A unified description of fermion mass matrices based on the cyclic group C_3

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CKM and PMNS matrices

- CKM matrix -

  $\theta_{12} = 13 \pm 0.05^{\circ}$, $\theta_{23} = 2.38 \pm 0.06^{\circ}$, $\theta_{13} = 0.201 \pm 0.011^{\circ}$, $\delta = 1.20 \pm 0.08$

  Small mixing angles $\approx$ Identity

$$CKM \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

CKM and PMNS matrices

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$$CKM \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- PMNS matrix -

  $31.3^\circ \leq \theta_{sol} \leq 36.9^\circ$, $38.6^\circ \leq \theta_{atm} \leq 53.1^\circ$, $\theta_{reactor} \leq 11.4^\circ$

  Two large mixing angles $\approx$ Tribimaximal mixing

$$PMNS \approx \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Form of the mass matrices

- Diagonalisation

$$\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \bar{\omega}\\ 1 & \bar{\omega} & \omega\end{pmatrix}\begin{pmatrix}0 & \epsilon & \bar{\epsilon}\\ \bar{\epsilon} & 0 & \epsilon\\ \epsilon & \bar{\epsilon} & 0\end{pmatrix}\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \bar{\omega}\\ 1 & \bar{\omega} & \omega\end{pmatrix}^{\dagger}\frac{1}{\sqrt{3}}$$

Form of the mass matrices

- Diagonalisation

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}a & 0 & 0\\0 & b & 0\\0 & 0 & c\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}^{\dagger}$$

Form of the mass matrices

- Diagonalisation

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}^\dagger$$

- A logical starting point to build the model:
The fermion mass matrices are composed of diagonal as well as circulant parts.

The flavour group

- $c.M_{circ}.\bar{c} = M_{circ}$ where $c = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

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- $d.M_{diag}.\bar{d} = M_{diag}$ where $d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \bar{\omega} \end{pmatrix}$
- The group with c and d as generators - $C_3 \times C_3 \rtimes C_3$ - Direct and semidirect product of C_3s

$C_3 \times C_3 \rtimes C_3$ - character table

- The defining representation(3) consists of the conjugacy classes - $\{I\}$, $\{\omega I\}$, $\{\bar{\omega} I\}$, $\{c, \omega c, \bar{\omega} c\}$, $\{\bar{c}, \omega\bar{c}, \bar{\omega}\bar{c}\}$, $\{d, \omega d, \bar{\omega} d\}$, $\{\bar{d}, \omega\bar{d}, \bar{\omega}\bar{d}\}$, $\{cd, \omega cd, \bar{\omega} cd\}$, $\{\bar{c}\bar{d}, \omega\bar{c}\bar{d}, \bar{\omega}\bar{c}\bar{d}\}$, $\{c\bar{d}, \omega c\bar{d}, \bar{\omega} c\bar{d}\}$ and $\{\bar{c}d, \omega\bar{c}d, \bar{\omega}\bar{c}d\}$.
- All irreducible representations except 3 and 3^* are one dimensional.
- Character table

	I	C_ω	$C_{\bar{\omega}}$	C_c	$C_{\bar{c}}$	C_d	$C_{\bar{d}}$	C_{cd}	$C_{\bar{d}\bar{c}}$	$C_{c\bar{d}}$	$C_{d\bar{c}}$
I	1	1	1	1	1	1	1	1	1	1	1
1_c	1	1	1	1	1	$\bar{\omega}$	ω	$\bar{\omega}$	ω	ω	$\bar{\omega}$
1_c^*	1	1	1	1	1	ω	$\bar{\omega}$	ω	$\bar{\omega}$	$\bar{\omega}$	ω
1_d	1	1	1	$\bar{\omega}$	ω	1	1	$\bar{\omega}$	ω	$\bar{\omega}$	ω
1_d^*	1	1	1	ω	$\bar{\omega}$	1	1	ω	$\bar{\omega}$	ω	$\bar{\omega}$
1_{cd}	1	1	1	ω	$\bar{\omega}$	$\bar{\omega}$	ω	1	1	$\bar{\omega}$	ω
1_{cd}^*	1	1	1	$\bar{\omega}$	ω	ω	$\bar{\omega}$	1	1	ω	$\bar{\omega}$
$1_{c\bar{d}}$	1	1	1	$\bar{\omega}$	ω	$\bar{\omega}$	ω	ω	$\bar{\omega}$	1	1
$1_{c\bar{d}}^*$	1	1	1	ω	$\bar{\omega}$	ω	$\bar{\omega}$	$\bar{\omega}$	ω	1	1
3	3	3ω	$3\bar{\omega}$	0	0	0	0	0	0	0	0
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1_d	1	1	1	$\bar{\omega}$	ω	1	1	$\bar{\omega}$	ω	$\bar{\omega}$	ω
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1_{cd}	1	1	1	ω	$\bar{\omega}$	$\bar{\omega}$	ω	1	1	$\bar{\omega}$	ω
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$1_{c\bar{d}}$	1	1	1	$\bar{\omega}$	ω	$\bar{\omega}$	ω	ω	$\bar{\omega}$	1	1
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  $\{\bar{c}\bar{d}, \omega\bar{c}\bar{d}, \bar{\omega}\bar{c}\bar{d}\}, \{c\bar{d}, \omega c\bar{d}, \bar{\omega} c\bar{d}\}$ and $\{\bar{c}d, \omega\bar{c}d, \bar{\omega}\bar{c}d\}$.
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1_d	1	1	1	$\bar{\omega}$	ω	1	1	$\bar{\omega}$	ω	$\bar{\omega}$	ω
1_d^*	1	1	1	ω	$\bar{\omega}$	1	1	ω	$\bar{\omega}$	ω	$\bar{\omega}$
1_{cd}	1	1	1	ω	$\bar{\omega}$	$\bar{\omega}$	ω	1	1	$\bar{\omega}$	ω
1_{cd}^*	1	1	1	$\bar{\omega}$	ω	ω	$\bar{\omega}$	1	1	ω	$\bar{\omega}$
$1_{c\bar{d}}$	1	1	1	$\bar{\omega}$	ω	$\bar{\omega}$	ω	ω	$\bar{\omega}$	1	1
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Constructing the Invariant

- Tensor product expansion
  $3 \times 3^* = \mathrm{I} + 1_c + 1_c^* + 1_d + 1_d^* + 1_{cd} + 1_{cd}^* + 1_{c\bar{d}} + 1_{c\bar{d}}^*$

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  $3 \times 3^* = \mathsf{I} + 1_c + 1_c^* + 1_d + 1_d^* + 1_{cd} + 1_{cd}^* + 1_{c\bar{d}} + 1_{c\bar{d}}^*$
- If $\psi =$ triplet(3) then
  $\psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_3^*\psi_3 = \mathsf{I}$
  $\psi_1^*\psi_3 + \psi_2^*\psi_1 + \psi_3^*\psi_2 = 1_c$
  $\psi_1^*\psi_1 + \bar{\omega}\psi_2^*\psi_2 + \omega\psi_3^*\psi_3 = 1_d$

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  $\psi_1^* \psi_1 + \bar{\omega} \psi_2^* \psi_2 + \omega \psi_3^* \psi_3 = 1_d$
- Define flavons $\phi_\iota = \mathsf{I}$, $\phi_c = 1_c$ and $\phi_d = 1_d$

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- Tensor product expansion
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- Define flavons $\phi_\iota = \mathrm{I}$, $\phi_c = 1_c$ and $\phi_d = 1_d$
- Invariant - $\phi_\iota(\psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_3^*\psi_3) +$
  $\phi_c(\psi_1^*\psi_3 + \psi_2^*\psi_1 + \psi_3^*\psi_2)^* + \phi_c^*(\psi_1^*\psi_3 + \psi_2^*\psi_1 + \psi_3^*\psi_2) +$
  $\phi_d(\psi_1^*\psi_1 + \bar{\omega}\psi_2^*\psi_2 + \omega\psi_3^*\psi_3)^* + \phi_d^*(\psi_1^*\psi_1 + \bar{\omega}\psi_2^*\psi_2 + \omega\psi_3^*\psi_3)$
  In matrix form -
  $$\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix}^\dagger \begin{pmatrix}\phi_\iota + \phi_d + \phi_d^* & \phi_c & \phi_c^* \\ \phi_c^* & \phi_\iota + \omega\phi_d + \bar{\omega}\phi_d^* & \phi_c \\ \phi_c & \phi_c^* & \phi_\iota + \bar{\omega}\phi_d + \omega\phi_d^*\end{pmatrix} \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix}$$

A model for leptons

- All fermions = Triplet (3)
- Flavons $\phi_\iota, \phi_c, \phi_d$
- The mass term for charged leptons (In Standard Model framework)

$$
\Big(\frac{m_\iota \phi_\iota}{\Lambda}(L_1^* e_R + L_2^* \mu_R + L_3^* \tau_R) +
\frac{m_d \phi_d}{\Lambda}(L_1^* e_R + \bar{\omega} L_2^* \mu_R + \omega L_3^* \tau_R)^* + \frac{m_d \phi_d^*}{\Lambda}(L_1^* e_R + \bar{\omega} L_2^* \mu_R + \omega L_3^* \tau_R)
\frac{m_c \phi_c}{\Lambda}(L_1^* \tau_R + L_2^* e_R + L_3^* \mu_R)^* + \frac{m_c \phi_c^*}{\Lambda}(L_1^* \tau_R + L_2^* e_R + L_3^* \mu_R)\Big) H
$$

- Similar Dirac mass term can be written for neutrinos as well with real constants n_ι, n_d, n_c.

Phenomenology

- After spontaneous symmetry breaking
  Charged lepton mass matrix -

$$\begin{pmatrix} \mathbf{m}_\iota + \mathbf{m}_d Cos(\alpha) & \mathbf{m}_c e^{i\beta} & \mathbf{m}_c e^{-i\beta} \\ \mathbf{m}_c e^{-i\beta} & \mathbf{m}_\iota + \mathbf{m}_d Cos(\alpha + \frac{2\pi}{3}) & \mathbf{m}_c e^{i\beta} \\ \mathbf{m}_c e^{i\beta} & \mathbf{m}_c e^{-i\beta} & \mathbf{m}_\iota + \mathbf{m}_d Cos(\alpha + \frac{4\pi}{3}) \end{pmatrix}$$

Neutrino mass matrix -

$$\begin{pmatrix} \mathbf{n}_\iota + \mathbf{n}_d Cos(\alpha) & \mathbf{n}_c e^{i\beta} & \mathbf{n}_c e^{-i\beta} \\ \mathbf{n}_c e^{-i\beta} & \mathbf{n}_\iota + \mathbf{n}_d Cos(\alpha + \frac{2\pi}{3}) & \mathbf{n}_c e^{i\beta} \\ \mathbf{n}_c e^{i\beta} & \mathbf{n}_c e^{-i\beta} & \mathbf{n}_\iota + \mathbf{n}_d Cos(\alpha + \frac{4\pi}{3}) \end{pmatrix}$$

α and β are phases of the vevs of ϕ_d and ϕ_c respectively.

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  Neutrino mass matrix -

$$\begin{pmatrix} \mathbf{n}_\iota + \mathbf{n}_d Cos(\alpha) & \mathbf{n}_c e^{i\beta} & \mathbf{n}_c e^{-i\beta} \\ \mathbf{n}_c e^{-i\beta} & \mathbf{n}_\iota + \mathbf{n}_d Cos(\alpha + \frac{2\pi}{3}) & \mathbf{n}_c e^{i\beta} \\ \mathbf{n}_c e^{i\beta} & \mathbf{n}_c e^{-i\beta} & \mathbf{n}_\iota + \mathbf{n}_d Cos(\alpha + \frac{4\pi}{3}) \end{pmatrix}$$

  α and β are phases of the vevs of ϕ_d and ϕ_c respectively.

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  Neutrino mass matrix -

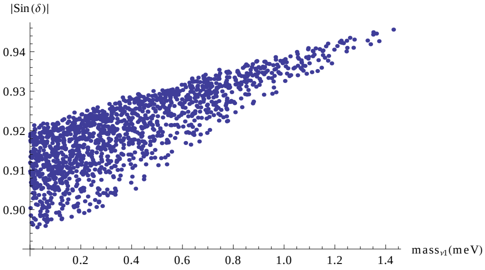
$$\begin{pmatrix} \mathbf{n}_\iota + \mathbf{n}_d Cos(\alpha) & \mathbf{n}_c e^{i\beta} & \mathbf{n}_c e^{-i\beta} \\ \mathbf{n}_c e^{-i\beta} & \mathbf{n}_\iota + \mathbf{n}_d Cos(\alpha + \frac{2\pi}{3}) & \mathbf{n}_c e^{i\beta} \\ \mathbf{n}_c e^{i\beta} & \mathbf{n}_c e^{-i\beta} & \mathbf{n}_\iota + \mathbf{n}_d Cos(\alpha + \frac{4\pi}{3}) \end{pmatrix}$$

  α and β are phases of the vevs of ϕ_d and ϕ_c respectively.

- Their diagonal and circulant parts differ only with respect to constant ratios.
- 8 independent parameters.

Fitting with the data

- Leptonic Yukawa sector has 10 experimental observables.
- Masses of charged leptons, mass squared differences of the neutrinos and solar, atmospheric and reactor mixing angles are known.† The model can predict the light neutino mass and the CP phase.



†Renormalised values of lepton masses from Table IV arXiv:0712.1419. Neutrino mixing angles with 1 sigma error from Table 2 arXiv:1103.0734

Concluding remarks

- A new flavour group $C_3 \times C_3 \rtimes C_3$ was introduced..
- A model based on this group to obtain the leptonic Yukawa sector was presented.
- This approach might be applied to quark sector as well (by adding extra flavons etc).

Extra

- Charged lepton(MeV)
$$\begin{pmatrix} 70.087 & 50.408+7.714i & 50.408-7.714i \\ 50.408-7.714i & 38.194 & 50.408+7.714i \\ 50.408+7.714i & 50.408-7.714i & 1776.6 \end{pmatrix}$$
$$\begin{pmatrix} 0.496 & 0 & 0 \\ 0 & 104.68 & 0 \\ 0 & 0 & 1779.7 \end{pmatrix}$$
$Trace/3 = 628.295$, $Abs(-279.104+501.835i) = 574.228$, $Abs(50.408-7.714i) = 50.995$
- Neutrino(meV)
$$\begin{pmatrix} 20.228 & 16.533+2.530i & 16.533-2.530i \\ 16.533-2.530i & 20.153 & 16.533+2.530i \\ 16.533+2.530i & 16.533-2.530i & 24.259 \end{pmatrix}$$
$$\begin{pmatrix} 0.402 & 0 & 0 \\ 0 & 9.55 & 0 \\ 0 & 0 & 54.69 \end{pmatrix}$$
$Trace/3 = 21.547$, $Abs(-0.660+1.185i) = 1.356$, $Abs(16.534+2.530i) = 16.726$
- $\alpha = 119.081^\circ$, $\beta = 8.700^\circ$