Speeding up Analytic Calculations in Theoretical Particle Physics via Machine Learning

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[2502.05121] with M. von Hippel
[2606.10698] with J. Berman, F. Charton, A. Luna and M. Zeng

DANMARKS FRIE FORSKNINGSFOND
QM Centre for Quantum Mathematics
ħQTC
VILLUM FONDEN

Fundamental Physics

Aim: Understand fundamental constituents of **matter** & **interactions**!

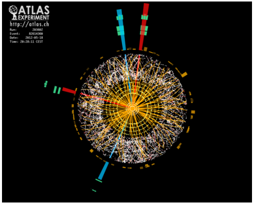

Image: ATLAS

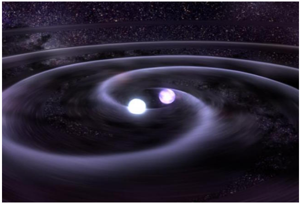

Image: NASA/Goddard Space Flight Center

New experiments $\Rightarrow$ Need for high-precision theory predictions!

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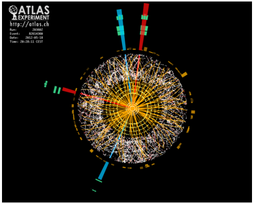

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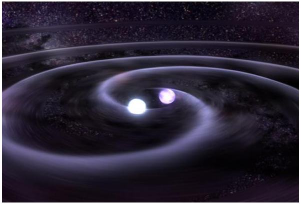

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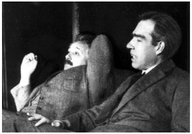
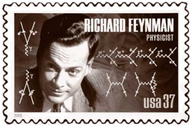
Theoretical framework:

Quantum Field Theory $=$ Special relativity $+$ Quantum Mechanics



Pipeline for calculating precision predictions

Feynman diagrams → Feynman integrals → Master integrals → Predictions



$\sim 10^5$

IBP reduction

$\sim 10^2$

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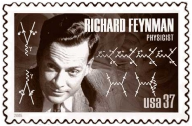

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What is integration-by-parts (IBP) reduction?

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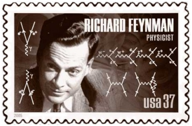


$\sim 10^5$ | IBP reduction | $\sim 10^2$

What is integration-by-parts (IBP) reduction?

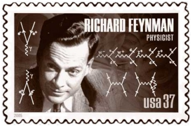
$\langle$Feynman integrals$\rangle$ = vector space of finite dimension

[Smirnov, Petukhov (2010)]

$\Rightarrow \exists$ finite basis $\{I_1, \ldots, I_N\}$ a.k.a. master integrals

Pipeline for calculating precision predictions

Feynman diagrams $\rightarrow$ Feynman integrals $\rightarrow$ Master integrals $\rightarrow$ Predictions



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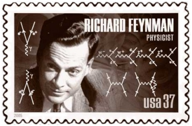

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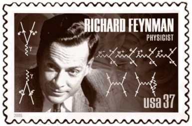

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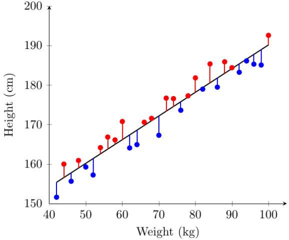
$\Rightarrow$ Interesting target for improvements via ML!

[Hippel, **MW** (2025)], [Song, Yang, Cao, Luo, Zhu (2025)], [Zeng (2025)], [Shih (2026)], [Berman, Charton, Luna, **MW**, Zeng (2026)]

ML for analytic calculations in theoretical physics

Typical machine learning applications: Noisy numeric real-world data



Theoretical physics: Exact analytic calculations

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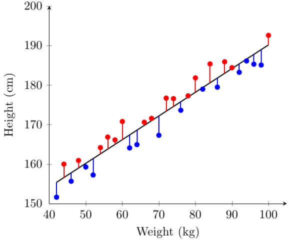

Theoretical physics: Exact analytic calculations

Solutions hard to calculate but easy to check
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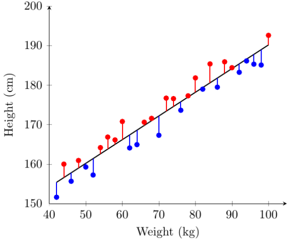

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Similar to ML for mathematics

Feynman integrals and IBP identities

General family of Feynman integrals
$=$ one integral for every $(a_1, \ldots, a_n) \in \mathbb{Z}^n$

$$I_{a_1,\ldots,a_n} = \int \frac{\prod_{l=1}^{L} d^D k_l}{\prod_{i=1}^{n} [D_i(k_1, \ldots, k_L)]^{a_i}}$$

where $L \in \mathbb{N}$, $k_l \in \mathbb{R}^D$, and D_i polynomials

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Integration-by-part identities [Chetyrkin, Tkachov (1981)]

$$0 = \int dk_l \frac{d}{dk_l}\left(\cdots \right) = \text{linear combination of } I_{a'_1,\ldots a'_n} \text{ with } a'_i \in \{a_i, a_i \pm 1\}$$

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$\Rightarrow$ **Infinitely** many sparse linear equations for **infinitely** many integrals

Integration-by-parts reduction Solve for a target integral $I_{a_1,\ldots,a_n}$ in terms of fixed master integrals $I_{a'_1,\ldots,a'_n}$!

Strategy Pick finite subset $s \subset \mathbb{Z}^n$ & use Gauss elimination [Laporta (2000)]

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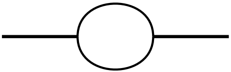
$=$ **Hard optimization problem $\rightarrow$ ML**

A simple example

Integral family example: Bubble integral



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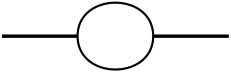

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Two integration-by-part equations for every $(a_1, a_2) \in \mathbb{Z}^2$:

$$0 = (D-2a_1-a_2)I_{a_1,a_2} + 2a_1 m^2 I_{a_1+1,a_2} - a_2 I_{a_1-1,a_2+1} + a_2(2m^2-p^2)I_{a_1,a_2+1}$$
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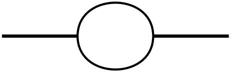

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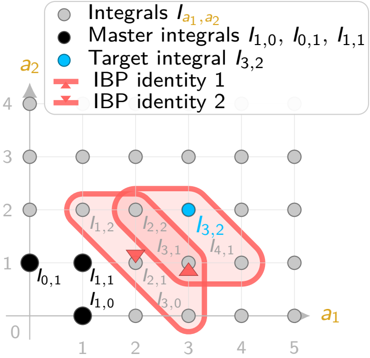
Integration-by-part reduction
Write any I_{a_1,a_2} as linear combination of e.g. $I_{1,1}$ & $I_{1,0} = I_{0,1}$!

Visualization on the lattice:



The simple example and heuristics

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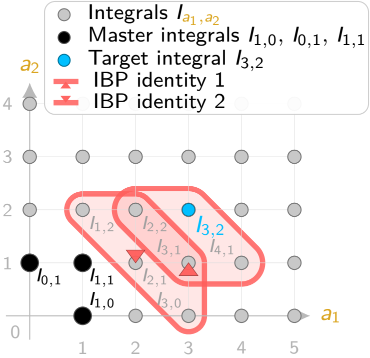

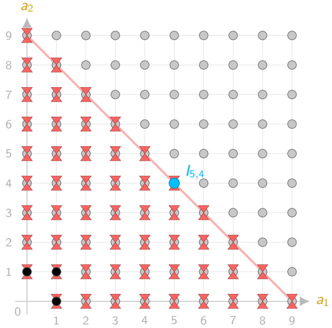

Laporta's golden rule [Laporta (2000)]

⇒ pick equations with $a_1 + a_2 \leq a_1 + a_2 - 1$ of target integral

⇒ Number of equations grows quadratically!

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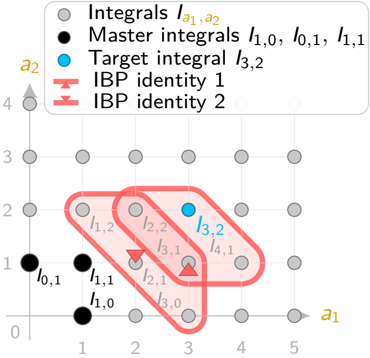

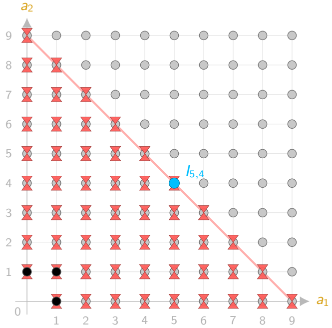

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State-of-the-art integrals: $(a_1, a_2) \to (a_1, \ldots, a_{22})$

$\Rightarrow$ Hard to visualize & Number of equations grows with power 22!

Machine learning for simple example

Optimization problem Pick equations to minimize arithmetic cost

[Berman, Charton, Luna, **MW**, Zeng (2026)]

Machine learning for simple example

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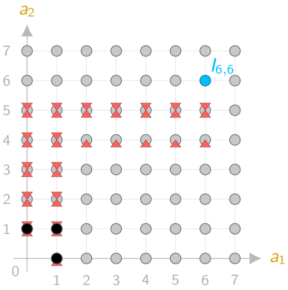
- Reinforcement learning with CNN



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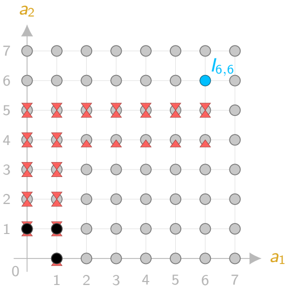

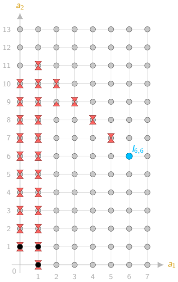

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Optimization problem Pick equations to minimize arithmetic cost

- Reinforcement learning with CNN
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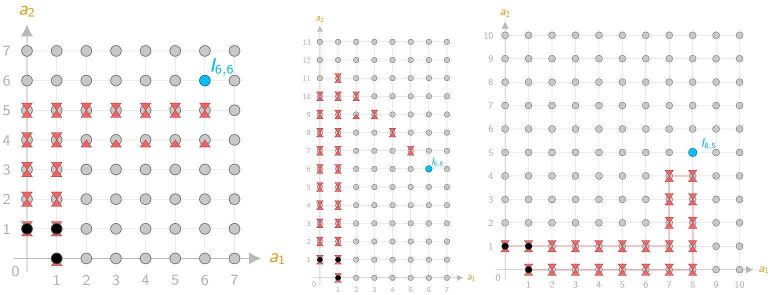

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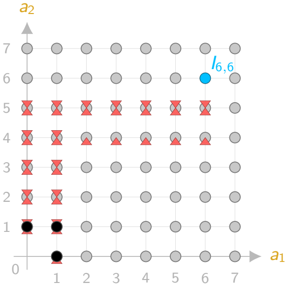
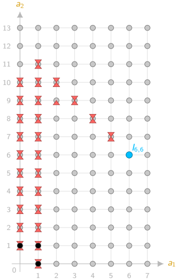
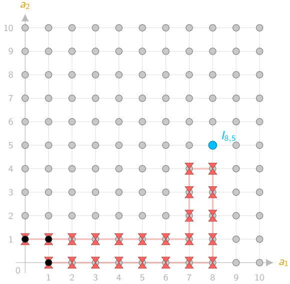






Common feature

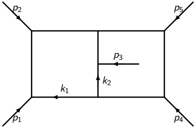
- Picks equations along thin tube between target integral and masters

$\Rightarrow$ Quadratic scaling $\rightarrow$ Linear scaling

[Berman, Charton, Luna, **MW**, Zeng (2026)]

Machine learning and human generalization

State of the art problem from two years ago



Eleven dimensional: $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}$

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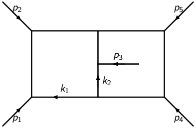

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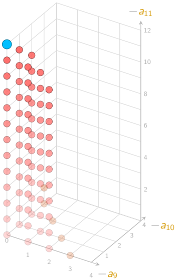
NN with gradient-less training for 3D subproblem



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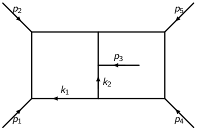
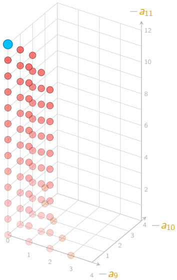


Eleven dimensional: $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}$

NN with gradient-less training for 3D subproblem



Human generalization to 11D
$\Rightarrow$ Eleventh-order scaling $\rightarrow$ Linear scaling

[Berman, Charton, Luna, **MW**, Zeng (2026)]

Conclusion & Outlook

- **Mathematics** and **analytic calculation** of precision predictions in **theoretical physics**

  Bottle neck: IBP reductions

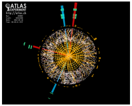


Image: ATLAS

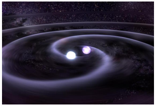


Image: NASA/Goddard Space Flight Center

- **Challenge:** Pick a subset s of sparse linear equations
- **Machine-Learning inspired heuristics**
  High-polynomial scaling $\rightarrow$ Linear scaling $\Rightarrow$ Drastic improvements!
  [Berman, Charton, Luna, **MW**, Zeng (2026)]
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  $\rightarrow$ Bright future ahead for ML assisted discovery!

Image: Road Travel America

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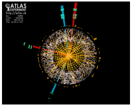

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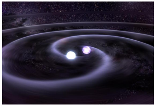

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Thank you!