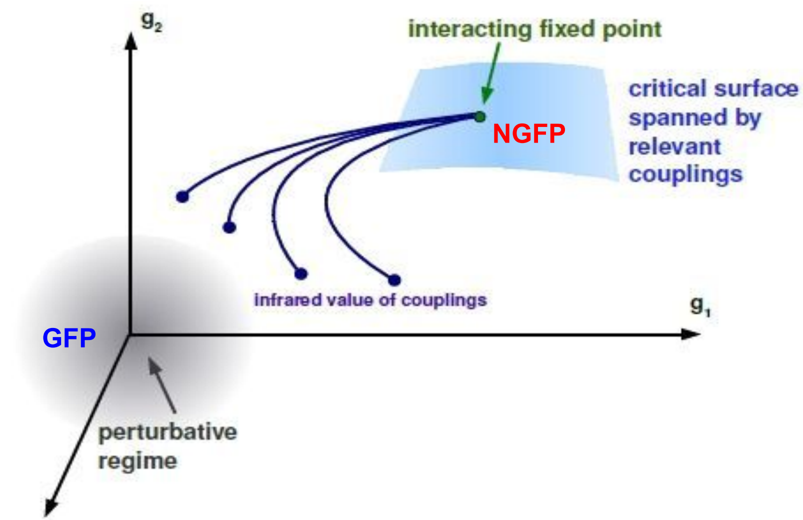
Landscapes

in Asymptotically Safe Gravity

Asymptotic Safety in a Nutshell

Let us start conservative: can we have a consistent QG theory fully within QFT?





Idea: *gravity non-perturbatively renormalizable, interacting UV-completion* (Weinberg, '76)

Predictivity: number of free parameters = number of relevant directions minus one fixing the scale

Methodology to test the idea: functional RG (semi-analytical) + Montecarlo simulations akin to lattice QCD.

Asymptotic Safety in a Nutshell

UV completeness and renormalizability: shown within a plethora of different analytical and lattice approximations, high confidence level

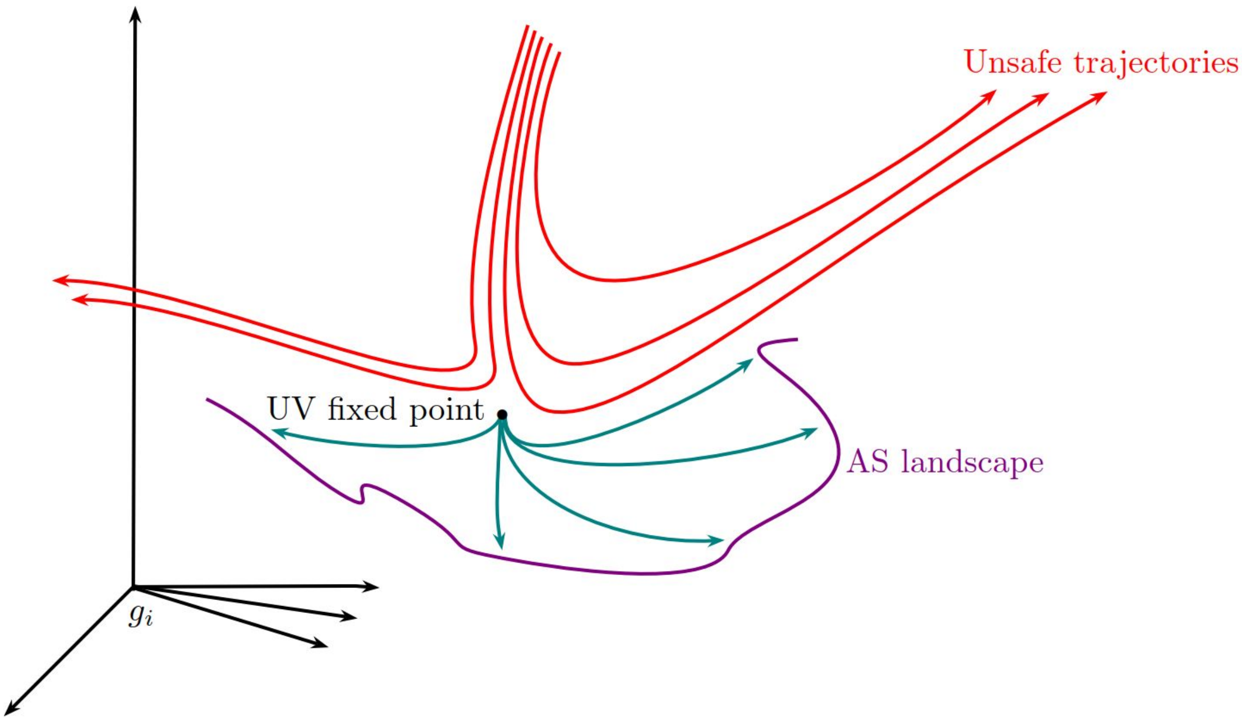
Promising results: when coupled with matter yields Higgs mass, top mass, quark mass differences compatible with experimental data + resolves Landau poles (review: Eichhorn, Schiffer, '22)

⇒ **ultimate theory? maybe not, but interesting** + maybe effective theory bridging EFT and a more fundamental description (de Alwis et al, '21, Basile, AP, '21)

Much more to be done:

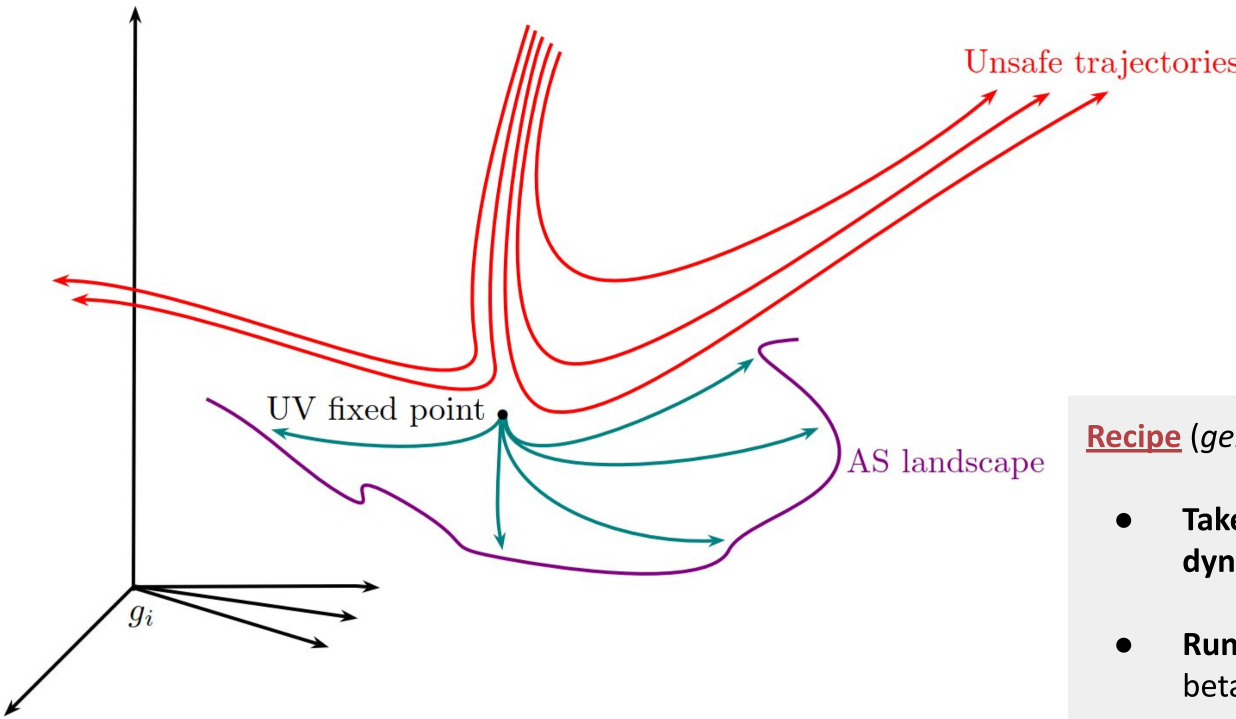
- **So far, much focus on the UV (completeness)**
- **Unitarity, causality, stability to be proven**
  [preliminary works show no inconsistencies; AP '20, Pawlowski et al '21, Knorr, Schiffer, '21, Knorr, AP, '24]
- ⇒ **constraining the landscape of gravitational EFTs stemming from AS**

Defining the asymptotic safety landscape



[Basile, AP, ‘21, Knorr, AP, ‘24]

Defining the asymptotic safety landscape

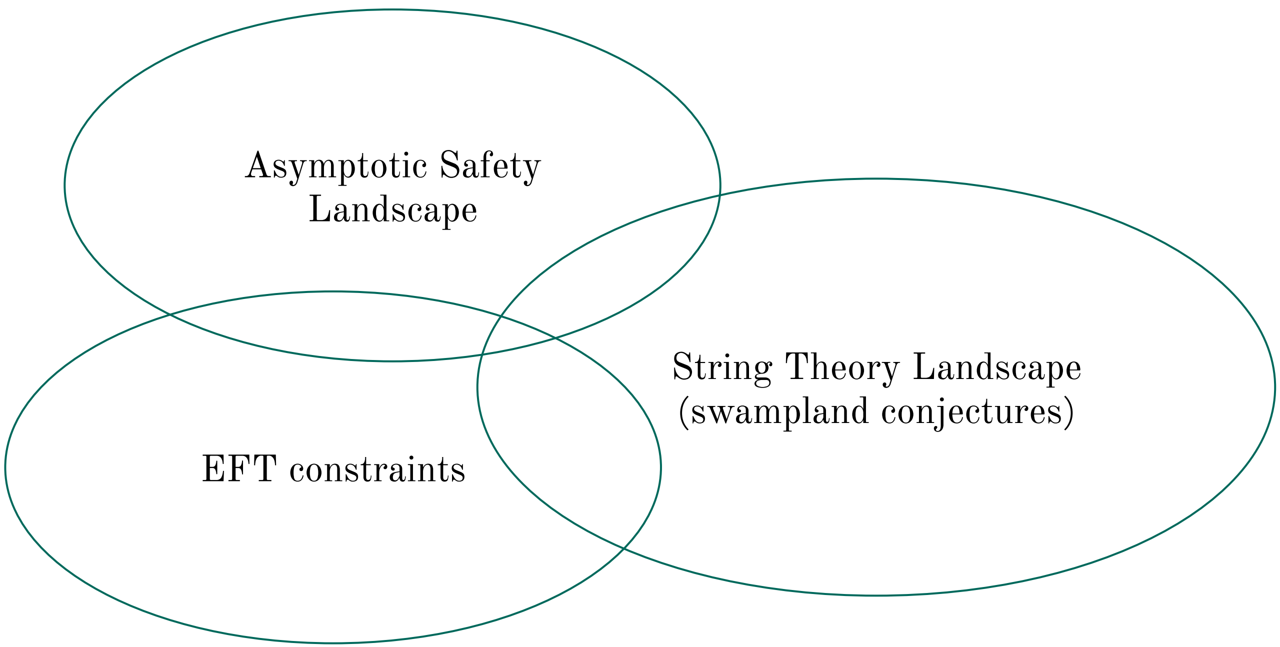


[Basile, AP, '21, Knorr, AP, '24]

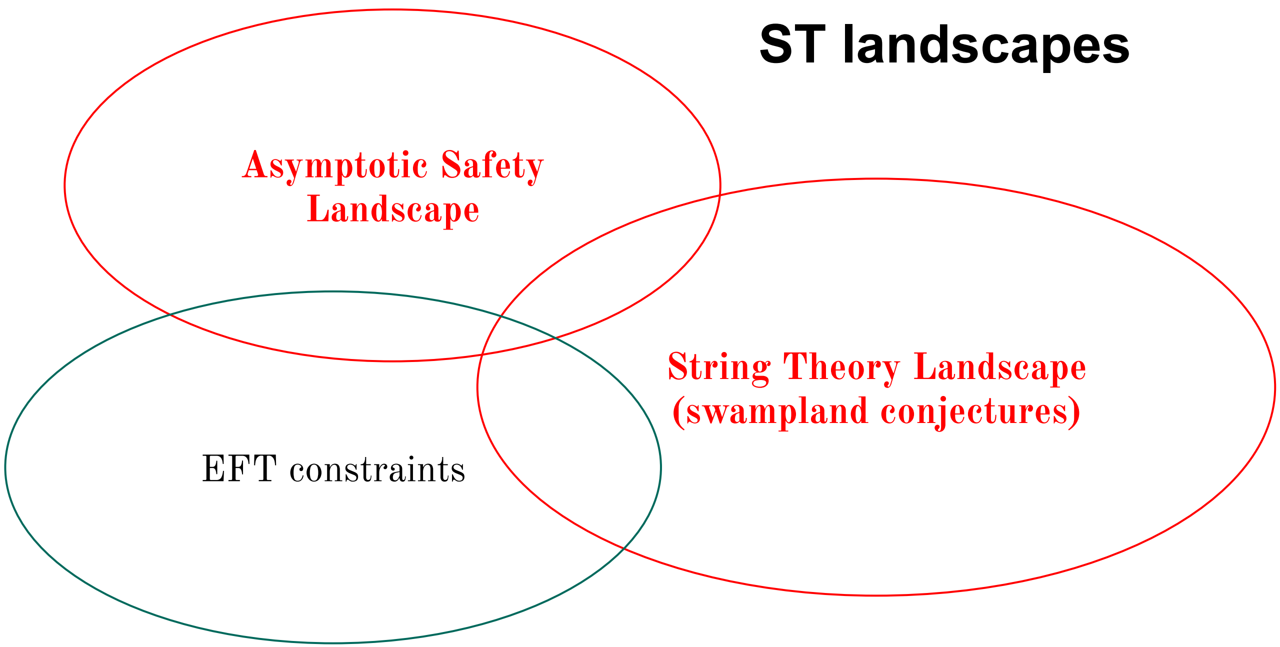
Recipe (*generalizable to other approaches?*)

- **Take a model of AS (truncation of the dynamics)**
- **Run (functional) RG machinery**: compute beta functions, solve beta functions for a sample of UV-complete trajectories
- Identify the "**AS landscape**" **in terms of dimensionless Wilson coefficients in the effective action** ***[some caveats in the def]***

Lessons from the intersections



AS landscapes
vs
ST landscapes

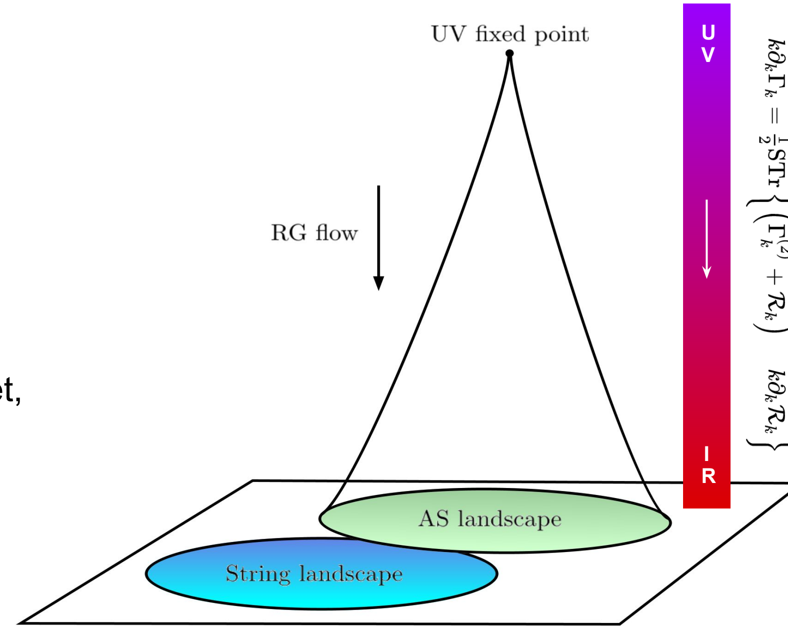


- **AS approximation**: one-loop quadratic gravity

$$\mathcal{L} = \frac{2\Lambda - R}{16\pi G} + \frac{1}{2\lambda} C^2 - \frac{\omega}{3\lambda} R^2 + \frac{\theta}{\lambda} E$$

- **Three dimensionless Wilson coefficients** (+ gauss-bonnet, but decoupled)
  One dimensionful coupling sets the mass unit scale!

$$G\Lambda, \qquad g_R = -\frac{\omega}{3\lambda}, \qquad g_C = \frac{1}{2\lambda}$$



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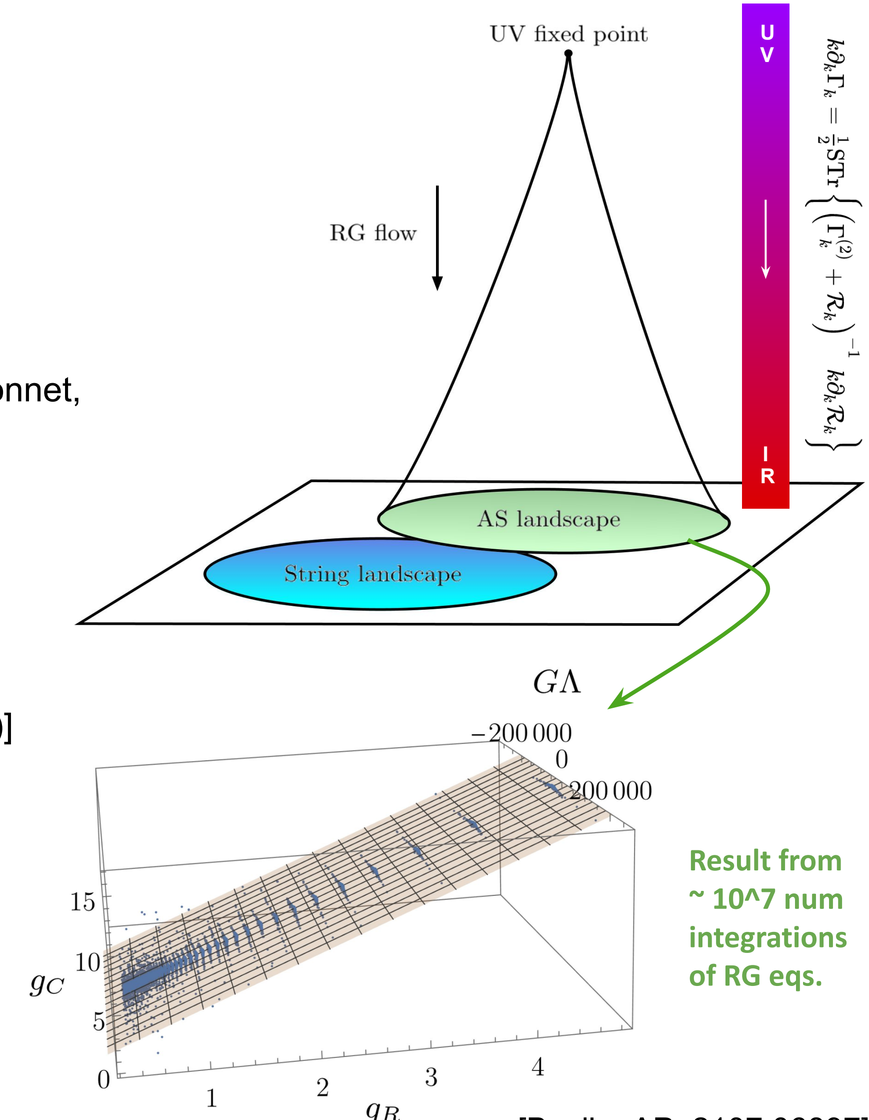
- Beta function and fixed points [(Codello, Percacci, 2006)]



[Basile, AP. 2107.06897]

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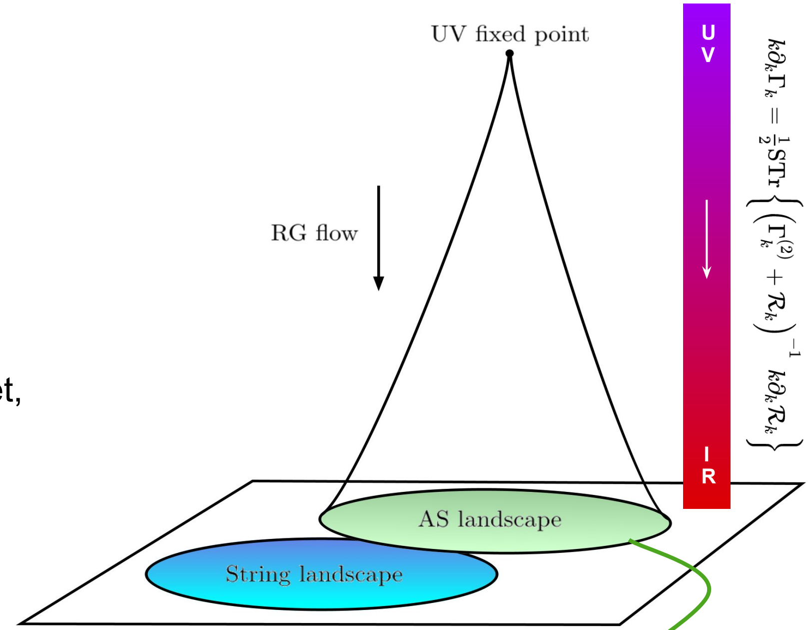
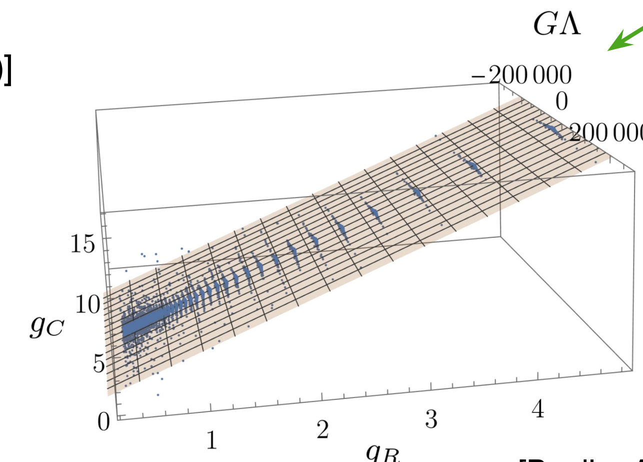
The Wilson coefficients stemming from an AS fixed point lie on a plane

$$\mathrm{EFT}_{\mathrm{AS}} \approx \{g_R = -0.74655 - \frac{2}{3}\,\omega_-\, g_C\}$$

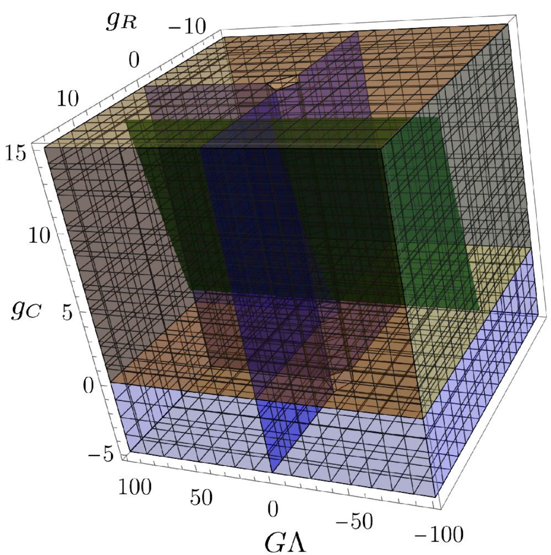
$$g_C > 0$$

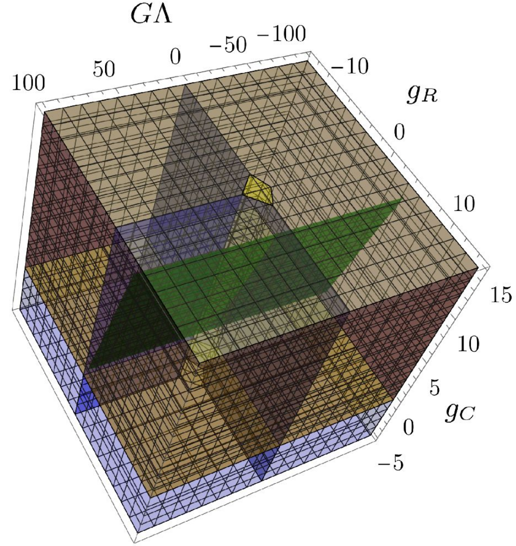


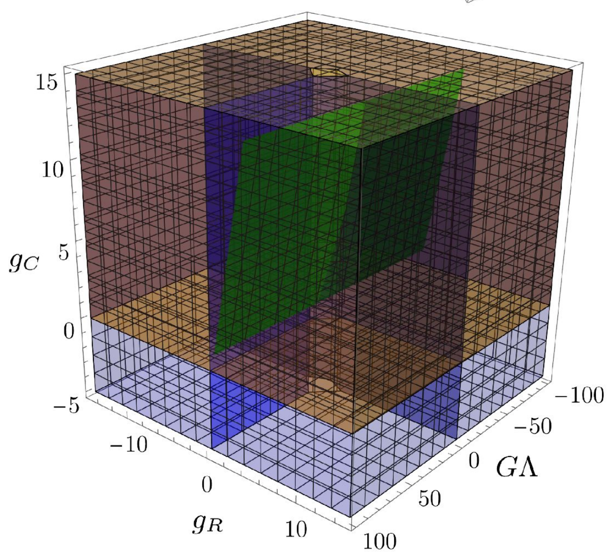


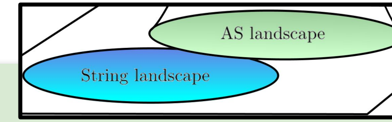
Result from ~ 10^7 num integrations of RG eqs.

[Basile, AP. 2107.06897]









Green plane:

AS landscape [one-loop quadratic approx]

$$\mathrm{EFT}_{\mathrm{AS}} \approx \{g_R = -0.74655 - \frac{2}{3}\,\omega_-\, g_C\} \qquad g_C > 0$$

Blue hyperplane:

Stringy “no de Sitter” conjecture

[~ no positive cosmological constant]

Yellow hyperplane:

Weak gravity conjecture

[~ gravity is the weakest force]

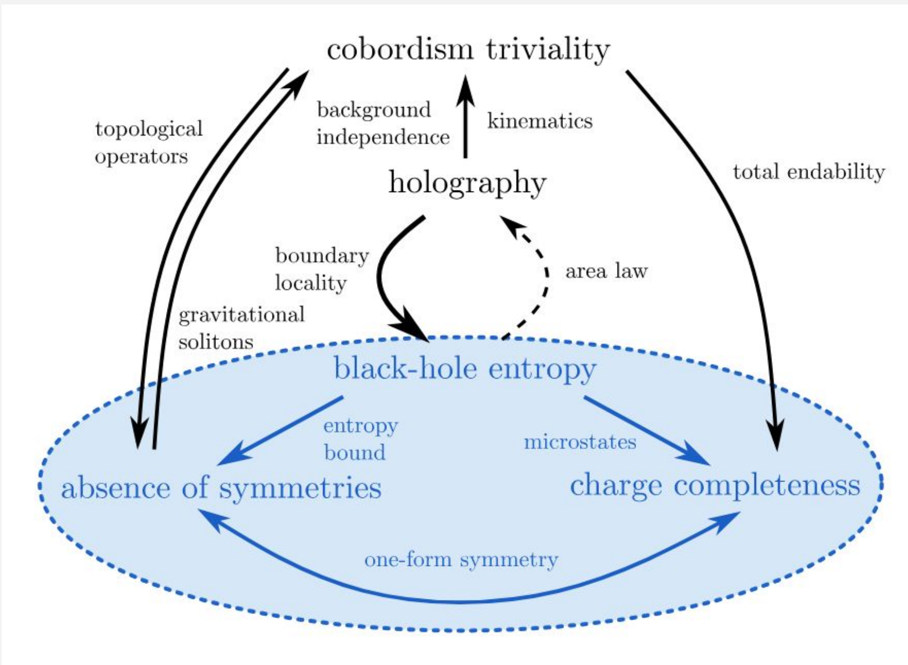
Within this simple model of AS, and only some swampland conjectures

⇒ **non-trivial intersection (partial compatibility?)**

[Basile, AP. 2107.06897]

Not all swampland conjectures need to hold in all approaches, they may only hold in ST.
Non-trivial intersection between different landscapes is possible, but conceptually, there may be differences

Beyond models and approximations:
On a more conceptual level (no model, no approximations), assuming standard EFT expansion and principles, topology change is key to satisfy the most solid swampland conjectures!



ST vs fundamental version of AS:

Non-trivial intersections [Basile, AP, '21], *but conceptual difference* [Basile, Knorr, AP, Schiffer, '25]

⇒ no full overlap of landscapes

[Basile, Knorr, AP, Schiffer, '25]

But what about Effective Asymptotic Safety?

- EFT holds at low energies
- AS holds at intermediate energies (hence, predictions for Higgs, top, quarks still hold)
- At higher energies QFT breaks down and a more fundamental description takes over

In this case there can be a full overlap, since the AS would inherit the properties of the more fundamental theory

Practical advantages:

- Can use AS to make universal predictions, with relatively simple calculations
- Can use the fixed point and RG with elements from the more fundamental dynamics to extract more specific predictions

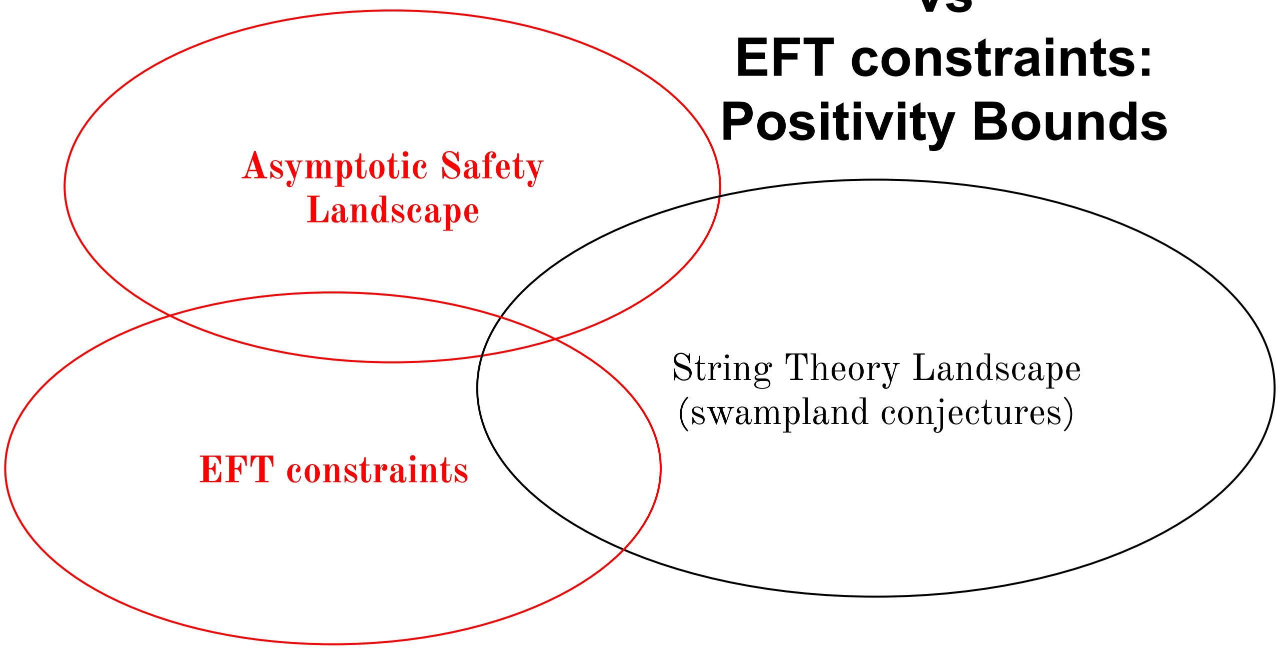
  ⇒ e.g., with effective AS, one can show that even non-pert dS solutions are hard to get in ST
  [Basile, AP, '21]
  ⇒ e.g., with effective AS, one shows that LQG can largely violate parity
  [Borissova, Dittrich, Eichhorn, Schiffer, '25]

How to test whether AS is fundamental or effective?

E.g., effective if it makes the right pre/post-dictions but breaks unitarity

⇒ need to test its fundamental details

AS landscapes
vs
EFT constraints:
Positivity Bounds



- **AS model**: **photon-graviton** systems at quadratic order, only **essential couplings** included

$$\mathcal{L} = -\frac{R}{16\pi G_N} + \Theta_E\, E + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + G_2\, (F^{\mu\nu} F_{\mu\nu})^2 + G_4\, F^{\mu}{}_{\nu} F^{\nu}{}_{\rho} F^{\rho}{}_{\sigma} F^{\sigma}{}_{\mu} + G_{CFF}\, C^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- **Three dimensionless Wilson coefficients (redefined for convenience; only one log-presc. ambiguity)**

$$w_+ = \frac{1}{2} \frac{G_2 + G_4}{(16\pi G_N)^2}\,, \quad w_- = \frac{1}{2} \frac{G_2 - G_4}{(16\pi G_N)^2} + b\, \ln\!\left[16\pi G_N k^2\right], \quad w_C = \frac{G_{CFF}}{16\pi G_N}$$

[Knorr, AP, 2405.08860]

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- **IR: positivity bounds and weak gravity conjecture**

Positivity bounds:

$$w_+ > w_-\,, \quad 3w_+ - w_- - 2|w_C| > 0$$

[Carrillo González, de Rham, Jaitly, Pozsgay, Tokareva, ‘23]

Electric WGC in the presence of higher derivatives

$$3w_+ - w_- + 2w_C > 0$$

[Cheung, Liu, Remmen, ‘18]

- **Ambiguity in removing the logs**
- **Positivity bounds typically identified in theories with massive DOF that are integrated out, not in the presence of massless poles**
- **NOTE: Standard positivity bounds may be violated in the presence of gravity**

$$c > 0 \quad \rightarrow \quad c > -\mathcal{O}(1)\, M^{-2} M_{Pl}^{-2}$$

[Alberte, de Rham, Jaitly, Tolley, ‘20+’21)]

[Knorr, AP, 2405.08860]

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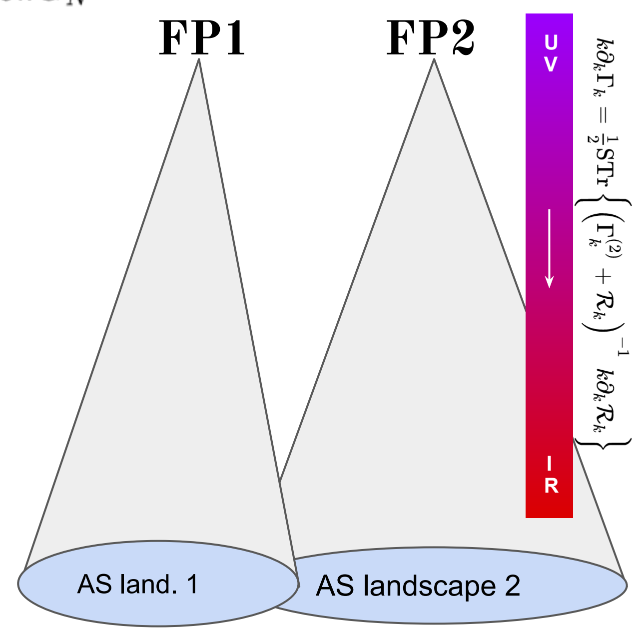
- **Two UV fixed points:**

FP1: one relevant direction (most predictive!)

⇒ once the QG scale is fixed, this is a zero-parameter theory =
1 point in the space of dimensionless Wilson coefficients

FP2: two relevant directions

⇒ effective action parametrized by 1 dimensionless parameter
(line of EFTs)



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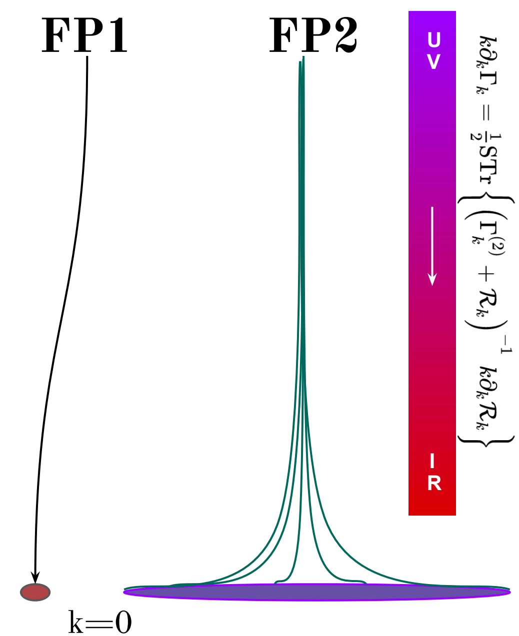
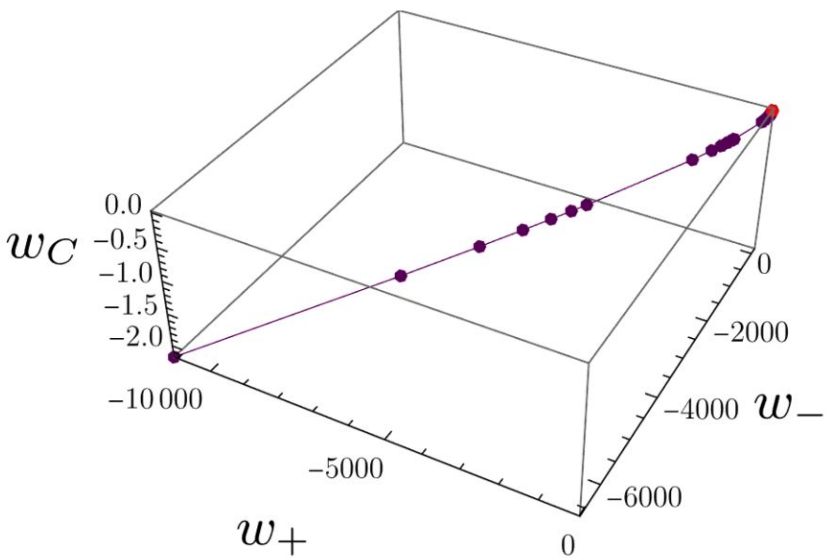
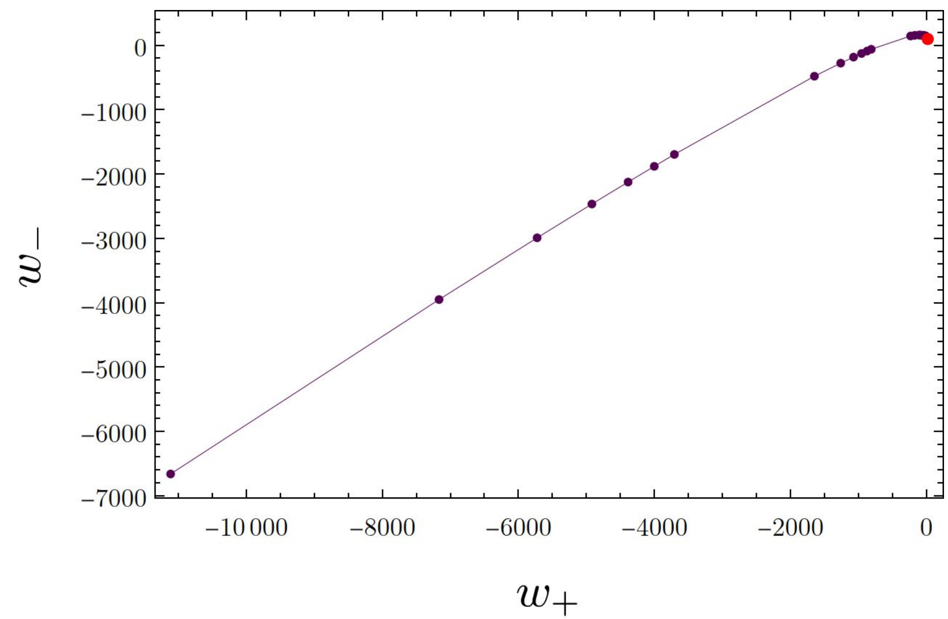


Asymptotic Safety Landscapes

[Knorr, AP, 2405.08860]





AS landscape from FP1: 1 single point

AS landscape from FP2: almost straight line

Asymptotic Safety Landscapes

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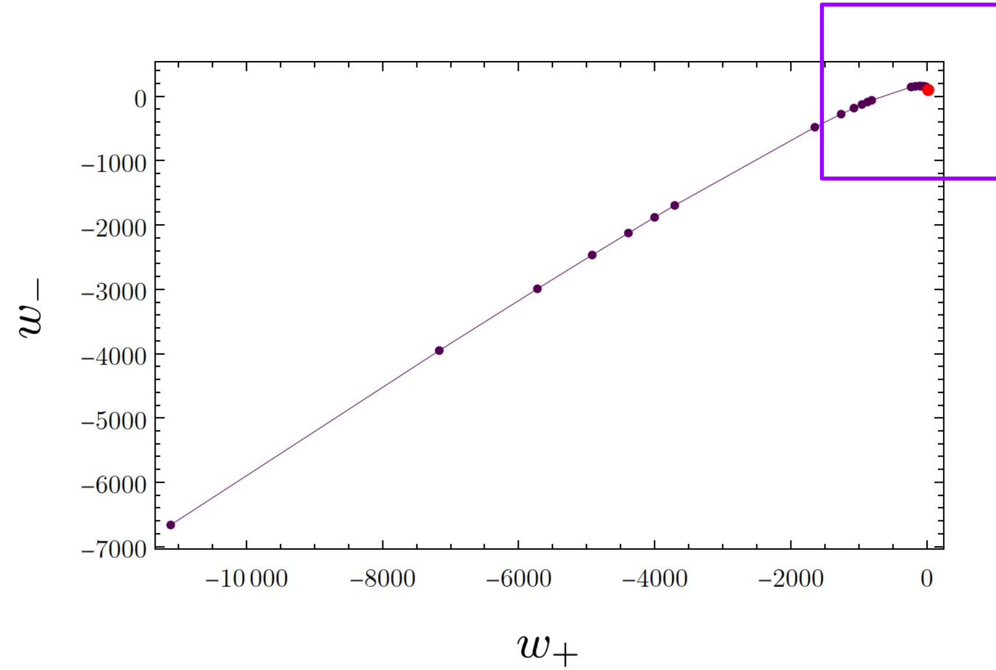


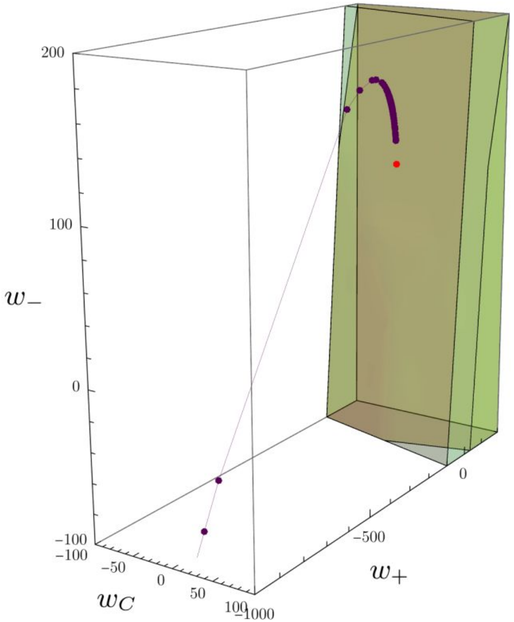
AS landscape from FP1: 1 single point

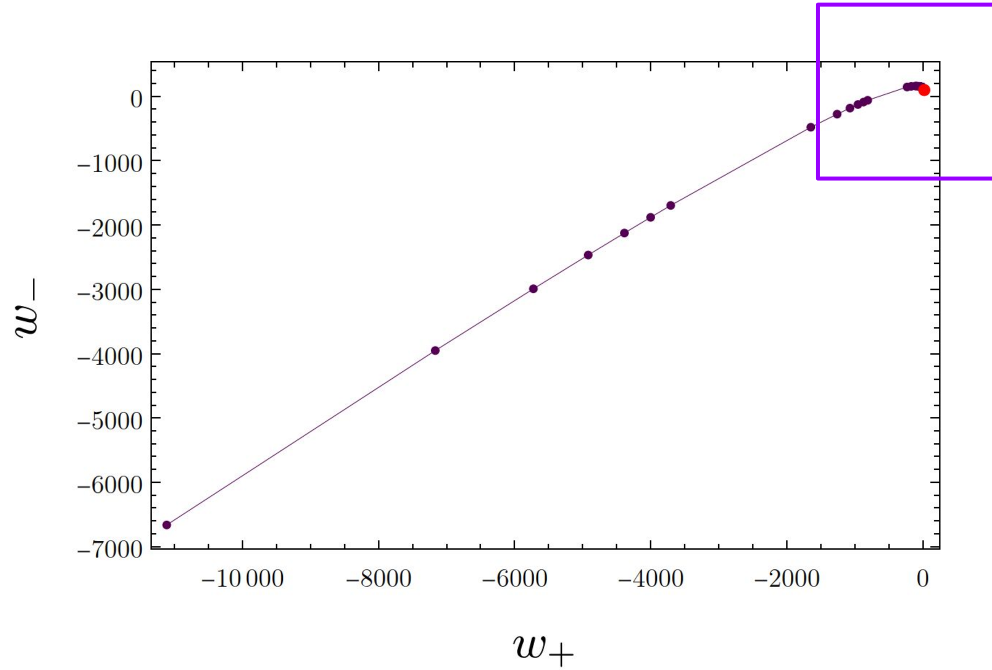
AS landscape from FP2: almost straight line

+ small "candy cane" regime which connects the two

Asymptotic Safety Landscapes

[Knorr, AP, 2405.08860]





Planck-scale suppressed violations of WGC and positivity bounds [B. Knorr, AP, '24]

Compatible with expectations/conjectures from EFT in the presence of massless poles:
[Alberte, de Rham, Jaitly, Tolley, '20]

$$c > -\mathcal{O}(1)\, M^{-2} M_{Pl}^{-2}$$

But may be a sign that AS holds only approximately

AS landscape from FP1: 1 single point

AS landscape from FP2: almost straight line

+ small "candy cane" regime which connects the two

AS landscapes and
Black Hole Mimickers

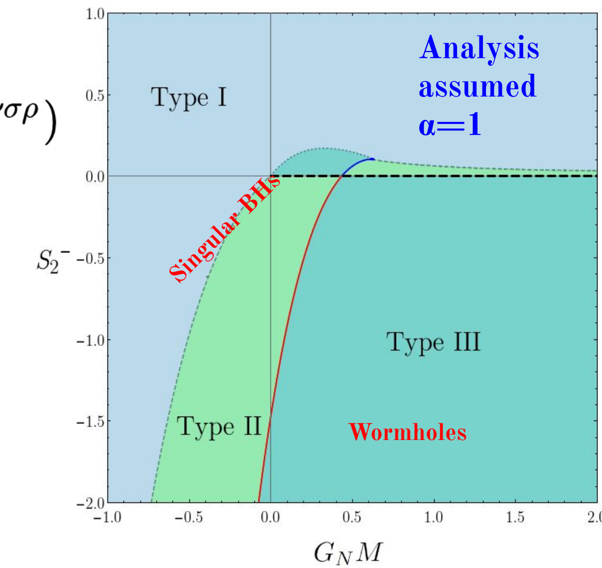
Towards Black Holes from First Principles

- Several phenomenological models, unclear relation to fundamental physics
- Some progress in charting "black hole phase diagrams": mapping solutions for certain actions as function of Wilson coefficients and integration constants [Bonanno, Silveravalle, Zuccotti...]
- Quantum gravity can predict Wilson coefficients
- **Idea**: connect the to worlds. **Compute Wilson coefficients from given UV completion of QG and associate specific black hole phase diagram**

Simple example: **Asymptotically Safe Einstein-Weyl gravity**

$$L = \frac{1}{16\pi G}\left(R + C2\, C_{\mu\nu\sigma\rho}C^{\mu\nu\sigma\rho}\right)$$

Only one dimensionless Wilson coefficient: α=C2/G



[Silveravalle, Zuccotti, '22]

Towards Black Holes from First Principles

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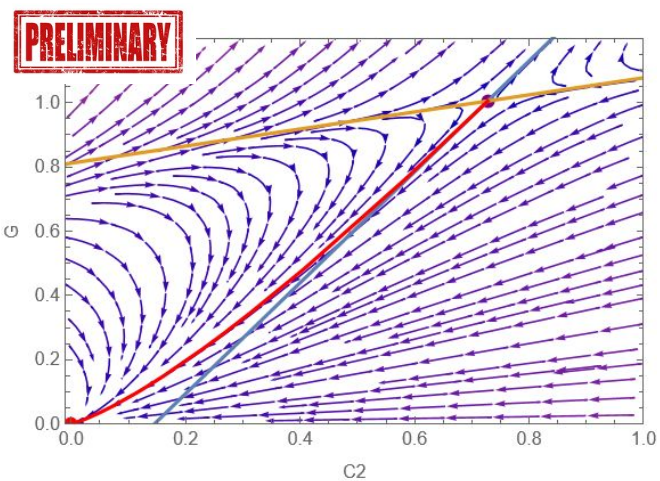

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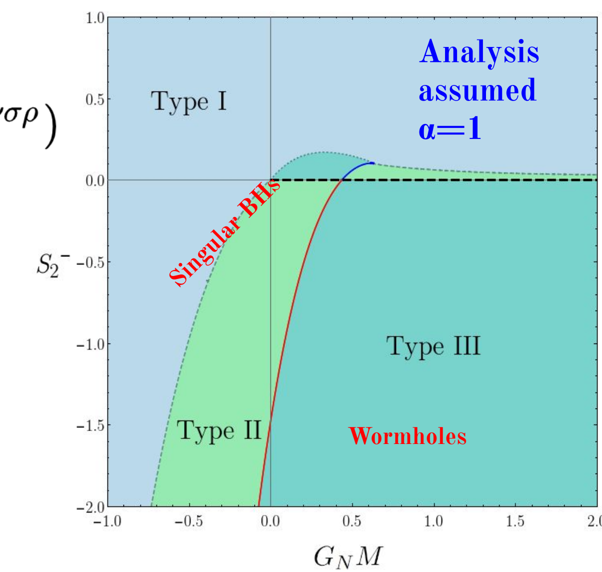


UV complete trajectory gives α=0.509249
⇒ complete BH phase diagram from UV complete model (truncation)!
[WIP with F. Del Porro, J. Pfeiffer, S. Silveravalle]

[Silveravalle, Zuccotti, '22]

Summary

- Testing quantum gravity and comparing theories: **QG through the lens of Effective Field Theory**

- One promising framework beyond ST: **asymptotically safe gravity, aka, QG as a QFT**

- *Computing QG landscapes: “killing N birds with one stone”*
  Testing swampland conjectures in other approaches to quantum gravity, e.g., asymptotic safety
  Testing consistency of QG predictions (from different approaches): positivity bounds
  ST vs AS landscape (vs others?): comparing predictions
  String Lamppost Principle: do swampland conjectures identify the string landscape or are more general?

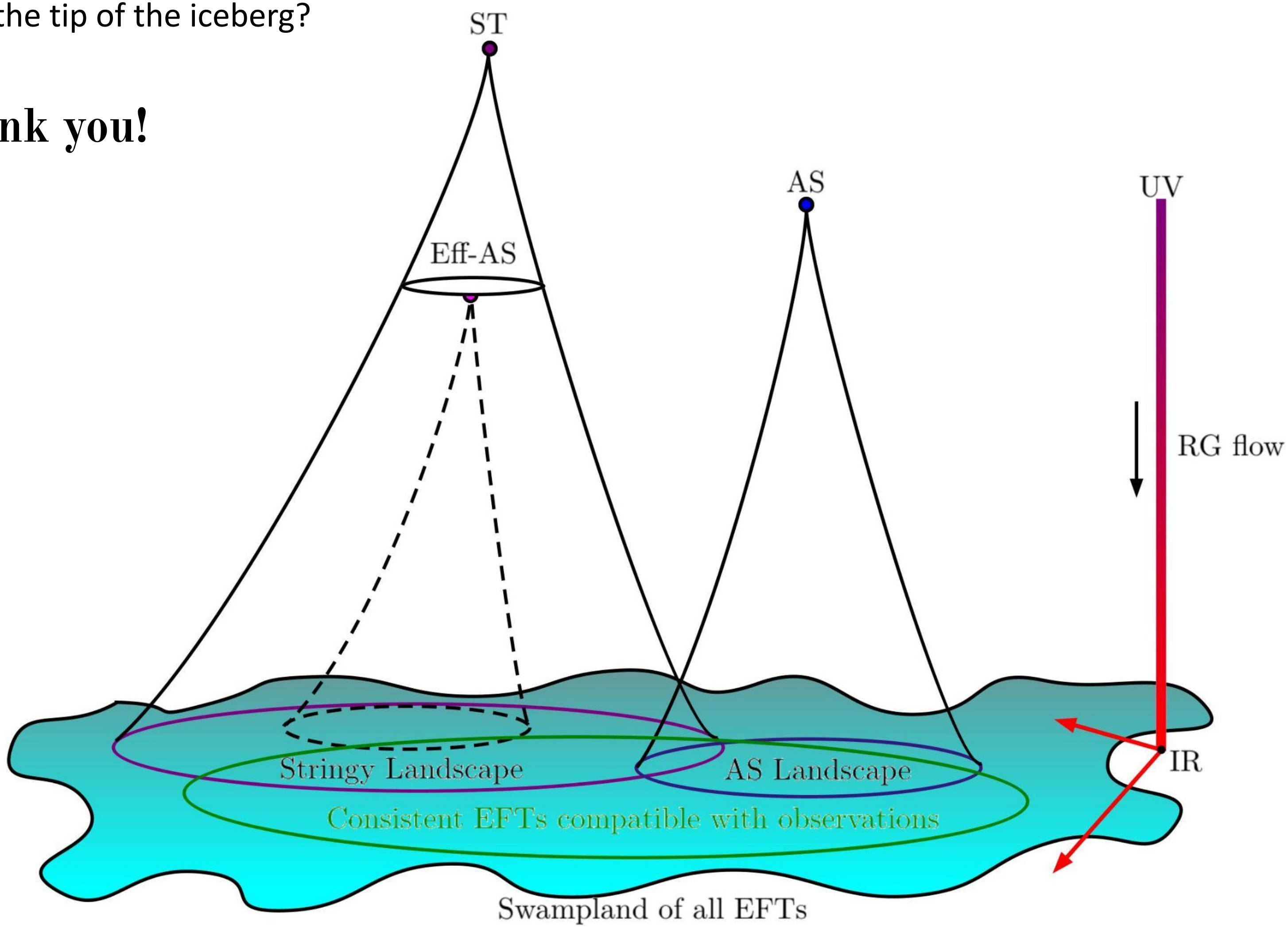
- **Very clear recipe in asymptotic safety**:
  - Start from UV fixed point, integrate the RG flow down to the IR ⇒ AS landscape
  - Find intersections: swampland constraints, positivity bounds, other QG landscapes
  - Quantum spacetimes from effective action

- **Exciting research directions opening up:**
  String and Asymptotic Safety Landscapes: non-trivial intersection? Effective AS?
  Positivity and causality bounds: (almost) satisfied by the landscape?
  Emerging feature: flatness of the Asymptotic Safety Landscape?
  Quantum black holes from first principles?

Thank you!

…merely the tip of the iceberg?

Thank you!



Functional Renormalization Group

Solving the **quantum theory** is equivalent to solve the functional **renormalization group equation**

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right\}$$

C. Wetterich. *Phys. Lett. B* 301:90 (1993)
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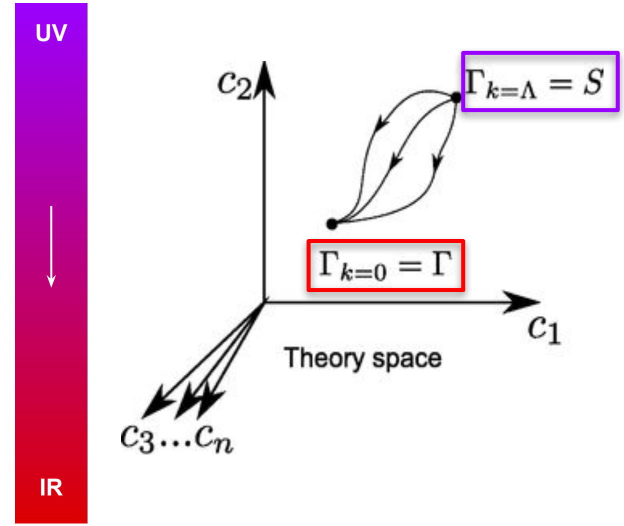


S_b **Fundamental (bare) action**, k→∞ → **Fixed points** = bare action, N relevant directions

Fast fluctuating modes are integrated out

Γ_k **Effective action at the energy scale k** → All terms compatible with symmetry and field content of the theory are generated

Γ_0 **Ordinary effective action**, k→0 → **Effective action** (limit k→0), infinitely many terms parametrized by N free parameters

All quantum fluctuations are integrated out → non-locality

Incorporates all quantum effects → fully-dressed quantities

Access to **S-matrix**, **Wilson coefficients**, **observables**

Implementation in AS: defining the Wilson coefficients

Defining the Wilson Coefficients (+ caveats)

- **Defining the dimensionful Wilson coefficients with the FRG:**

  $$W_{G_i} \equiv \lim_{k\to 0} G_i(k)$$

- But, actually:

  We only measure dimensionless quantities, thus we need one unit mass scale (e.g., Newton coupling) and N-1 dimensionless Wilson coefficients to parametrize the landscape of EFTs (N=number of relevant directions)

  $$w_{G_i} \equiv \lim_{k\to 0} G_i(k) M_{Pl}^p$$

- **CAVEAT: Defining Wilson coefficients in the presence of Log running in the IR is ambiguous, and one needs a prescription to subtract logs. Our prescription: use the transition scale to QG.**

  $$w = a + b\,\log(k^2/M_{Pl}^2) + b(\log(k_0^2) - \log(k_0^2))$$
  $$= \tilde{a} + \tilde{b}\log(k/k_0^2)$$

[Basile, AP '21]
[Knorr, AP '24]

Computing the AS landscape

<u>Constraining the theory-space landscape</u>

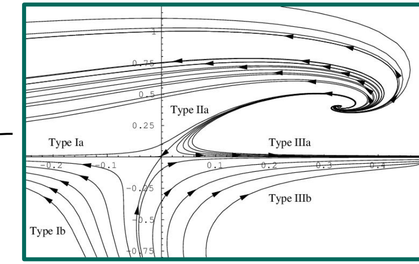
- Only relevant directions ⇒ *landscape = entire theory space (-inf,+inf)*
- General systems: focus on a few representative trajectories
- Matter content of the AS landscape and constraints on matter couplings from AS

[most recent review: *Eichhorn, Schiffer, '22*]

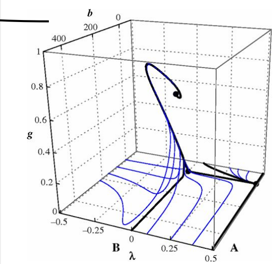
<u>Computing the AS landscape</u>

- Makes sense when there are irrelevant directions ⇒ predictions
- Use sufficiently high number of initial conditions to evenly cover the entire theory-space landscape—boundary to boundary (or use **spectral methods**, as in *[Saueressig, Silva, '24]*)
- Switch to space of dimensionless Wilson coefficients:
  - Log prescription
  - Set the units
  - Limit k->0 of dimensionless ratios of couplings
- Ideally: determine <u>geometry/equation of the AS landscape</u> ⇒ easier comparisons

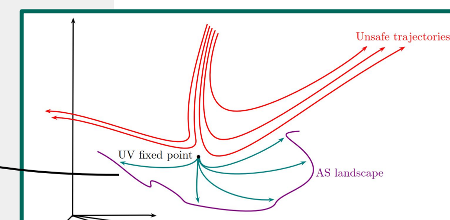
[*Basile, <u>AP</u>, '21; Knorr, <u>AP</u>, '24*]



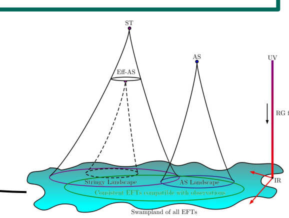
[Reuter, Saueressig, '02]

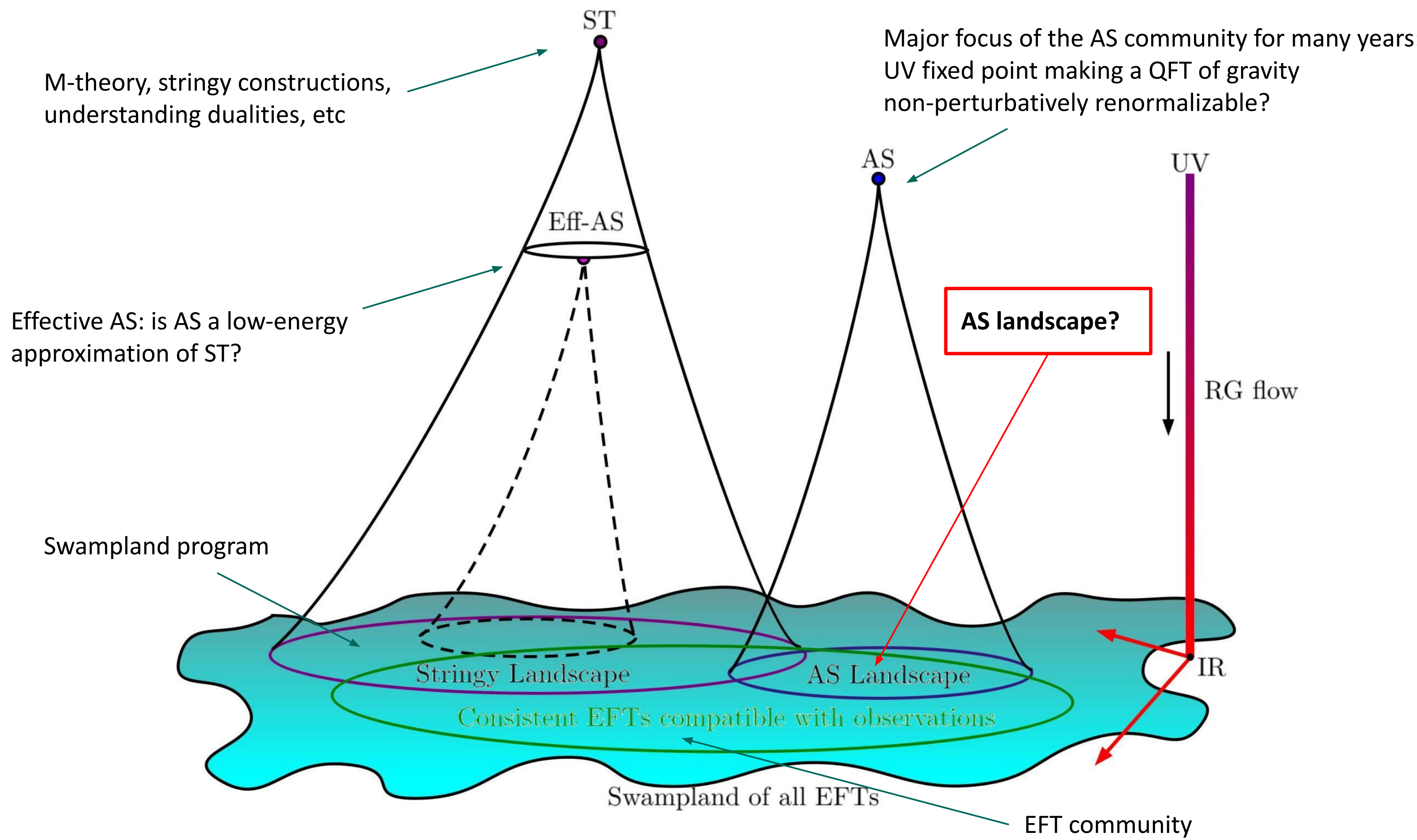


[Rechenberger, Saueressig, '12]



[Knorr, <u>AP</u>, '24]





ST
M-theory, stringy constructions,
understanding dualities, etc
Major focus of the AS community for many years:
UV fixed point making a QFT of gravity
non-perturbatively renormalizable?
AS
UV
Eff-AS
Effective AS: is AS a low-energy
approximation of ST?
AS landscape?
RG flow
Swampland program
Stringy Landscape
AS Landscape
IR
Consistent EFTs compatible with observations
Swampland of all EFTs
EFT community

Concretely:

- **EFT parametrizes and constrains.** (e.g. Wilson coefficients, amplitudes)

- **QG computes.** (ideally as many parameters as possible: predictivity!)
  (e.g., via renormalization group, simulations, amplitude techniques)

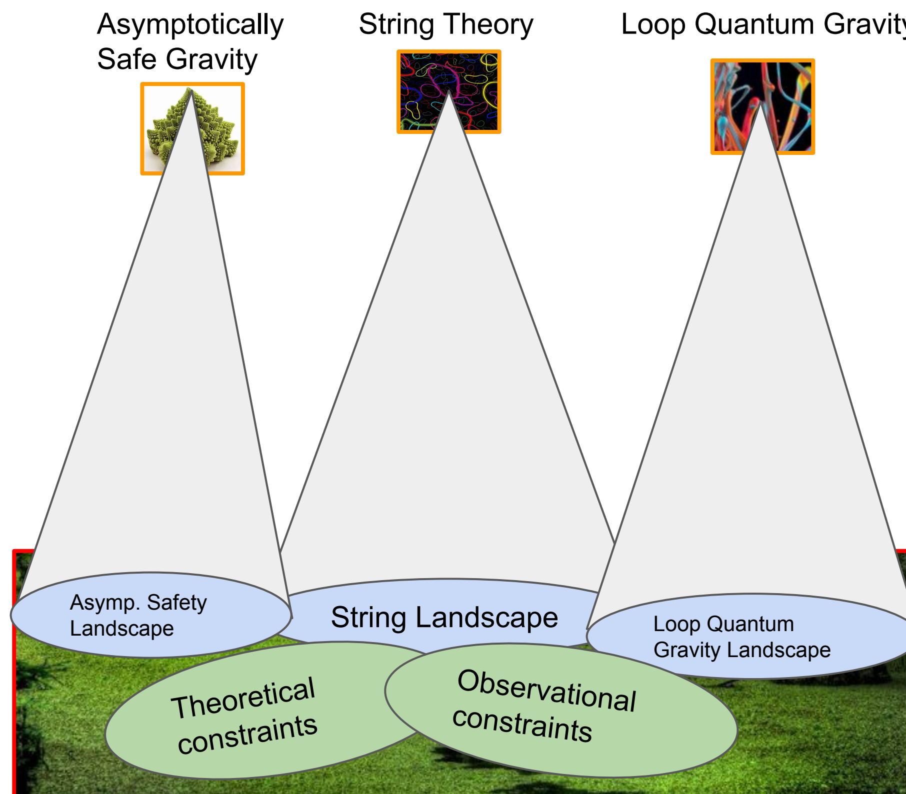
Concretely, in terms of effective actions (can be mapped onto amplitudes):

$$\Gamma_{\mathrm{eff}} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + \mathcal{L}_{\mathrm{HD}} \right)$$

$$\mathcal{L}_{\mathrm{HD}} = \frac{1}{16\pi G} \left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)R^{\mu\nu\rho\sigma} + \mathcal{O}(R^3) \right)$$

From a given **UV completion**, one may map a *specific* set of Wilson coefficients. This is one way to define a **landscape**.

Then **compare**, extract **physical results**



- ***QG is a multi-scale problem***

    - Different theories / UV completions ⇒ different fundamental properties (and different conceptual and technical problems). Details relevant at trans-Planckian scales.
    - Observations spanning intermediate to large distances (cosmology, dark energy, gravitational waves)
    - EFT: consistency constraints in the IR

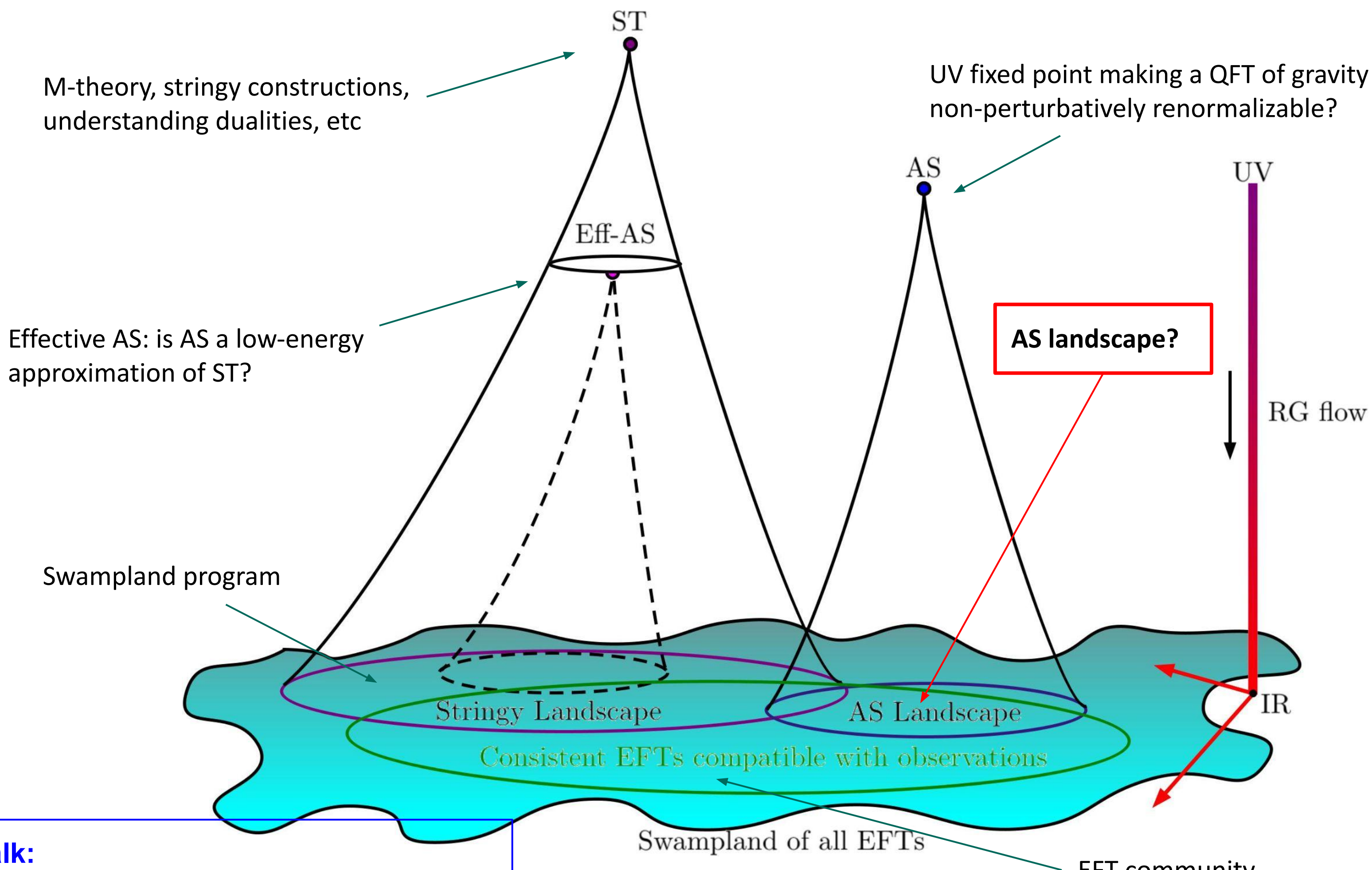
- ***Technical and conceptual interrelated difficulties in connecting UV and IR, and different UVs***

    - Theory is not driven by experiment (scale separation)
    - Difficult to make predictions from scratch
    - Equivalent theories?

      Comparing approaches in the UV is like comparing apples with bananas!

- ***A "decoupling phenomenon" in gravity***

    - "Formal" QG communities: mostly focus on the UV
    - Pheno & EFT communities: mostly focus on the IR



This talk:

- **AS vs ST (some swamp conjs)**
- **AS vs EFT (positivity)**

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Solving the **quantum theory** is equivalent to solve the functional **renormalization group equation**

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right\}$$

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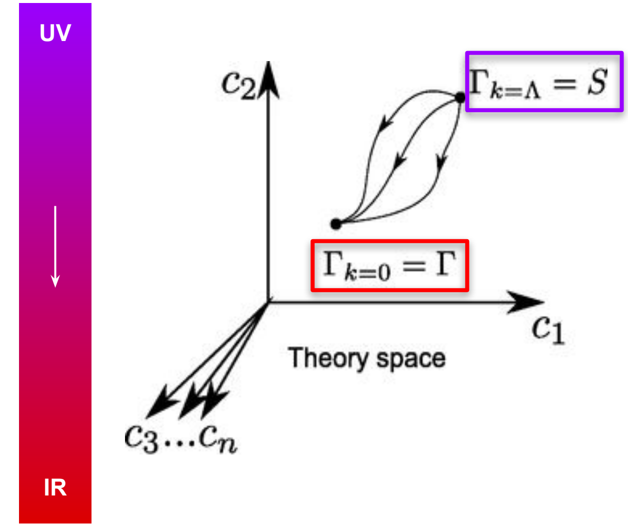
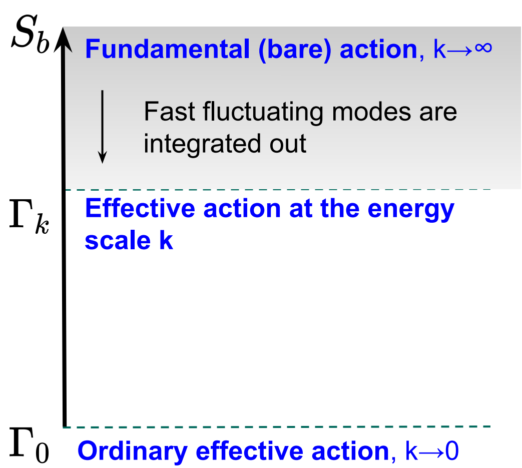




→ <u>UV fixed points</u> = bare actions, <u>N relevant directions</u>

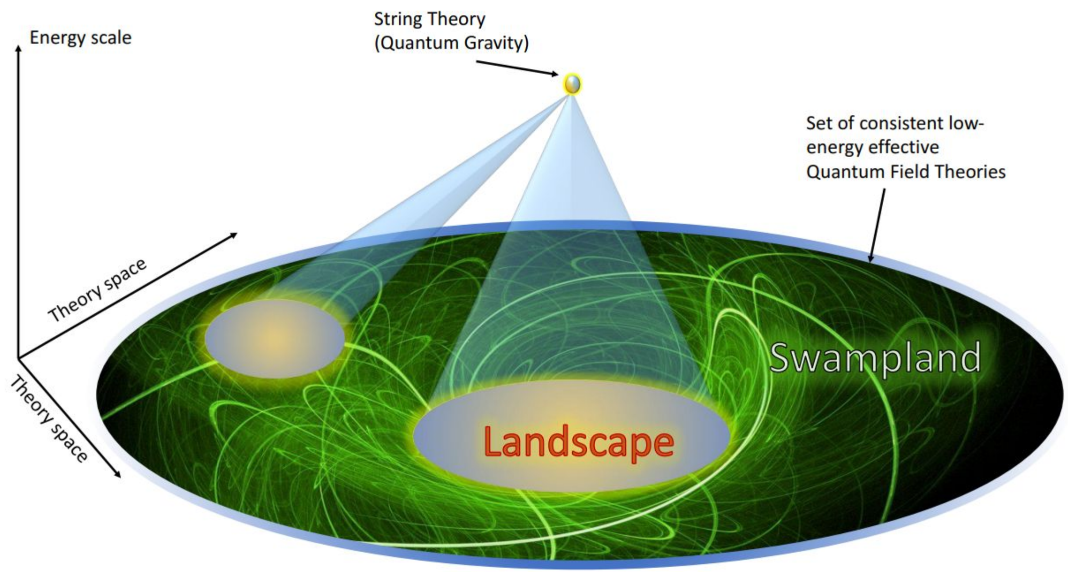
→ <u>Effective action</u> (limit k→0), infinitely many terms parametrized by <u>N free parameters</u>

⇒ **S-matrix**, **Wilson coefficients**, **observables**

One Attempt within String Theory: The Swampland Program

- **What: Swampland Program**: aims at identifying the "string landscape" of EFTs coming from its UV completion

- **How: via Swampland "Criteria"**, tied to string (mostly susy) constructions:
  - Partially inspired by ST (but also from general considerations, e.g., BH physics and cosmology);
  - Tested within string models, no counterexamples

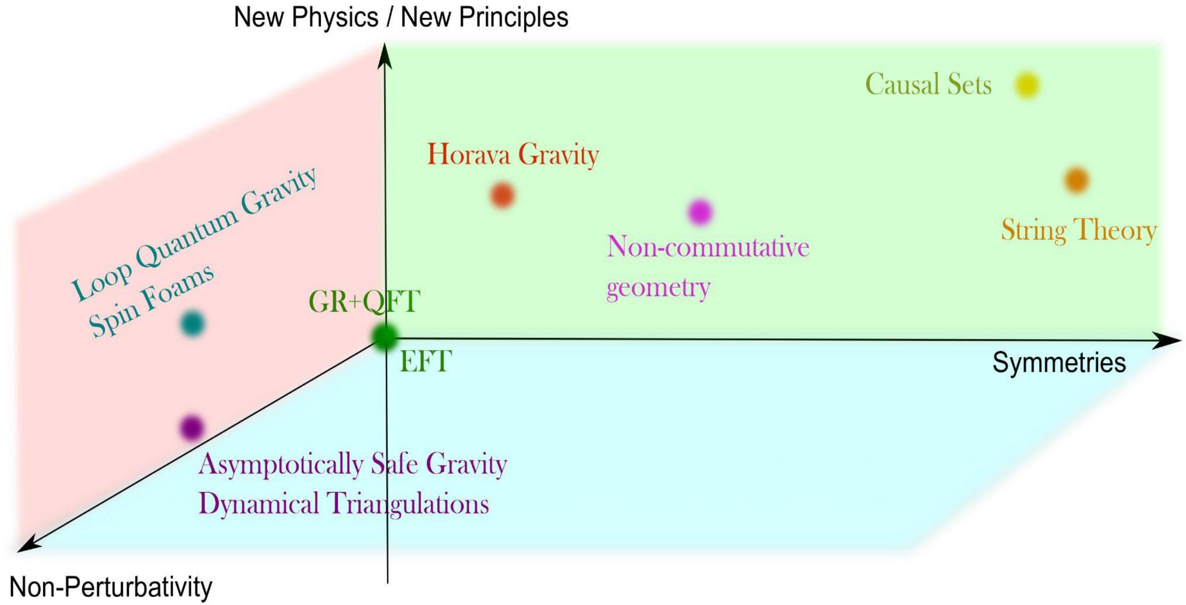


E. Palti (2019)

The realm of Quantum Gravity

Several theories:

- String Theory
- Asymptotically Safe Gravity
- Dynamical Triangulation
- Non-local gravity
- Loop quantum gravity
- Group field theory
- Causal sets
- Horava gravity
- …

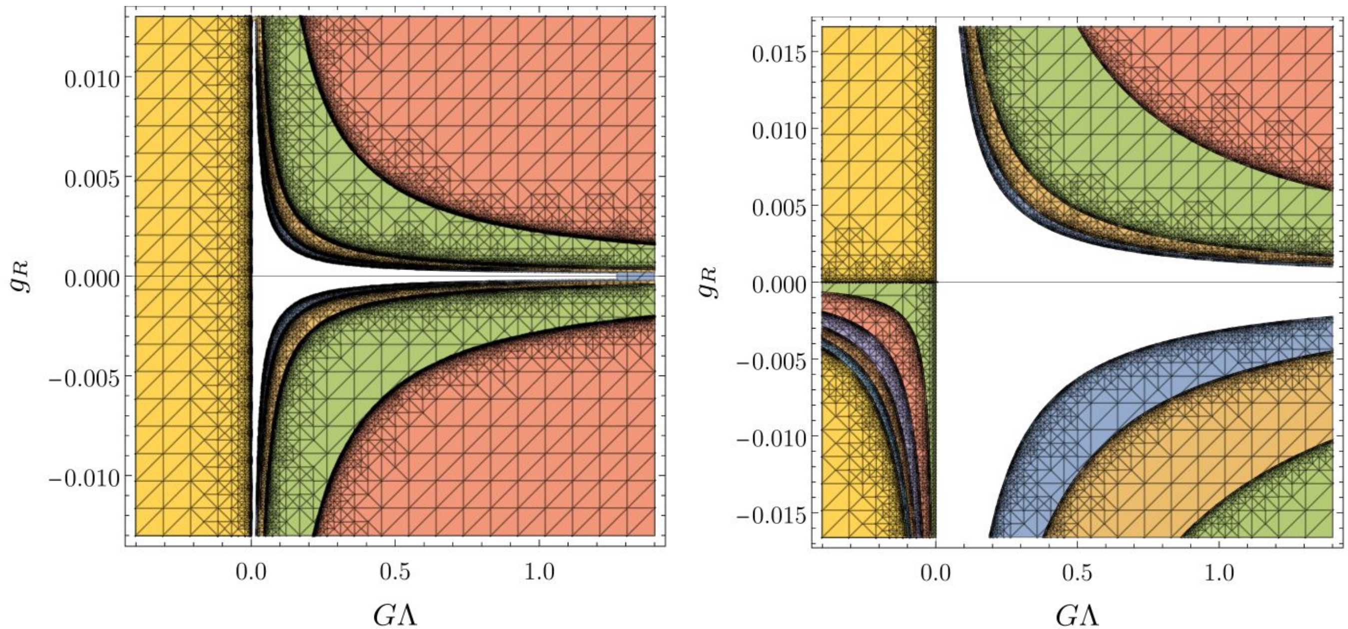


Goals:

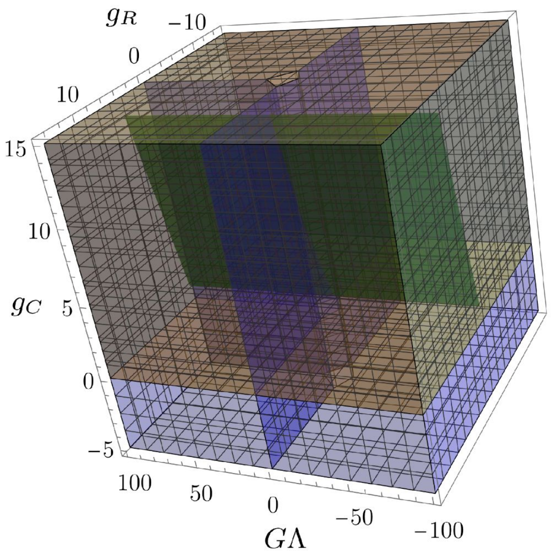
- Consistency: Renormalizability, unitarity, compatibility with large scale physics & observations
- Predictions: quantum cosmology, quantum black holes, scattering amplitudes, grav. Waves
- Comparison between approaches?

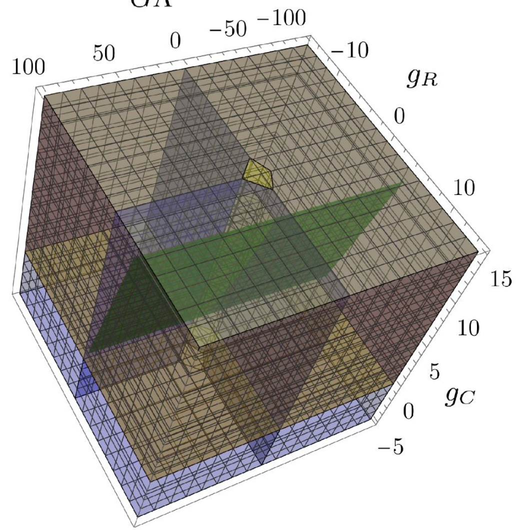
- **Swampland conjectures**:

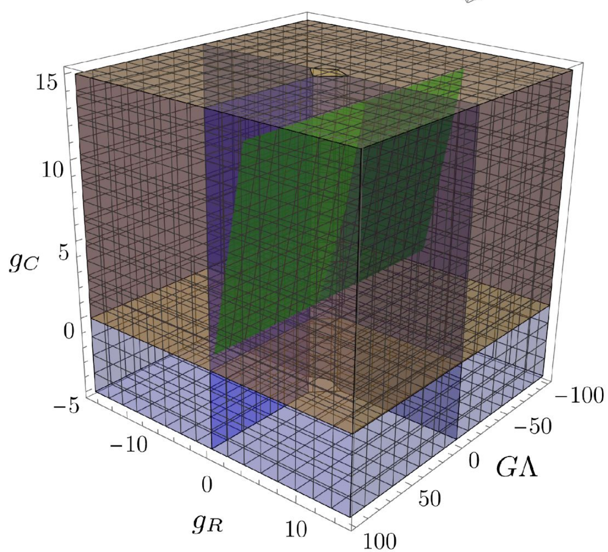
  ➔ **De Sitter and trans-Planckian conjectures**

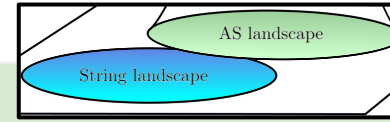


$$0 \leq c \leq 3.5, \quad f = 0.1 \text{ (left)}, \quad f = 1 \text{ (right)}$$









Green plane:

AS landscape [one-loop quadratic approx]

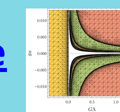
$$\mathrm{EFT}_{\mathrm{AS}} \approx \{g_R = -0.74655 - \frac{2}{3}\,\omega_-\, g_C\} \qquad g_C > 0$$

$\forall g_C$

Blue hyperplane:

Stringy “no de Sitter” conjecture

[~ no positive cosmological constant]



Yellow hyperplane:

Weak gravity conjecture

[~ gravity is the weakest force] $g_C > 0$

Within this simple model of AS, and only some swampland conjectures

⇒ **non-trivial intersection (partial compatibility?)**

[Basile, AP. 2107.06897]

- **Swampland conjectures**:

  - **De Sitter conjecture** [(Obied, Ooguri, Spoyneiko, Vafa, 2018), (Ooguri, Palti, Shiu, Vafa, 2019)]

$$M_{Pl}||\nabla V|| \geq cV \quad \text{for } \Delta\phi \leq f M_{Pl} \qquad f, c \sim \mathcal{O}(1)$$

  - **Trans-Planckian conjecture** [(Bedroya, Vafa, 2020)]

    Relevant for early-universe cosmology. Special value of c:

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$\Rightarrow$ Non-trivial bounds for different f and c.

[Knorr, AP, 2405.08860]

- **AS model**: photon-graviton systems at quadratic order, only **essential couplings** included

[see Knorr's talk!]

$$\mathcal{L} = -\frac{R}{16\pi G_N} + \Theta_E\, E + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + G_2\, (F^{\mu\nu} F_{\mu\nu})^2 + G_4\, F^{\mu}{}_{\nu} F^{\nu}{}_{\rho} F^{\rho}{}_{\sigma} F^{\sigma}{}_{\mu} + G_{CFF}\, C^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

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- **Two UV fixed points:**

FP1: one relevant direction (most predictive!)

$$g^* = 0.131\,, \quad g_+^* = 0.351\,, \quad g_-^* = 3.327\,, \quad g_{CFF}^* = 0.00375$$

$$\theta_1 = 1.845\,, \quad \theta_{2,3} = -0.239 \pm 0.0155\mathbf{i}\,, \quad \theta_2 = -0.291$$

FP2: two relevant directions

$$g^* = 0.126\,, \quad g_+^* = -0.308\,, \quad g_-^* = 4.001\,, \quad g_{CFF}^* = -0.00410$$

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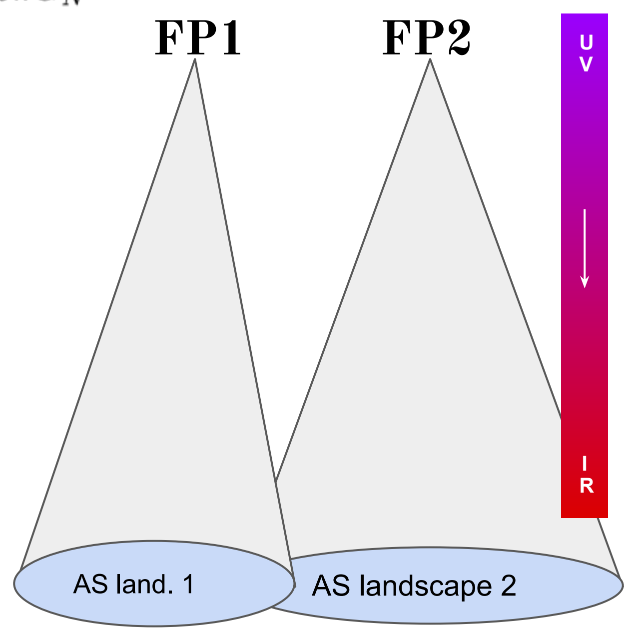

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  → **Weak gravity conjecture** (Arkani-Hamed, Motl, Nicolis, Vafa, 2006)

  $$m/M_{Pl} \leq q\,\mathcal{O}(1)$$

  Black holes remain sub-extremal:

  $$Q/M \leq (Q/M)_{extr}$$

  Higher derivative corrections [(Kats, Motl, Padi, 2007), (Charles, Larsen, Mayerson, 2017), (Cheung, Liu, Remmen, 2018), (Hamada, Noumi, Shiu, 2019), (Charles, 2019)]:

  $$Q/M \leq (Q/M)_{extr}\left(1 - \frac{\Delta}{M^2}\right) \qquad \mathcal{L}_{HD} = c_1\, R^2 + c_2\, R_{\mu\nu} R^{\mu\nu} + c_3\, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

  $$\Delta \propto (1-\xi)^2\,(c_2 + 4\,c_3) + 10\,\xi\,(1+\xi)\,c_3 \overset{\text{WGC}}{>} 0\,, \qquad \xi \equiv \sqrt{1 - \frac{Q^2}{M^2}}$$

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  In terms of dimensionless couplings, this condition yields

  $g_C > 0$ **Satisfied by AS-EFT**

  [Basile, AP. 2107.06897]

Note:
WGC also satisfied at the AS fixed point [de Alwis et al, '21]

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Can be violated in AS:
deSitter solutions can be found in AS

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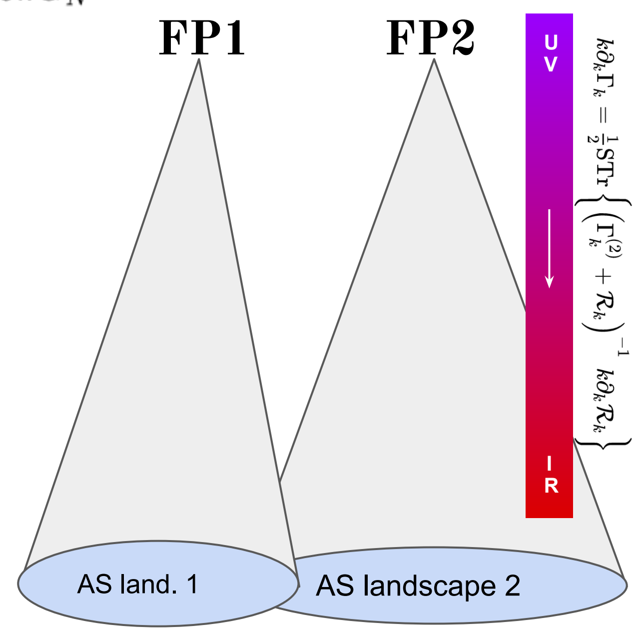

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