Dynamical simulation of N=1 supersymmetric Yang-Mills theory with domain wall fermions

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Lattice SUSY and Beyond
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IN THE CITY OF NEW YORK

Supersymmetry (SUSY) on the lattice

- Much effort devoted toward formulating supersymmetric lattice theories, motivated by
  - fascinating theoretical challenge
  - potential role of SUSY in beyond the standard model physics
  - desire to understand various nonperturbative aspects of SUSY theories via numerical simulation

Incompatibility of SUSY and the lattice

- SUSY algebra
  - SUSY is an extension of Poincare group
  - $[P,P] \sim 0 \;, \quad \{Q,Q\} \sim 0 \;, \quad [P,Q] \sim 0 \;, \quad \{Q,\bar{Q}\} \sim P \;, \quad etc.$
- Lattice fermion “doublers”
  - fermion discretizations may yields more continuum degrees of freedom than expected
- Violation of “leibniz rule”
  - $\partial(ab) \neq (\partial a)b + a(\partial b)$ on the lattice
  - cannot easily construct SUSY lattice actions using superfields

Incompatibility of SUSY and the lattice

- Standard lattice formulations explicitly break SUSY
- SUSY violating operators may arise radiatively
  - scalar mass terms are additively renormalized
  - depending on fermion discretization, fermion masses may be additively renormalized
- Fine tuning of operators required to reach SUSY point
  - extremely difficult in practice
  - large numerical cost

Incompatibility of SUSY and the lattice

- Is it possible to achieve SUSY accidentally?
  - e.g. rotational symmetry restoration in lattice QCD
- Advances in formulating SUSY lattice theories
  - theories which preserve a sub-algebra of SUSY on the lattice
    - require little fine tuning
    - exotic lattices
  - N=1 SYM in four dimensions can be achieved with conventional lattice formulations

N=1 super Yang-Mills (SYM)

$$L = \frac{1}{g^2}\left[\bar{\lambda}\gamma_\mu D_\mu \lambda + \frac{1}{4} v_{\mu\nu} v_{\mu\nu}\right] \ , \qquad \bar{\lambda} = \lambda^T C$$

- One of the simplest of SUSY gauge theories in terms of field content
  - 1 vector field and 1 adjoint Majorana fermion (gluino)
  - a single input parameter, the gauge coupling (g)
- Anomalous $U(1)_R$ (chiral) symmetry: $\lambda \to e^{-\ \alpha\gamma_5}\lambda \Rightarrow \theta \to \theta - 2N\ \alpha$
- Z_{2N} subgroup of $U(1)_R$ survives at quantum level
  - partition function invariant for $\alpha = \dfrac{\pi k}{N_c}\ , \qquad k = 0, \ldots, 2N_c - 1$

N=1 SYM

$$L = \frac{1}{g^2}\left[\bar{\lambda}\gamma_\mu D_\mu \lambda + \frac{1}{4} v_{\mu\nu} v_{\mu\nu}\right] \,, \qquad \bar{\lambda} = \lambda^T C$$

- Gluino condensation $\langle \lambda\lambda \rangle \neq 0$
  - discrete chiral symmetry breaking $Z_{2N} \to Z_2$
- Confinement
  - colorless bound states
- No SUSY breaking (non-vanishing Witten index)

N=1 SYM on the lattice

- From numerical point of view N=1 SYM ideal starting point for studying SUSY
  - possesses interesting nonperturbative physics
  - is QCD-like in some respects
    - chiral symmetry breaking (although discrete)
    - confinement
  - unlike QCD, no Goldstone bosons

SUSY restoration in N=1 SYM on the lattice

- Only SUSY violating relevant operator which may arise from quantum fluctuations (consistent with gauge and lattice symmetries) is a gluino mass term
  - chiral symmetry realized on the lattice implies SUSY restoration in the continuum limit
- Key to the idea is the absence of scalars
  - only SUSY and shift symmetry can prevent radiative corrections to scalar masses
  - unable to introduce matter multiplets without additional fine tuning

Some numerical studies of N=1 SYM

- Wilson
  - DESY-Muenster-Roma Collaboration (summarized in hep-lat/0112007 and references therein, arXiv:0811.1964)
- DWF
  - G. T. Fleming, et. al. (hep-lat/0008009)
  - M. G. Endres (arXiv:0810.0431)
  - Giedt, et. al. (arXiv:0807.2032, arXiv:0810.5746)

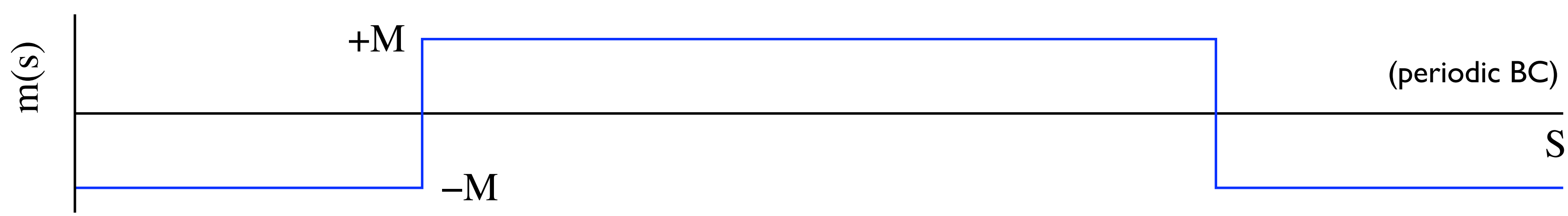
N=1 SYM with Domain wall fermions (DWFs)

- DWF formulation ideal for N=1 SYM — D. B. Kaplan and M. Schmaltz (1999)
  - good chiral properties (no fine tuning)
  - no fermion doublers
  - positive definite fermion determinant (no "sign problem")
- More expensive compared to other lattice discretizations (e.g. Wilson) in terms of computing resources

Kaplan's DWFs

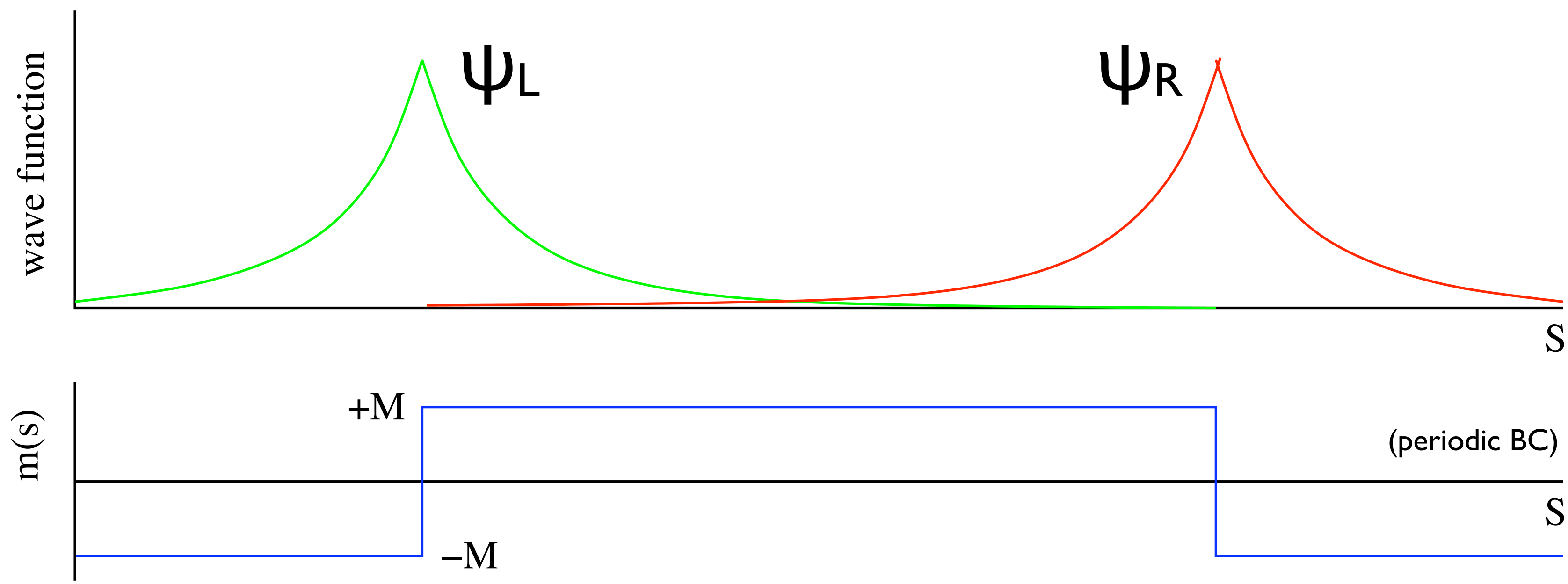
- Exact chiral symmetry on the lattice — D. B. Kaplan (1992)
- Introduce a 5th dimension (s) and an s-dependent mass term with a step profile

$$S = \int dx ds \, \bar{\Psi}(x,s) \left[\gamma_\mu \partial_\mu + \gamma_5 \partial_s + m(s) \right] \Psi(x,s)$$

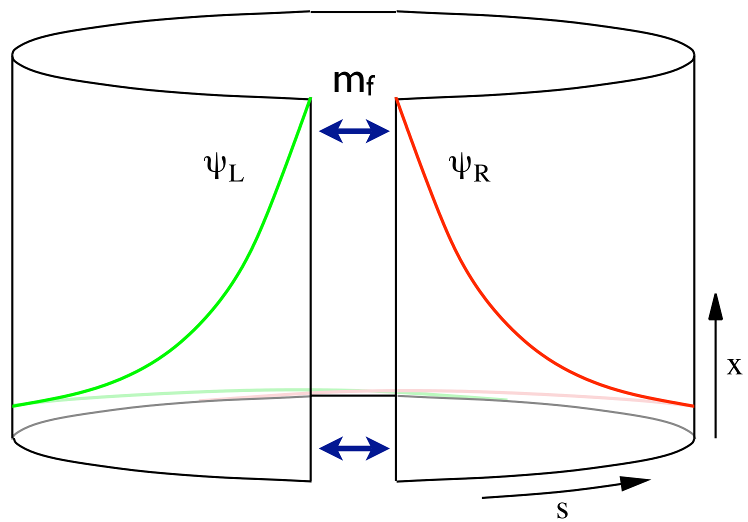


Kaplan's DWFs

- Solutions of the Dirac equation yield chiral modes bound to the fifth dimension boundaries of a 5-dimensional theory
- All other modes have O(1/a) masses on the lattice



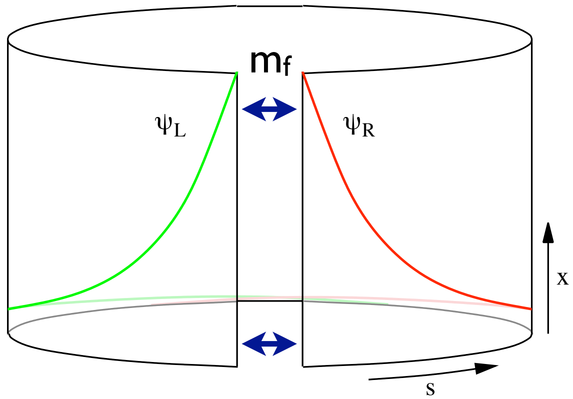
Shamir's DWFs



Y. Shamir (1993)
V. Furman and Y. Shamir (1994)

- Only consider half of the fifth dimension (M > 0 region)
- Chiral modes bound to the boundaries of fifth dimension
- Explicit coupling of left- and right-handed modes with strength m_f

N=1 SYM with DWFs



$$m_{res} \sim \# \frac{e^{-\# L_s}}{L_s} + \# \frac{\rho(0)}{L_s}$$

- Two sources for nonvanishing gluino mass:
  - explicit coupling of domain walls via m_f
  - nonzero overlap of wave functions gives rise to a residual mass (m_{res}) at finite L_s
- Chiral (m_f=0, L_s=∞) and SUSY limits coincide

Numerical simulations

- Gluino condensate
- Static potential
- Residual mass
- Spectrum
- Domain walls arising from discrete chiral symmetry breaking
- Effects of nonzero gluino mass

Numerical simulations

- Gluino condensate — Fleming, Kogut and Vranas (2001)
- Static potential
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- Effects of nonzero gluino mass

N=1 SYM with DWFs

- Questions not addressed by Fleming, et. al.
  - lattice scale
  - what is the residual mass (how close to SUSY point)?
  - continuum limit of gluino condensate

N=1 SYM with DWFs

- Since then
  - improved algorithms (e.g. RHMC algorithm)
  - much faster computers (therefore larger lattices)
  - better understanding of DWFs (e.g. L_s dependence of residual mass)
    - can do better job extrapolating to chiral limit

N=1 SYM with DWFs

- Recent renewed interest in N=1 SUSY simulations with DWFs:
  - M. G. Endres (arXiv:0810.0431)
    - simulate at a variety of m_f values
    - extrapolate to $m_f=0$
  - J. Giedt, et. al. (arXiv:0810.5746)
    - simulate at $m_f=0$
    - computationally more costly
    - no need to perform $m_f=0$ extrapolation

Independent studies!

Numerical simulations

- Gluino condensate
- Static potential
- Residual mass
- Spectrum
- Domain walls arising from discrete chiral symmetry breaking
- Effects of nonzero gluino mass

Numerical simulations

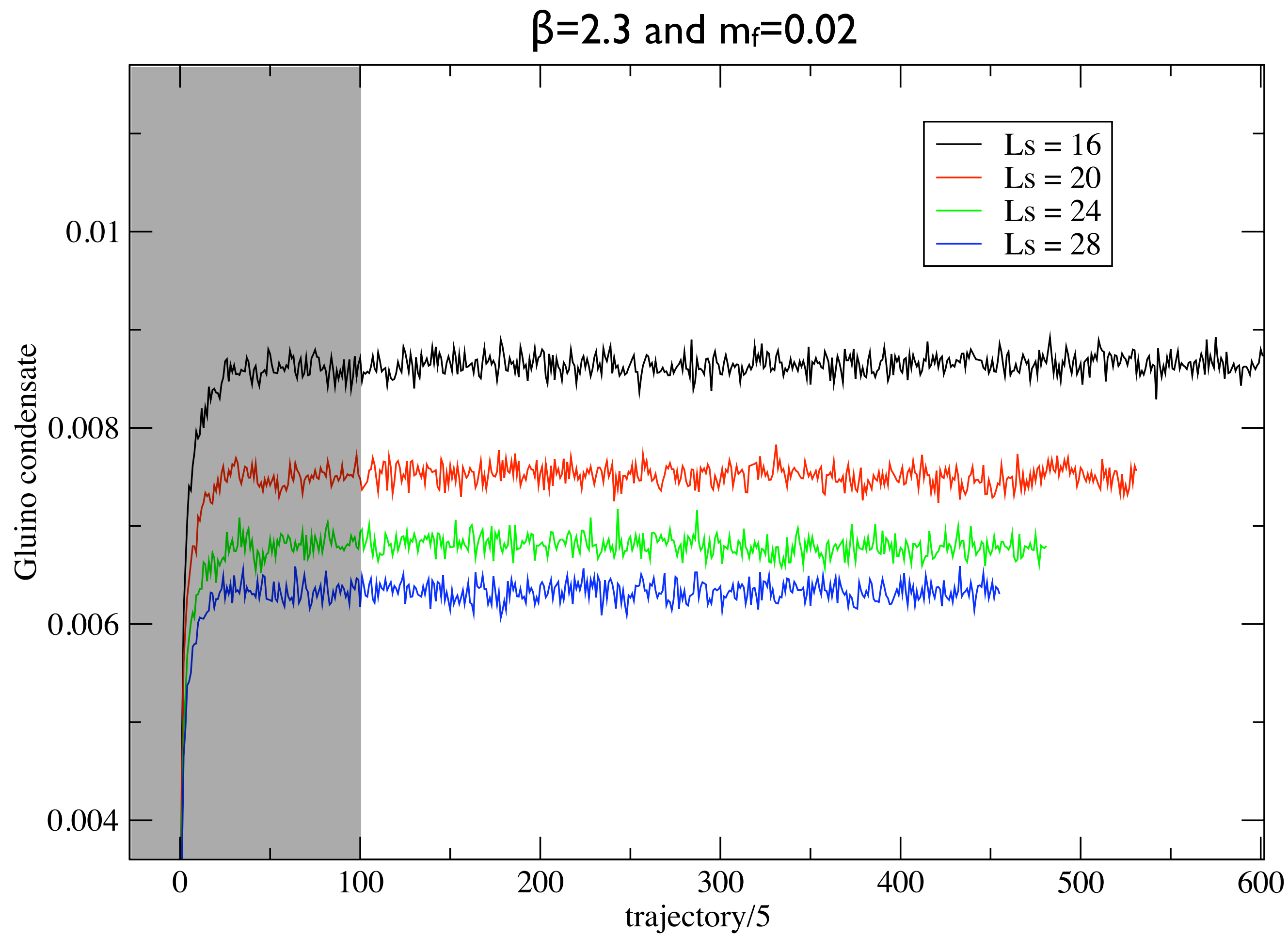
- Wilson gauge action with domain wall fermions
- SU(2) gauge group with adjoint Majorana fermions
- Simulations performed on an appropriately modified version of the Columbia Physics System (CPS)
- $8^3 \times 8 \times L_s$ ensembles were generated and measurements made on QCDOC at Columbia University
- $16^3 \times 32 \times L_s$ ensembles were generated and measurements made on New York Blue (BlueGene/L) at Brookhaven National Laboratory

Simulation parameters for 16^3x32 lattices

Lattice 2008 proceedings

β	L_s	m_f	N_{traj}	acc. rate	$\sqrt{\langle \Delta H^2 \rangle}$	$\langle e^{-\Delta H} \rangle$
2.3	16	0.01	3125	0.758	0.673	1.016
		0.02	3195	0.762	0.625	0.991
		0.04	2790	0.776	0.577	1.003
	20	0.01	2895	0.745	0.685	1.008
		0.02	2655	0.753	0.674	0.993
		0.04	2760	0.731	0.722	0.995
	24	0.01	2855	0.775	0.578	1.019
		0.02	2620	0.792	0.593	1.020
		0.04	2610	0.760	0.690	1.050
	28	0.01	2740	0.817	0.474	1.022
		0.02	2855	0.796	0.536	0.974
		0.04	2880	0.784	0.577	0.996
	32	0.02	0720	0.745	0.686	1.020
	40	0.02	0564	0.603	1.163	1.002
	48	0.02	0220			
2.3533	16	0.02	2575	0.782	0.583	0.996
	28	0.01	1085	0.837	0.389	0.992
		0.02	1125	0.836	0.426	1.033
		0.04	1165	0.833	0.448	1.000
2.4	16	0.02	2710	0.824	0.432	0.996
	28	0.01	1420	0.874	0.340	1.012
		0.02	1540	0.879	0.332	1.027
		0.04	1155	0.855	0.346	0.987

Thermalization

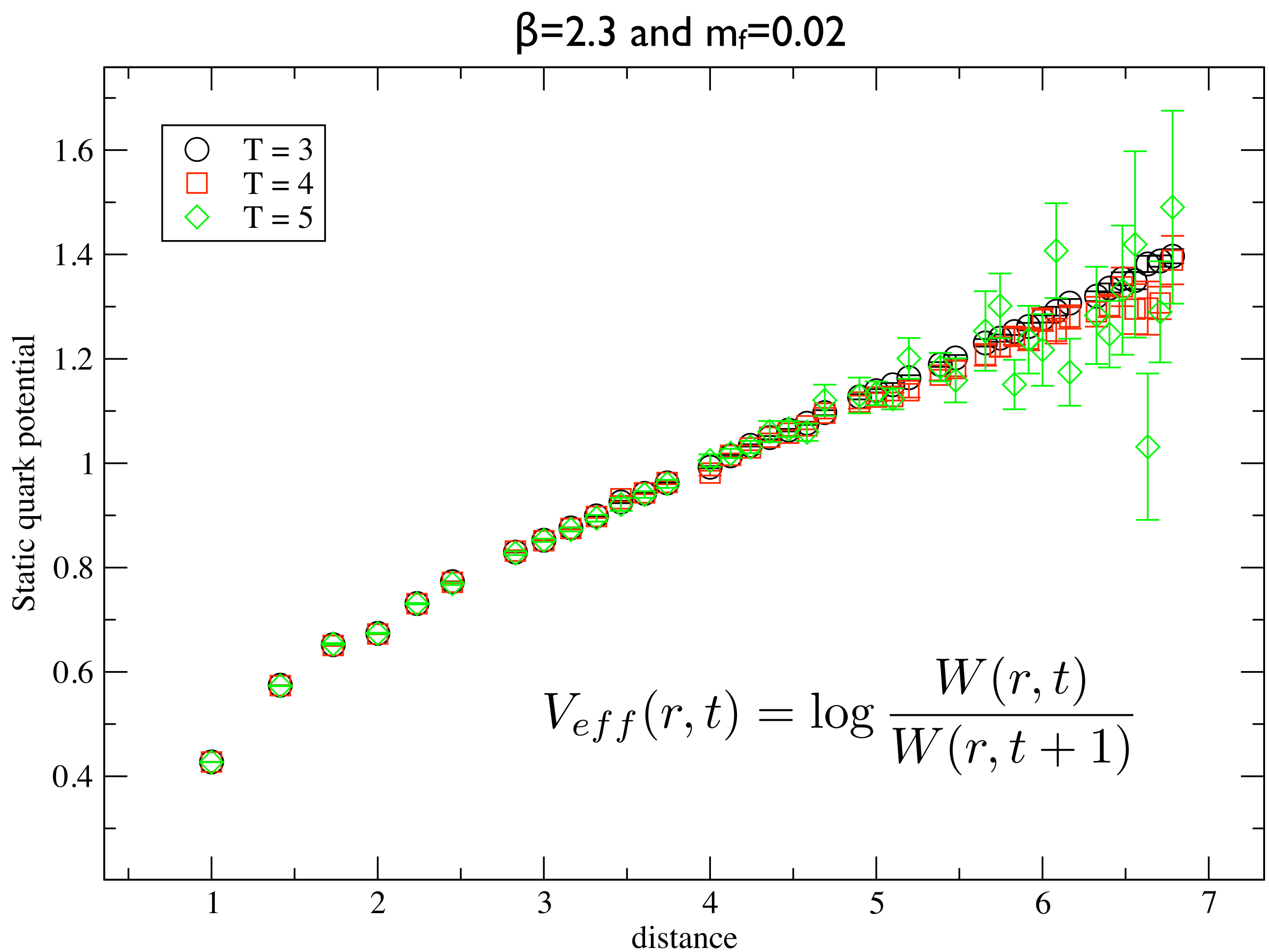


- All configurations generated from a cold start

Static potential

- Wilson loops measured on Coulomb gauge-fixed gauge configurations
  - space-time dependence: $W(t,r) = C(r)e^{-V(r)t} + \text{excited states}$
  - effective potential given by: $V_{eff}(r,t) = \log W(r,t)/W(r,t+1)$

Static potential



Static potential

- Fit Wilson loops to an exponential over plateau region

$$W(t,r) = C(r)e^{-V(r)t} + \text{excited states}$$

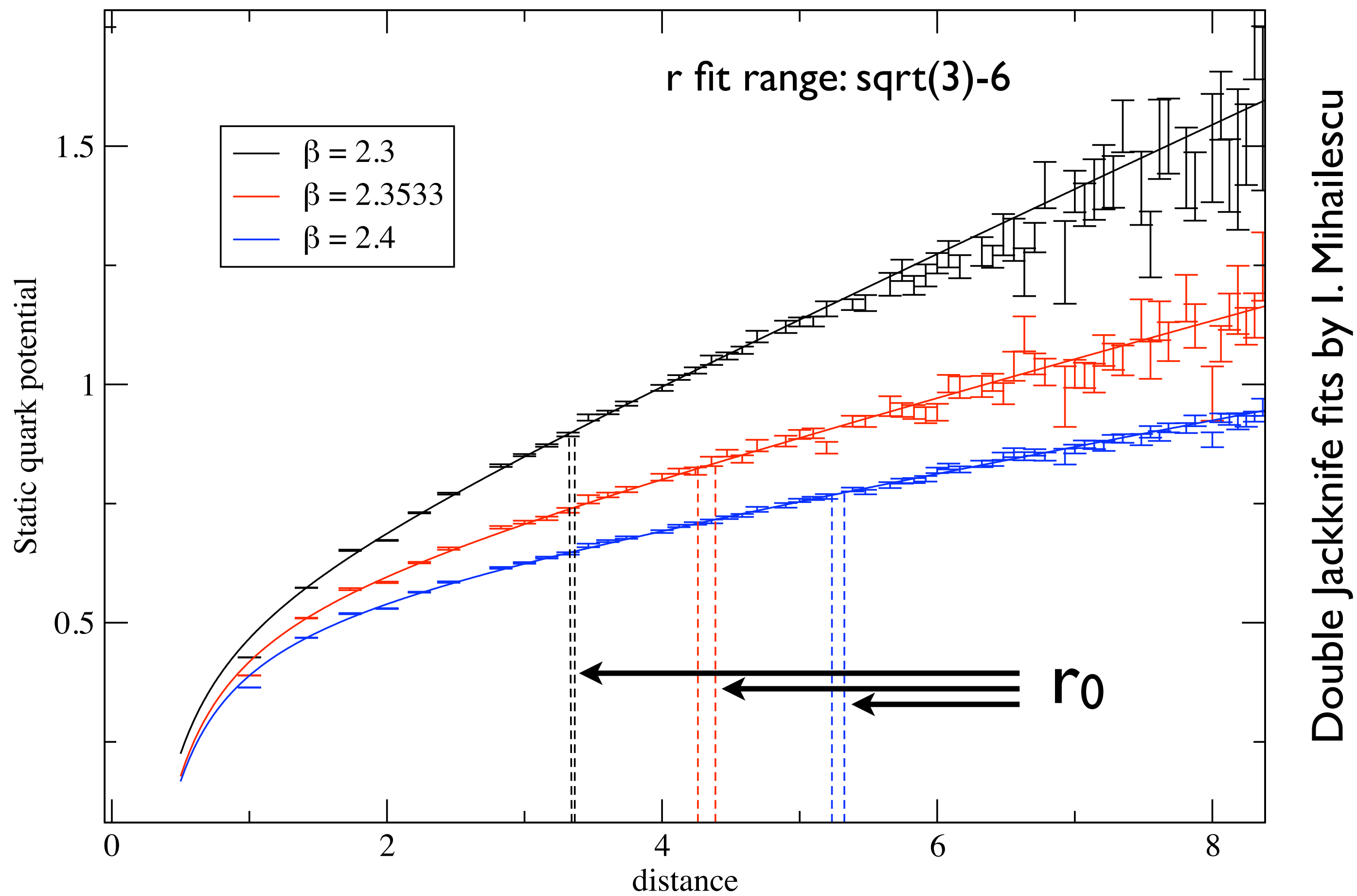
- Potential was fit to the Cornell form:

$$V(r) = V_0 - \frac{\alpha}{r} + \sigma r$$

- Lattice scale set via Sommer scale:

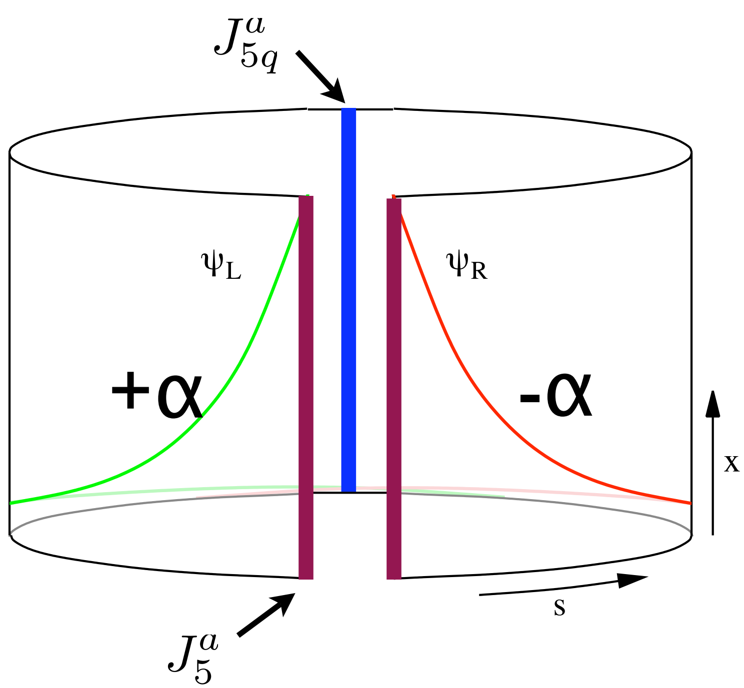
$$r_0^2 V'(r_0) = 1.65 \qquad r_0 = \sqrt{\frac{1.65 - \alpha}{\sigma}}$$

Coupling dependence of static potential



Residual mass: a lesson from in QCD

- Define 4-d axial current which corresponding to opposite phase rotations on each half of the fifth dimension: $\mathcal{A}_\mu^a$



$$\Delta_\mu \mathcal{A}_\mu(x) = 2m_f J_5^a(x) + 2J_{5q}^a(x)$$

forward derivative

"wall" pseudo-scalar density

"mid-point" pseudo-scalar density

Residual mass: a lesson from in QCD

- Axial Ward-Takahashi (WT) identity:

$$\Delta_\mu \langle \mathcal{A}^a_\mu(x)\mathcal{O}(y)\rangle = 2m_f\langle J^a_5(x)\mathcal{O}(y)\rangle + 2\langle J^a_{5q}(x)\mathcal{O}(y)\rangle + i\langle \delta^a \mathcal{O}(y)\rangle$$

Pseudo-scalar density defined on wall

Pseudo-scalar density defined at midpoint

- In effective continuum Lagrangian, chiral symmetry breaking effects appear to lowest order in lattice spacing (i.e. a^{-1}) as:

$$L_{symanzik} = \underbrace{\bar\psi(D+m_f)\psi}_{L_0} + \underbrace{m_{res}\bar\psi\psi}_{L_{-1}} + \underbrace{c\,\bar\psi\sigma_{\mu\nu}F_{\mu\nu}\psi}_{L_1} + \dots$$

Residual mass: a lesson from in QCD

- Axial Ward-Takahashi (WT) identity:

$$\Delta_\mu \langle \mathcal{A}^a_\mu(x)\mathcal{O}(y)\rangle = 2m_f \langle J^a_5(x)\mathcal{O}(y)\rangle + 2\langle J^a_{5q}(x)\mathcal{O}(y)\rangle + i\langle \delta^a \mathcal{O}(y)\rangle$$

Pseudo-scalar density defined on wall

Pseudo-scalar density defined at midpoint

- In effective continuum Lagrangian, chiral symmetry breaking effects appear to lowest order in lattice spacing (i.e. a^{-1}) as:

$$L_{symanzik} = \bar{\psi}(D + m_f)\psi + m_{res}\bar{\psi}\psi + \cancel{c\,\bar{\psi}\sigma_{\mu\nu}F_{\mu\nu}\psi} + \ldots$$

- Agreement of axial WT identity between lattice and continuum effective theory implies the low energy identity:

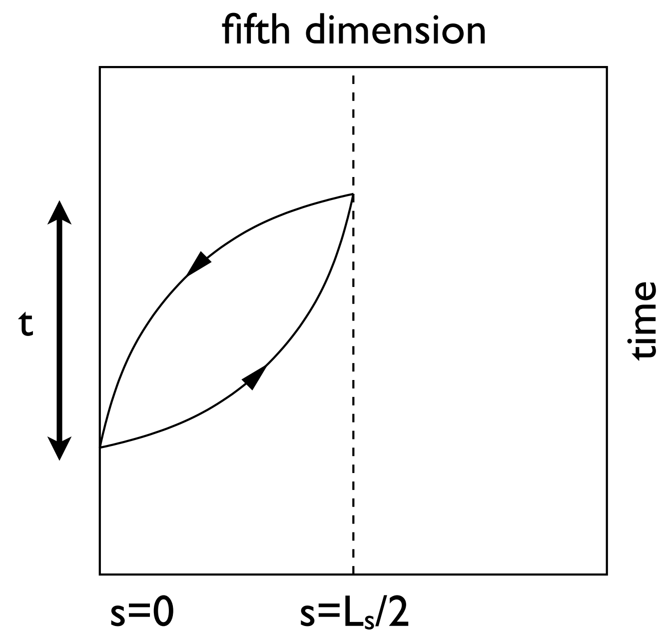
$$J^a_{5q}(x) \approx m_{res}\, J^a_5(x)$$

Residual mass: a lesson from in QCD

- Residual mass obtained from the ratio:

$$R(t) = \frac{\langle \sum_{\mathbf{x}} J^a_{5q}(\mathbf{x},t) J^a_5(0) \rangle}{\langle \sum_{\mathbf{x}} J^a_5(\mathbf{x},t) J^a_5(0) \rangle} \to m_{res} \ , \qquad t >> 1$$

Numerator:



Residual mass in SYM

- "1/2" flavor instead of 2 flavors
  - no flavor non-singlet axial currents
  - need to worry about anomalous contribution to axial WT identity:
  - low energy identity: $J_{5q}(x) \approx m_{res}\, J_5(x) + \rho_{top}(x)$
- Wall-midpoint and wall-wall pseudo-scalar correlation functions involve both "connected" and "disconnected" contributions.

Residual mass in SYM

- Open question: can one extract m_{res} from R(t) in the N_f<2 case (i.e. using only connected contributions to wall-midpoint and wall-wall correlators)?
  - possible, but details needs to be worked out...

$$\langle J_{5q}(\mathbf{x},t) J_5(0) \rangle \approx m_{res} \, \langle J_q(\mathbf{x},t) J_5(0) \rangle + \langle \rho_{top}(\mathbf{x},t) J_5(0) \rangle$$

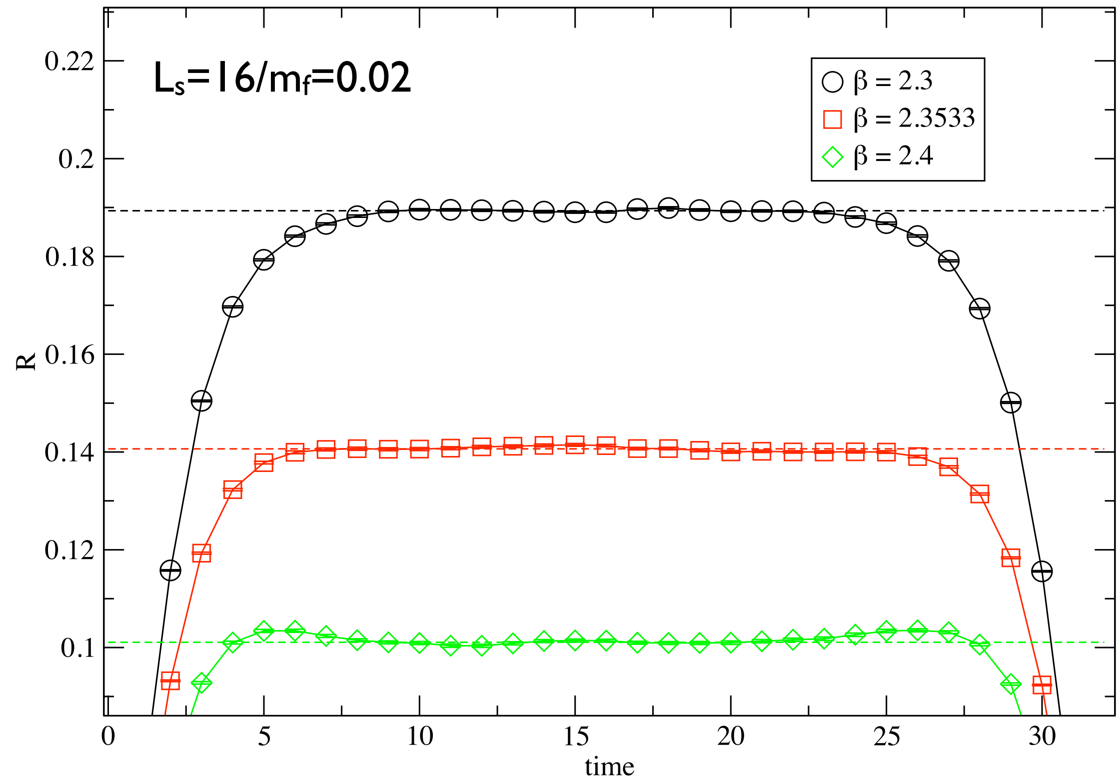
$$\Big\Downarrow \, ?$$

$$\langle J_{5q}(\mathbf{x},t) J_5(0) \rangle_{connected} \approx m_{res} \, \langle J_q(\mathbf{x},t) J_5(0) \rangle_{connected}$$

$$\langle J_{5q}(\mathbf{x},t) J_5(0) \rangle_{disconnected} \approx m_{res} \, \langle J_q(\mathbf{x},t) J_5(0) \rangle_{disconnected} + \langle \rho_{top}(\mathbf{x},t) J_5(0) \rangle$$

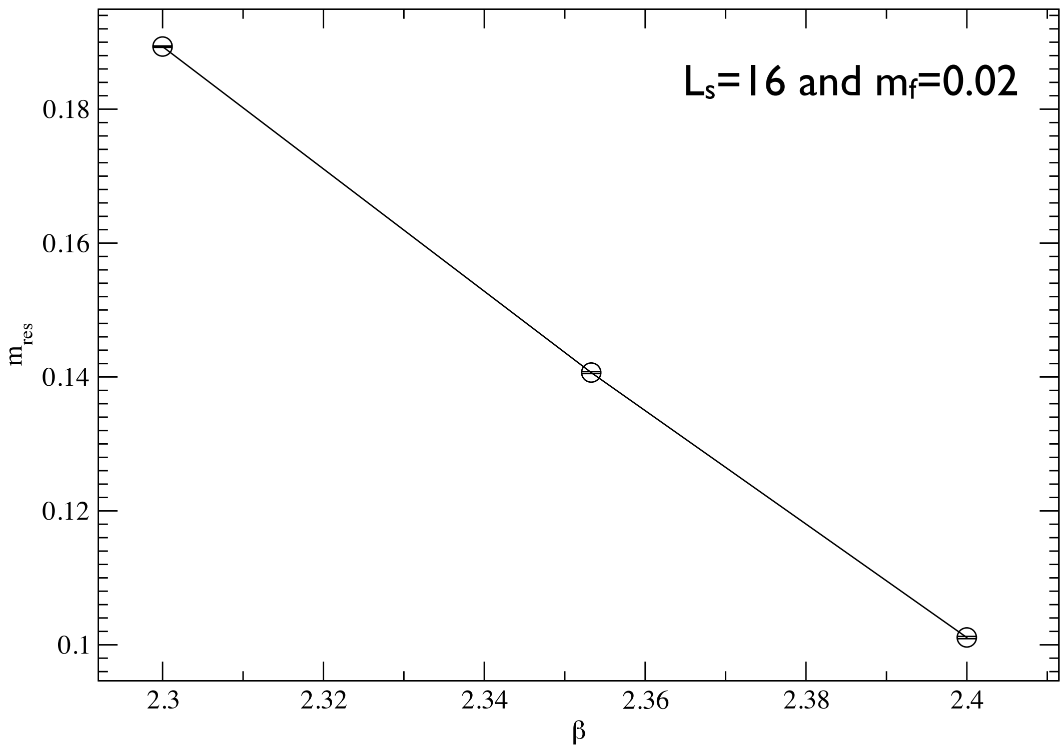
Ratio R(t)

t fit range: 8-23



- R(t) appears time-independent for large times
- Connected contributions tend to a constant (m_{res}?)
- Independent measure of m_{res} is desirable for consistency check

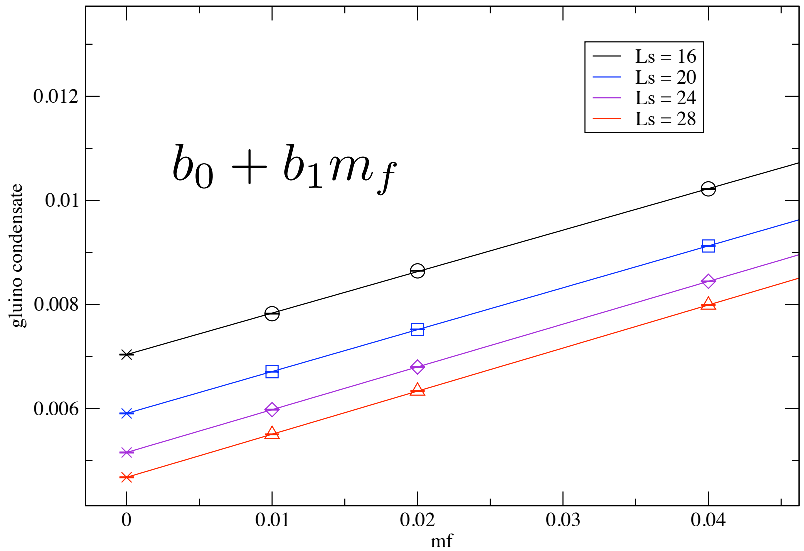
Coupling-dependence of residual mass



$$m_{res} \sim \# \frac{e^{-\# L_s}}{L_s} + \# \frac{\rho(0)}{L_s}$$

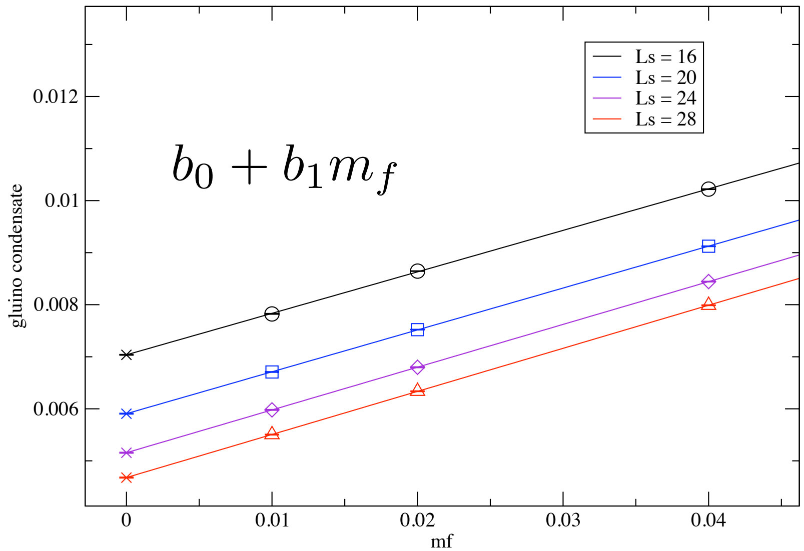
- Residual mass is 5-10 times larger than current m_f values
- Depends strongly on the coupling
- $1/L_s$ term dominates residual mass

Gluino condensate in the chiral limit



- Two parameter extrapolation: m_f and L_s
- L_s dependence is due to residual mass
- Extrapolation performed by first taking $m_f=0$ then $L_s=\infty$

Gluino condensate in the chiral limit



beta	L_s	M.G.E. (m_f=0 extrapolation)	Giedt, et. al. (m_f=0 simulation)
2.3	16	0.007037(8)	0.007051(5)
	24	0.005156(10)	0.00556(5)
2.4*	28	0.002981(14)	0.003452(45)

*not plotted

- m_f=0 extrapolation appears inconsistent with Geidt, et. al. for L_s>16 (currently under investigation)
- Discrepancy in m_f=0 results appears to increase with decreasing m_{res}

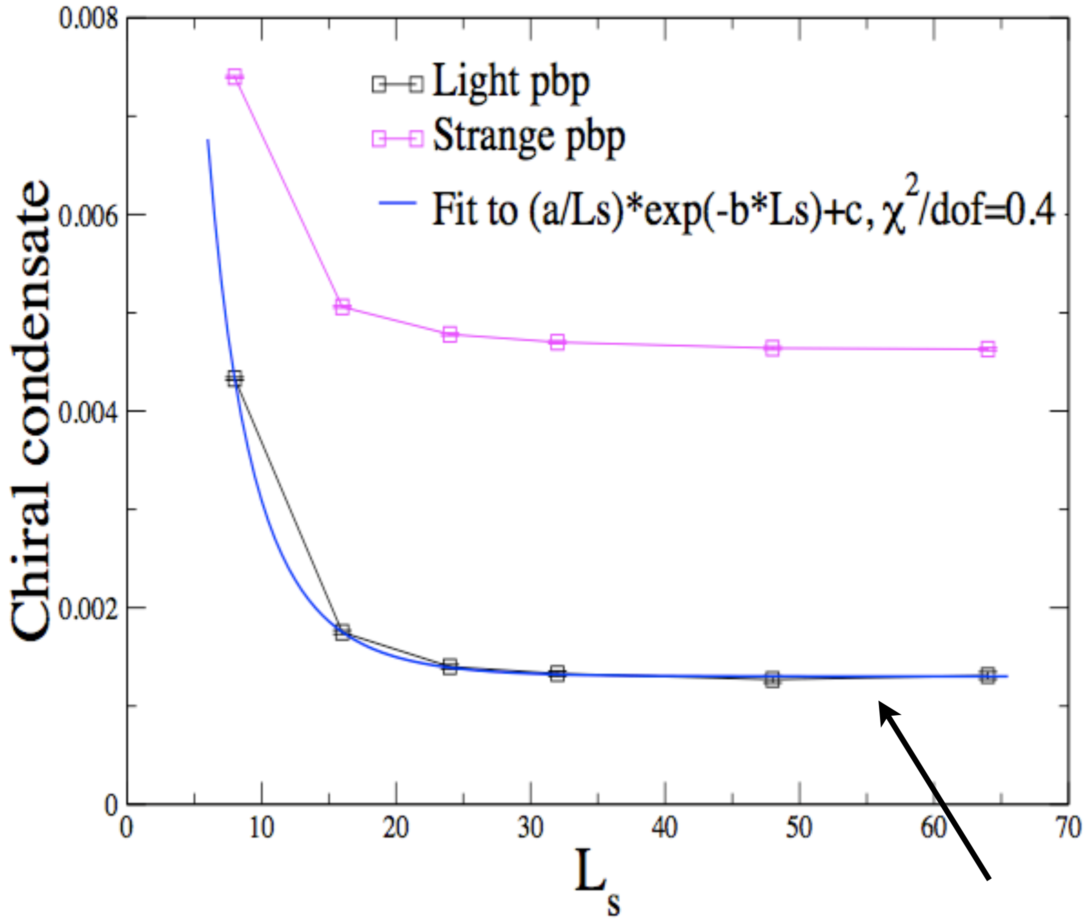
L_s-dependence of the gluino condensate

$$\langle\psi\psi\rangle \sim a_0 + a_1 \frac{e^{-a_2 L_s}}{L_s} + \frac{a_3}{L_s}$$ (fixed m_f)

need not be proportional to m_{res}...

- As of Lattice 2008 proceedings:
  - limited data (cannot perform 4 parameter fit)
  - set a_3=0 (motivated by finite temperature simulations by M. Cheng, et. al.)

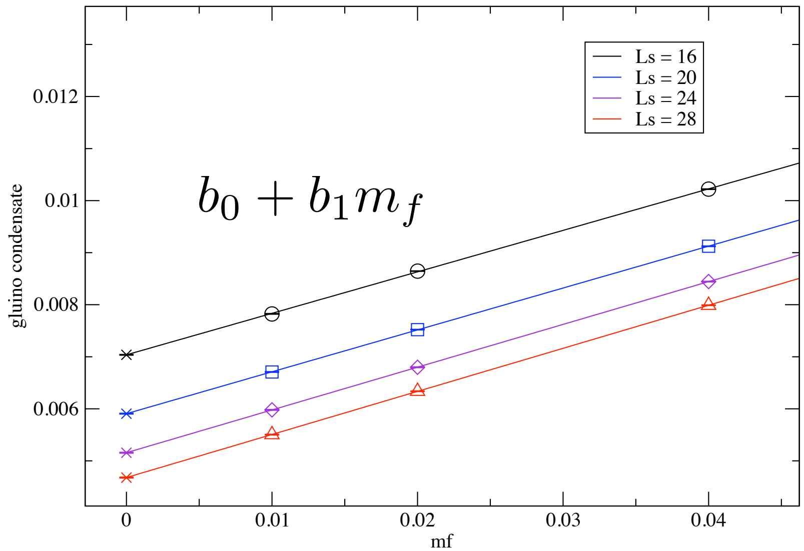
L_s-dependence of the gluino condensate

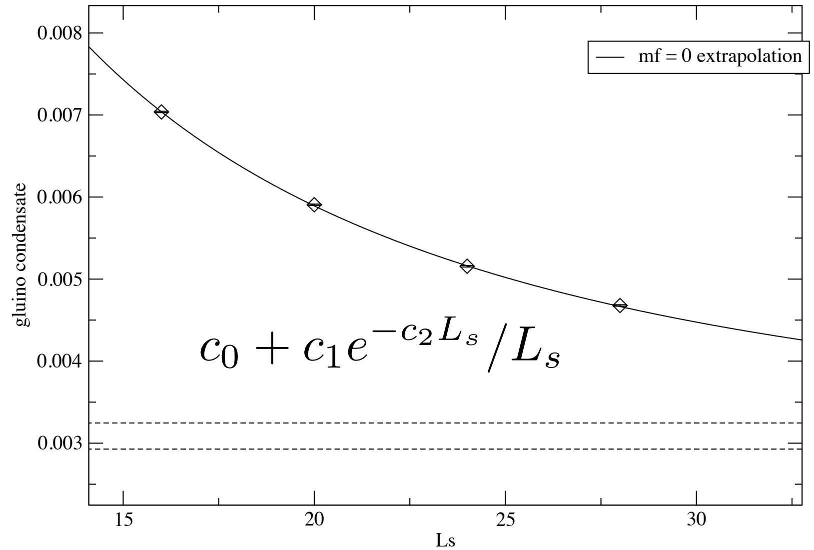


M. Cheng, Lattice 2008 (talk)
M. Cheng, RBC Collaboration (in preparation)

Found large variation in m_{res} at large L_s

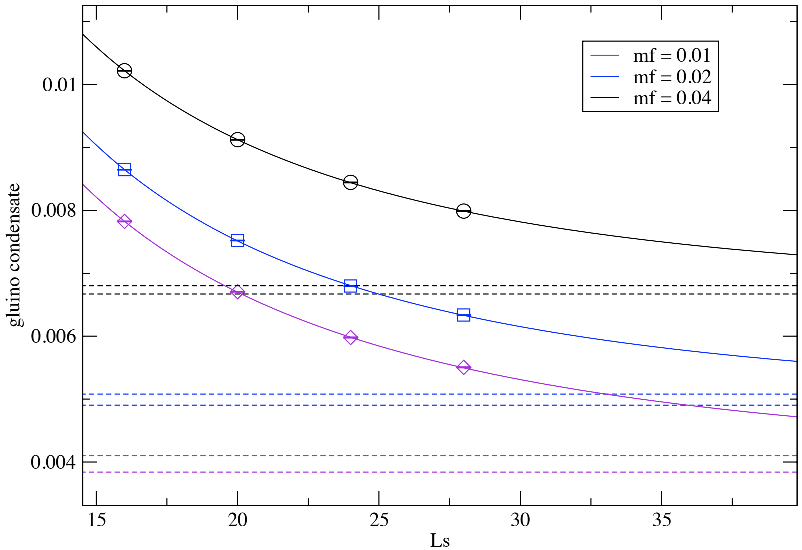
Gluino condensate in the chiral limit

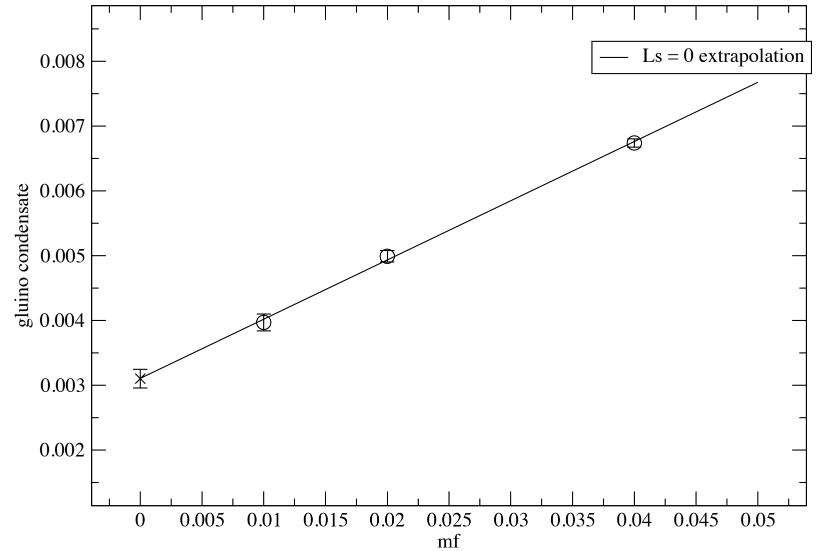




- Two parameter extrapolation: m_f and L_s
- L_s dependence due to residual mass
- Extrapolation performed by first taking m_f=0 then L_s=inf

Gluino condensate in the chiral limit





	Gluino Condensate	
extrapolation order	m_f then L_s	L_s then m_f
16^3x32	0.003087(159)	0.003102(144)
Giedt, et. al	0.00083(19)*	

*linear m_{res} extrapolation of m_f=0 simulated results

L_s-dependence of the gluino condensate

$$\langle \psi\psi \rangle \sim a_0 + a_1 \frac{e^{-a_2 L_s}}{L_s} + \frac{a_3}{L_s} \qquad \text{(fixed } m_f\text{)}$$

need not be proportional to m_{res}...

- In Lattice 2008 proceedings:
  - limited data (can not perform 4 parameter fit)
  - set a_3=0 (motivated by finite temperature simulations by M. Cheng, et. al.) (Is this assumption valid?)
- Need more data for larger L_s at a fixed coupling and m_f

L_s-dependence of the gluino condensate

β	L_s	m_f	N_{traj}	acc. rate	$\sqrt{\langle \Delta H^2 \rangle}$	$\langle e^{-\Delta H} \rangle$
2.3	16	0.01	3125	0.758	0.673	1.016
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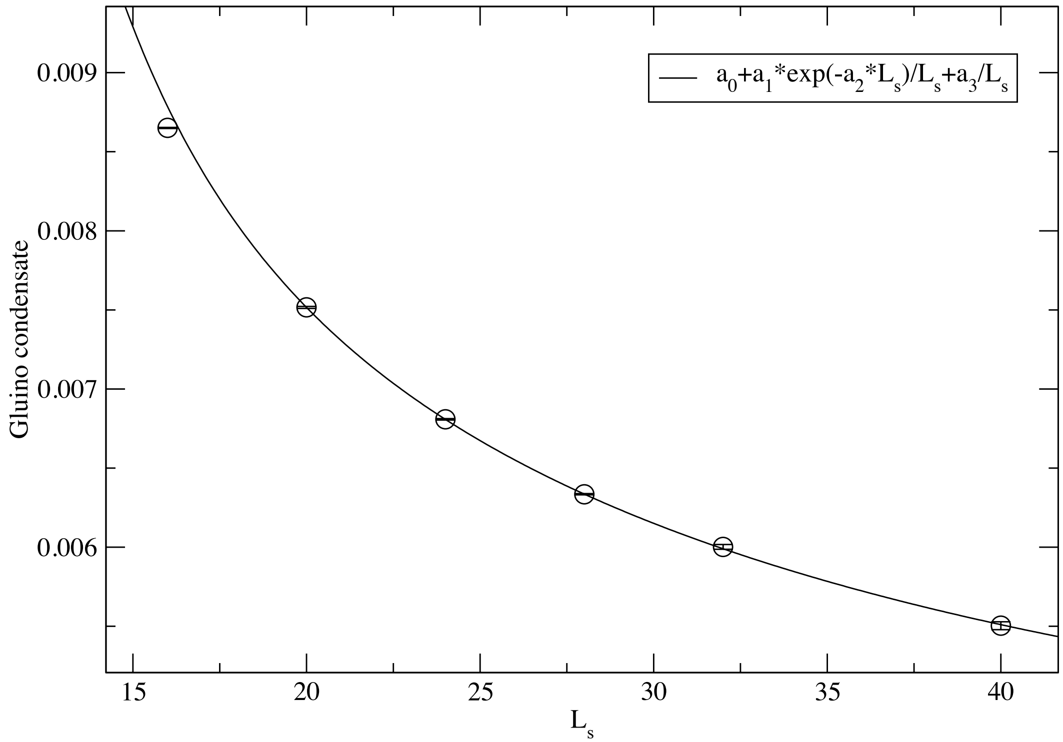
		0.04	1155	0.855	0.346	0.987

In progress...

L_s-dependence of the gluino condensate

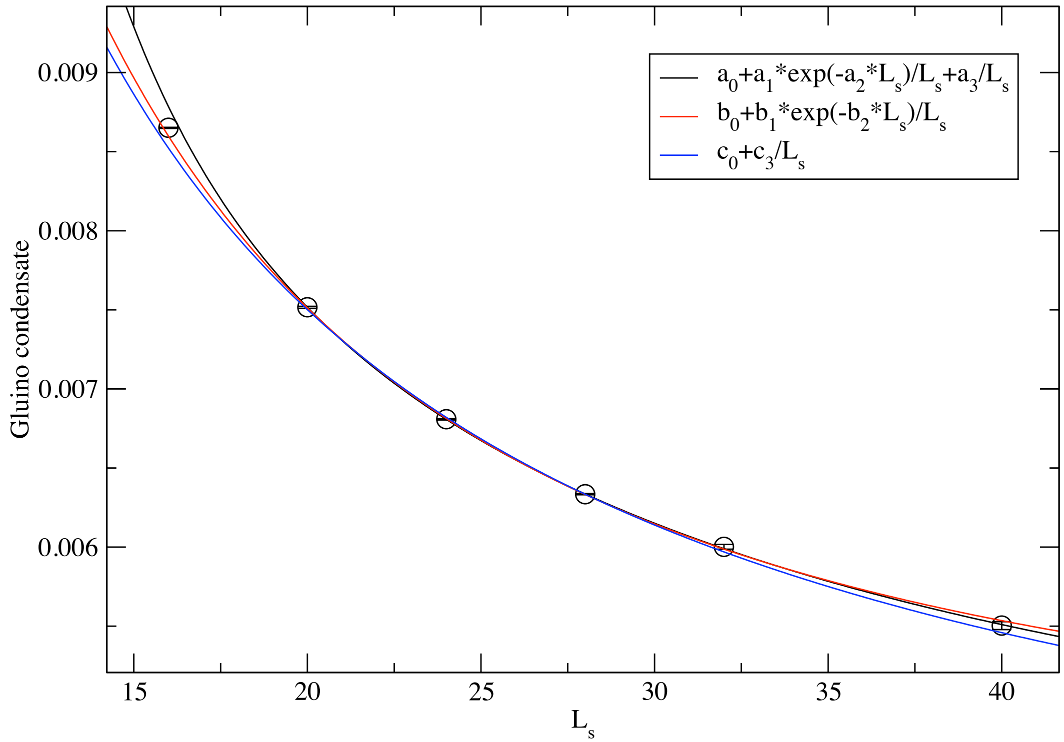


$$\langle\psi\psi\rangle \sim a_0 + a_1\frac{e^{-a_2 L_s}}{L_s} + \frac{a_3}{L_s}$$

Fit to data for L_s>16:

	χ_2/d.o.f.	0	1	2	3
a	0.8	0.00360	0.972	0.313	0.0765

L_s-dependence of the gluino condensate



$$\langle\psi\psi\rangle \sim a_0 + a_1 \frac{e^{-a_2 L_s}}{L_s} + \frac{a_3}{L_s}$$

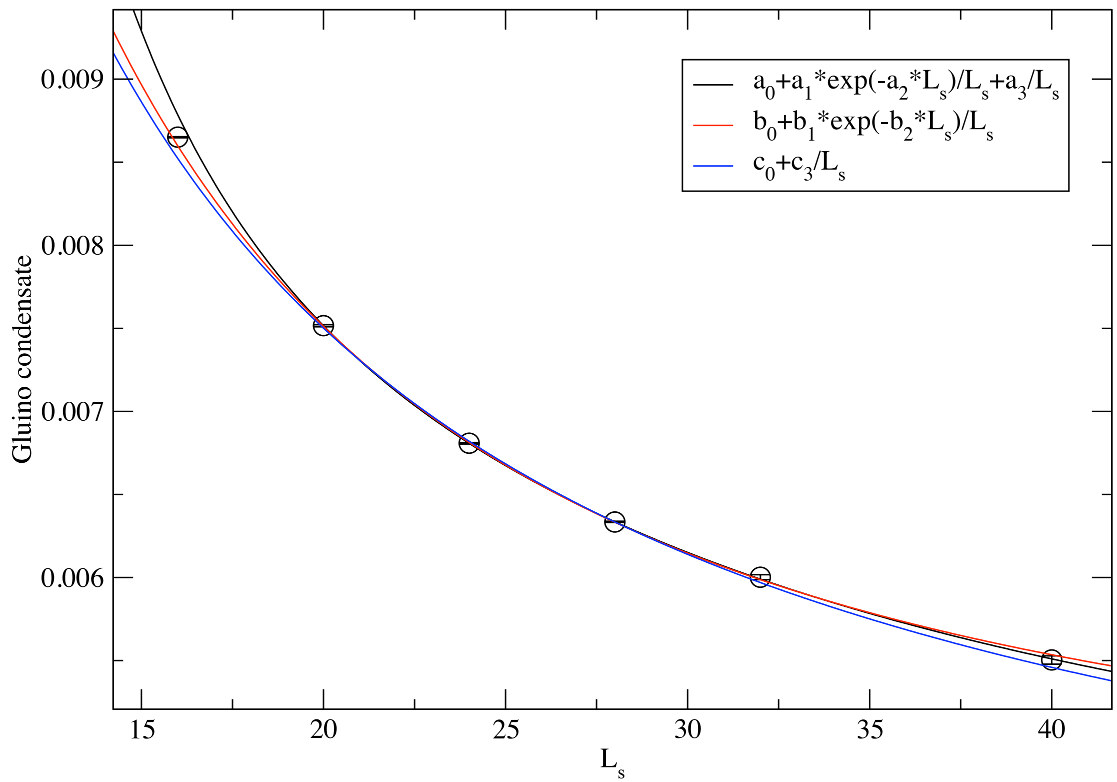
$$\langle\psi\psi\rangle \sim b_0 + b_1 \frac{e^{-b_2 L_s}}{L_s}$$

$$\langle\psi\psi\rangle \sim c_0 + \frac{c_3}{L_s}$$

Fit to data for L_s>16:

	χ_2/d.o.f.	0	1	2	3
a	0.8	0.00360	0.972	0.313	0.0765
b	2.2	0.00467	0.0941	0.0255	
c	6.8	0.00342			0.0817

L_s-dependence of the gluino condensate



- Measurements of m_{res} will help to understand these results and are currently underway
- Expect extrapolated results to vary by 20% depending on the fit function

Spectrum of N=1 SYM

- Low energy effective theories predict colorless composite states:

Veneziano and Yankilowicz (1982)
Farrar, Gabadadze and Schwetz (1998)

  - gluino-gluino
  - glue-gluino
  - glue-glue
- States form chiral super-multiplets (a complex scalar, weyl fermion and an auxiliary field of equal mass)

Spectrum of N=1 SYM

N=1 SYM	QCD
$Tr\,\bar{\psi}\psi$	f_0
$Tr\,\bar{\psi}\gamma_5\psi$	η'
$Tr\,F_{\mu\nu}\sigma_{\mu\nu}\psi$	no analogue

Spectrum of N=1 SYM

N=1 SYM		Multiplet
$Tr\,\bar{\psi}\psi$ $Tr\,\bar{\psi}\gamma_5\psi$	$\Rightarrow$	a complex scalar
$Tr\,F_{\mu\nu}\sigma_{\mu\nu}\psi$	$\Rightarrow$	a Majorana fermion

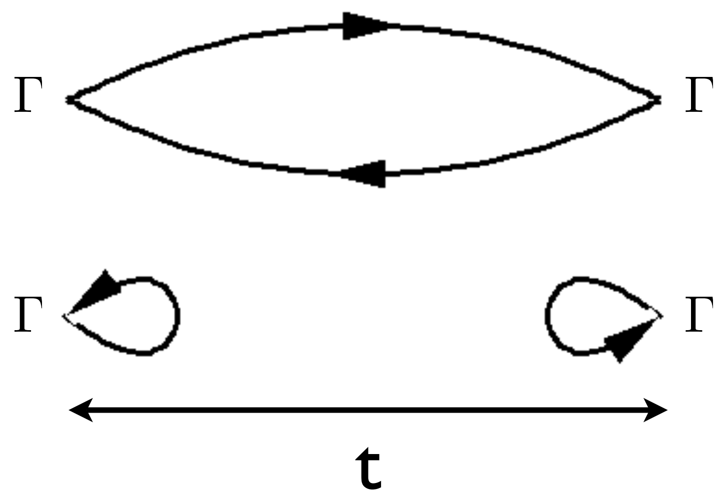
Veneziano and Yankilowicz (1982)

- Low energy described by supermultiplets
  - mass splittings away from SUSY point
  - how big is the residual mass?

Spectrum: pseudo-scalar and scalar

$C(t) =$

$D(t) =$



$\Gamma = \{1, \gamma_5\}$

$$G(t) = 2C(t) - D(t)$$

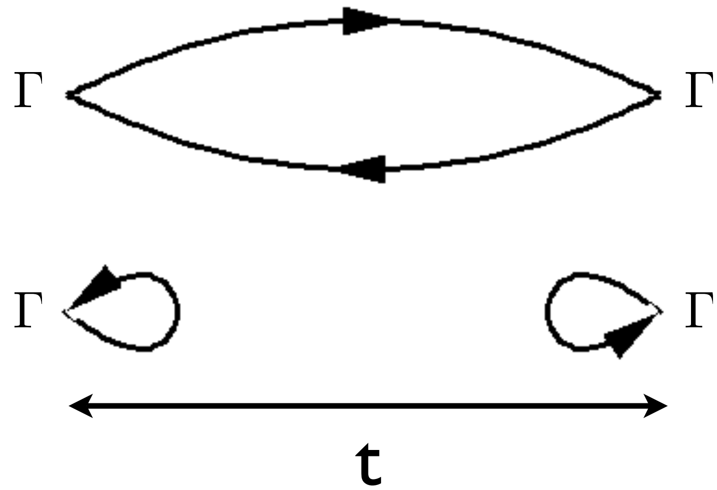
$$C(t) = \sum_x Tr\left[D^{-1}(x,t;x,0)\Gamma D^{-1}(x,0;x,t)\Gamma\right]$$

$$D(t) = \sum_x Tr\left[D^{-1}(x,t;x,0)\Gamma\right]^2 - \text{vacuum contribution}$$

Spectrum: pseudo-scalar and scalar

$C(t) =$

$D(t) =$



$$\Gamma = \{1, \gamma_5\}$$

- Connected (C) and disconnected (D) quark contractions
- Stochastic estimator to measure disconnected diagram
    - 1 hit for connected
    - 5 hits for disconnected

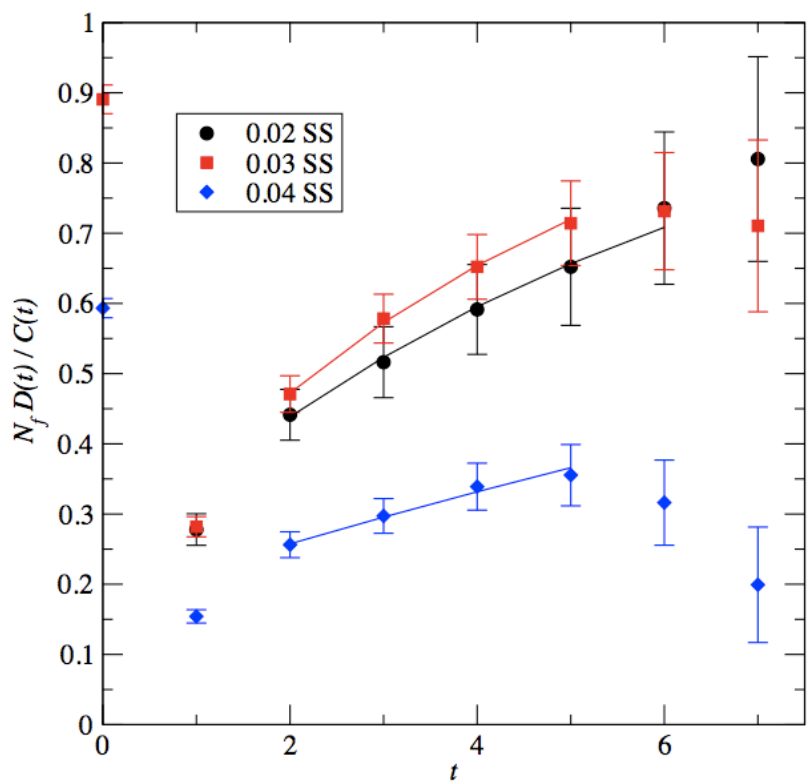
Lessons from the eta-prime in QCD

$$G_{\eta'}(t) = C(t) - 2D(t) = e^{-m_\eta t} + \text{excited states}$$

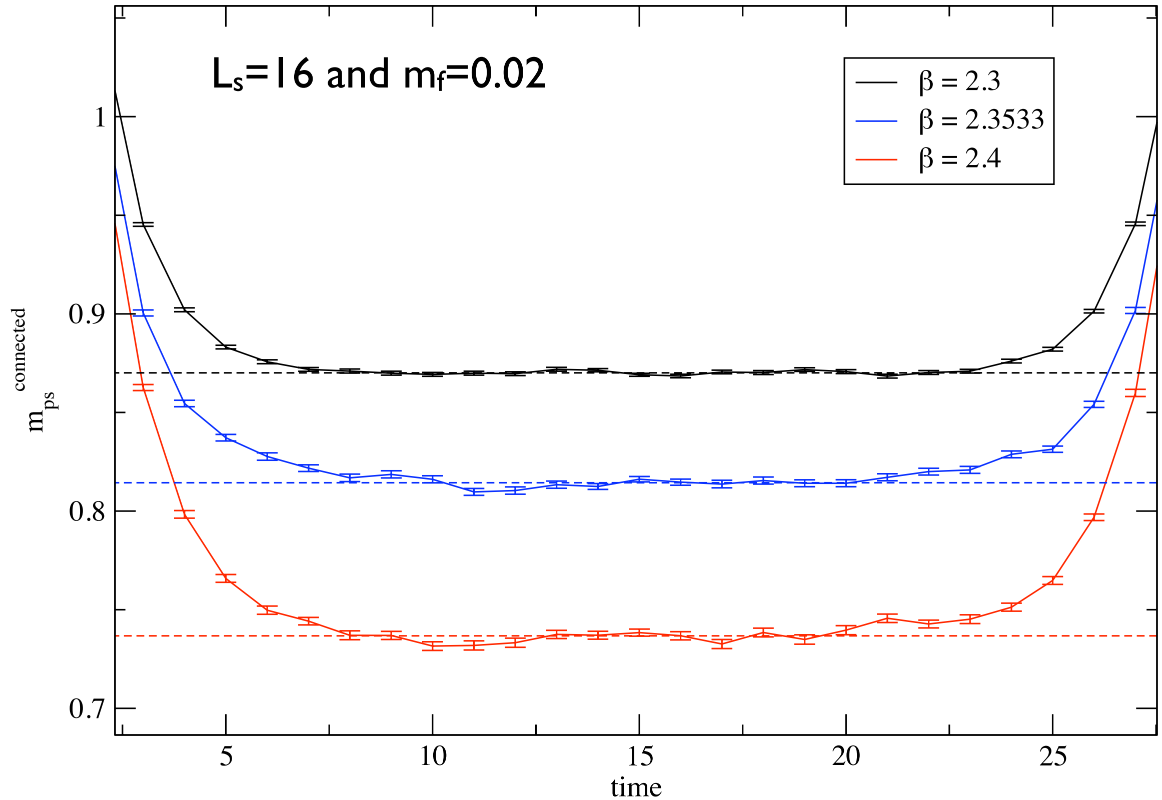
$$G_\pi(t) = C(t) = e^{-m_\pi t} + \text{excited states}$$

$$2\frac{D(t)}{C(t)} = 1 - e^{-(m_{\eta'} - m_\pi)\, t} + \ldots$$

Hashimoto, Izubuchi (2008)

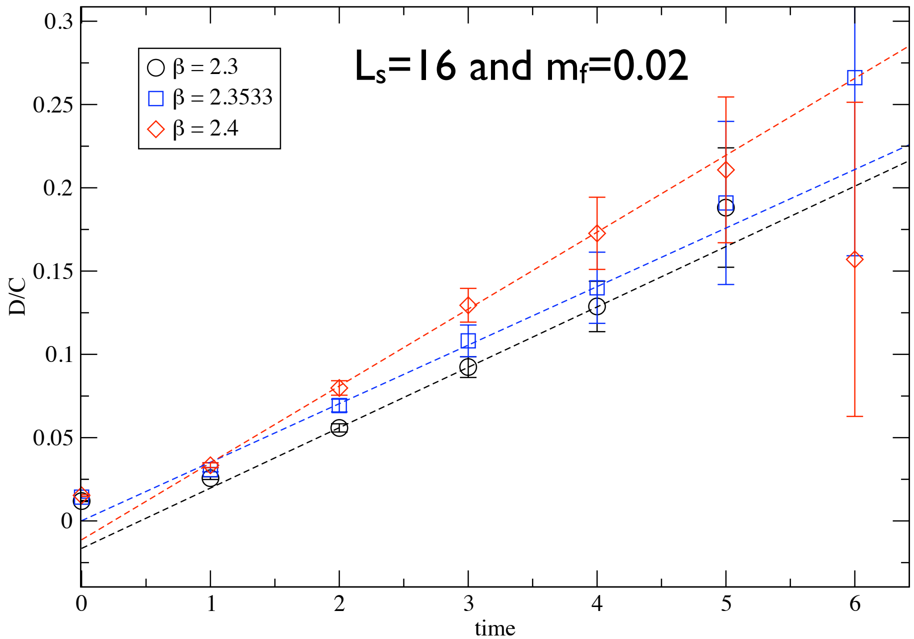


Spectrum: connected pseudo-scalar mass



- Effective mass extracted from the connected part of the pseudo-scaler correlator C(t)
- Connected part exhibits exponential-like behavior

Ratio D(t)/C(t) for pseudo-scalar



$$\frac{D_{ps}(t)}{C_{ps}(t)} = 2 - d\, e^{-\Delta m_{ps} t}$$

$$\Delta m_{ps} = m_{ps} - m_{ps}^{connected}$$

- Linearity suggests $\Delta m_{ps}\, t \ll 1$
  - linear fit to data yields $\Delta m_{ps} \sim 0.02$ which is consistent with this assumption
  - results suggest that m_{res} is very large

Future tasks and directions

- Better understanding of residual mass in the context of SYM
  - theoretical
  - independent measurement of m_{res}
- m_{res} appears to be large, may be reduced with
  - larger L_s
  - smaller coupling
  - improved actions (e.g. DBW2/Iwasaki, auxillary determinant)

Future tasks and directions

- Continuum extrapolation of gluino condensate
  - requires several couplings
- Continue spectrum measurements
  - requires increased statistics and smaller gluino masses
  - measure mass of
    - fermion super-partner (code has been written)
    - glue-balls

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