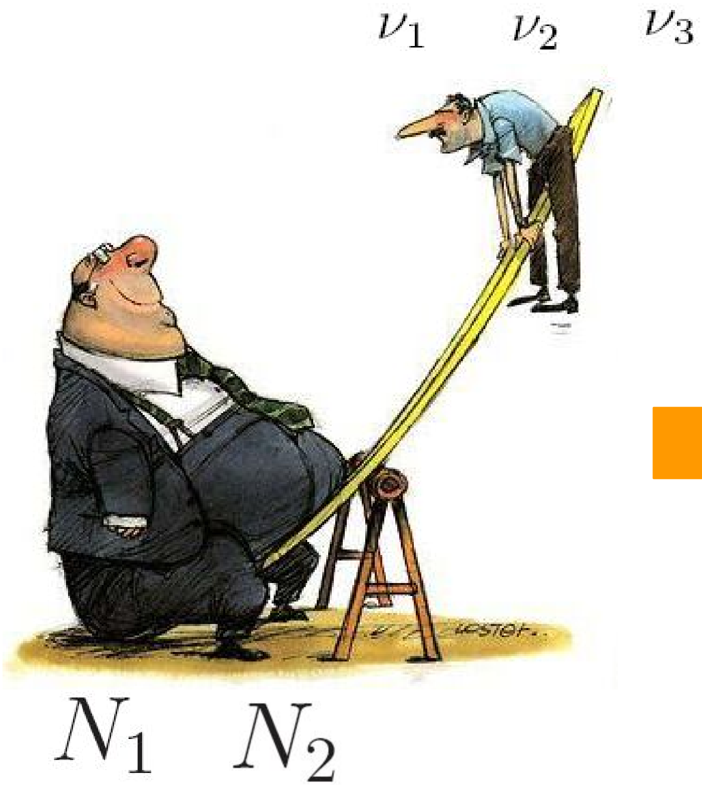
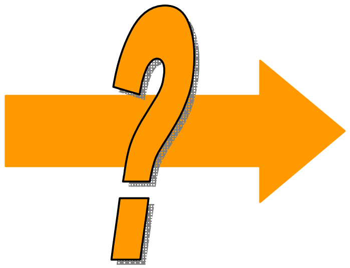
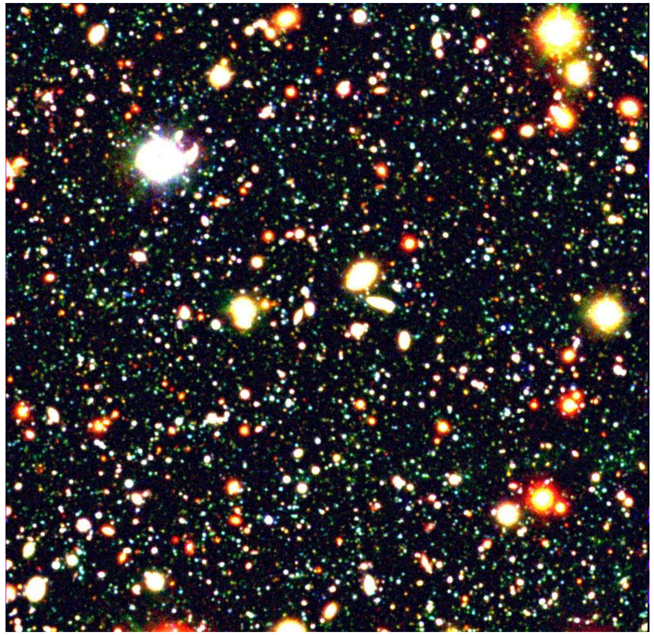
Leptogenesis in the Two Right-Handed Neutrino Model

David Jones, University of Southampton, May 4th 2011





$$\eta = (6.21 \pm 0.16) \times 10^{-10}$$



The talk is based on our paper, out soon!

See S. Antusch, P. Di Bari, D.A. Jones, S.F. King, should be on **arXiv** soon!

Ground I'll cover…

Introduction to Leptogenesis.

Flavoured Leptogenesis in Two Right-Handed Neutrino Models. (Light and Heavy Flavour effects.)

Sequential Dominance.

The importance of N_2 decays in Light Sequential Dominance.

Leptogenesis – a quick reminder

To make a Baryon number asymmetry we need to satisfy three conditions:

1) B number violation
2) C and CP violation
3) Departure from Thermal Equilibrium (DTE)

} Sakharov's conditions for Baryogenesis

Leptogenesis uses the Seesaw Mechanism to produce the Baryon Asymmetry of the Universe (BAU):

$$\eta \equiv \left.\frac{n_B - n_{\bar{B}}}{n_\gamma}\right|_0 = (6.21 \pm 0.16) \times 10^{-10}$$

In the **Type I Seesaw**, this happens because the **decays of heavy N_R violate L** (hence B, more later…) and **may violate CP**. The expansion and cooling of the universe gives **DTE when N go non-relativistic at $T \approx M_1$**

Type I Seesaw Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i\overline{N}_{Ri}\gamma_\mu\partial^\mu N_{Ri} - h_{\alpha i}\overline{\ell}_{L\alpha}N_{Ri}\tilde{\Phi} - \frac{1}{2}M_i\overline{N}^c_{Ri}N_{Ri} + h.c.$$

Example: “Vanilla” Leptogenesis:

The **Boltzmann equation** is:

$$\hat{L}[f] = C[f]$$

For an FRW metric the **Liouville operator** is:

$$\hat{L}[f(E,t)] = \frac{\partial f}{\partial t} - H(t)\, E^2 \frac{\partial f}{\partial E}$$

The **Collision operator** for $N_i \rightleftharpoons l_i + \phi^\dagger$ is:

$$C[f] = \int d\Pi_l \int d\Pi_\phi \, (2\pi)^4 \delta^{(4)}(p_N - p_l - p_\phi)$$

$$\times \left\{ |M|^2_{N_i \to l_i + \phi^\dagger} \; f_N (1 - f_l)(1 + f_\phi) \; - \; |M|^2_{l_i + \phi^\dagger \to N_i} \; f_l f_\phi (1 - f_N) \right\}$$

To find the **number density** of a species from $f(E,t)$ we integrate over its 3-momenum:

$$n(t) = \int \frac{d^3 p}{2\pi^3} f(E,t)$$

With some simplifying assumptions, we can get equations for **B – L evolution**:

$$\frac{dN_{N_i}}{dz_i} = -D_i \left(N_{N_i} - N_{N_i}^{\rm eq}\right)$$

$$\frac{dN_{B-L}}{dz_i} = \varepsilon_i \, D_i \left(N_{N_i} - N_{N_i}^{\rm eq}\right) - W_i N_{B-L}$$

$$z_i = \frac{M_i}{T}$$

Production term: decays of N_R produce B – L asymmetry.
Inverse decays erase existing B – L asymmetry via the **Washout term**

Washout

Washout of a coin asymmetry:

A heads / tails asymmetry.

many flips

$\longrightarrow$

A random equal mixture:

Will any Lepton asymmetry survive washout by inverse decays?

When the reaction $N_i \rightleftharpoons l_i + \phi^\dagger$ is in equilibrium, # decays (D) and # inverse decays (ID) balance, so: $\Gamma_D \, n_{N_i}^{eq} = \Gamma_{ID} \, n_{l_i}^{eq}$

Then $n_{N_i}^{eq} \propto e^{-z_i}$ implies $\Gamma_{ID} \ll \Gamma_D$ for $z_i > 1$

$$W_i(z_i) \equiv \frac{\Gamma_{ID}}{z_i \, H(z_i)}$$

$$D_i(z_i) \equiv \frac{\Gamma_D}{z_i \, H(z_i)}$$

The **washout term** $\propto \Gamma_{ID}$ cuts off faster than the **production term** $\propto \Gamma_D$ – we expect some final lepton asymmetry to survive ☺

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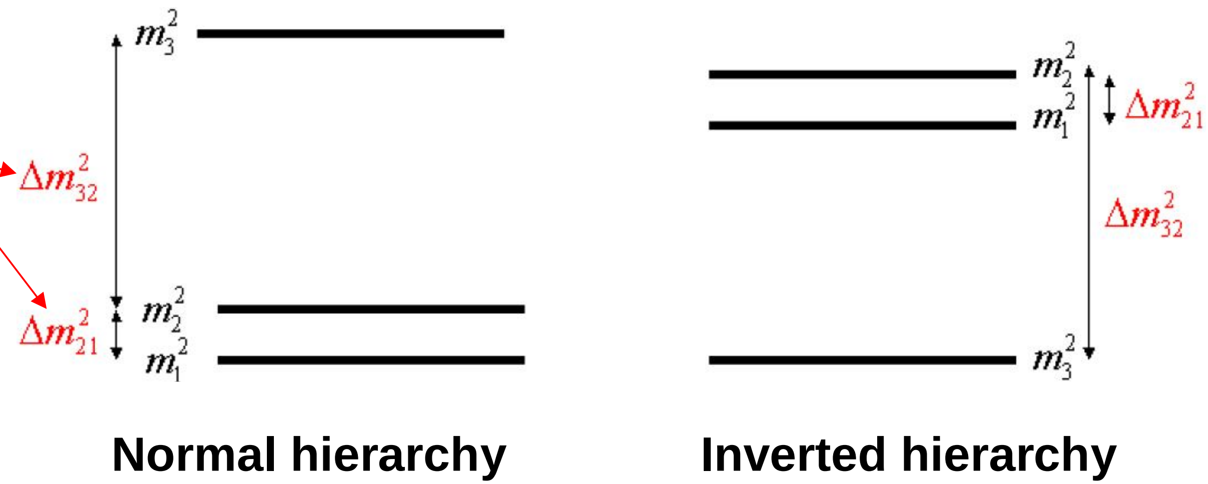
Two Right Handed Neutrino Model (2RHNM)

We measure a **solar** and an **atmospheric** **mass splittitng** in ν oscillation experiments.

From solar neutrino matter effects, we know $m_2 > m_1$. This leaves two possible mass orderings:



3RHNM: 3 non-zero M_R @ high energy → 3 non-zero m_L in low-energy EFT

2RHNM: 2 non-zero M_R @ high energy → 2 non-zero m_L + $m_{lightest}$ = 0

Pros: more predictive: has a seesaw with less unknown "high energy" parameters

Cons: 2RHNM is killed if $m_\nu << m_{sol}$ is not found in future ν experiments.

Light Flavour effects

In the **2RHNM** the N_2s decay first, producing some B – L asymmetry

$$|l_2\rangle \equiv \sum_\alpha |l_\alpha\rangle \langle l_\alpha | l_2\rangle$$

If $\Gamma_{ID} < \Gamma_{l_\alpha + \phi \to \alpha}$ the universe can "measure" the flavour α of a lepton doublet

Our M_R mass spectrum is: $10^9 GeV < M_1 \lesssim M_2 / 3 < 10^{12} GeV$ ← Recovers hierarchical limit

Hence the $N_{1,2}$ decay at temperature: $10^9 GeV < T < 10^{12} GeV$

$\Gamma_{l_\alpha + \phi \to \alpha} \approx 5 \times 10^{-3}\, y_\alpha^2\, T$ ⟶ α = τ is "measured", but { e, μ } stays coherent.

The lepton doublet state is projected into a **two flavour basis** { l_2 } ↦ { 2γ, τ }.

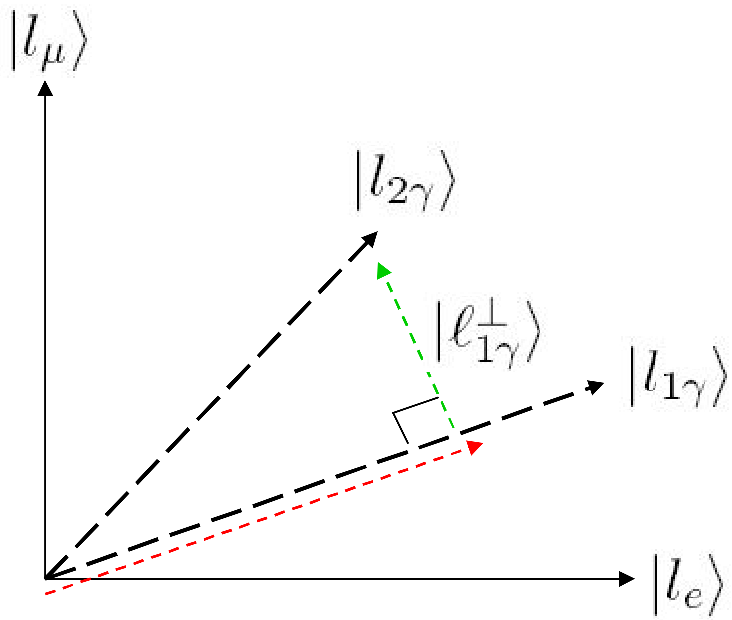
$$\begin{aligned}
\frac{dN_{N_2}}{dz} &= -D_2\,(N_{N_2} - N_{N_2}^{\rm eq})\,, \\
\frac{dN_{\Delta_{2\gamma}}}{dz} &= \varepsilon_{2\gamma}\, D_2\,(N_{N_2} - N_{N_2}^{\rm eq}) - P_{2\gamma}^0\, W_2\, N_{\Delta_\gamma}\,, \\
\frac{dN_{\Delta_\tau}}{dz} &= \varepsilon_{2\tau}\, D_2\,(N_{N_2} - N_{N_2}^{\rm eq}) - P_{2\tau}^0\, W_2\, N_{\Delta_\alpha}\,.
\end{aligned}$$

Heavy flavour effects

Q: Will all the asymmetry made by N_2 decays be washed out by N_1 decays?

A: We expect an **orthogonal component** in { e, μ } plane to survive
N_1 inverse decays can only wash out the **parallel component**.

(we would also expect the τ asymmetry from N_2 to be washed out by N_1)



$$
\begin{aligned}
\frac{dN_{N_1}}{dz} &= -D_1\,(N_{N_1}-N_{N_1}^{\rm eq})\,,\\
\frac{dN_{\Delta_{1\gamma}}}{dz} &= \varepsilon_{1\gamma}\,D_1\,(N_{N_1}-N_{N_1}^{\rm eq})-P_{1\gamma}^0\,W_1\,N_{\Delta_{1\gamma}}\,,\\
\frac{dN_{\Delta_{1\gamma}^{\perp}}}{dz} &= 0,\\
\frac{dN_{\Delta_\tau}}{dz} &= \varepsilon_{1\tau}\,D_1\,(N_{N_1}-N_{N_1}^{\rm eq})-P_{1\tau}^0\,W_1\,N_{\Delta_\tau}\,,
\end{aligned}
$$

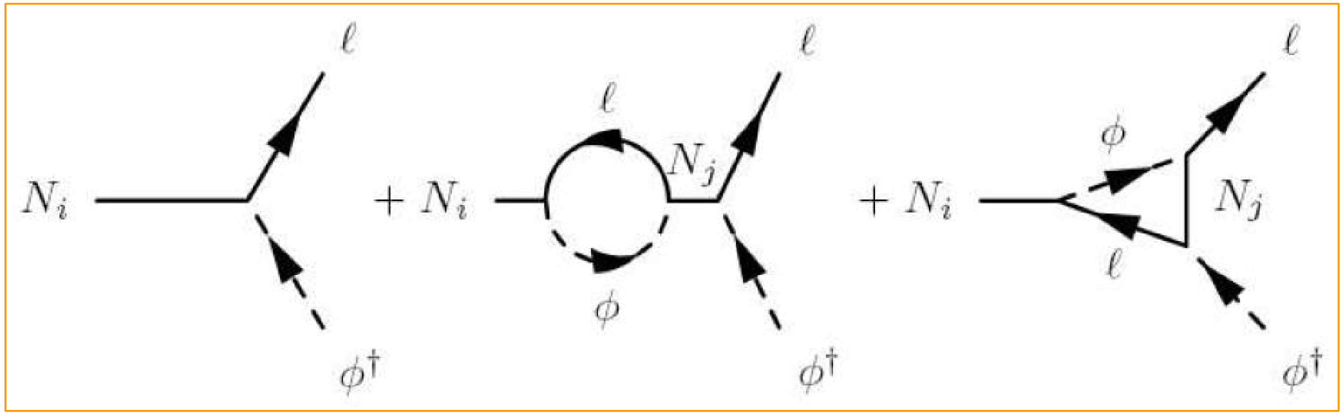
← **Boltzmann equations** for $\Delta_\tau = B/3 - L_\tau$ $\Delta_{1\gamma} = 2B/3 - L_{1\gamma}$ produced by N_1 decays.
The **orthogonal component** is unchanged by N_1

$\not{CP}$ in the 2RHNM

Previous studies of the 2RHNM have neglected N_2 decays.

From **heavy flavour effects**, if some asymmetry is made by N_2 decays an orthogonal part will survive N_1 washout. How much is made? This depends on their CP asymmetry:

CP violation comes from interferences between diagrams **1** and **2**, **3**



The **1 + 3** interference gives: $\varepsilon_2^{(1+3)} \lesssim 10^{-6} \left(\frac{M_1}{10^{10} GeV}\right) \left(\frac{M_1}{M_2}\right)$

while the **1 + 2** interference gives: $\varepsilon_2^{(1+2)} \lesssim 10^{-6} \left(\frac{M_1}{10^{10} GeV}\right)$

Part of ε_2 is un-suppressed by a mass ratio $\longrightarrow$ **ε_2 same order of mag. as** ε_1

Conclusion: N_2 decays should be included.

Ground I’ll cover…

~~Introduction to Leptogenesis.~~

~~Flavoured Leptogenesis in Two Right-Handed Neutrino Models. (Light and Heavy Flavour effects.)~~

The 2RHNM parameter space & Sequential Dominance.

The importance of N_2 decays in Light Sequential Dominance.

Exploring 2RMNM parameter space

# of N_R	Independent "Low energy" parameters (PMNS-Matrix and v masses)	Independent "High energy" parameters (R-Matrix and v masses)	Total
3	3 light v masses + 3 light v mixing angles **+ 1 Dirac phase** **+ 2 Majorana phases = 6 + 3**	3 heavy v masses + 3 heavy v lifetimes **+ 3 total CP asym.** = **6 + 3**	**12 + 6** = 18
2	2 light v masses + 3 light v mixing angles **+ 1 Dirac phase** **+ 1 Majorana phase = 5 + 2**	2 heavy v masses + 1 heavy v lifetime } z **+ 1 total CP asym.** **= 3 + 1**	**8 + 3** = 11

Leptonic ~~CP~~ can come from the "low" and "high-energy" **phases**, coloured red.

In the **2RHNM** we parameterise "high energy" ~~CP~~ using **complex angle** **z = x + i y**

Sequential Dominance

The seesaw was invented to "naturally" explain why $\mathbf{m_\nu << m_e}$.

It was **not** invented to explain large θ_{12} and θ_{23} measured much later in ν oscillation expts.

Taking $\lambda_\nu = \begin{pmatrix} A & B & C \end{pmatrix}$ we can see how each N_R contributes to mixing:

$$\mathcal{L}^{\nu}_{eff} = \frac{(\nu_i^T A_i)(A_j^T \nu_j)}{M_A} + \frac{(\nu_i^T B_i)(B_j^T \nu_j)}{M_B} + \frac{(\nu_i^T C_i)(C_j^T \nu_j)}{M_C}$$

A **hierarchy on how N_R contribute to mixing**, →

$$\frac{A_i A_j}{M_A} \gg \frac{B_i B_j}{M_B} \gg \frac{C_i C_j}{M_C}$$

gives us two large mixing angles θ_{12} θ_{23}

This ansatz is known as **Sequential Dominance.**

So-called **Constrained Sequential Dominance** (CSD) gives exact **TBMM**:

$$\begin{aligned} |A_1| &= 0, \\ |A_2| &= |A_3|, \\ |B_1| &= |B_2| = |B_3|, \\ A^\dagger B &= 0. \end{aligned}$$

→

$$U_{PMNS} = \sqrt{\frac{1}{6}} \begin{pmatrix} \sqrt{2} & 0 & 2 \\ \sqrt{2} & \sqrt{3} & -1 \\ \sqrt{2} & -\sqrt{3} & -1 \end{pmatrix}$$

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Light Sequential Dominance (LSD)

For **SD** there are 3! = 6 ways of associating $M_1 < M_2 < M_3$ with M_A , M_B , M_C
For two of these ways $M_1 = M_A$ and the **lightest** N_R contributes dominantly to mixing.

For these **LSD** models the light ν masses are **hierarchical** and are given in terms of A, B, C by: →

$$m_3 \approx \frac{(|A_2|^2 + |A_3|^2)v^2}{M_A},$$

$$m_2 \approx \frac{|B_1|^2 v^2}{s_{12}^2 M_B},$$

$$m_1 \approx \mathcal{O}(|C|^2 v^2/M_C).$$

For the **CP** asymmetry we can use the formulae:

$$\varepsilon_{1\alpha} \approx -\frac{3}{16\pi}\frac{M_A}{M_B}\frac{1}{A^\dagger A}\mathrm{Im}\left[A_\alpha^*(A^\dagger B)B_\alpha\right] \qquad \varepsilon_{2\alpha} \approx -\frac{2}{16\pi}\frac{1}{B^\dagger B}\mathrm{Im}\left[B_\alpha^*(A^\dagger B)A_\alpha\right]$$

$$\varepsilon_{1\mu,\tau} \approx -\frac{3}{16\pi}\frac{m_2 M_1}{v^2}, \quad \varepsilon_{1e} \approx \frac{A_1}{A_2}\varepsilon_{1\mu,\tau}.$$

$$\varepsilon_{2\mu,\tau} \approx -\frac{1}{16\pi}\frac{m_3 M_1}{v^2}, \quad \varepsilon_{2e} \approx \frac{A_1}{A_2}\varepsilon_{2\mu,\tau}$$

Comparing the two, we find that **ε_2 is larger than ε_1** by a factor ≈ $m_3 / 3m_2$

An R-Matrix dictionary

"High energy" physics is in the **R-matrix** with complex angle **z** to parameterise C̸P̸

LSD is done in terms of **Y_ν = (A, B, C).** We want to compare the two parametrisations.

From the definition of the R-matrix:

$$R = D^{-1}_{\sqrt{M}} U^\dagger_M m^T_D U^* D^{-1}_{\sqrt{k}}$$

$$R^N \approx \begin{pmatrix} 0 & -z_{23} & 1 \\ 0 & -1 & -z_{23} \\ 1 & 0 & 0 \end{pmatrix}$$

$$z \approx \frac{\pi}{2} + z_{23}$$

$$Re(z_{23}) \approx \frac{Re(A^\dagger B)v^2}{(m_3 - m_2)M_1^{1/2}M_2^{1/2}}$$

$$Im(z_{23}) \approx \frac{Im(A^\dagger B)v^2}{(m_3 + m_2)M_1^{1/2}M_2^{1/2}}$$

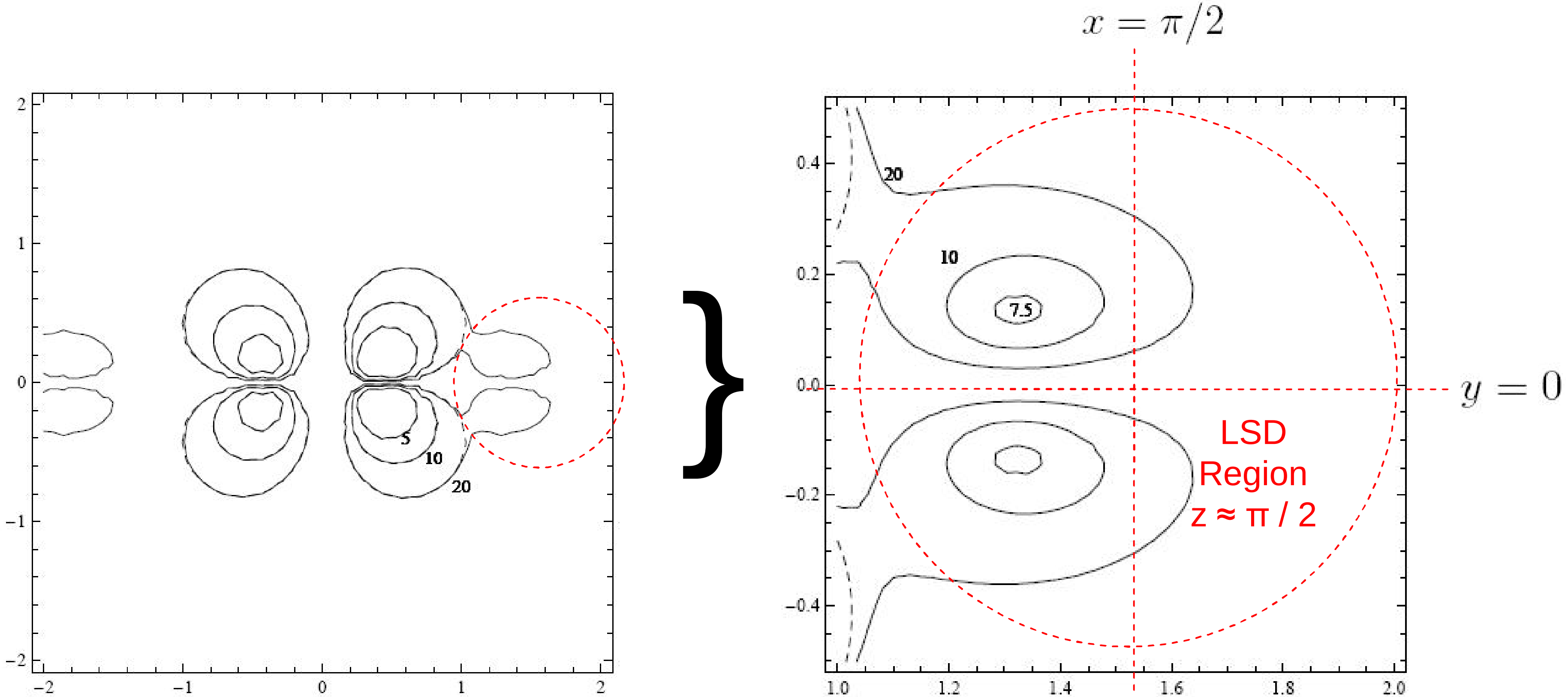
The main point is this: **it is possible to translate z info into Y_ν (+ mass) info..**

The **LSD region is found in the small z_{23} region.** $z_{23} \approx 0 \rightarrow$ **$z \approx \pi / 2$**

⟶ We expect a region close to (π / 2, 0) in the z-plane where N_2 decays dominate.

Our main result:

Recap: from analytic arguments we expect a region close to (π / 2, 0) in the z-plane where N2 decays dominate**. Z-plane contour plots confirm this:**



With $\theta_{13} = 0.2$ $\delta = 0$, $\alpha_{21} = 0$, the circled "lobe" regions are thanks to N_2 decays

Conclusion: LSD regions of 2RHNM need N_2 decay for enough Baryon asymmetry

Summary

- **Leptogenesis** can explain the Baryon Asymmetry of the Universe (BAU) of $\eta = (6.21 \pm 0.16) \times 10^{-10}$
- **Two Right-Handed Neutrino Models** (2RHNM) can explain the ν oscillation data thus far.
- For Leptogenesis in 2RHNM, **light and heavy flavour effects** can make N_2 decays become relevant.

- **Sequential Dominance** lets us focus on relevant regions of the seesaw parameter space.
- In the case of **Light Sequential Dominance** (LSD), N_2 decays significantly enhance the final asymmetry.
- We are currently investigating the sensitivity to U_{PMNS}. We find that the LSD region **prefers phases α, δ to be off**

Thank you for inviting me to talk ☺

Conclusion: N2 likes LSD!

Leptogenesis – extra slides

To make a Baryon number asymmetry we need to satisfy three conditions:

1) B number violation
2) C and CP violation
3) Departure from Thermal Equilibrium (DTE)

} Sakharov's conditions for Baryogenesis

Leptogenesis uses the Seesaw Mechanism to produce the Baryon Asymmetry of the Universe (BAU):

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In the **Type I Seesaw**, this happens because the **decays of heavy N_R violate L** (hence B, more later…) and **may violate CP**. The expansion and cooling of the universe gives **DTE when N go non-relativistic at T ≈ M_1**

Type I Seesaw Lagrangian:

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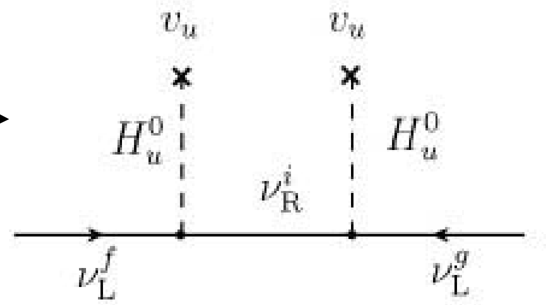
L violation -> Sakharov I

The idea of **Leptognenesis**: B violation from L violation

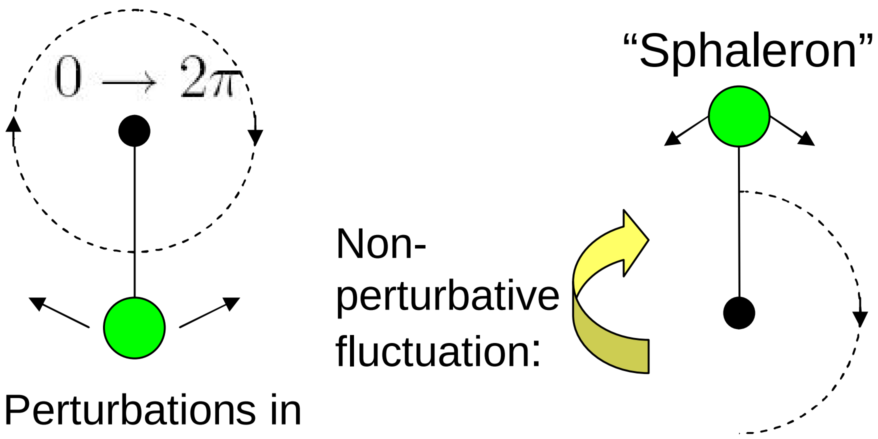
A **Majorana mass term** for N_R **violates L** by two units: $\longmapsto$

B – L is conserved, but B and L aren't conserved separately.

About 1/3 of initial $\not{L}$ is re-distributed into $\not{B}$ by **sphalerons**



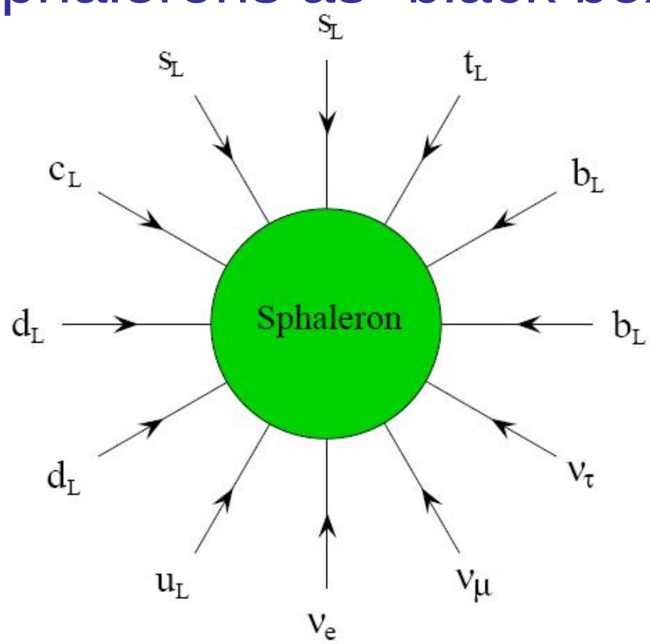
Analogy with a pendulum:



Perturbations in a degenerate vacuum state

Decay into "new" vacuum state

Sphalerons as "black box":

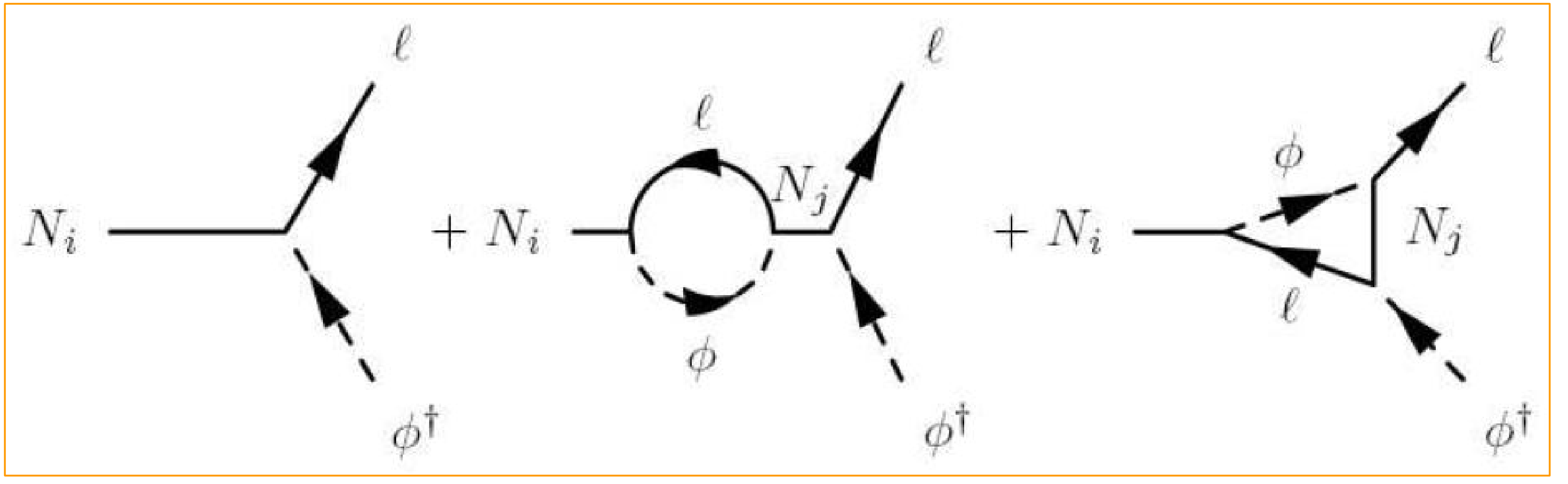


$\not{CP}$ in leptons -> Sakharov II

We define the CP asymmetry as:

Loops interfere with the tree:

$$\varepsilon_i \equiv -\frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i}$$



Explicitly:

$$\varepsilon_i = \frac{3}{16\pi(\lambda^\dagger\lambda)_{ii}} \sum_{j\neq i} \mathrm{Im}\left[(\lambda^\dagger\lambda)^2_{ij}\right] \frac{\xi(x_j)}{\sqrt{x_j}}$$

$$\xi(x) = \frac{2}{3}\, x \left[(1+x)\,\ln\left(\frac{1+x}{x}\right) - \frac{2-x}{1-x}\right] \qquad x_j \equiv (M_j/M_i)^2$$

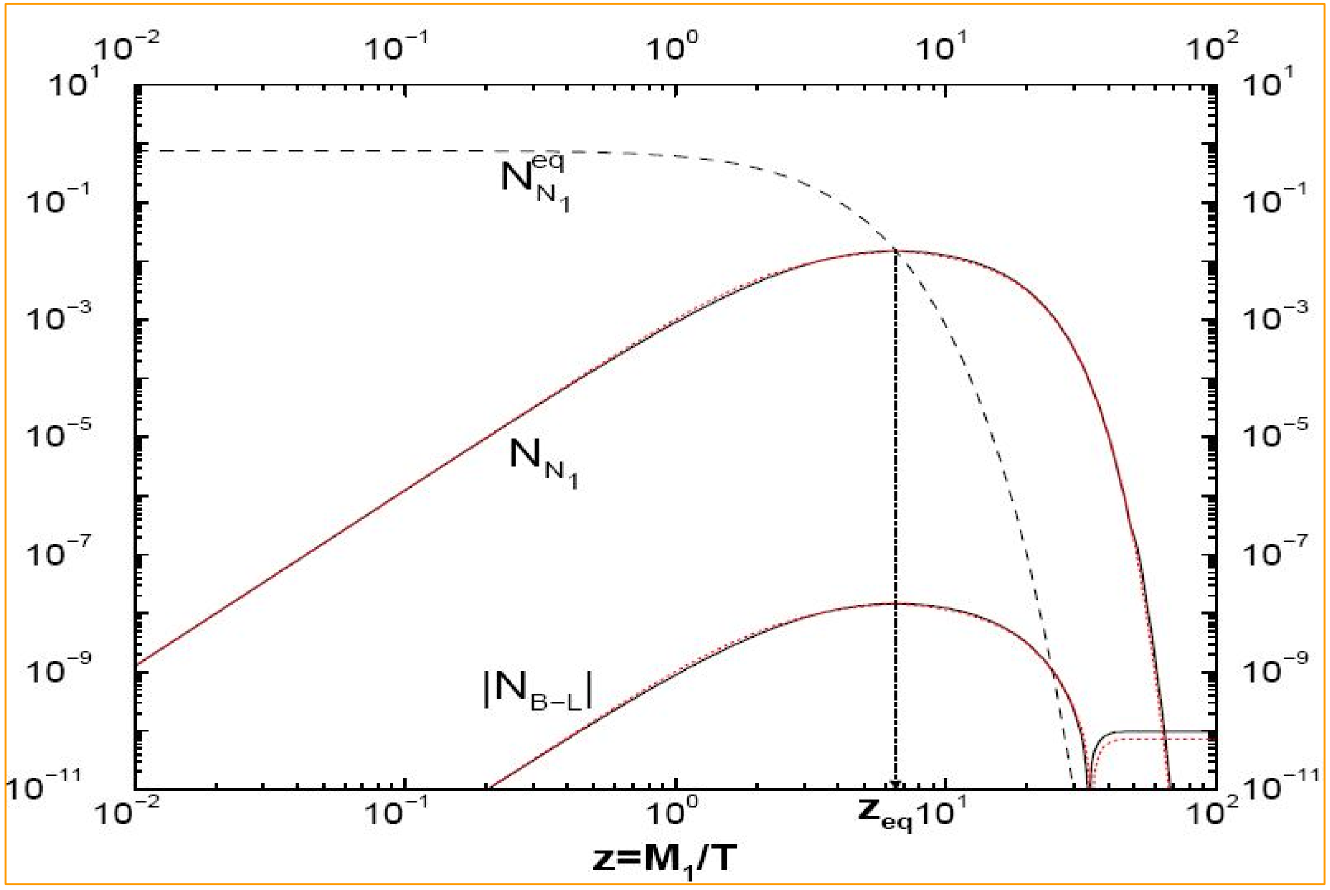
Heavy N_R decays -> Sakharov III

We can define N_R mass relative to temperature as:

$$z_i = \frac{M_i}{T}$$

For $z_i > 1$, N_i is suppressed like: $n^{eq}_{N_i} \propto e^{-z_i}$

The B – L producing decays of N_i look like this:



The final Baryon asymmetry

To work out the final **B – L asymmetry** we integrate the Boltzmann equations:

$$N_{B-L}^{\rm f} = N_{B-L}^{\rm in} \exp\left(-\sum_i \int {\rm d}z'\, W_i(z')\right) + \sum_i \varepsilon_i\, \kappa_i^{\rm f}$$

Where the **efficiency factor** is given explicitly as:

$$\kappa_i^{\rm f} = -\int_{z_{\rm in}}^{\infty} {\rm d}z'\, \frac{{\rm d}N_{N_i}}{{\rm d}z'}\, \exp\left(-\sum_i \int_{z'}^{z} {\rm d}z''\, W_i(z'')\right)$$

Basically we are **parameterise the solution** to separate the effects of ~~**CP**~~ (contained in ε) and washout (contained in κ) on any new asymmetry produced during the $N_i \rightleftharpoons l_i + \phi^\dagger$ reactions.

The integral can be done **numerically**, or there are some very accurate **analytic** approximations to it (see “Leptogenesis for Pedestrians” for examples).

The final **Baryon asymmetry** is given from the final B – L asymmetry as:

$$\eta_B = a_{\rm sph} \frac{N_{B-L}^{\rm f}}{N_\gamma^{\rm rec}} \simeq 0.96 \times 10^{-2} N_{B-L}^{\rm f} \qquad a_{\rm sph} = n_B/n_{B-L} = 28/79$$

Sphalerons put about 1/3 of initial L violation into B violation, while conserving B – L. This happens continuously before the EWPT.