Holographic fixed points in D=3

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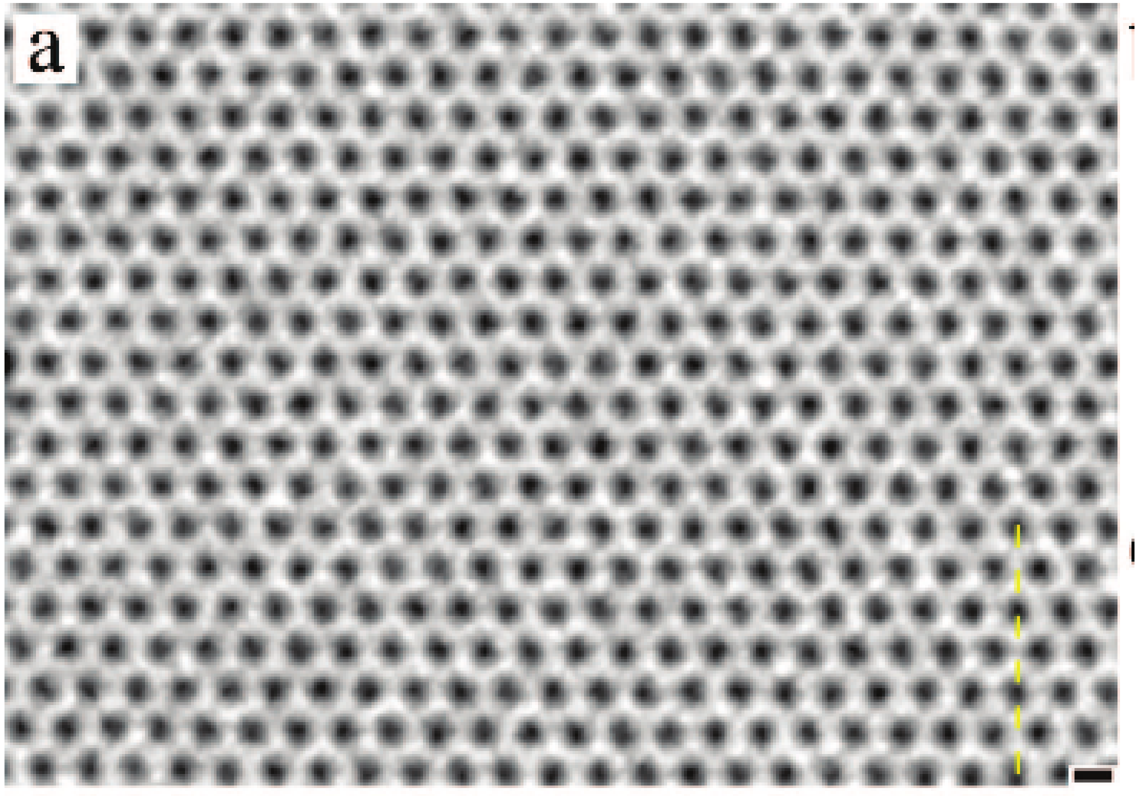
Construct a holographic model of 2+1-dimensional relativistic fermions

Construct a holographic model of 2+1-dimensional relativistic fermions

Condensed matter applications

- p-wave high T_c superconductors
- graphene
- topological insulators
- optical lattices

Graphene is a 2-dimensional array of carbon atoms



Jannik C. Meyer, C. Kisielowski, R. Erni, Marta D. Rossell, M. F. Crommie, and A. Zettl, Nano Letters 8, 3582 (2008).

Graphene with Coulomb interaction up to $\sim 1ev$

$$S = \int d^3x \sum_{k=1}^{4} \bar{\psi}_k \left[\gamma^t (i\partial_t - A_t) + v_F \vec{\gamma} \cdot (i\vec{\nabla} - \vec{A}) \right] \psi_k$$

$$+ \frac{\epsilon}{2e^2} \int d^3x \; F_{0i} \frac{1}{2\sqrt{-\partial^2}} F_{0i} - \frac{1}{4e^2} \int d^3x \; F_{ij} \frac{1}{2\sqrt{-\partial^2}} F_{ij}$$

U(4) symmetry

Speeds of light differ, $v_F \sim c/300$ $(c = 1)$, $\quad \to$ non-relativistic

Graphene fine structure constant

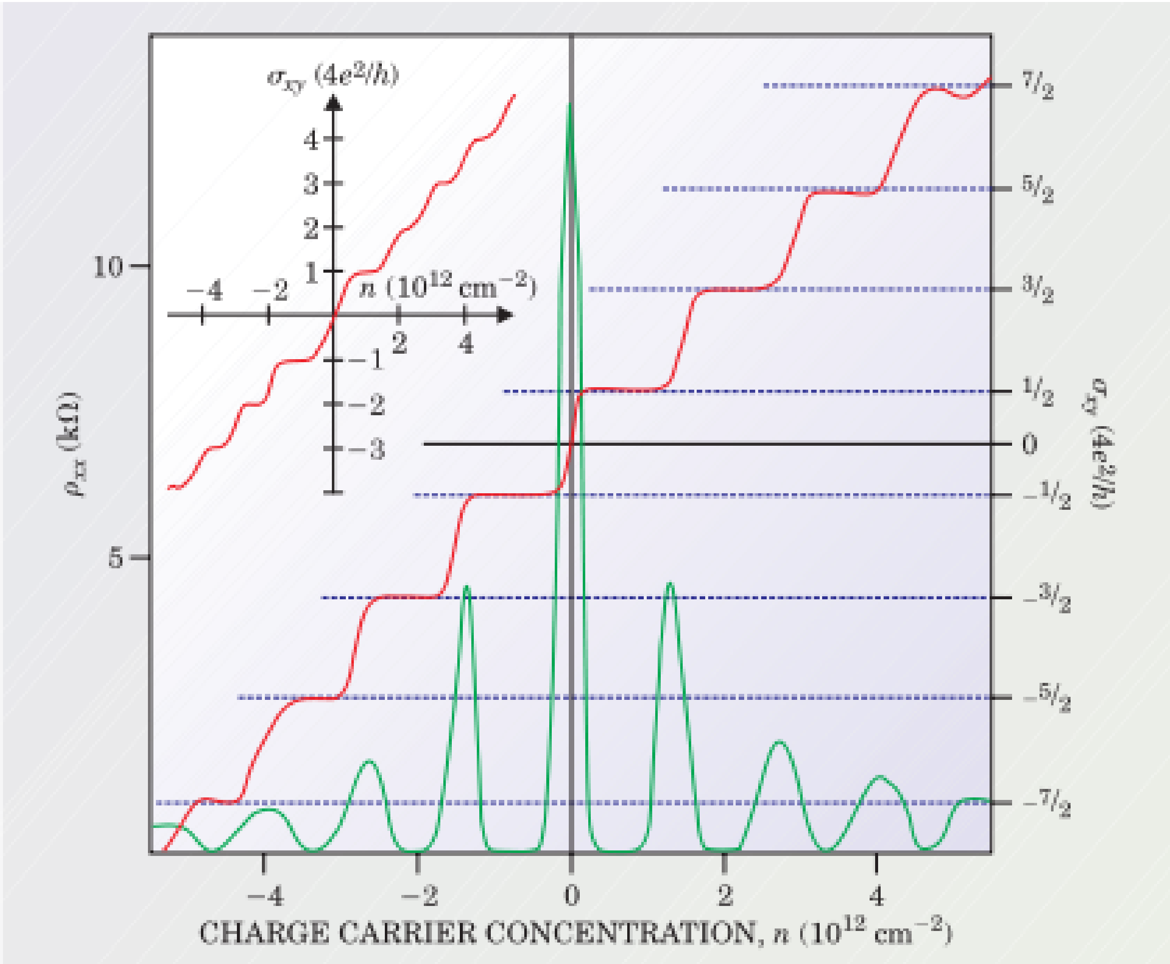
$$\alpha_{\rm graphene} = \frac{e^2}{4\pi\hbar\epsilon v_F} = \frac{e^2}{4\pi\hbar c} \frac{c}{v_F} \frac{1}{\epsilon} \approx \frac{300}{137} \frac{1}{\epsilon}$$

Chiral symmetry breaking? .. $\langle \bar{\psi}\psi \rangle$

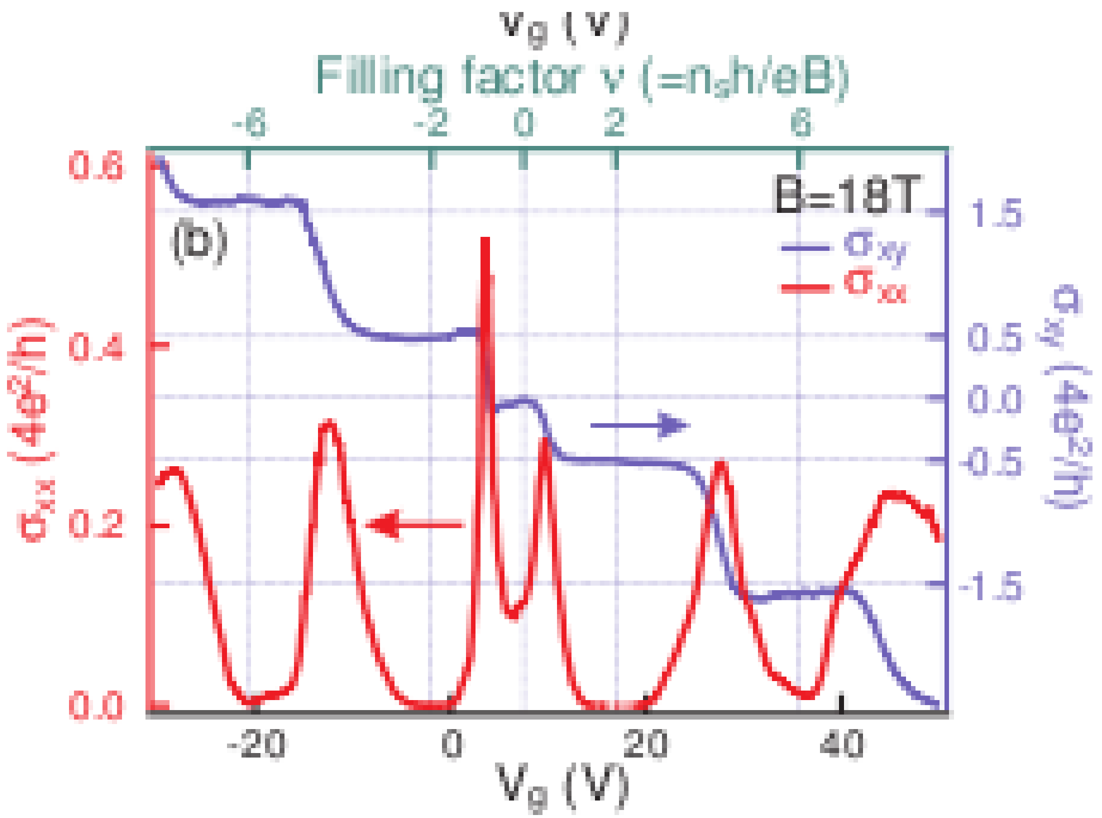
Integer Hall effect in Graphene

K. Novoselov et. al. *Nature 438, 197 (2005)*

Y. Zhang et. al. *Nature 438, 201 (2005)*



Splitting of $\nu = 0$ Landau level **Zhang et.al. arXiv:1003.2738**



QHE data as a function of the gate voltage V_g, for B = 18 T at T = 0.25 K

Graphene

with Coulomb interaction

$$S=\int d^3x\ \sum_{k=1}^{4}\bar\psi_k\left[\gamma^t(i\partial_t-A_t)+v_F\vec\gamma\cdot(i\vec\nabla-\vec A)\right]\psi_k$$

$$+\frac{\epsilon}{2e^2}\int d^3x\ F_{0i}\frac{1}{2\sqrt{-\partial^2}}F_{0i}-\frac{1}{4e^2}\int d^3x\ F_{ij}\frac{1}{2\sqrt{-\partial^2}}F_{ij}$$

Kinetic terms have U(4)×SO(2,1) symmetry, $v_F\sim c/300$ $(c=1)$

Interaction is non-relativistic with U(4) symmetry

Graphene fine structure constant

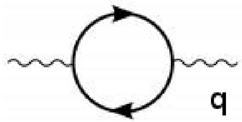
$$\alpha_{\rm graphene}=\frac{e^2}{4\pi\hbar\epsilon v_F}=\frac{e^2}{4\pi\hbar c}\frac{c}{v_F}\frac{1}{\epsilon}\approx\frac{300}{137}\frac{1}{\epsilon}$$

Chiral symmetry breaking? .. $\langle\bar\psi\psi\rangle$

2-component relativistic fermion in 2+1-D

$$S = \int \sum_{k=1}^{N} \bar{\psi}_k \left[\gamma^\mu i\partial_\mu - m\right] \psi_k + \text{interactions}$$

$$\langle j_\mu(x) j_\nu(0)\rangle = \int \frac{d^3q\ e^{iqx}}{(2\pi)^3} \left[\Delta_{\mathrm{T}}(q) \left(q^2 g_{\mu\nu} - q_\mu q_\nu\right) + \Delta_{\mathrm{CS}}(q)\ i\epsilon_{\mu\nu\lambda} q^\lambda\right]$$

At one loop order 

$$q >> m: \quad \Delta_{\mathrm{CS}}(q) = \frac{N}{4}\frac{m}{q} + \ldots \quad , \quad \Delta_T(q) = \frac{N}{16q} + \ldots$$

$$q << m: \quad \Delta_{\mathrm{CS}}(q) = \frac{N}{4\pi}\frac{m}{|m|} + \ldots \quad , \quad \Delta_T(q) = \frac{N}{12\pi|m|} + \ldots$$

No charge gap: $\Delta_{\mathrm{CS}}(q \sim 0) = \text{ const. } + \ldots$, $\Delta_{\mathrm{T}}(q \sim 0) \sim \frac{\text{const.}}{|q|}$

Charge gap: $\Delta_{\mathrm{CS}}(q \sim 0) = \frac{N}{4\pi}\text{sign}(m) + \ldots$, $\Delta_{\mathrm{T}}(q \sim 0) \sim \text{const.}$

Higgs: $\Delta_{\mathrm{CS}}(q \sim 0) = \text{ const. } + \ldots$, $\Delta_{\mathrm{T}}(q \sim 0) \sim \frac{\text{const.}}{q^2}$

D3-D7 system

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

brane extends in directions X

brane sits at single point in directions O

$\#ND = 6$ system – no supersymmetry – no tachyon – only zero modes of 3-7 strings are in R-sector and are 2-component fermions (N_7 flavors and N_3 colors).

Mass = separation in x_9-direction.

$$S = \int d^3x \sum_{\sigma=1}^{N_7} \sum_{\alpha=1}^{N_3} \bar{\psi}_\alpha^\sigma [i\gamma^\mu \partial_\mu - m] \psi_\alpha^\sigma + \text{interactions}$$

$N_3 \to \infty$, $\lambda = 4\pi g_s N_3$ fixed $\to$ replace D3's by $AdS_5 \times S^5$, large λ

D3-D7 system

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

brane extends in directions X

brane sits at single point in directions O

$AdS_5 \times S^5$ metric is $(R^2 = \sqrt{\lambda}\alpha')$

$$dS^2 = \frac{r^2}{R^2}(-dt^2 + dx^2 + dy^2 + dz^2) + \frac{R^2}{r^2}dr^2 + R^2(d\Theta^2 + \cos^2\Theta d\Omega_4^2)$$

Embed D7-brane as

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx^2 + dy^2) + \frac{R^2}{r^2}(1 + \frac{r^2}{R^2}(\Theta')^2)dr^2 + R^2\cos^2\Theta(r)d\Omega_4^2$$

As $r \to \infty$, $\Theta \sim \frac{m}{r^{\Delta_-}} + \frac{<\bar{\psi}\psi>}{r^{\Delta_+}} + \ldots$

D3-D7 system

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

$$\frac{ds^2}{R^2} = r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}(1 + r^2(\Theta')^2) + \cos^2\Theta(r)d\Omega_4^2$$

S. J. Rey, Talk at Strings 2007;

Prog. Theor. Phys. Suppl. 177, 128 (2009) arXiv:0911.5295

D-brane construction of graphene

This embedding is unstable.

Fluctuation of Θ violates BF bound for AdS_4

$$\Theta(r \to \infty) \sim \frac{m}{r^{\Delta_-}} + \frac{< \bar{\psi}\psi >}{r^{\Delta_+}} + \ldots$$

$\Delta_\pm$ are complex

D. Kutasov, J. Lin, A.Parnachev, arXiv:1107.2324

D3-D7 system

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

R. C. Myers and M. C. Wapler, JHEP 0812, 115 (2008) [arXiv:0811.0480 [hepth]].

Stabilize by putting instanton bundle on S^4.

O. Bergman, N. Jokela, G. Lifschytz and M. Lippert, JHEP 1010 (2010) 063 [arXiv:1003.4965 [hep-th]].

U(1) fluxes $f, \tilde{f}$ on 2-spheres in

$$d\Omega_5^2 = d\psi^2 + \sin^2\psi d\Omega_2^2 + \cos^2\psi d\tilde{\Omega}_2^2$$

Stable when f or $\tilde{f}$ large enough.

D3-D7 system

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

Embed D7-brane as

$$\frac{ds^2}{R^2} = r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}(1 + r^4 z'^2 + r^2(\psi')^2) \\ + \sin^2\psi d\Omega_2^2 + \cos^2\psi d\tilde{\Omega}_2^2$$

with fluxes $f, \tilde{f}$ on $\Omega_2, \tilde{\Omega}_2$.

Bergman et.al. constructed Hall states.

Hall plateaus described by "Minkowski" embeddings where brane does not reach horizon. Brane can end when one of the spheres shrinks, at $\psi = 0$ or $\pi/2$. $\to$ Either f or $\tilde{f}$ must be zero.

However, to get discrete symmetries, we need $f = \tilde{f}$.

J.Davis, H.Omid and G.S., arXiv:1107.4397 [hep-th]

One flavor, 2-component fermion in 2+1-D

$$S = \int \bar{\psi}[\gamma^\mu(i\partial_\mu + A_\mu) - m]\psi + \text{interactions}$$

$$\gamma^0 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\ ,\ \gamma^1 = i\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\ ,\ \gamma^2 = i\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Dicrete symmetries:

Parity **P**: $x = (t, x, y) \to x' = (t, -x, y)$

P: $\psi(x) \to \gamma^1\psi(x')$ $(A_0, A_1, A_2) \to (A_0(x'), -A_1(x'), A_2(x'))$

Charge conjugation **C**: $\psi(x) \to \psi^*(x)$ $A_\mu \to -A_\mu$

CP: $\psi(x) \to \gamma^1\psi^*(x')$

P: $\mu \to \mu$, $B \to -B$, $\bar{\psi}\psi \to -\bar{\psi}\psi$

C: $\mu \to -\mu$, $B \to -B$, $\bar{\psi}\psi \to \bar{\psi}\psi$

CP: $\mu \to -\mu$, $B \to B$, $\bar{\psi}\psi \to -\bar{\psi}\psi$

DBI + WZ Action for D7-Brane

$$L = N_7 T_7 \left[-\sqrt{-\det(g + 2\pi\alpha' F)} + F \wedge F \wedge \omega^{(4)} \right]$$

$$\frac{ds^2}{R^2} = r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2} + d\psi^2 + \sin^2\psi d\Omega_2^2 + \cos^2\psi d\tilde{\Omega}_2^2$$

$$\frac{1}{R^4}\omega^{(4)} = r^4 dt \wedge dx \wedge dy \wedge dz + \frac{1}{2}c(\psi) d\Omega_2^2 \wedge d\tilde{\Omega}_2^2$$

$$c(\psi) = \psi - \frac{\pi}{4} - \frac{1}{4}\sin 4\psi$$

C: $F \to -F$ + reverse orientation of $\Omega_2, \tilde{\Omega}_2$ to preserve background $f d\Omega + \tilde{f} d\tilde{\Omega}$.

P: $(t, x, y) \to (t, -x, y)$ also needs $c(\psi) \to -c(\psi)$ or $\psi \to \frac{\pi}{2} - \psi$ requires $f = \tilde{f}$.

Action for D7-Brane

With $2\pi\alpha' F = f d\Omega_2 + \tilde{f} d\tilde{\Omega}_2$

$$\frac{ds^2}{R^2} = r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}$$

$$+ d\psi^2 + \sin^2\psi d\Omega_2^2 + \cos^2\psi d\tilde{\Omega}_2^2$$

$$\frac{ds^2}{R^2} = r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}(1 + r^4(z')^2 + r^2(\psi')^2)$$

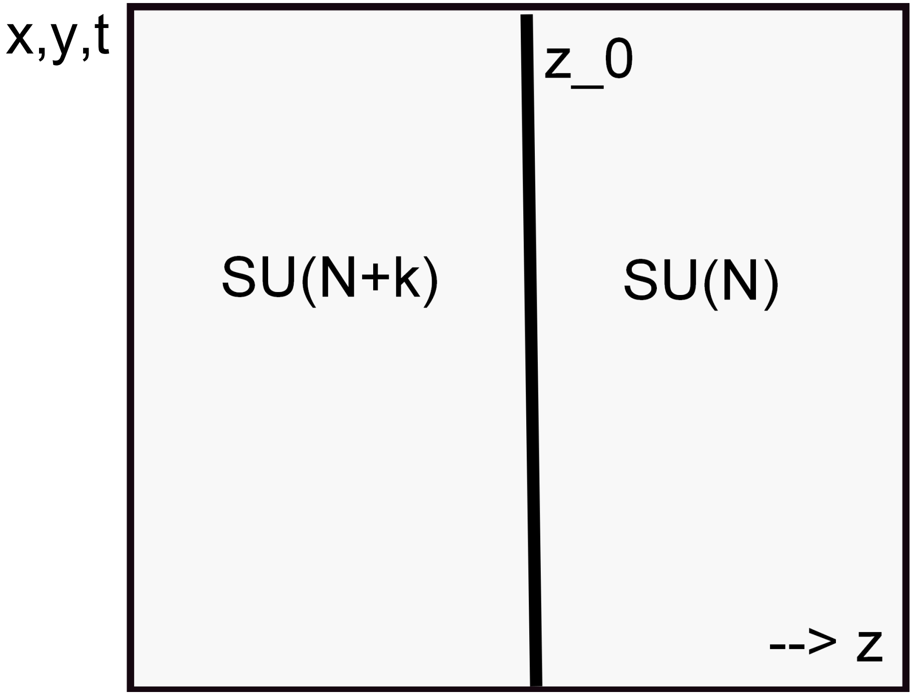
$$+ \sin^2\psi(r) d\Omega_2^2 + \cos^2\psi(r) d\tilde{\Omega}_2^2$$

$$L \sim r^2\sqrt{(f^2 + 4\sin^4\psi)(f^2 + 4\cos^4\psi)(1 + r^4 z'^2 + r^2\psi'^2)} - f^2 r^4 z'$$

Parity invariant solution is constant $\psi = \pi/4$

Defect conformal field theory with $SO(3,2)$ symmetry.

Defect conformal field theory



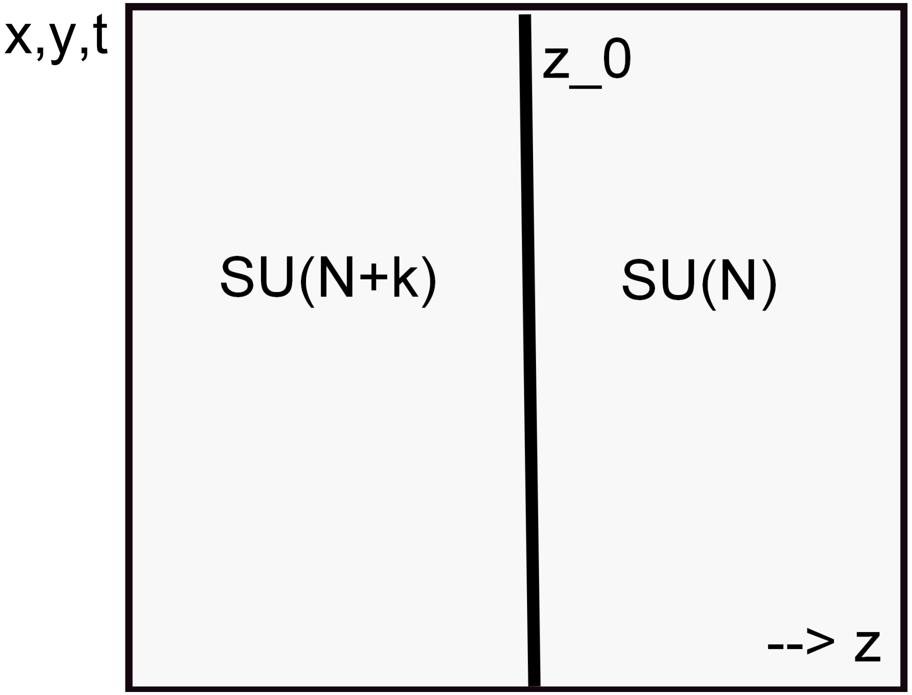
$$\left\langle \bar\Psi\Psi(x)\ \bar\Psi\Psi(0)\right\rangle = \frac{\text{const.}}{x^{2\Delta}}\ ,\ \Delta = \frac{3}{2}+\frac{3}{2}\sqrt{1-\frac{32}{9}\frac{1-f^2}{1+2f^2}}$$

$f^2 \geq \frac{23}{50}$

Tree level dimension $\Delta = 2$ at $f^2 = 1/2$

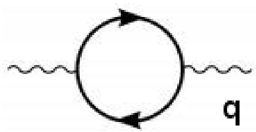
$\Delta = 3$ when $f^2 = 1$

Defect conformal field theory

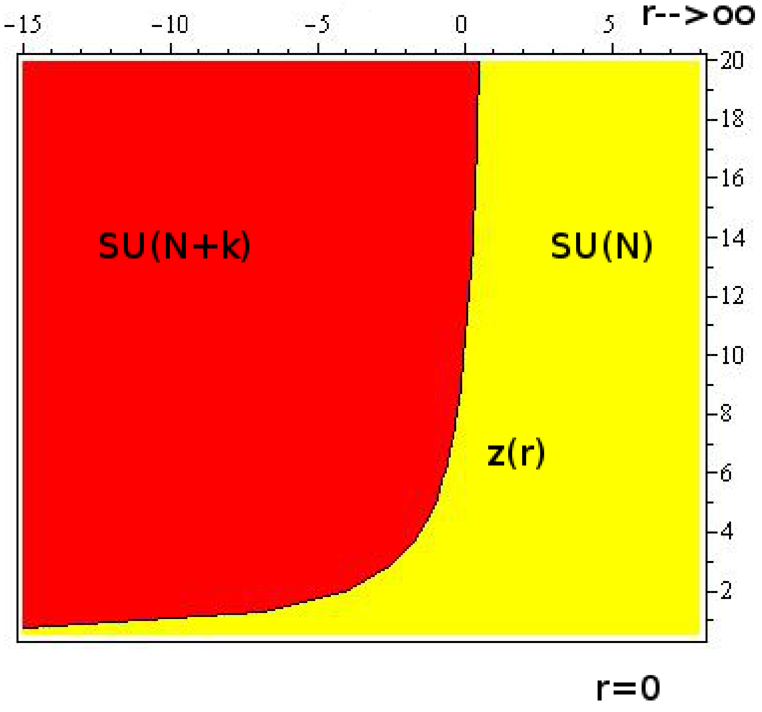


$$\left\langle \bar{\Psi}\gamma_\mu \Psi \;\; \bar{\Psi}\gamma_\nu \Psi \right\rangle = \frac{N_3 N_7}{2\pi^2} \frac{(f^2+1)}{q} \left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)$$

Compare with $\frac{N_3 N_7}{16}\frac{1}{q}$ at weak coupling (1-loop)



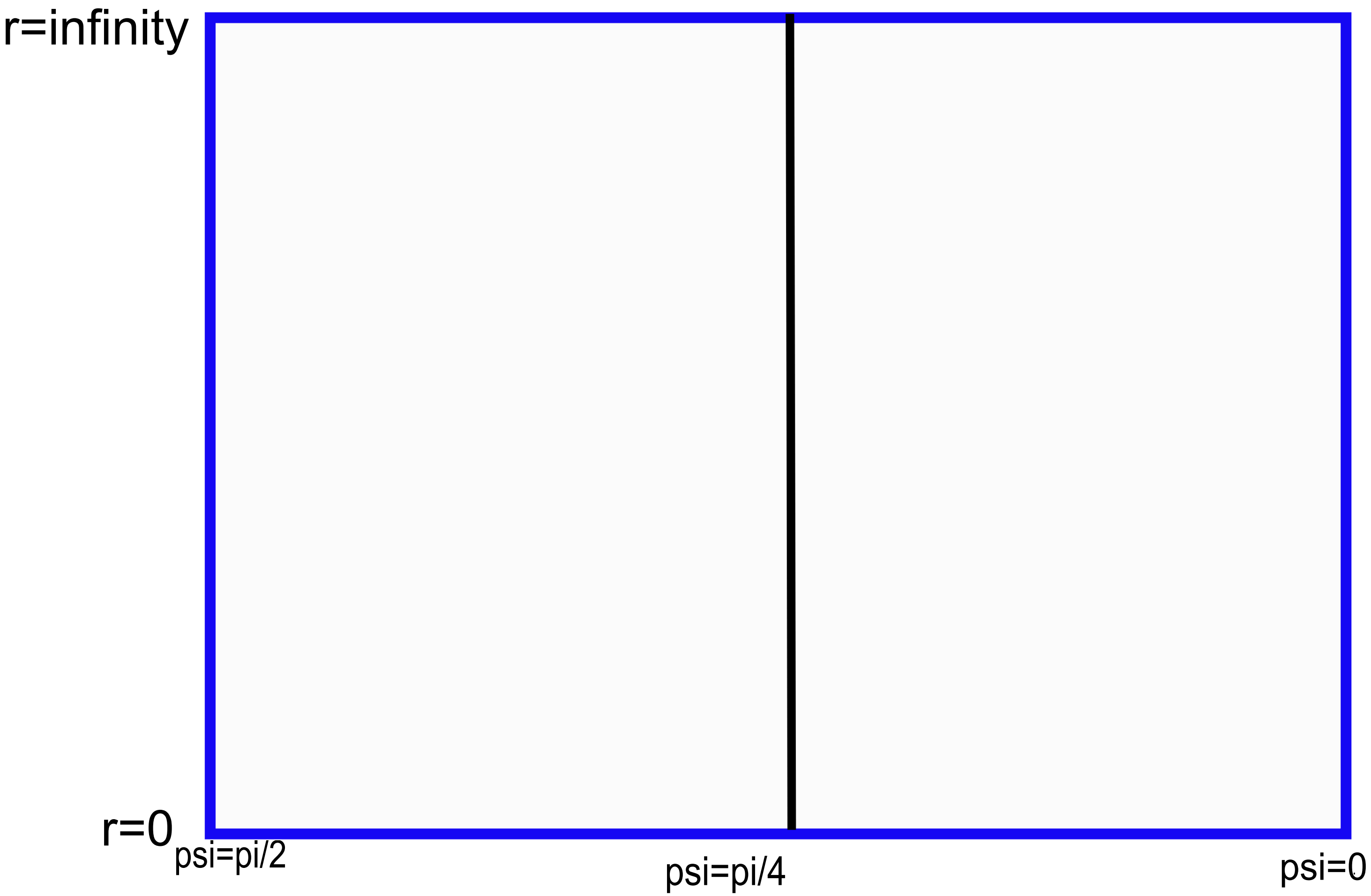
Defect conformal field theory



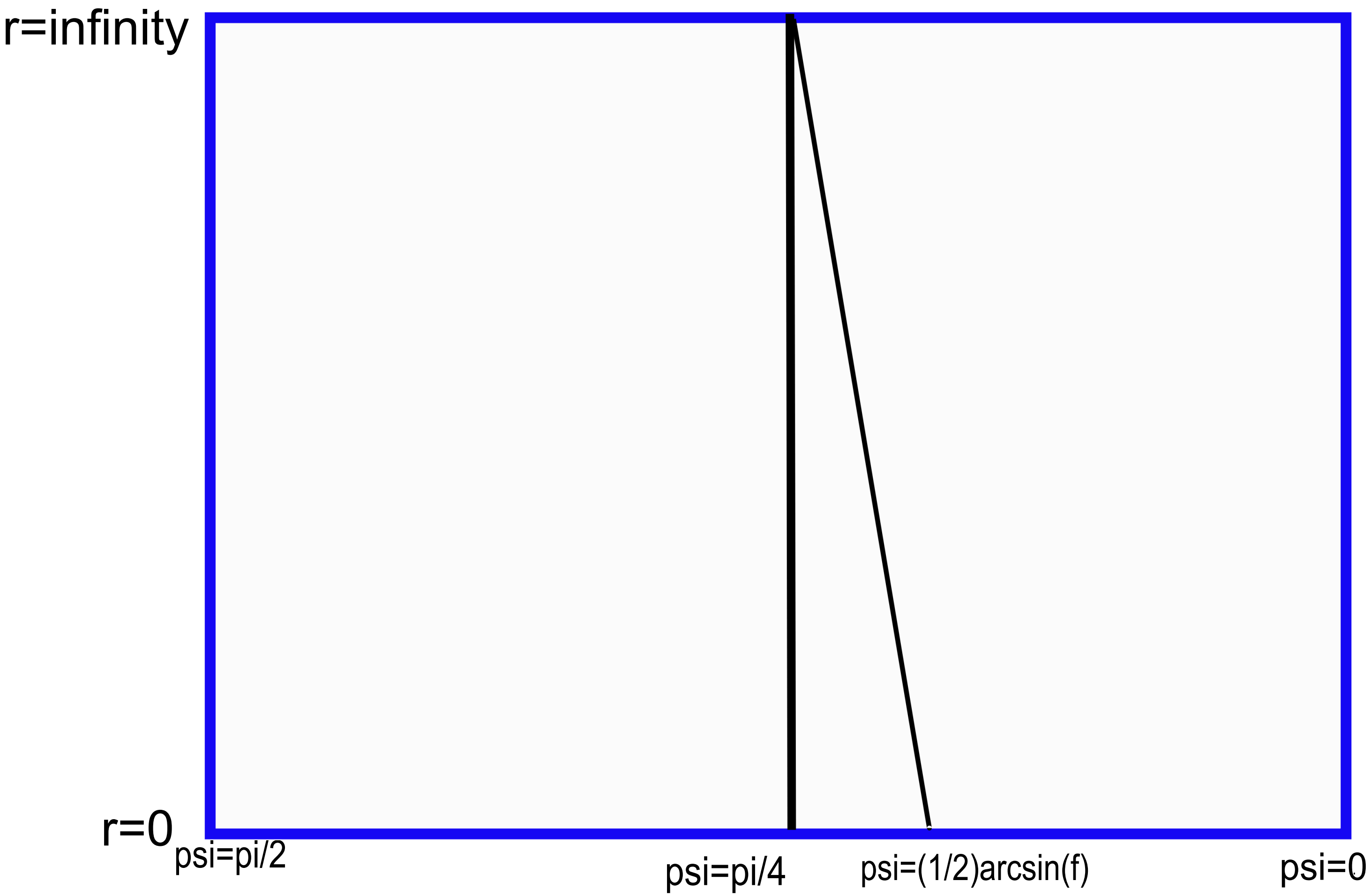
$k = n^2$ where $n = \sqrt{\lambda} f$

$$z(r) = -\frac{f^2}{\sqrt{1+2f^2}} \frac{1}{r}$$

Conformal field theory



Turn on mass operator → Parity violating solution



Turn on mass operator → Parity violating solution

Consider the case where ψ depends on r (and therefore breaks $\psi \to \frac{\pi}{2} - \psi$ parity symmetry). Numerical solution with boundary behavior

$$\psi(r \sim \infty) = \frac{\pi}{4} + \frac{m}{r^{\Delta_-}} + \frac{\langle \bar{\psi}\psi \rangle}{r^{\Delta_+}} + \dots$$

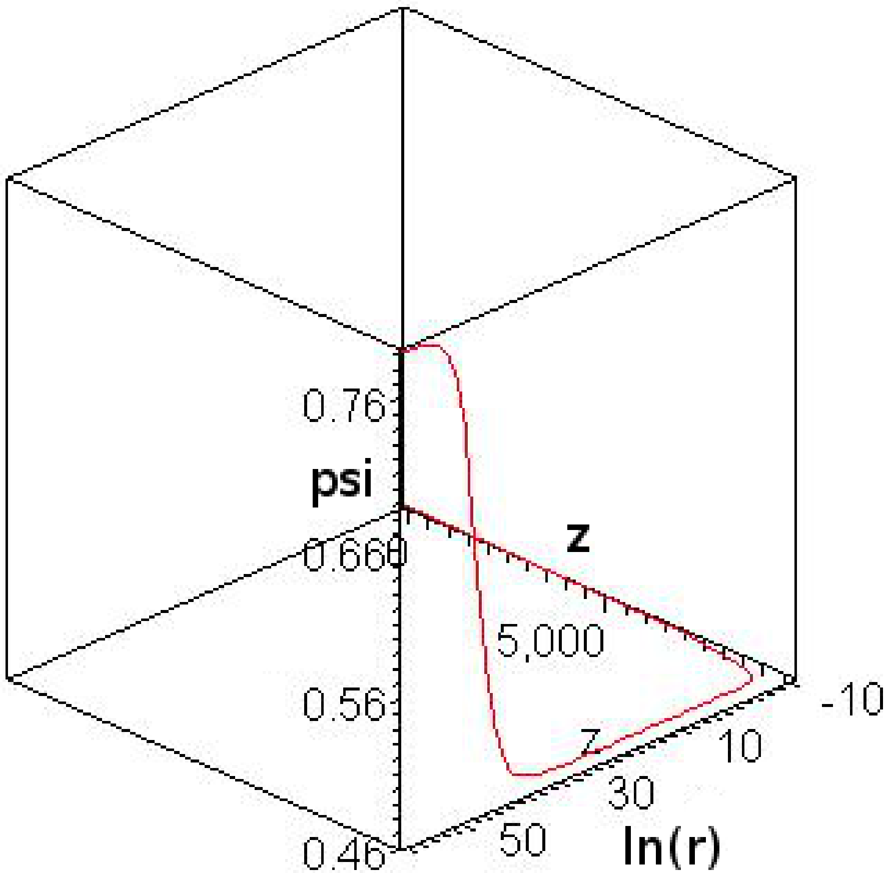
$$\Delta_\pm = \frac{3}{2} \pm \frac{3}{2}\sqrt{1 - \frac{32}{9}\frac{1-f^2}{1+2f^2}} \ , \quad \frac{23}{50} \le f^2 \le 1$$

$$\psi(r \sim 0) = \frac{1}{2}\arcsin f + \psi_+ r^\nu + \dots \quad , \ \nu = -\frac{3}{2} + \frac{3}{2}\sqrt{1 + \frac{64}{9}\frac{1-f^2}{4-f^2}}$$

Fix m, then $< \bar{\psi}\psi >$ and ψ_+ determined and are functions of m.

$$< \bar{\psi}\psi >= c(f) m^{\Delta_+/\Delta_-}$$

No chiral symmetry breaking.


0.76
psi
0.660
z
5,000
0.56
z
-10
10
30
50
ln(r)
0.46

Turn on mass operator → Parity violating solution

Consider the case where ψ depends on r (and therefore breaks $\psi \to \frac{\pi}{2} - \psi$ parity symmetry). Numerical solution with boundary behavior

$$\psi(r \sim \infty) = \frac{\pi}{4} + \frac{m}{r^{\Delta_-}} + \frac{\langle \bar{\psi}\psi \rangle}{r^{\Delta_+}} + \ldots \ , \ \Delta_\pm = \frac{3}{2} \pm \frac{3}{2}\sqrt{1 - \frac{32}{9}\frac{1-f^2}{1+2f^2}}$$

$$\psi(r \sim 0) = \frac{1}{2}\arcsin f + \psi_+ r^\nu + \ldots \quad , \ \nu = -\frac{3}{2} + \frac{3}{2}\sqrt{1 + \frac{64}{9}\frac{1-f^2}{4-f^2}}$$

$$\text{one loop :} \quad < j_\mu j_\nu >= \frac{N_3 N_7}{16}\frac{1}{q}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$$

$$\text{large } q \text{ :} \quad < j_\mu j_\nu >= \frac{N_3 N_7}{2\pi^2}\frac{f^2+1}{q}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$$

small q:

$$< j_\mu j_\nu >= \frac{N_3 N_7}{2\pi^2}\frac{2f}{q}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \frac{N_3 N_7}{\pi^2}(f\sqrt{1-f^2} - \cos^{-1} f) i\epsilon_{\mu\nu\lambda}p^\lambda$$

Turn on charge density and magnetic field

$$L \sim -\sqrt{(f^2 + 4\sin^4\psi)(f^2 + 4\cos^4\psi)(r^4 + b^2)}\,\cdot$$
$$\cdot\sqrt{(1 + r^4 z'^2 + r^2 \psi'^2 - {a_0'}^2)} + f^2 r^4 z' + 2c(\psi) b a_0$$

Turning on any two of a_0, b, m yields a source for the third.

With charge density and magnetic field, ψ cannot be constant.

Routhians

$$\frac{\partial \mathcal{L}}{\partial F_{r0}} = Q$$

$$\frac{\partial \mathcal{L}}{\partial z'} = p_z (= 0)$$

$$R_7 = b\sqrt{1 + r^2\psi'^2}\,\cdot$$

$$\cdot\sqrt{(r^4+1)(f^2+4\cos^4\psi)(f^2+4\sin^4\psi) - f^2r^4 + (\frac{Q}{b} - 2c(\psi))^2}$$

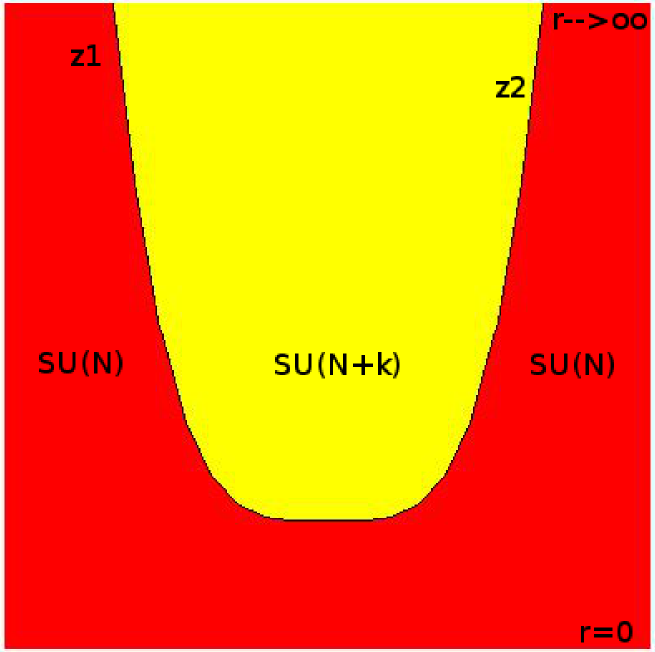
near $r = 0$, $\psi(r)$ must approach

$$f^2\cot 2\psi(0) = \frac{Q}{b} + 2\psi(0) - \frac{\pi}{2}$$

Landau levels have spread into a single band

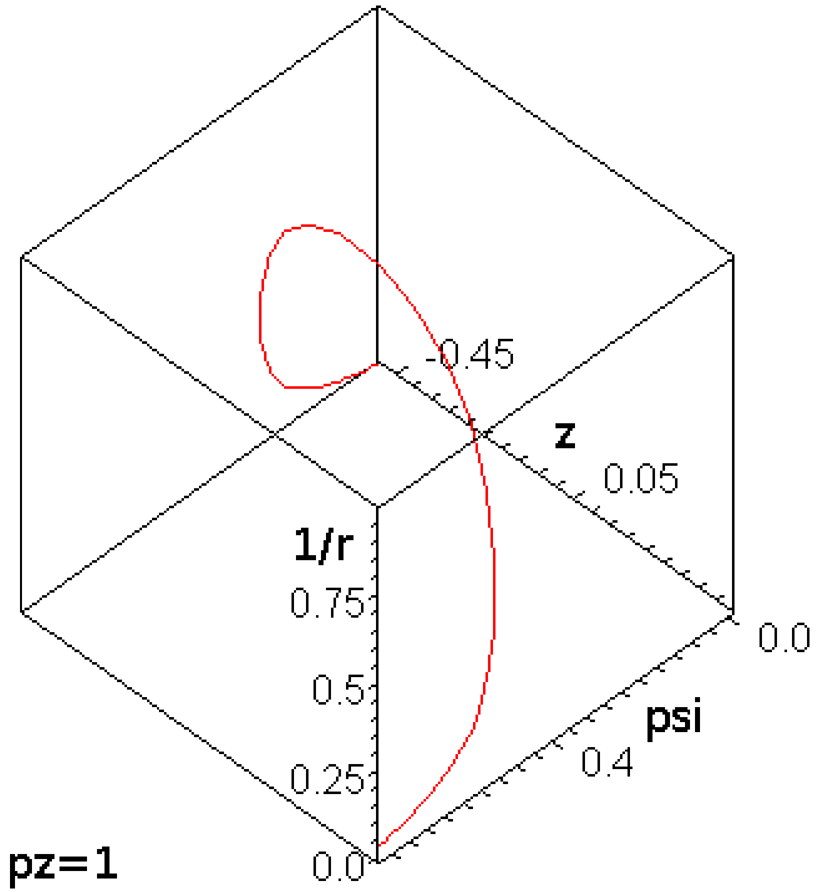
What about solutions with a charge gap?

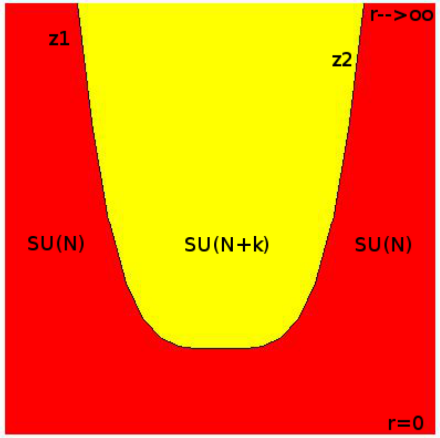
Suspended brane solutions D7-D5 brane join



$$\longleftarrow z \longrightarrow$$

Suspended brane solutions D7-D5 brane join



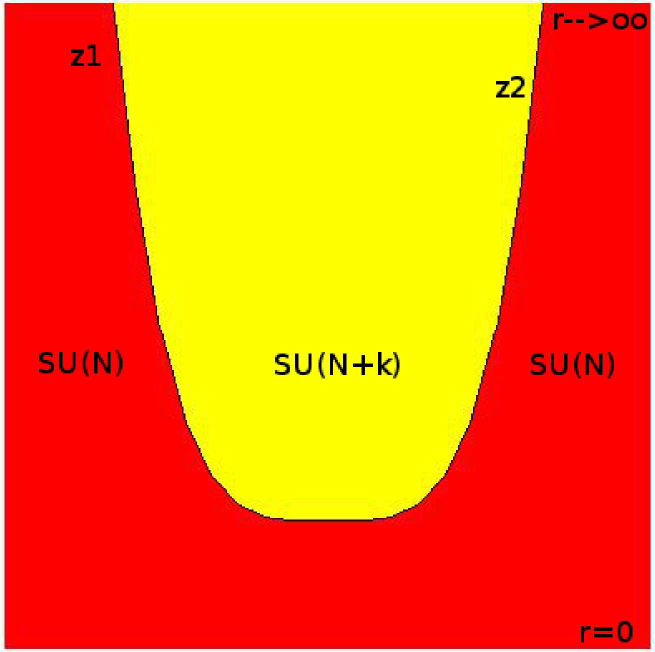


$$
\begin{aligned}
\langle j_{+a} j_{+b} \rangle &= \frac{N_3 N_7}{4\pi} \epsilon_{acb} q_c + \mathcal{O}(q^2) \\
\langle j_{-a} j_{-b} \rangle &= \frac{N_3 N_7}{\pi^2 \rho_m} (\delta_{ab} - \frac{q_a q_b}{q^2}) + \epsilon_{acb} q_c \Delta_{\text{CS}}^{(-)}(0) + \dots
\end{aligned} \quad (1)
$$

where

$$
\begin{aligned}
\rho_m &= \int_{r_{\min}}^{\infty} \frac{d\tilde{r}}{\tilde{r}^2} \frac{\sqrt{(f^2 + 4\sin^4\psi)(f^2 + 4\cos^4\psi)}}{\sqrt{1 + \tilde{r}^2 \psi'^2 + \tilde{r}^4 z'^2}} \\
\Delta_{\text{CS}}^{(-)}(0) &= \frac{N_3 N_7}{\pi^2} \int_0^{\pi/4} d\psi (1 - \cos 4\psi) \left(1 - \frac{\rho(\psi)}{\rho_m}\right)^2
\end{aligned} \quad (2)
$$

Suspended brane solutions: D7-$\bar{\mathrm{D}}7$



Dynamical symmetry breaking

C.Vafa, E.Witten, Phys.Rev.Lett. 53 (1984): 535536

With a single brane, there is no dynamical mass generation.
Euclidean Action

$$L = \tau \left\{ \sqrt{(f^2 + 4\sin^4\psi)(f^2 + 4\cos^4\psi)(r^4 + b^2)} \cdot \right.$$

$$\left. \cdot \sqrt{1 + r^4 z'^2 + r^2 \psi'^2 + (a_0')^2} \right\} - \mathbf{i} f^2 r^4 z' + \mathbf{i} 2 c(\psi) b a_0'$$

The $2N_7$ D7-branes with $U(2N_7) \to U(N_7) \times U(N_7)$ symmetry breaking (or non-breaking) pattern.

Conclusions

- D7-D3 system as strongly coupled 2+1-dimensional relativistic fermions
- Conformal field theory at strong coupling
- gapless state with explicitly broken P and T symmetry
- only gapped states are joined branes D7-D5 and D7-D7 with $U(N_7) \times U(N_7) \to U(N_7)$ symmetry breaking pattern
- evidence for no renormalization of Chern-Simons at strong coupling