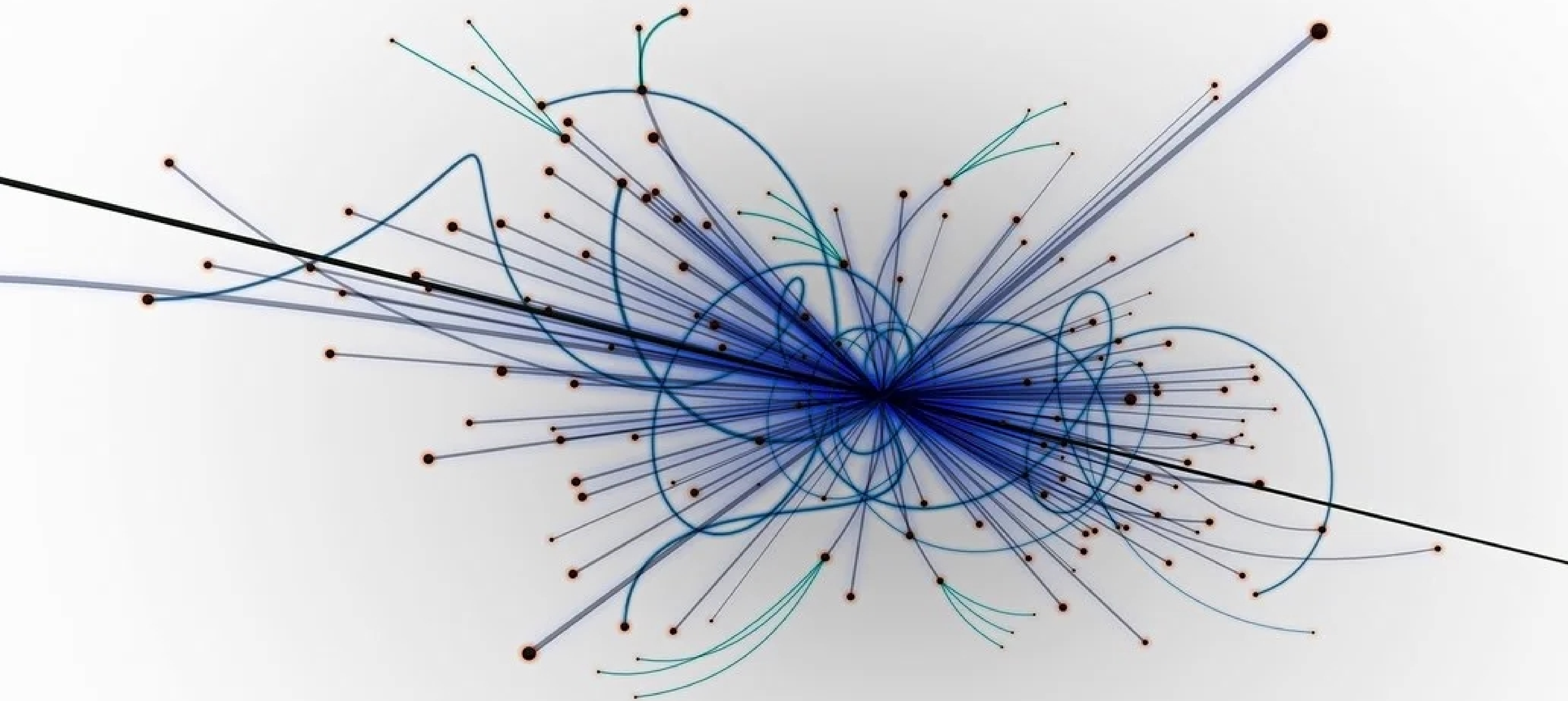
Amplitudes, intersection theory and Higgs physics



ZHEJIANG UNIVERSITY

Hjalte Frellesvig

Particle Physics – A Brief History

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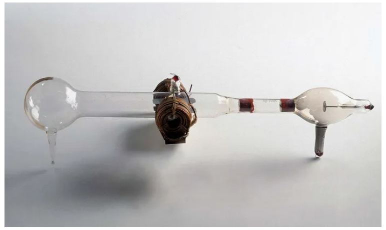
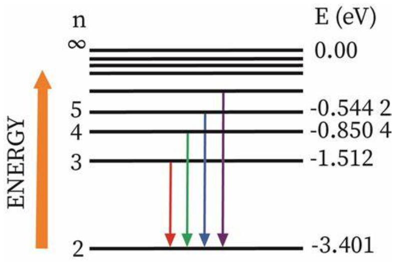
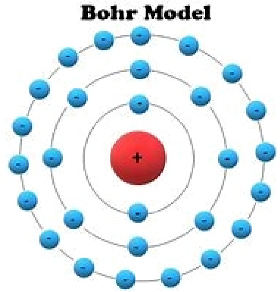
Electron discovery 1897

Bohr Model 1913







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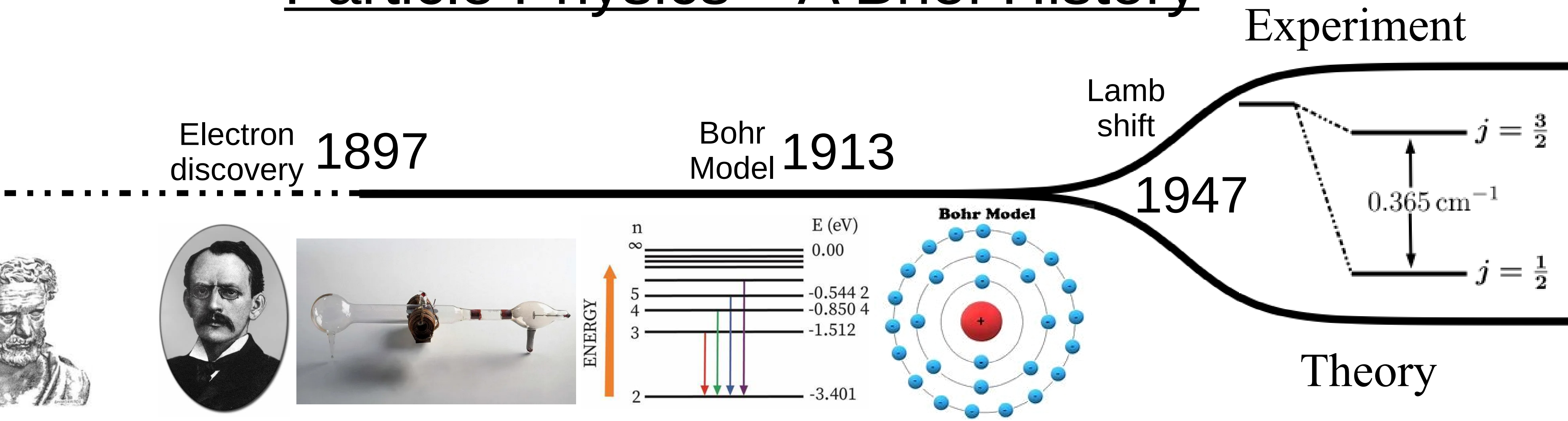


Particle Physics – A Brief History

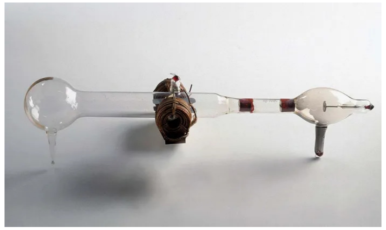
Electron discovery 1897

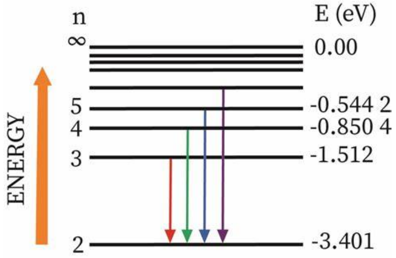
Bohr Model 1913

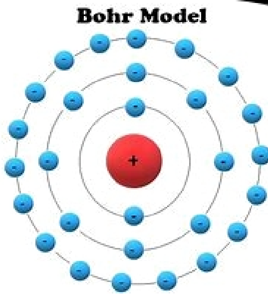
Lamb shift 1947

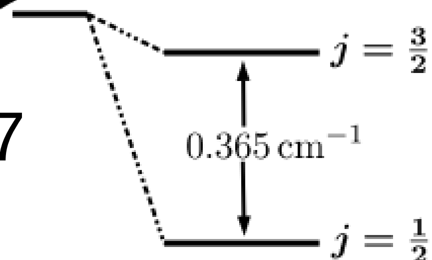
Experiment

Theory



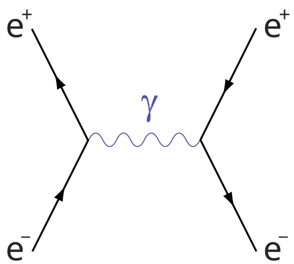


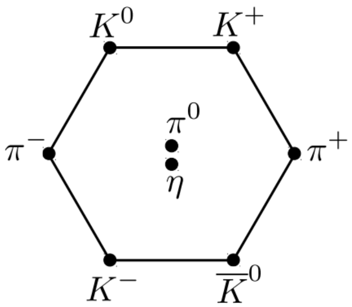




Theory

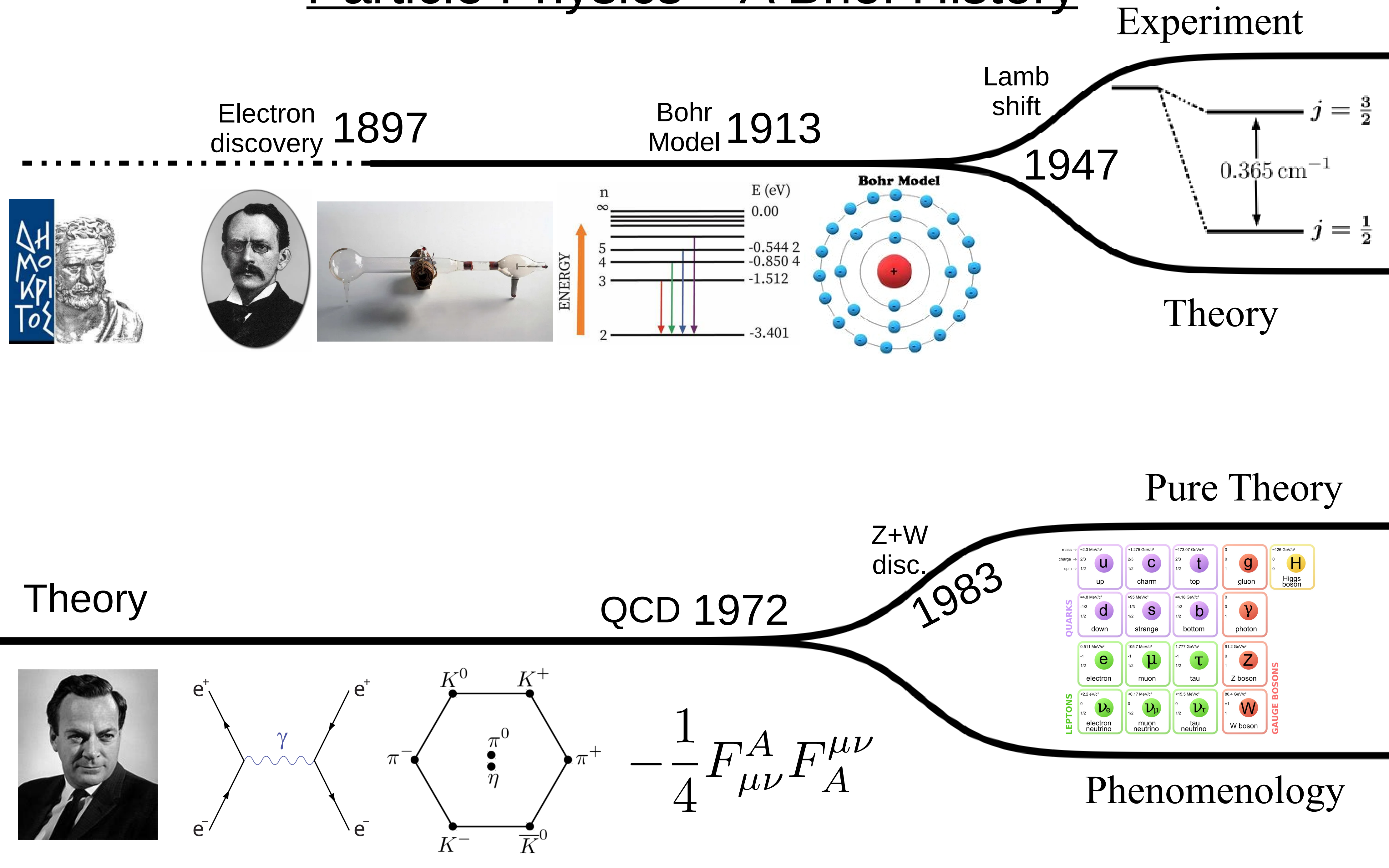
QCD 1972





$$-\frac{1}{4}F^{A}_{\mu\nu}F^{\mu\nu}_{A}$$

Particle Physics – A Brief History



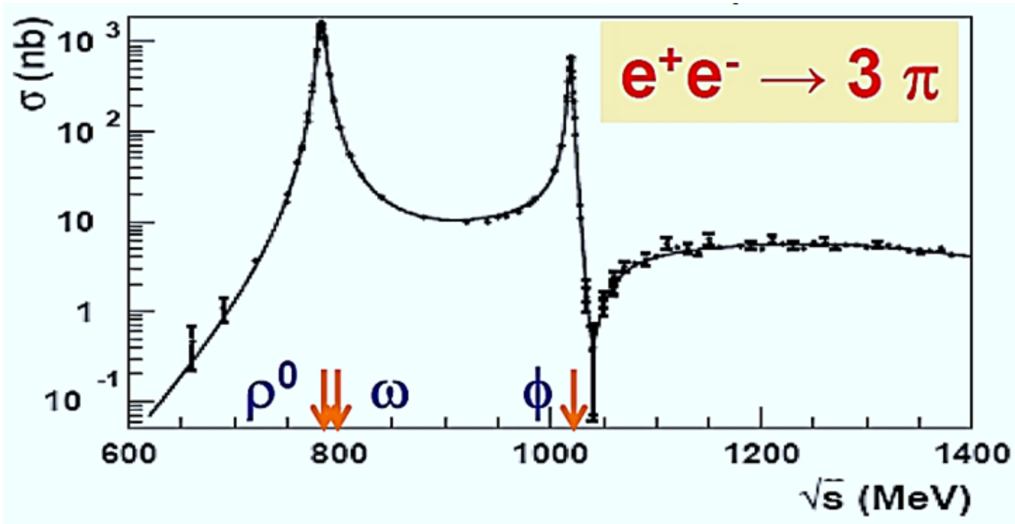
Particle Physics – A Brief History

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \sum_{\text{flavors}} \bar{q}_i (i \not{D} - m)_{ij} q_j$$

$$F^{\mu\nu}_a := \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g_s f_{abc} A^\mu_b A^\nu_c$$

$$D^\mu := \partial^\mu - i g_s A^\mu_a T_a$$

The QCD Lagrangian



Plot taken from arXiv:2507:21144
SND collaboration at CERN

Left: The Lagrangian of QCD , Right: A theory plot from QCD.

Surely it's easy to go from one to the other...

Particle Physics – A Brief History

- Feynman rules
- Feynman diagrams
- Color algebra
- Dirac algebra
- Regularization
- Feynman integrals
- Scattering amplitude
- Renormalization
- Infrared subtraction
- Cross section
- Parton distr. functions
- Hadronization
- Jet algorithms
- Event generation
- ….

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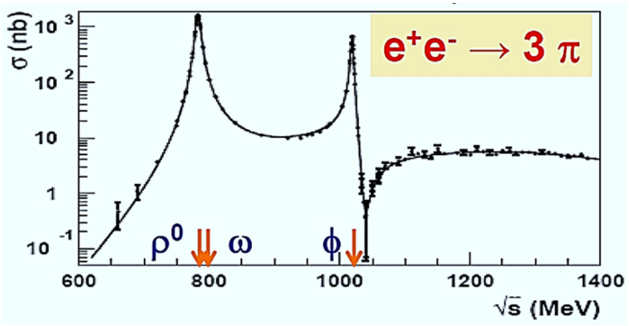

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$pp \to 2j$ at NLO takes a whole course

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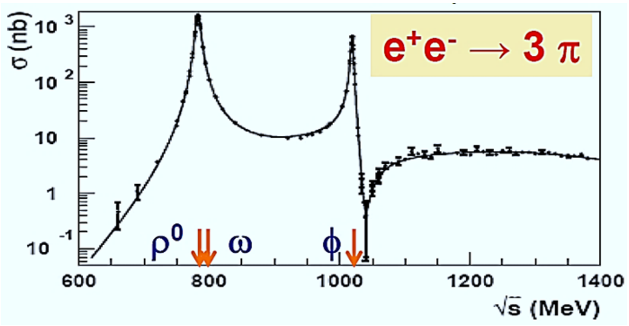

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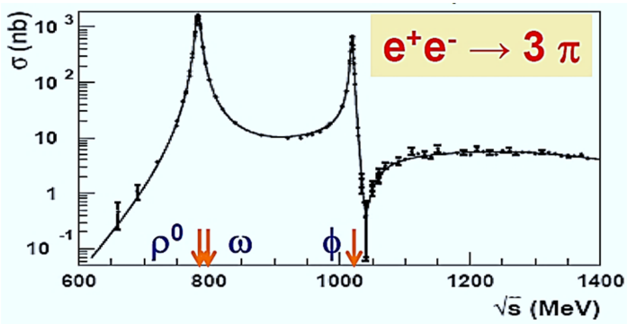

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I'd claim that no other field of science
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Perhaps there is a better way?

Amplitudes is a suggestion of
a different path

Amplitudes

$$\mathcal{A} = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

The Parke-Taylor amplitude

Unexpected simplifications appear when you do QFT calculations

The structure of that $\mathcal{A}$ reflects the underlying physics

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Perturbativity: Tree-level, loop-level, non-perturbative

Theory: QCD, $\mathcal{N}=4$ SYM, SM, quantum gravity, string theory, classical gravity (PN, PM, …), inflationary cosmology, theory independent, ...

Approach: From mathematical to phenomenological

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Is *Amplitudes* another bifurcation?

I would say no. It is more of a different mindset...

My own work

Perturbativity: Tree-level, loop-level, non-perturbative

My own work

2% 98%

Perturbativity: Tree-level, loop-level, non-perturbative

My own work

Perturbativity: Tree-level (2%), loop-level (**98%**), non-perturbative

Theory: QCD (5%), $\mathcal{N}=4$ SYM, QED, SM (15%), Quantum gravity, Classical gravity (PN, PM, …) (15%), Theory independent (**65%**), ...

My own work

2% **98%**

Perturbativity: Tree-level, loop-level, non-perturbative

5% 15%

QCD, $\mathcal{N}=4$ SYM, QED, SM, Quantum gravity,

Theory:

15% **65%**

Classical gravity (PN, PM, …), Theory independent, ...

80% 20%

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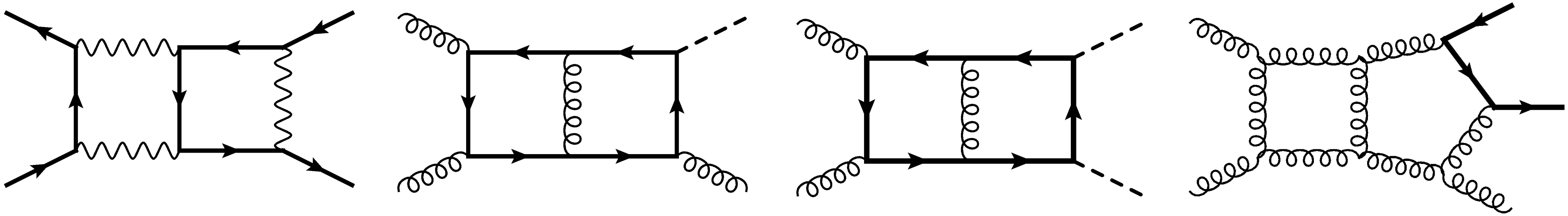
My main projects are

- Feynman integrals and intersection theory
- Geometries in Feynman integrals
- Phenomenology of Higgs production
- Geometries in post-Minkowskian classical gravity
- Computer algebra
- Hypergeometric functions
- …….

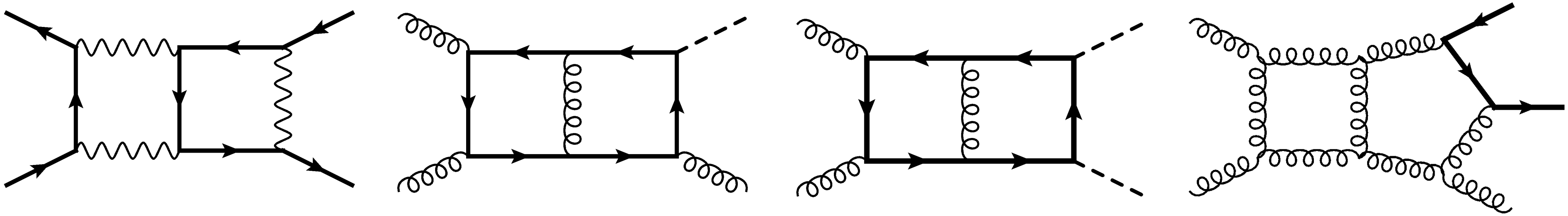
Intersection Theory and Feynman Integrals



For state-of-the-art two-loop scattering amplitude calculations

$\mathcal{O}(10\,000)$ Feynman diagrams $\rightarrow$ $\mathcal{O}(100\,000)$ Feynman integrals

Intersection Theory and Feynman Integrals



For state-of-the-art two-loop scattering amplitude calculations

$\mathcal{O}(10\,000)$ Feynman diagrams $\rightarrow$ $\mathcal{O}(100\,000)$ Feynman integrals

Linear relations bring this down to $\mathcal{O}(300)$ *master integrals*

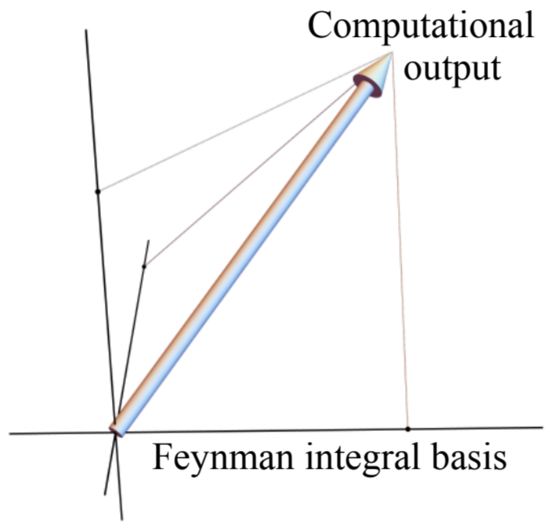
Such relations may be derived using IBP (integration by part) identities

$$\int \frac{d^d k}{\pi^{d/2}} \frac{\partial}{\partial k^\mu} \frac{q^\mu N(k)}{D_1^{a_1}(k) \cdots D_P^{a_P}(k)} = 0$$

Systematic by *Laporta's algorithm*: Solve a huge linear system

The linear relations form a *vector space*

$$I = \sum_{i \in \text{master integrals}} c_i \, I_i$$



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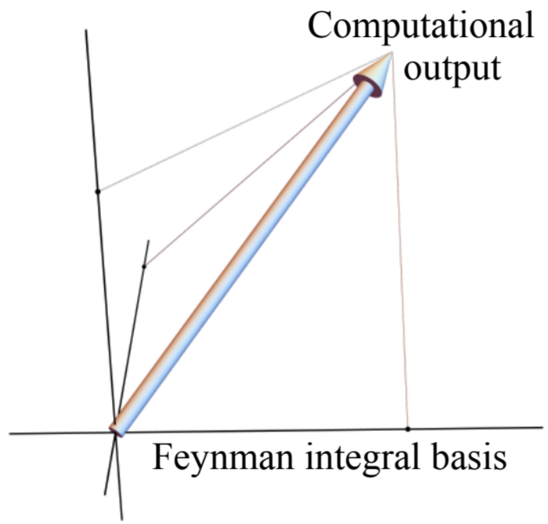

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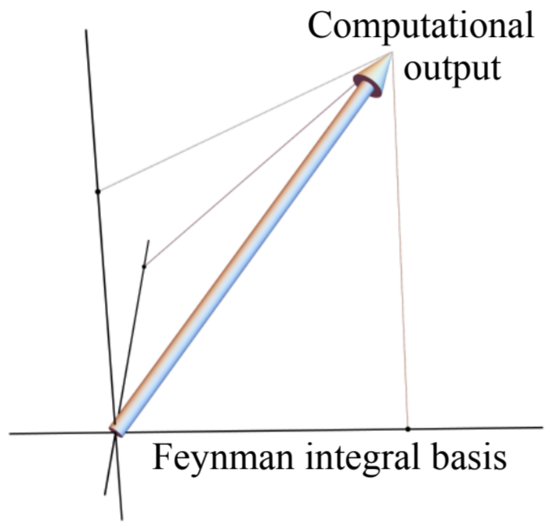
Not all vector spaces are *inner product spaces*

$$\langle v| = \sum_i c_i \langle v_i| \quad \Rightarrow \quad c_i = \sum_j \langle v | v_j^* \rangle (\mathbf{C}^{-1})_{ji}$$

$$\mathbf{C}_{ij} = \langle v_i | v_j^* \rangle \qquad \left(\mathbf{C} = \mathbf{I} \ \Rightarrow \ c_i = \langle v | v_i^* \rangle \right)$$

If only there were a way to define an inner product on Feynman integrals….

The linear relations form a *vector space*



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If only there were a way to define an inner product on Feynman integrals….

The *intersection number* forms such an inner product

We need a parametric representation of Feynman Integrals

The *Baikov* representation: $$I = \int_{\mathcal{C}} d^n x \frac{\mathcal{B}^\gamma N(x)}{x_1^{a_1} \cdots x_P^{a_P}} = \int_{\mathcal{C}} u\phi$$

$u = \mathcal{B}^\gamma$ is the *twist* – a multivalued function

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This generates an equivalence class of integrands

$I = \langle\phi|\mathcal{C}]$ becomes a *pairing* between form and contour

$\langle\phi|$ is a *twisted cocycle*, $|\mathcal{C}]$ is a *twisted cycle*

elements of a twisted *cohomology* group H^n and *homology* group H_n respectively

The Intersection Number

The intersection number $\langle\varphi|\check{\varphi}\rangle$ between twisted cocycles
Developed by mathematicians since the 90s

$u = \prod_i \mathcal{B}_i^{\gamma_i}$: Multivalued function

$\varphi = \prod_i x_i^{-a_i} \mathrm{d}^d x$: Rational differential form

$$I = \int_{\mathcal{C}} u\,\varphi = \langle\varphi|\mathcal{C}]$$

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$$\check{I} = \int_{\mathcal{C}} u^{-1}\check{\varphi} = [\mathcal{C}|\check{\varphi}\rangle$$

Dual integrals

The intersection number:

$$\langle\varphi|\check{\varphi}\rangle := \int (u\varphi)_{\text{reg}}\,(u^{-1}\check{\varphi}) = \ldots$$

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$$\boxed{\langle \varphi | \check{\varphi} \rangle = \sum_{p \in \mathcal{P}} \mathrm{Res}_{z=p}(\psi \check{\varphi})} \quad \text{with} \quad (\mathrm{d} + \mathrm{dlog}(u))\psi = \varphi$$

We also need the *multivariate* intersection number

Univariate:

$$\langle \varphi | \check{\varphi} \rangle = \sum_{p \in \mathcal{P}} \mathrm{Res}_{z=p}(\psi \check{\varphi}) \quad \text{with} \quad (\mathrm{d} + \mathrm{dlog}(u))\psi = \varphi$$

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Multivariate:

$$_{\mathbf{n}}\langle \varphi^{(\mathbf{n})} | \check{\varphi}^{(\mathbf{n})} \rangle = \sum_{p \in \mathcal{P}_n} \mathrm{Res}_{z_n=p}\Big(\psi_i^{(n)} \,_{\mathbf{n-1}}\langle e_i^{(\mathbf{n-1})} | \check{\varphi}^{(\mathbf{n})} \rangle \Big)$$

$$(\delta_{ij}\partial_{z_n} + \mathbf{\Omega}_{ij}^{(n)})\psi_j^{(n)} = \varphi_i^{(n)}$$

It is a recursive formula

There are also alternative approaches, such as the use of multivariate residues

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Multivariate:

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There are also alternative approaches, such as the use of multivariate residues

Also we allow the use of *delta-forms* from *relative* cohomology

$$\langle \varphi | \delta_z \rangle := \mathrm{Res}_{z=0}(\varphi)$$

Intersection summary:

The intersection number can help bypass the bottleneck

Gargantuan linear system becomes $$I = \sum_i c_i I_i \ \Rightarrow \ c_i = \langle \varphi | \check{\varphi}_i \rangle$$

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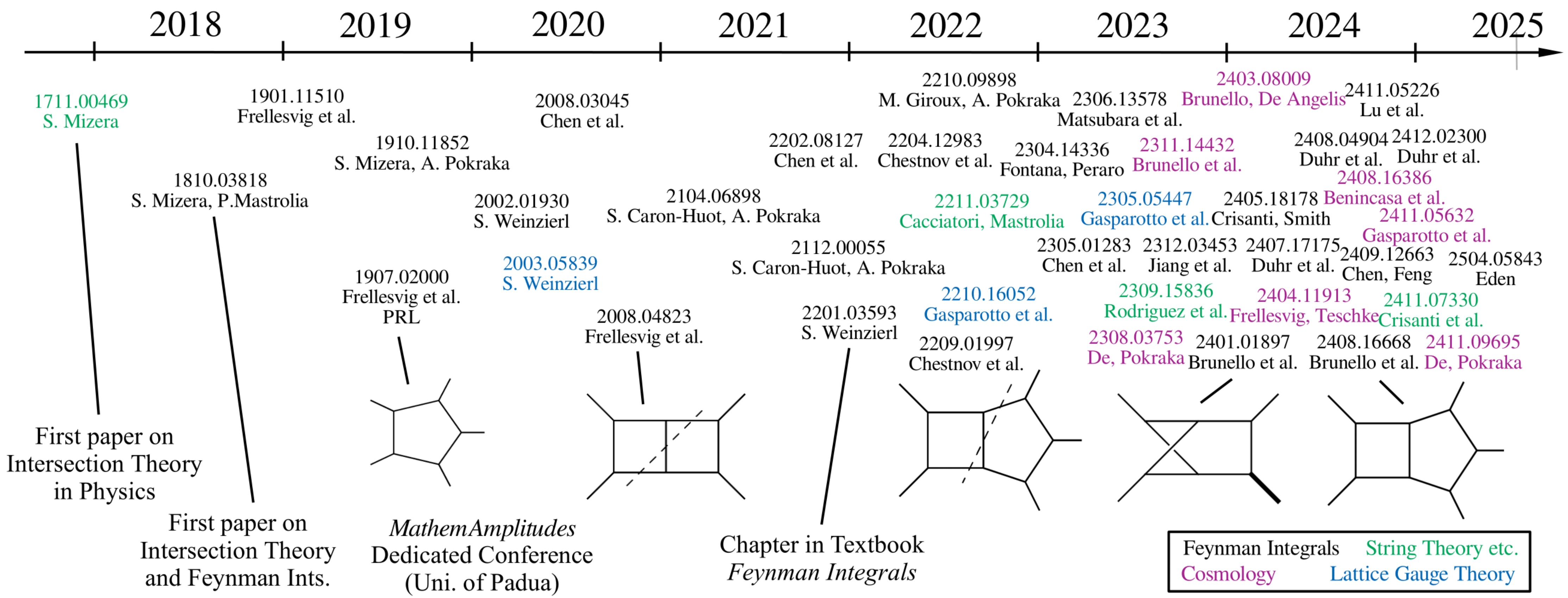


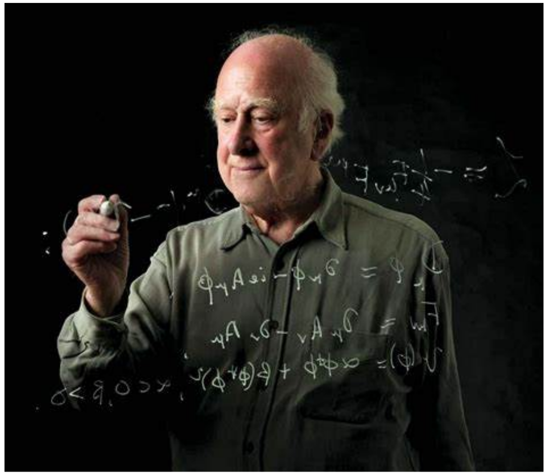
Intersection perspectives:

Intersection theory has (in my opinion) the potential to outpace IBPs as the preferred method to derive FI relations

Yet there are still a number of open problems and loose ends

- Find a good approach to integrals related by *magic identities*
- Better understanding of multivariate intersection numbers
- Replace the fibration with a fully multivariate approach?
- Further investigate connections to *symbols* and Landau singularities
- Make an Extremely Fast Code

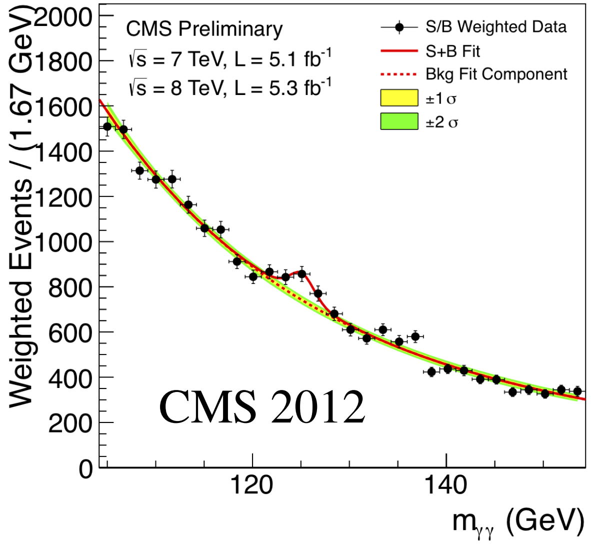
Particle physics phenomenology



The Higgs sector

This is the least investigated part of the Standard Model

Promising for new physics



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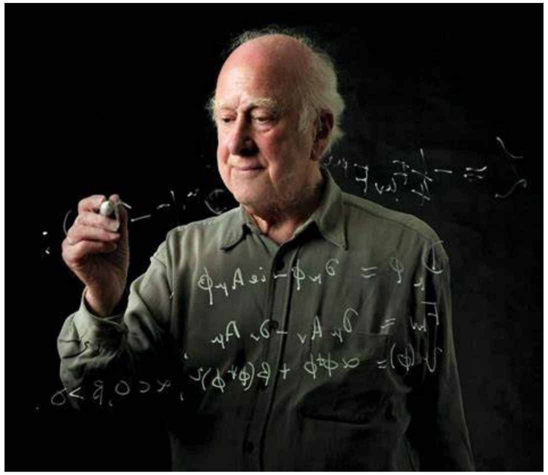

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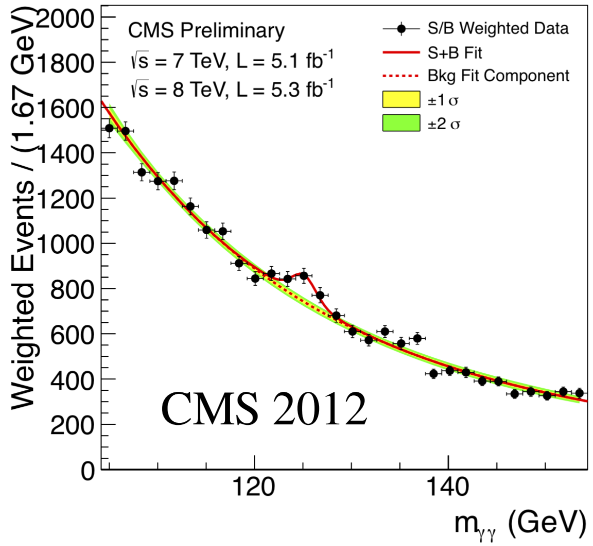
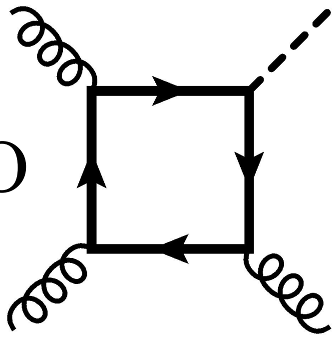
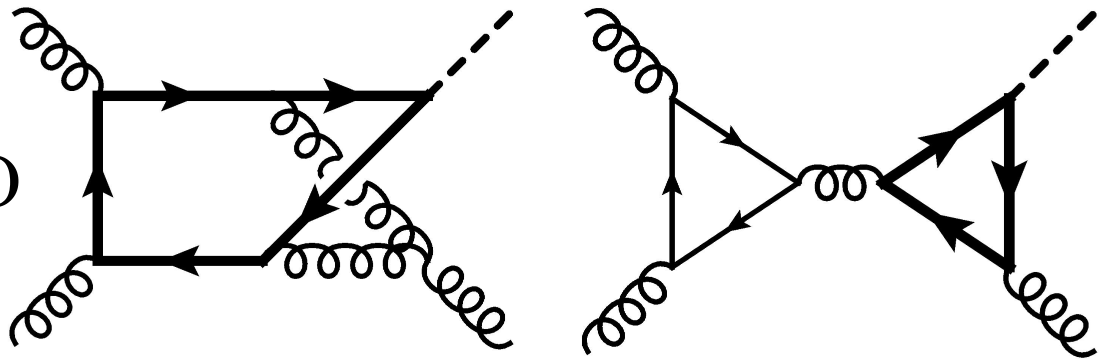


Higgs + *jet* production:

LO

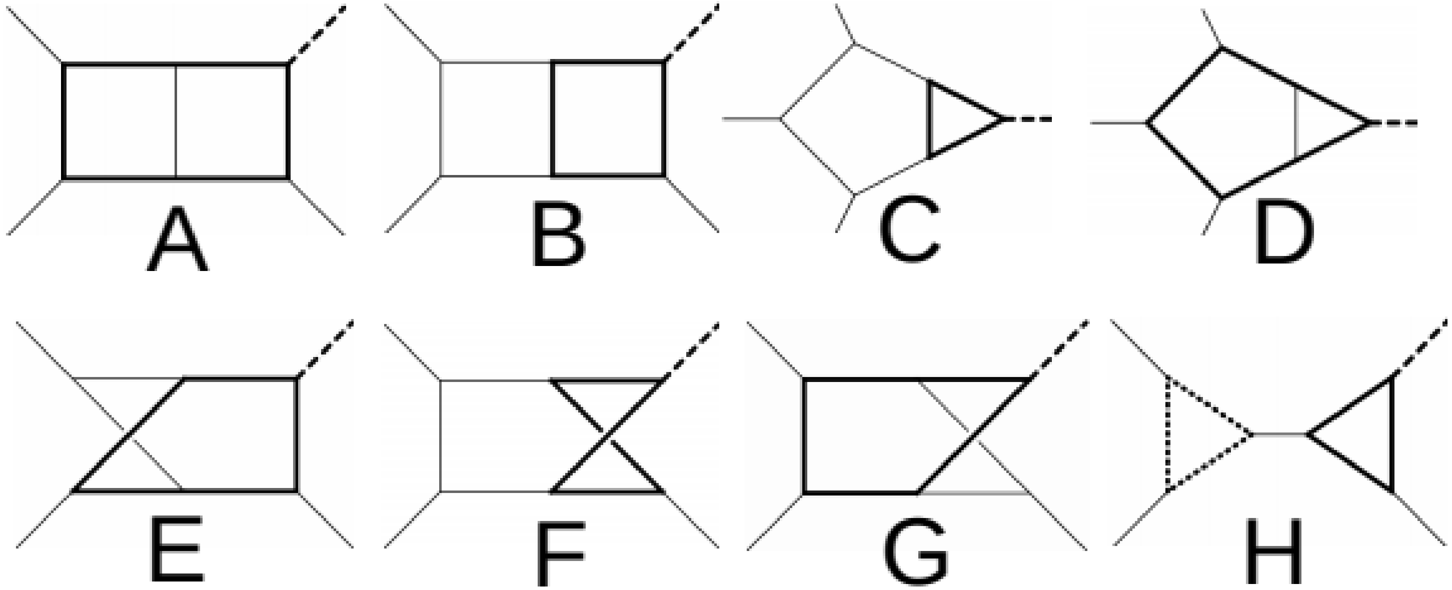


NLO



The biggest challenge was the Feynman integrals

$$I^f = \iint \frac{\mathrm{d}^d k_1 \mathrm{d}^d k_2}{D_{f;1}(k) \cdots D_{f;7}(k)}$$



Bonciani, Del Duca, **HF**, Henn, Moriello, Smirnov
JHEP, vol. 12(2016), p. 096 [arXiv:1609.06685]

Bonciani, Del Duca, **HF**, Henn, et al.
JHEP, vol. 11(2020), p. 132 [arXiv:1907.13156]

HF, Hidding, Maestri, Moriello, Salvatori
JHEP, vol. 16(2020), p. 093 [arXiv:1911.06308]

$$I \propto \int_{-1}^{1} \frac{dz}{\sqrt{(1-z^2)(1-k^2 z^2)}} = K(k)$$

Elliptic structures appear

Slide from “Danish PANP meeting”

Results

$$\partial_s f = \epsilon A f$$

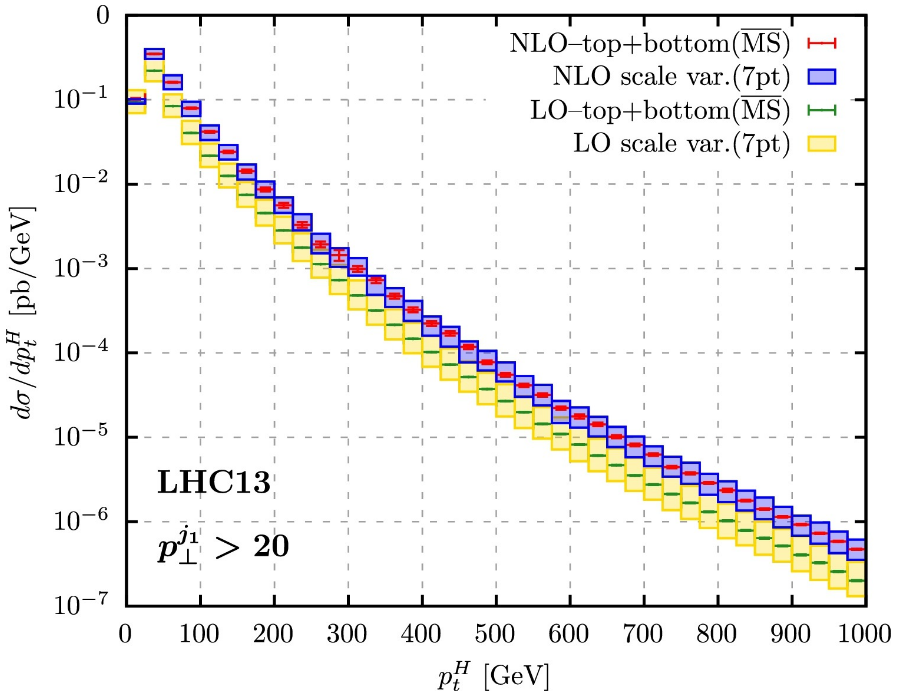
What we actually do, is solve the diff-eqs numerically.

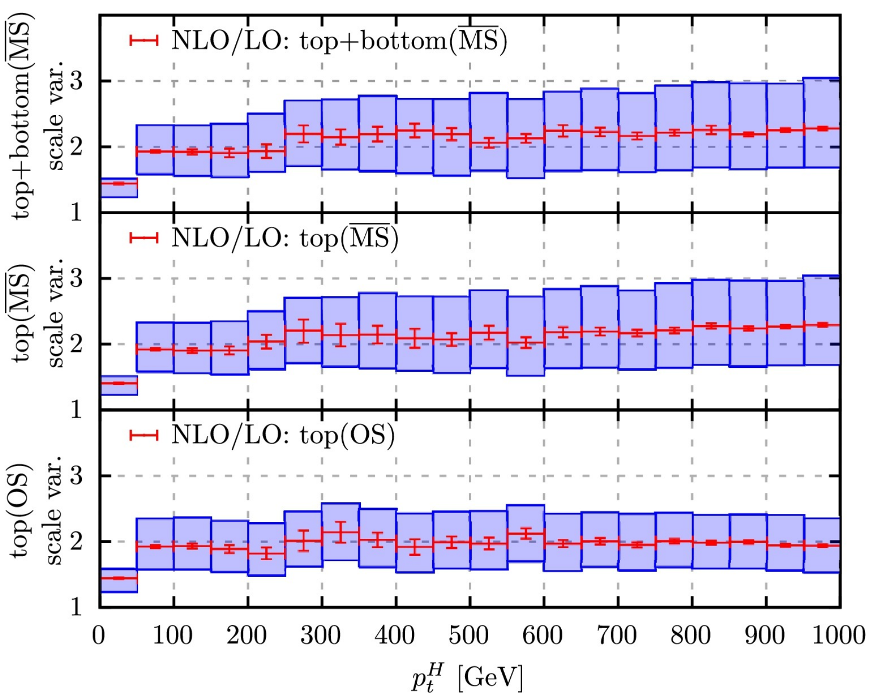
We use the Frobenius method: sequential series expansions near critical points
Moriello [2020], Hidding [2020]

This can be done to arbitrary precision, also close to branch points.

Plots for the final NLO cross section will be published this year!

Higgs + *jet* production





Results from arXiv: 2206.10490: R. Bonciani, V. Del Duca, HF, M. Hidding, V. Hirschi, F. Moriello, G. Salvatori, G. Somogyi, F. Tramontano

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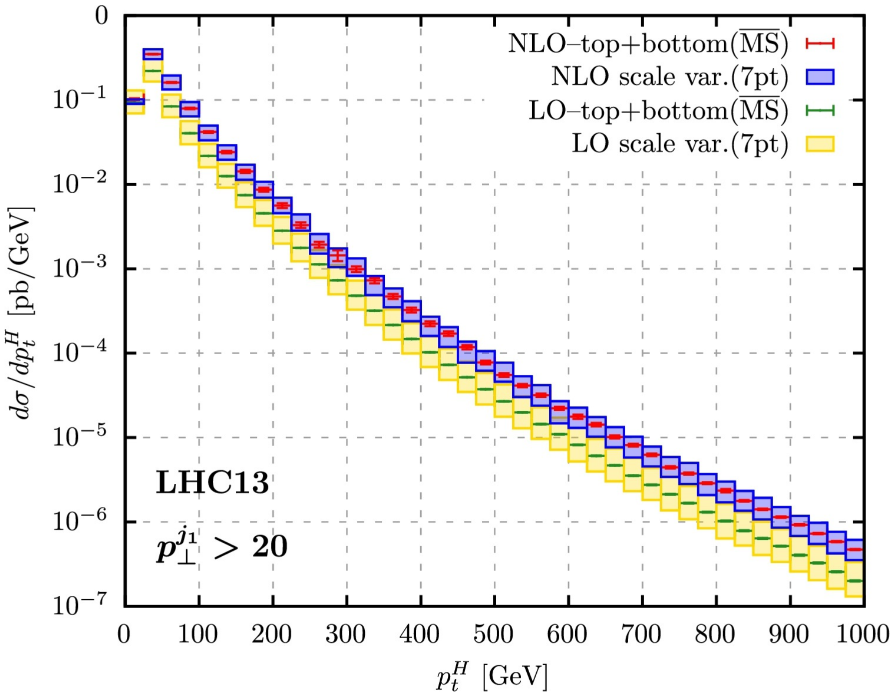

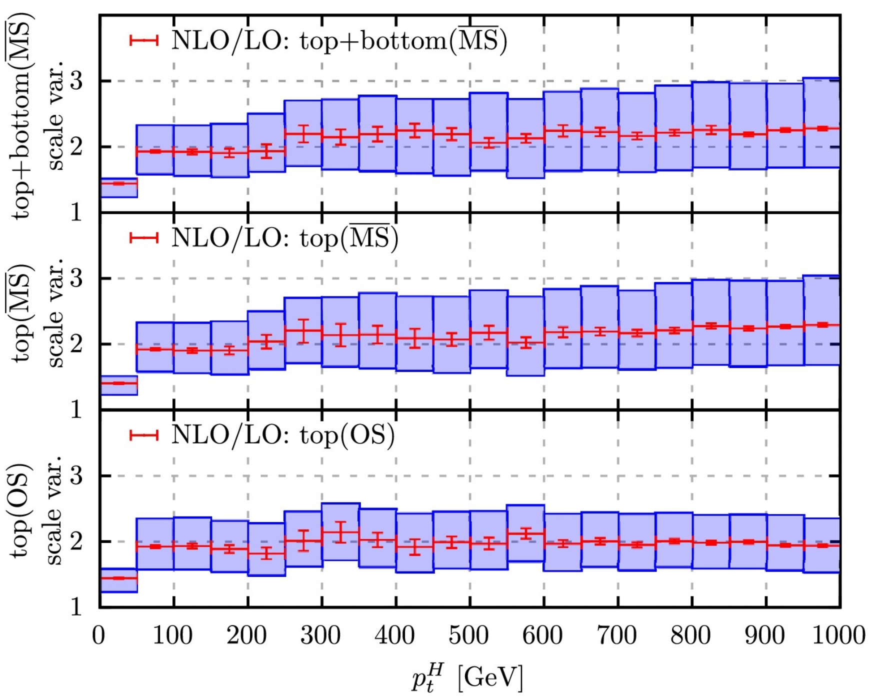

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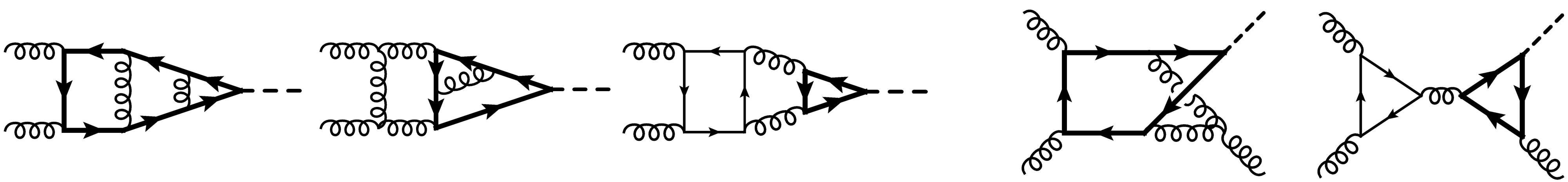
NNLO *H*-production is next



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Thank you for inviting me to speak and thank you for listening

Hjalte Frellesvig