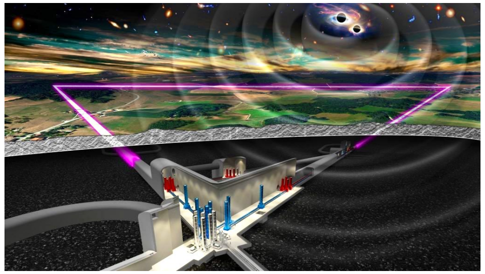
# ERA OF GRAVITATIONAL WAVE PHYSICS: NEED FOR HIGH-PRECISION PREDICTIONS

- Upcoming 3rd generation of gravitational wave observatories with $10^2$ sensitivity increase
- Need for accurate waveform predictions well beyond state-of-the art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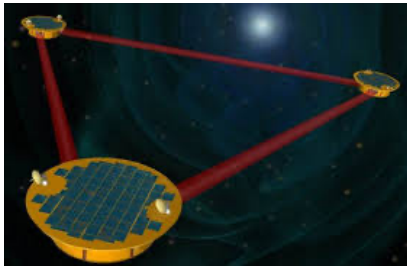
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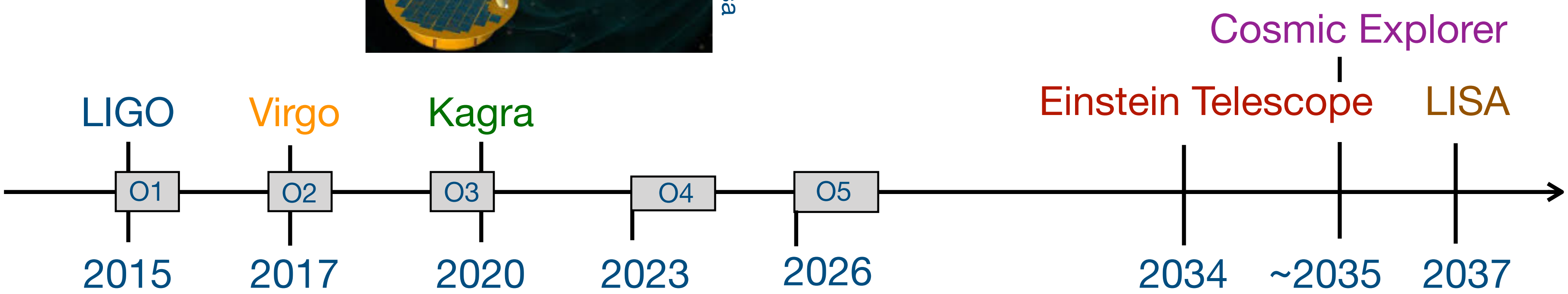
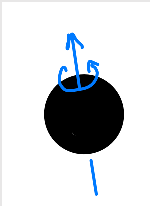
High-precision predictions necessary basis to study fundamental questions in physics:

- Is Einstein's theory correct?
- Black hole formation & population?
- Neutron star properties?
- Physics beyond the standard model?



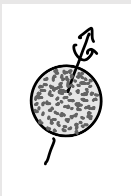
# GRAVITATIONAL TWO-BODY PROBLEM

Black Hole



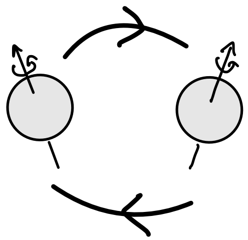
mass, spin

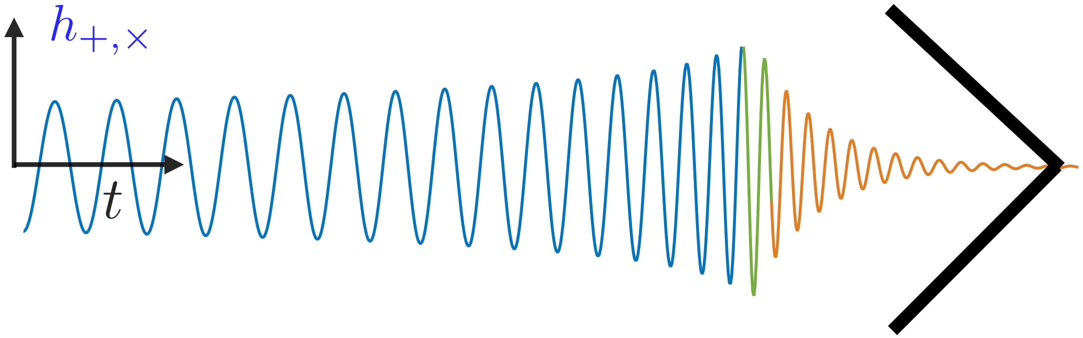
Neutron Star



mass, spin, radius, tidal deformability

Black Hole/Neutron Star Binaries:






Bound state

**inspiral**

**merger**

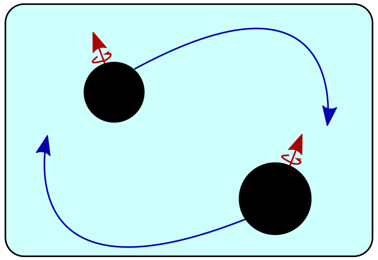
Einstein's eqs. cannot be solved exactly $\Rightarrow$ approximation schemes

- **Numerical relativity:** good for **merger** (strong gravity, short duration)
- **Perturbation theory**: good for **inspiral** (weak fields, long duration)

$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$ $\Leftrightarrow$ import **Quantum Field Theory** tools!

# PERTURBATIVE SCHEMES

## Post-Newtonian (PN)

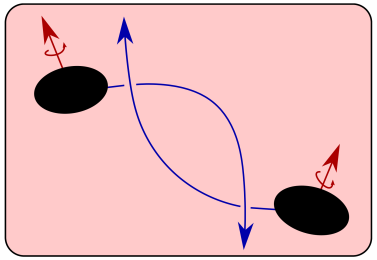


- Large-separation & slow-velocity expansion

$$\epsilon_{\rm PN} = \frac{GM}{r} \sim v^2$$

- Good for quasi-circular bound orbits

## Post-Minkowskian (PM)

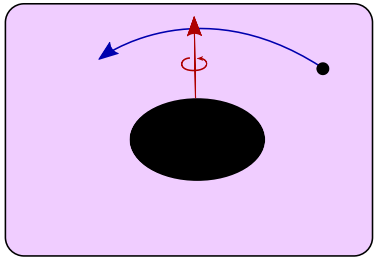


- Large-separation expansion

$$\epsilon_{\rm PM} = \frac{GM}{r} \quad \text{exact in } v!$$

- Good for scattering & eccentric orbits

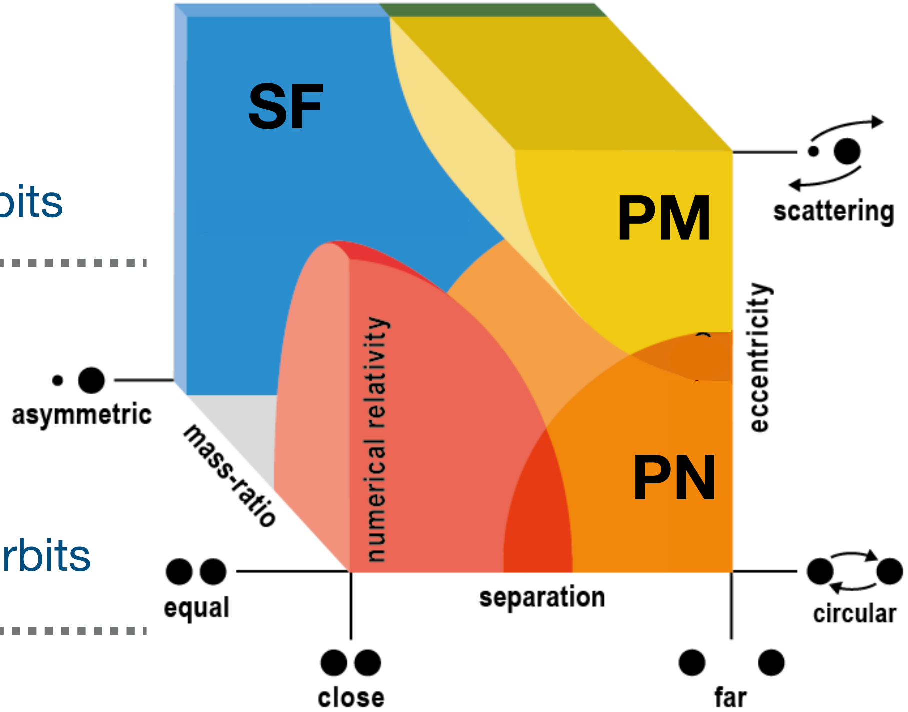
## Gravitational Self-Force (SF)



- Large-mass ratio expansion

$$\epsilon_{\rm GSF} = \frac{m_1}{m_2} \quad \text{semi-analytic, exact in } G \;\&\; v!$$

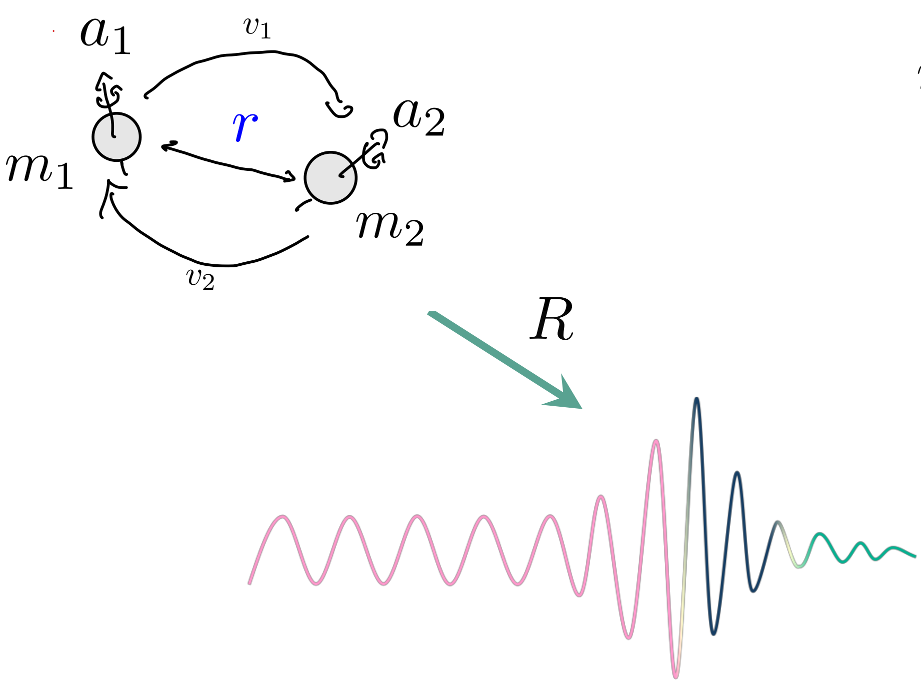
- Good for extreme-mass ratios, scattering and bound orbits



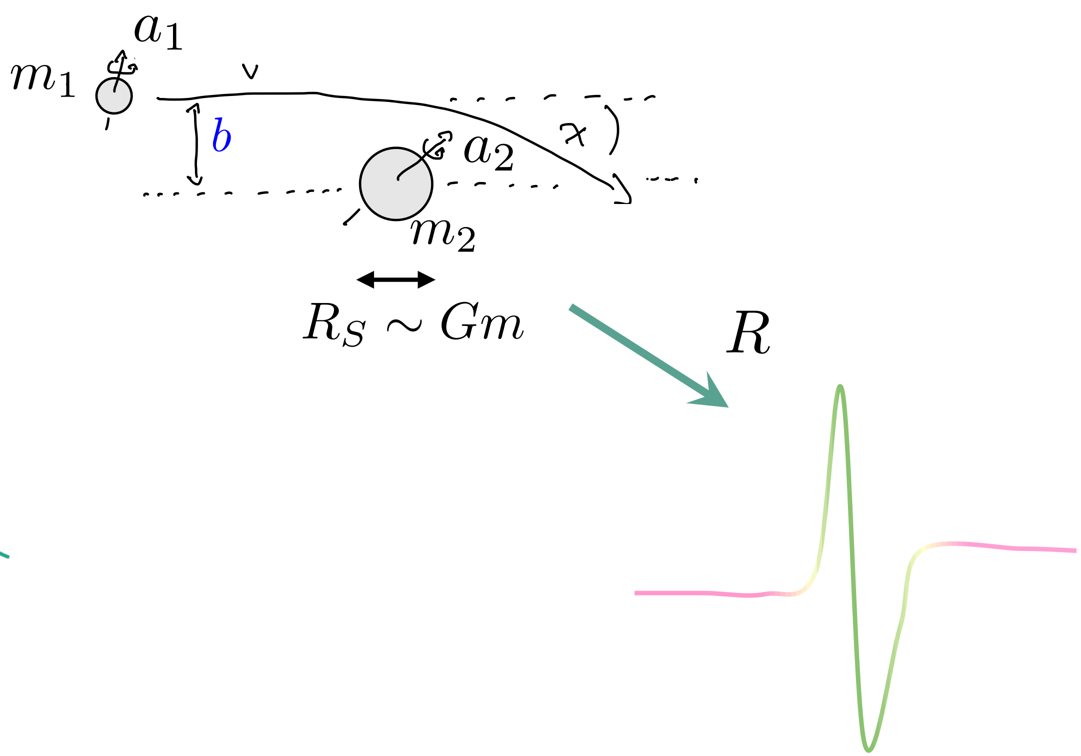

(credit: Ana Carvalho)

# WORLDLINE EFFECTIVE FIELD THEORY OF COMPACT OBJECTS [Goldberger,Rothstein][Porto][Levi,Steinhoff]

During **inspiral** or **scattering**: Separation of scales: $R_S \sim Gm \ll b$ or $r \ll R$




Far field waveform

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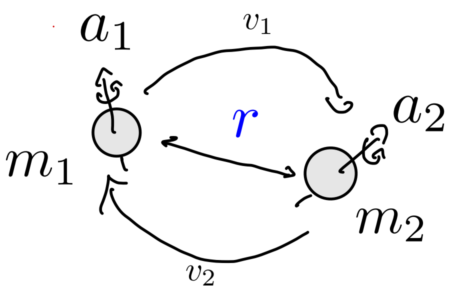
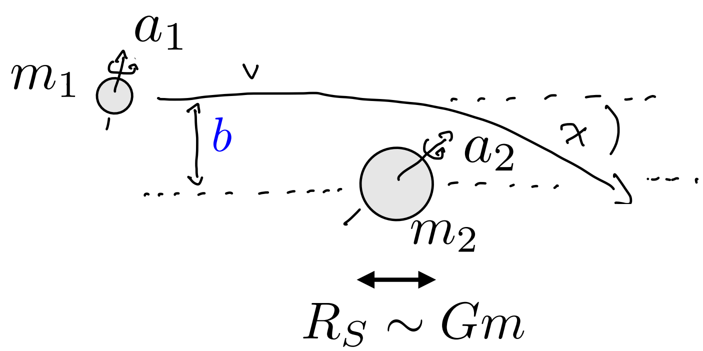



Effective Field Theory description: BH?NS as **spinning point particle** moving on its **worldline**

$$S_{\mathrm{EFT}} = -m \int d\tau \left( \sqrt{g_{\mu\nu}\dot{x}^\mu(\tau)\dot{x}^\nu(\tau)} + \sum_{n=1}^{\infty} \mathcal{L}_{\mathrm{EFT}}^{(n)} \right)$$

Black-hole spin $a^\mu$:

$$|a^\mu| \le m G_N$$

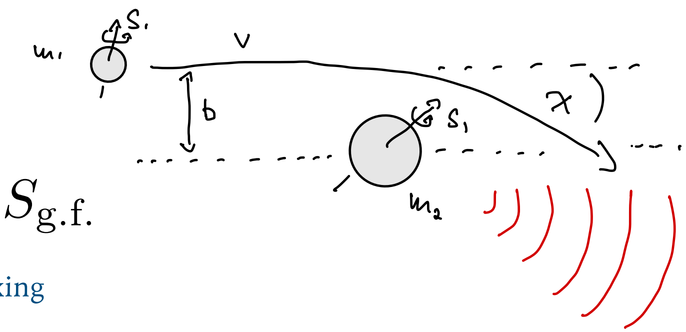
$$\mathcal{L}_{\mathrm{EFT}}^{(1)} \sim \frac{mG_N}{b} \sim a\,\partial g$$

$$\mathcal{L}_{\mathrm{EFT}}^{(2)} \sim \left(\frac{mG_N}{b}\right)^2 \sim a^2\partial^2 g \qquad \ldots$$

$$\mathcal{L}_{\mathrm{EFT}}^{(4)} \sim \left(\frac{mG_N}{b}\right)^4 \sim a^4\partial^4 g + (mG_N)^4(\partial^2 g)^2 + \ldots$$

# RELATIVISTIC TWO BODY PROBLEM IN PM: TRADITIONAL APPROACH

Point-particle approximation for BHs (or NSs) $|b| \gg Gm_*$



$$S = -\sum_{i=1}^{2} m_i \int d\tau_i \sqrt{g_{\mu\nu}\dot{x}_i^{\mu}(\tau_i)\dot{x}_i^{\nu}(\tau_i)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_{\text{g.f.}}$$

Point particle approximation — Bulk gravity & gauge fixing

1) Equations of motion:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = \frac{\kappa^2}{8}T_{\mu\nu}$$

Einstein's eqs.

$$\ddot{x}_i^{\mu} + \Gamma^{\mu}{}_{\nu\rho}\dot{x}_i^{\nu}\dot{x}_i^{\rho} = 0$$

Geodesic eqs.

2) Solve iteratively in $G$

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\sum_{n=0}^{\infty} G^n h_{\mu\nu}^{(n)}(x)$$

emitted radiation

$$x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\,\tau + \sum_{n=1}^{\infty} G^n z_i^{(n)\,\mu}(\tau)$$

straight line: „in" state — deflections

3) Construct observables

Far field waveform:

$$\lim_{r\to\infty} h_{\mu\nu} = \frac{f_{\mu\nu}(t-r,\theta,\varphi)}{r} + \mathcal{O}(\frac{1}{r^2})$$

„Impulse" (change in momentum):

$$\Delta p_i^{\mu} = m_i\dot{x}_i^{\mu}\Big|_{\tau=-\infty}^{\tau=+\infty} = m_i\int d\tau\,\ddot{x}_i^{\mu}(\tau)$$