Dynamical simulation of N=1 supersymmetric Yang-Mills theory with domain wall fermions

Michael G. Endres
Lattice SUSY and Beyond
November 25, 2008
arXiv:0810.0431
Supersymmetry (SUSY) on the lattice

- Much effort devoted toward formulating supersymmetric lattice theories, motivated by
  - fascinating theoretical challenge
  - potential role of SUSY in beyond the standard model physics
  - desire to understand various nonperturbative aspects of SUSY theories via numerical simulation
Incompatibility of SUSY and the lattice

- SUSY algebra
  - SUSY is an extension of Poincare group
  - $[P, P] \sim 0$, $\{Q, Q\} \sim 0$, $[P, Q] \sim 0$, $\{Q, Q\} \sim P$, etc.

- Lattice fermion “doublers”
  - fermion discretizations may yield more continuum degrees of freedom than expected

- Violation of “leibniz rule”
  - $\partial(ab) \neq (\partial a)b + a(\partial b)$ on the lattice
  - cannot easily construct SUSY lattice actions using superfields
Incompatibility of SUSY and the lattice

- Standard lattice formulations explicitly break SUSY
- SUSY violating operators may arise radiatively
  - scalar mass terms are additively renormalized
  - depending on fermion discretization, fermion masses may be additively renormalized
- Fine tuning of operators required to reach SUSY point
  - extremely difficult in practice
  - large numerical cost
Incompatibility of SUSY and the lattice

• Is it possible to achieve SUSY accidentally?
  • e.g. rotational symmetry restoration in lattice QCD

• Advances in formulating SUSY lattice theories
  • theories which preserve a sub-algebra of SUSY on the lattice
    • require little fine tuning
    • exotic lattices

• N=1 SYM in four dimensions can be achieved with conventional lattice formulations
N=1 super Yang-Mills (SYM)

\[ L = \frac{1}{g^2} \left[ \bar{\lambda} \gamma_{\mu} D_{\mu} \lambda + \frac{1}{4} v_{\mu\nu} v_{\mu\nu} \right], \quad \bar{\lambda} = \lambda^T C \]

- One of the simplest of SUSY gauge theories in terms of field content
- 1 vector field and 1 adjoint Majorana fermion (gluino)
- a single input parameter, the gauge coupling (g)
- Anomalous U(1)\(_R\) (chiral) symmetry: \( \lambda \to e^{-\alpha \gamma_5} \lambda \Rightarrow \theta \to \theta - 2N \alpha \)
- \( Z_{2N} \) subgroup of U(1)\(_R\) survives at quantum level
  - partition function invariant for \( \alpha = \frac{\pi k}{N_c} \), \( k = 0, \ldots, 2N_c - 1 \)
\[ L = \frac{1}{g^2} \left[ \bar{\lambda} \gamma_\mu D_\mu \lambda + \frac{1}{4} v_{\mu\nu} v_{\mu\nu} \right], \quad \bar{\lambda} = \lambda^T C \]

- Gluino condensation \( \langle \lambda \lambda \rangle \neq 0 \)
- discrete chiral symmetry breaking \( Z_{2N} \rightarrow Z_2 \)
- Confinement
- colorless bound states
- No SUSY breaking (non-vanishing Witten index)
N=1 SYM on the lattice

- From numerical point of view N=1 SYM ideal starting point for studying SUSY
- possesses interesting nonperturbative physics
- is QCD-like in some respects
  - chiral symmetry breaking (although discrete)
  - confinement
- unlike QCD, no Goldstone bosons
SUSY restoration in N=1 SYM on the lattice

- Only SUSY violating relevant operator which may arise from quantum fluctuations (consistent with gauge and lattice symmetries) is a gluino mass term

- Chiral symmetry realized on the lattice implies SUSY restoration in the continuum limit

- Key to the idea is the absence of scalars

- Only SUSY and shift symmetry can prevent radiative corrections to scalar masses

- Unable to introduce matter multiplets without additional fine tuning
Some numerical studies of N=1 SYM

- Wilson
  - DESY-Muenster-Roma Collaboration (summarized in hep-lat/0112007 and references therein, arXiv:0811.1964)

- DWF
  - G.T. Fleming, et. al. (hep-lat/0008009)
  - M. G. Endres (arXiv:0810.0431)
N=1 SYM with Domain wall fermions (DWFs)

- DWF formulation ideal for N=1 SYM
  - good chiral properties (no fine tuning)
  - no fermion doublers
  - positive definite fermion determinant (no “sign problem”)
- More expensive compared to other lattice discretizations (e.g. Wilson) in terms of computing resources

Kaplan’s DWFs

- Exact chiral symmetry on the lattice

- Introduce a 5th dimension \((s)\) and an \(s\)-dependent mass term with a step profile

\[
S = \int dx ds \bar{\Psi}(x, s) \left[ \gamma_\mu \partial_\mu + \gamma_5 \partial_s + m(s) \right] \Psi(x, s)
\]
Kaplan’s DWFs

- Solutions of the Dirac equation yield chiral modes bound to the fifth dimension boundaries of a 5-dimensional theory
- All other modes have $O(1/a)$ masses on the lattice
• Only consider half of the fifth dimension ($M > 0$ region)
• Chiral modes bound to the boundaries of fifth dimension
• Explicit coupling of left- and right-handed modes with strength $m_f$
Two sources for nonvanishing gluino mass:

- explicit coupling of domain walls via $m_f$
- nonzero overlap of wave functions gives rise to a residual mass ($m_{res}$) at finite $L_s$

- Chiral ($m_f=0, L_s=\infty$) and SUSY limits coincide
Numerical simulations

- Gluino condensate
- Static potential
- Residual mass
- Spectrum
- Domain walls arising from discrete chiral symmetry breaking
- Effects of nonzero gluino mass
Numerical simulations

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Fleming, Kogut and Vranas (2001)
N=1 SYM with DWFs

- Questions not addressed by Fleming, et. al.
  - lattice scale
  - what is the residual mass (how close to SUSY point)?
  - continuum limit of gluino condensate
N=1 SYM with DWFs

• Since then
  • improved algorithms (e.g. RHMC algorithm)
  • much faster computers (therefore larger lattices)
  • better understanding of DWFs (e.g. $L_s$ dependence of residual mass)
  • can do better job extrapolating to chiral limit
N=1 SYM with DWFs

- Recent renewed interest in N=1 SUSY simulations with DWFs:
  - M. G. Endres (arXiv:0810.0431)
    - simulate at a variety of $m_f$ values
    - extrapolate to $m_f=0$
  - J. Giedt, et. al. (arXiv:0810.5746)
    - simulate at $m_f=0$
    - computationally more costly
    - no need to perform $m_f=0$ extrapolation

Independent studies!
Numerical simulations

- Gluino condensate
- Static potential
- Residual mass
- Spectrum
  - Domain walls arising from discrete chiral symmetry breaking
  - Effects of nonzero gluino mass
Numerical simulations

- Wilson gauge action with domain wall fermions
- SU(2) gauge group with adjoint Majorana fermions
- Simulations performed on an appropriately modified version of the Columbia Physics System (CPS)
- $8^3 \times 8 \times L_s$ ensembles were generated and measurements made on QCD OC at Columbia University
- $16^3 \times 32 \times L_s$ ensembles were generated and measurements made on New York Blue (BlueGene/L) at Brookhaven National Laboratory
## Simulation parameters for $16^3 \times 32$ lattices

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Thermalization

$\beta=2.3$ and $m_f=0.02$

• All configurations generated from a cold start
Static potential

- Wilson loops measured on Coulomb gauge-fixed gauge configurations

- space-time dependence: \( W(t, r) = C(r)e^{-V(r)t} + \text{excited states} \)

- effective potential given by: \( V_{eff}(r, t) = \log \frac{W(r, t)}{W(r, t + 1)} \)
Static potential

\[ V_{\text{eff}}(r, t) = \log \frac{W(r, t)}{W(r, t + 1)} \]

- \( \beta = 2.3 \) and \( m_f = 0.02 \)
- Distance range: 1 to 7
Static potential

- Fit Wilson loops to an exponential over plateau region
  \[ W(t, r) = C(r)e^{-V(r)t} + \text{excited states} \]

- Potential was fit to the Cornell form:
  \[ V(r) = V_0 - \frac{\alpha}{r} + \sigma r \]

- Lattice scale set via Sommer scale:
  \[ r_0^2 V'(r_0) = 1.65 \]
  \[ r_0 = \sqrt{\frac{1.65 - \alpha}{\sigma}} \]
Coupling dependence of static potential

\[ V(r) = 2.3 \] 
\[ V(r) = 2.3533 \] 
\[ V(r) = 2.4 \]

Double Jackknife fits by I. Mihailescu

Distance range: \( \sqrt{3} - 6 \)
Residual mass: a lesson from in QCD

- Define 4-d axial current which corresponding to opposite phase rotations on each half of the fifth dimension: \( A_\mu^a \)

\[
\Delta_\mu A_\mu(x) = 2m_f J_5^a(x) + 2J_{5q}^a(x)
\]

forward derivative

“wall” pseudo-scalar density

“mid-point” pseudo-scalar density
Residual mass: a lesson from in QCD

- Axial Ward-Takahashi (WT) identity:

\[ \Delta_\mu \langle A_\mu^a(x)O(y) \rangle = 2m_f \langle J_5^a(x)O(y) \rangle + 2\langle J_{5q}(x)O(y) \rangle + i\langle \delta^a O(y) \rangle \]

- In effective continuum Lagrangian, chiral symmetry breaking effects appear to lowest order in lattice spacing (i.e. a^{-1}) as:

\[ L_{\text{symzik}} = \bar{\psi}(D + m_f)\psi + m_{\text{res}} \bar{\psi}\psi + c \bar{\psi}\sigma_{\mu\nu}F_{\mu\nu}\psi + \ldots \]
Residual mass: a lesson from in QCD

- Axial Ward-Takahashi (WT) identity:

\[
\Delta_\mu \langle A^a_\mu(x) \mathcal{O}(y) \rangle = 2m_f \langle J^a_5(x) \mathcal{O}(y) \rangle + 2 \langle J^a_{5q}(x) \mathcal{O}(y) \rangle + i \langle \delta^a \mathcal{O}(y) \rangle
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- In effective continuum Lagrangian, chiral symmetry breaking effects appear to lowest order in lattice spacing (i.e. \( a^{-1} \)) as:

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L_{\text{symannzik}} = \bar{\psi}(D + m_f)\psi + m_{\text{res}} \bar{\psi}\psi + c \bar{\psi} \sigma_{\mu \nu} F_{\mu \nu} \psi + \ldots
\]

- Agreement of axial WT identity between lattice and continuum effective theory implies the low energy identity:

\[
J^a_{5q}(x) \approx m_{\text{res}} J^a_5(x)
\]
Residual mass: a lesson from in QCD

- Residual mass obtained from the ratio:

\[
R(t) = \frac{\langle \sum_x J^a_{5q}(x, t) J^a_{5}(0) \rangle}{\langle \sum_x J^a_{5}(x, t) J^a_{5}(0) \rangle} \rightarrow m_{res}, \quad t >> 1
\]

Numerator:
Residual mass in SYM

- “1/2” flavor instead of 2 flavors
- no flavor non-singlet axial currents
- need to worry about anomalous contribution to axial WT identity:
- low energy identity:
  \[ J_{5q}(x) \approx m_{res} J_5(x) + \rho_{top}(x) \]
- Wall-midpoint and wall-wall pseudo-scalar correlation functions involve both “connected” and “disconnected” contributions.
Residual mass in SYM

- Open question: can one extract $m_{\text{res}}$ from $R(t)$ in the $N_f<2$ case (i.e. using only connected contributions to wall-midpoint and wall-wall correlators)?

- possible, but details needs to be worked out...

\[
\langle J_{5q}(x, t) J_5(0) \rangle \approx m_{\text{res}} \langle J_q(x, t) J_5(0) \rangle + \langle \rho_{\text{top}}(x, t) J_5(0) \rangle
\]

\[
\langle J_{5q}(x, t) J_5(0) \rangle_{\text{connected}} \approx m_{\text{res}} \langle J_q(x, t) J_5(0) \rangle_{\text{connected}}
\]

\[
\langle J_{5q}(x, t) J_5(0) \rangle_{\text{disconnected}} \approx m_{\text{res}} \langle J_q(x, t) J_5(0) \rangle_{\text{disconnected}} + \langle \rho_{\text{top}}(x, t) J_5(0) \rangle
\]
• \( R(t) \) appears time-independent for large times

• Connected contributions tend to a constant \((m_{\text{res}}?)\)

• Independent measure of \( m_{\text{res}} \) is desirable for consistency check
Coupling-dependence of residual mass

- Residual mass is 5-10 times larger than current $m_f$ values
- Depends strongly on the coupling
- $1/L_s$ term dominates residual mass

\[ m_{\text{res}} \sim \# e^{-\# L_s} + \# \frac{\rho(0)}{L_s} \]

$L_s=16$ and $m_f=0.02$
Gluino condensate in the chiral limit

- Two parameter extrapolation: $m_f$ and $L_s$
- $L_s$ dependence is due to residual mass
- Extrapolation performed by first taking $m_f=0$ then $L_s=\infty$
• $m_f=0$ extrapolation appears inconsistent with Geidt, et. al. for $L_s > 16$ (currently under investigation)

• Discrepancy in $m_f=0$ results appears to increase with decreasing $m_{\text{res}}$
Ls-dependence of the gluino condensate

\[ \langle \psi \psi \rangle \sim a_0 + a_1 \frac{e^{-a_2 L_s}}{L_s} + \frac{a_3}{L_s} \]  

(fixed \( m_f \))

need not be proportional to \( m_{\text{res}} \)...

- As of Lattice 2008 proceedings:
  - limited data (cannot perform 4 parameter fit)
  - set \( a_3 = 0 \) (motivated by finite temperature simulations by M. Cheng, et. al.)
L_s-dependence of the gluino condensate

M. Cheng, Lattice 2008 (talk)
M. Cheng, RBC Collaboration (in preparation)

Found large variation in m_res at large L_s
Gluino condensate in the chiral limit

- Two parameter extrapolation: $m_f$ and $L_s$
- $L_s$ dependence due to residual mass
- Extrapolation performed by first taking $m_f=0$ then $L_s=\infty$
Gluino condensate in the chiral limit

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*linear m_res extrapolation of m_f=0 simulated results
\( L_s \)-dependence of the gluino condensate

\[ \langle \psi \psi \rangle \sim a_0 + a_1 \frac{e^{-a_2 L_s}}{L_s} + \frac{a_3}{L_s} \]  

(fixed \( m_f \))

- In Lattice 2008 proceedings:
  - limited data (can not perform 4 parameter fit)
  - set \( a_3 = 0 \) (motivated by finite temperature simulations by M. Cheng, et. al.) (Is this assumption valid?)
  - Need more data for larger \( L_s \) at a fixed coupling and \( m_f \)

need not be proportional to \( m_{\text{res}} \)...
### $L_s$-dependence of the gluino condensate

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<td>0.02</td>
<td>2710</td>
<td>0.824</td>
<td>0.432</td>
<td>0.996</td>
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<td>28</td>
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<td>1420</td>
<td>0.874</td>
<td>0.340</td>
<td>1.012</td>
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<tr>
<td></td>
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<td>1540</td>
<td>0.879</td>
<td>0.332</td>
<td>1.027</td>
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<td></td>
<td>0.04</td>
<td></td>
<td>1155</td>
<td>0.855</td>
<td>0.346</td>
<td>0.987</td>
</tr>
</tbody>
</table>
L_s-dependence of the gluino condensate

\[ \langle \psi \psi \rangle \sim a_0 + a_1 \frac{e^{-a_2 L_s}}{L_s} + \frac{a_3}{L_s} \]

Fit to data for \( L_s > 16 \):

<table>
<thead>
<tr>
<th>( \chi^2/d.o.f. )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.8</td>
<td>0.00360</td>
<td>0.972</td>
<td>0.313</td>
</tr>
</tbody>
</table>
**L_s-dependence of the gluino condensate**

\[ \langle \psi \psi \rangle \sim a_0 + a_1 \frac{e^{-a_2 L_s}}{L_s} + \frac{a_3}{L_s} \]

\[ \langle \psi \psi \rangle \sim b_0 + b_1 \frac{e^{-b_2 L_s}}{L_s} \]

\[ \langle \psi \psi \rangle \sim c_0 + \frac{c_3}{L_s} \]

**Fit to data for L_s > 16:**

<table>
<thead>
<tr>
<th></th>
<th>( \chi^2/d.o.f. )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.8</td>
<td>0.00360</td>
<td>0.972</td>
<td>0.313</td>
<td>0.0765</td>
</tr>
<tr>
<td>b</td>
<td>2.2</td>
<td>0.00467</td>
<td>0.0941</td>
<td>0.0255</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>6.8</td>
<td>0.00342</td>
<td></td>
<td></td>
<td>0.0817</td>
</tr>
</tbody>
</table>
L_s-dependence of the gluino condensate

- Measurements of m_{res} will help to understand these results and are currently underway

- Expect extrapolated results to vary by 20% depending on the fit function
Spectrum of N=1 SYM

- Low energy effective theories predict colorless composite states:
  - gluino-gluino
  - glue-gluino
  - glue-glue
- States form chiral super-multiplets (a complex scalar, weyl fermion and an auxiliary field of equal mass)
### Spectrum of N=1 SYM

<table>
<thead>
<tr>
<th>N=1 SYM</th>
<th>QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Tr \psi \psi$</td>
<td>$f_0$</td>
</tr>
<tr>
<td>$Tr \psi \gamma_5 \psi$</td>
<td>$\eta'$</td>
</tr>
<tr>
<td>$Tr F_{\mu \nu} \sigma_{\mu \nu} \psi$</td>
<td>no analogue</td>
</tr>
</tbody>
</table>
Spectrum of N=1 SYM

\( \text{N=1 SYM} \)

\[\begin{align*}
\text{Tr} \psi \psi \\
\text{Tr} \psi \gamma_5 \psi \\
\text{Tr} F_{\mu\nu} \sigma_{\mu\nu} \psi
\end{align*}\]

**Multiplet**

- a complex scalar
- a Majorana fermion

- Low energy described by supermultiplets
- mass splittings away from SUSY point
- how big is the residual mass?

Veneziano and Yankilowicz (1982)
Spectrum: pseudo-scalar and scalar

\[ C(t) = \Gamma \]

\[ D(t) = \Gamma \]

\[ \Gamma = \{1, \gamma_5\} \]

\[ G(t) = 2C(t) - D(t) \]

\[ C(t) = \sum_x \text{Tr} \left[ D^{-1}(x, t; x, 0) \Gamma D^{-1}(x, 0; x, t) \Gamma \right] \]

\[ D(t) = \sum_x \text{Tr} \left[ D^{-1}(x, t; x, 0) \Gamma \right]^2 - \text{vacuum contribution} \]
Spectrum: pseudo-scalar and scalar

\[ C(t) = \Gamma \quad \Gamma \]
\[ D(t) = \Gamma \quad \Gamma \]

- Connected (C) and disconnected (D) quark contractions
- Stochastic estimator to measure disconnected diagram
  - 1 hit for connected
  - 5 hits for disconnected

\[ \Gamma = \{1, \gamma_5\} \]
Lessons from the eta-prime in QCD

\[ G_{\eta'}(t) = C(t) - 2D(t) = e^{-m_{\eta}t} + \text{excited states} \]

\[ G_{\pi}(t) = C(t) = e^{-m_{\pi}t} + \text{excited states} \]

\[ 2 \frac{D(t)}{C(t)} = 1 - e^{-(m_{\eta'}-m_{\pi})t} + \ldots \]

Hashimoto, Izubuchi (2008)
Spectrum: connected pseudo-scalar mass

- Effective mass extracted from the connected part of the pseudo-scalar correlator $C(t)$
- Connected part exhibits exponential-like behavior
Ratio $D(t)/C(t)$ for pseudo-scalar

$D_{ps}(t) = 2 - d e^{-\Delta m_{ps} t}$

$\Delta m_{ps} = m_{ps} - m_{ps}^{connected}$

- Linearity suggests $\Delta m_{ps} t \ll 1$
  - linear fit to data yields $\Delta m_{ps} \sim 0.02$ which is consistent with this assumption
- results suggest that $m_{res}$ is very large
Future tasks and directions

- Better understanding of residual mass in the context of SYM
  - theoretical
  - independent measurement of $m_{\text{res}}$
- $m_{\text{res}}$ appears to be large, may be reduced with
  - larger $L_s$
  - smaller coupling
  - improved actions (e.g. DBW2/Iwasaki, auxiliary determinant)
Future tasks and directions

• Continuum extrapolation of gluino condensate
  • requires several couplings
• Continue spectrum measurements
  • requires increased statistics and smaller gluino masses
  • measure mass of
    • fermion super-partner (code has been written)
    • glue-balls
Acknowledgements

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