Schwarzschild Radius and Black Hole Thermodynamics with $\alpha'$ Corrections from Simulations of SUSY Matrix Quantum Mechanics

Jun Nishimura (KEK)

“Lattice Supersymmetry and Beyond”
at The Niels Bohr International Academy, Nov.24, ’08

Ref.) Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 (’08) 021601
based on collaborations with

Konstantinos Anagnostopoulos  
(National Technical University, Athens, Greece)

Masanori Hanada (Weizmann Inst., Israel)

Yoshifumi Hyakutake (Osaka Univ.)

Akitsugu Miwa  
(U. of Tokyo, Komaba  
→ Harish-Chandra Research Inst., India)

Shingo Takeuchi (KEK → APCTP, Korea)
0. Introduction
Supersymmetric large-N gauge theories

suitable formulation for describing superstrings non-perturbatively

- non-pert. formulation of superstring/M theories
  e.g.) Matrix Theory (Banks-Fischler-Shenker-Susskind ’97)
  IIB matrix model (Ishibashi-Kawai-Kitazawa-Tsuchiya ’97)

  ➢ dynamical origin of space-time dimensionality,
gauge group, matters, etc.

- gauge/string duality
  e.g.) AdS/CFT Maldacena (’97)

  ➢ quantum/stringy description of black holes
  in terms of gauge theories

Difficulty: Strongly coupled dynamics should be investigated!
Monte Carlo simulation

can be a powerful approach as in lattice QCD, but...

SUSY : broken on the lattice
\[ \{ Q, \bar{Q} \} \propto P_\mu \rightarrow \text{translational symmetries broken to discrete ones} \]

- Lattice formulation preserving a part of supersymmetry by using various ideas such as twisting, orbifolding, ...
  (Cohen-Kaplan-Katz-Unsal, Catterall, Sugino, Kanamori-Suzuki, Arianos-D’Adda-Feo-Kawamoto-Saito, Nagata, Damgaard-Matsuura,...)

- Non-lattice approach respecting SUSY maximally
  systems with 16 supercharges can be studied!

1d gauge theory (SUSY matrix QM) (This talk)
non-perturbative gauge fixing + Fourier mode cutoff
\[ \mathcal{N} = 4 \text{ SYM on } R \times S^2 \text{ and } R \times S^3 \]

fuzzy sphere
Supersymmetric matrix quantum mechanics

$1d \ U(N) \ gauge \ theory \ with \ 16 \ supercharges$

$$S = \frac{N}{\lambda} \int_0^\beta dt \ \text{tr} \left\{ \frac{1}{2} (DX_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 + \text{(fermionic part)} \right\}$$

non-perturbative formulation of M theory

BFSS conjecture  
Banks-Fischler-Shenker-Susskind ('97)

low energy effective theory of $N$ D0 branes

gauge/gravity correspondence (non-conformal ver.)

Itzhaki-Maldacena-Sonnenschein-Yankielowicz ('98)

SUSY mass deformation  \rightarrow  plane-wave matrix model

Berenstein-Maldacena-Nastase ('02)

Expanding around a (multi-)fuzzy-sphere background

$\mathcal{N} = 4 \ SYM$ on $R \times S^2$ and $R \times S^3$

Ishii-Ishiki-Shimasaki-Tsuchiya ('08)
Gauge-gravity duality for D0-brane system

Type IIA superstring

\( N \) D0 branes

1d U(\( N \)) SUSY gauge theory

at finite \( T \)

\( \lambda : 't \) Hooft coupling

Itzhaki-Maldacena-Sonnenschein-Yankielowicz (’98)

horizon

black 0-brane solution in type IIA SUGRA

near-extremal black hole

In the decoupling limit, the D0 brane system describes the black hole microscopically.

large \( N \) and large \( \lambda \) → SUGRA description: valid
Simulating superstrings inside a black hole

black hole thermodynamics

Anagnostopoulos-Hanada-J.N.-Takeuchi (’08)

\[ \frac{1}{N^2} \left( \frac{E}{\lambda^{1/3}} \right) = \frac{9}{14} \left( \frac{13 \cdot 15^2}{7} \right)^{1/5} \left( \frac{T}{\lambda^{1/3}} \right)^{14/5} \]

including \( \alpha' \) corrections

Hanada-Hyakutake-J.N.-Takeuchi, arXiv:0811.3102

Schwarzschild radius from Wilson loop


\[ W \equiv \text{tr} \mathcal{P} \exp \left[ i \int_0^\beta dt \{ A(t) + i X_9(t) \} \right] \sim \exp \left( \frac{\beta R_{\text{Sch}}}{2\pi \alpha'} \right) \]

\[ \ln W = \frac{\beta R_{\text{Sch}}}{2\pi \alpha'} = \frac{1}{2\pi} \left( \frac{16 \sqrt{15} \pi^{7/2}}{7} \right)^{2/5} \left( \frac{T}{\lambda^{1/3}} \right)^{-3/5} \]

1.89
Plan of the talk

0. Introduction

1. Simulating SUSY matrix QM with 16 supercharges

2. Dual gravity description and black hole thermodynamics

3. Higher derivative corrections to black hole thermodynamics from SUSY QM

4. Schwarzschild radius from Wilson loop

5. Summary
1. Simulating SUSY QM with 16 supercharges
SUSY matrix QM with 16 supercharges

\[
S_b = \frac{N}{\lambda} \int_0^\beta dt \text{ tr } \left\{ \frac{1}{2} (DX_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 \right\}
\]

\[
S_f = \frac{\bar{N}}{\lambda} \int_0^\beta dt \text{ tr } \left\{ \frac{1}{2} \psi_\alpha D \psi_\alpha - \frac{1}{2} \psi_\alpha (\gamma_i)_{\alpha\beta} [X_i, \psi_\beta] \right\}
\]

1d gauge theory

\[
D = \partial_t - i [A(t), \cdot]
\]

\[
\begin{cases}
X_j(t) & (j = 1, \cdots, 9) \quad \text{p.b.c.} \\
\psi_\alpha(t) & (\alpha = 1, \cdots, 16) \quad \text{anti p.b.c.}
\end{cases}
\]

\[
T = \beta^{-1} \quad \text{temperature}
\]

\[
\lambda = g^2 N \quad \text{'t Hooft coupling}
\]

\[
\lambda = 1 \quad \text{(without loss of generality)}
\]

\[
\lambda_{\text{eff}} = \frac{\lambda}{T^3}
\]

\[
\begin{cases}
\text{low T} & \Rightarrow \text{strongly coupled} \quad \text{dual gravity description} \\
\text{high T} & \Rightarrow \text{non-zero modes : weakly coupled (high T exp.)} \\
& \quad \text{(zero modes : integrated non-perturbatively)}
\end{cases}
\]

Kawahara-J.N.-Takeuchi, 
Fourier-mode simulation respecting SUSY maximally


\[ X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_{i,n} e^{i\omega nt} \]
\[ \omega = \frac{2\pi}{\beta} \]

Note: Gauge symmetry can be fixed non-perturbatively in 1d.

- **static diagonal gauge**:
  \[ A(t) = \frac{1}{\beta} \text{diag}(\alpha_1, \cdots, \alpha_N) \]
  \[ S_{FP} = -\sum_{a<b} 2 \ln \left| \sin \frac{\alpha_a - \alpha_b}{2} \right| \]

- **residual** gauge symmetry:
  \[ g(t) = \text{diag}(e^{i\omega \nu_1 t}, \cdots, e^{i\omega \nu_N t}) \]
  \[
  \begin{align*}
  \tilde{X}_{i,n}^{ab} &\mapsto \tilde{X}_{i,n-\nu_a+\nu_b}^{ab} \\
  \alpha_a &\mapsto \alpha_a + 2\pi \nu_a \\
  X_i &\mapsto gX_ig^\dagger \\
  A &\mapsto gAg^\dagger + ig\partial_t g^\dagger
  \end{align*}
  \]
  should be fixed by imposing
  \[ -\pi < \alpha_a \leq \pi \]

C.f.) lattice approach: Catterall-Wiseman, PRD78 (08) 041502
2. Dual gravity description and black hole thermodynamics

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 (’08) 021601 [arXiv:0707.4454]
Dual gravity description

After taking the decoupling limit: \( \alpha' \to 0 \)

\[
U \equiv \frac{r}{\alpha'}, \quad \lambda \equiv g_s N \alpha'^{-3/2} \quad \text{(fixed)}
\]

\[
f(U) \equiv \frac{U^{7/2}}{\sqrt{d_0} \lambda} \left( 1 - \left( \frac{U_0}{U} \right)^7 \right)
\]

\[
ds^2 = \alpha' \left\{ f(U) dt^2 + \frac{1}{f(U)} du^2 + \sqrt{d_0} \lambda U^{-3/2} d\Omega_5^2 \right\}
\]

range of validity: \( N^{-10/21} \ll \frac{T}{\lambda^{1/3}} \ll 1 \)

Black hole thermodynamics

\[
\begin{align*}
\text{Hawking temperature:} & \quad T = \frac{7}{16 \sqrt{15} \pi^{7/2}} \left( \frac{U_0}{\lambda^{1/3}} \right)^{5/2} \\
\text{Bekenstein-Hawking entropy:} & \quad S = \frac{1}{28 \sqrt{15} \pi^{7/2}} N^2 \left( \frac{U_0}{\lambda^{1/3}} \right)^{9/2}
\end{align*}
\]

\[
\frac{1}{N^2 \lambda^{1/3}} E = \frac{9}{14} \left( \frac{4^{13} 15^2}{7^{14}} \left( \frac{\pi}{7} \right)^{14} \right)^{1/5} \left( \frac{T}{\lambda^{1/3}} \right)^{14/5}
\]

Klebanov-Tseytlin ('96)

Jun Nishimura (KEK)
Schwarzschild radius and black hole...

2008.11.24 NBIA
Result: Internal energy

\[ E = \frac{\partial}{\partial \beta}(\beta F) \]

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 (’08) 021601 [arXiv:0707.4454]

\[ \frac{E}{N^2} \approx 7.41 T^{14/5} \]

Free energy

High T expansion (incl. next-leading order)

Result obtained from 10d BH

\[ \lambda_{\text{eff}} = \frac{\lambda}{T^3} = \frac{1}{0.46^3} \approx 10 \]
Result: Polyakov line

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 (’08) 021601 [arXiv:0707.4454]

High T expansion (including next-leading order)

Characteristic behavior of the deconfined phase

\[
\langle |P| \rangle = \exp\left( -\frac{a}{T} + b \right)
\]

no phase transition unlike in bosonic case

consistent with analyses on the gravity size (Barbon et al., Aharony et al.)
3. Higher derivative corrections to black hole thermodynamics from SUSY QM

α' corrections to type IIA SUGRA action

low energy effective action of type IIA superstring theory

leading term: type IIA SUGRA action

\[
S_{(0)} = \frac{1}{16\pi G_N} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left( R + 4\partial_\mu \phi \partial^\mu \phi \right) - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} \right\}
\]

\[G_N \sim \alpha'^4 g_s^2\]

explicit calculations of 2-pt and 3-pt amplitudes

\[S_{(1)} = S_{(2)} = 0\]

4-pt amplitudes

\[
S_{(3)} = \frac{\alpha'^3}{16\pi G_N} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} R^4 + \cdots \right\}
\]

Complete form is yet to be determined, but we can still make a dimensional analysis.
Black hole thermodynamics with $\alpha'$ corrections

Curvature radius of the dual geometry

$$\rho^2 \sim \left( \frac{\lambda^{1/3}}{U_0} \right)^{3/2} \alpha'$$

$\alpha'$ corrections

$$\frac{\alpha'}{\rho^2} \sim \left( \frac{U_0}{\lambda^{1/3}} \right)^{3/2} \sim \left( \frac{T}{\lambda^{1/3}} \right)^{3/5}$$

$$\frac{T}{\lambda^{1/3}} \sim \left( \frac{U_0}{\lambda^{1/3}} \right)^{5/2}$$

corrections at $\alpha'^3$ order gives

$$\frac{1}{N^2} \frac{E}{\lambda^{1/3}} = 7.41 \left( \frac{T}{\lambda^{1/3}} \right)^{14/5} \left\{ 1 + a \left( \frac{T}{\lambda^{1/3}} \right)^{9/5} \right\}$$

More careful treatment leads to the same conclusion. (Hanada-Hyakutake-J.N.-Takeuchi, arXiv:0811.3102)

Setting $\lambda = 1$,

$$\frac{E}{N^2} = 7.41 T^{14/5} - C T^{23/5}$$
Higher derivative corrections to black hole thermodynamics from SUSY QM

\[
\frac{E}{N^2} = 7.41 \frac{T^{14/5}}{5} - C T^{23/5}
\]

- slope = 4.6
- finite cutoff effects
- higher derivative corrections
- \(C = 5.28\)
- MC data at \(T \lesssim 0.7\) can be nicely fitted with \(C = 5.28\)
3. Schwarzchild radius from Wilson loop

Calculation of Wilson loop


\[ W = \text{tr} \mathcal{P} \exp \left[ i \int_0^\beta dt \{ A(t) + iX_9(t) \} \right] \]

\( N \) D0 branes

\( t \)

\( \text{horizon} \)

\( \text{probe D0 brane} \)

Rey-Yee (’98), Maldacena (’98)

\( \text{fundamental string} \)

gauge theory side:
propagation of a test particle coupled to \( A(t) \) and \( X_9(t) \)
Calculation of Wilson loop (cont’d)

Replace $N$ D0 branes by the black 0-brane background

$$ds^2 = \alpha' \left\{ f(U) dt^2 + \frac{1}{f(U)} dU^2 + (S^8 \text{ part}) \right\}$$

**gravity theory side:**
propagation of the string in the b.g. geometry

**string action for the minimal surface:**

$$S_{\text{string}} = \frac{1}{2\pi} \beta (U_\infty - U_0)$$
Calculation of Wilson loop (cont’d)

\[ W e^{-M\beta} = e^{-S_{\text{string}}} \]

at large \( N \) and large \( \lambda \)

perimeter-law suppression factor
due to propagation of a particle with mass \( M \)

\[ S_{\text{string}} = \frac{1}{2\pi} \beta (U_\infty - U_0) \]

\[
\log W - \beta M = \frac{\beta U_0}{2\pi} - \frac{\beta U_\infty}{2\pi}
\]

natural to identify

more sophisticated justification
a la Drukker-Gross-Ooguri (’99)

\[
\log W = \frac{\beta U_0}{2\pi} = \frac{\beta R_{\text{Sch}}}{2\pi\alpha'} = \frac{1}{2\pi} \left( \frac{16\sqrt{15}\pi^{7/2}}{7} \right)^{2/5} \left( \frac{T}{\lambda^{1/3}} \right)^{-3/5}
\]

1.89

Jun Nishimura (KEK)
Schwarzschild radius and black hole...
Results: Wilson loop

\[ W = \text{tr} \mathcal{P} \exp \left[ i \int_0^\beta dt \{ A(t) + iX_9(t) \} \right] \]


\[ \langle \log |W| \rangle \]

\[ T^{-3/5} \]

high T exp. (next-leading)

Schwarzschild radius from the Wilson loop

\[ log W = 1.89 T^{-3/5} - 4.58 \]

subleading term (perturbative corrections)
5. Summary
Summary

- Monte Carlo studies of supersymmetric large N gauge theories
  powerful method for superstring theory

- simulating superstrings inside a black hole
  based on gauge/string duality

  Black hole thermodynamics (E v.s. T relation)
  Schwarzschild radius reproduced from Wilson loop

- a highly nontrivial check of the duality
  microscopic origin of the black hole thermodynamics
  including higher derivative corrections!
Future prospects

- extension to lower SUSY case
  (Hanada-Matsuura-J.N., in progress)
  easier, and many things to explore

- higher dimensional case
  possible using mass deformation
  (Ishiki-Kim-J.N.-Tsuchiya, in progress)

- so far, planar limit
  important next step is to study non-planar limit
  Matrix theory, IIB matrix model

sign problem has to be treated carefully
  toy model  (Anagnostopoulos-Azuma-J.N., in prep.)
  6d IKKT model  (Aoyama-Azuma-Hanada-J.N., in progress)

SO(6) \rightarrow SO(3)  from Gaussian expansion  (Aoyama-J.N.-Okubo, in prep.)
6. Related on-going projects

SYM on $R \times S^2$, $R \times S^3$ from SUSY matrix QM

4d universe from 10d(11d) space-time ?
SYM on $R \times S^2$, $R \times S^3$ from SUSY matrix QM

respect SUSY maximally
c.f.) lattice approach

1) U(k) SYM on $R \times S^2$ in the planar limit

plane wave matrix model
(matrix QM with mass terms & CS terms)

around $k$ copies of the fuzzy sphere solution

$$X_a = L_a^{(n)} \otimes 1_k \quad (a = 1, 2, 3)$$

$k \to \infty$ limit removes fuzzyness
2) U(k) SYM on \( R \times S^3 \) in the planar limit

thermodynamics at strong coupling from the gravity side (Witten '98)

\( S^1 \) fibered to \( S^2 \)

plane wave matrix model
around \( k \) copies of the multi-fuzzy-sphere solution

\[
X_a = L_a \otimes 1_k \quad (a = 1, 2, 3) \quad \text{Ishii-Ishiki-Shimasaki-Tsuchiya ('08)}
\]

\[
L_a = \begin{pmatrix}
L_a^{(n)} \\
L_a^{(n+1)} \\
\vdots \\
L_a^{(n+s)}
\end{pmatrix}
\]

\( k \to \infty \) limit removes fuzzyness
Agreement at weak coupling

\[ S_{\text{PWMM}} = N \int_0^\beta dt \text{tr} \left\{ \frac{1}{2} (DX_i)^2 + \frac{1}{2} \mu^2 (X_a)^2 + \frac{1}{8} \mu^2 (X_m)^2 
+ \frac{1}{2} i \mu \epsilon_{abc} X_a [X_b, X_c] - \frac{1}{4} [X_i, X_j]^2 
+ \frac{1}{2} \psi^\dagger D \psi + \frac{3}{8} i \mu \psi^\dagger \gamma^{123} \psi 
- \frac{1}{2} \psi^\dagger \gamma^i [X_i, \psi] \right\} \]

\[ X_a = L_a \otimes 1_k \quad (a = 1, 2, 3) \]

\[ L_a = \begin{pmatrix} L_a^{(n)} \\ L_a^{(n+1)} \\ \vdots \\ L_a^{(n+s)} \end{pmatrix} \]

For \( \mu \gg 1 \) (weak coupling),

\[ \begin{cases} \text{fluctuations of } X_i \text{ and } \psi \\ \text{integrated out at } 1\text{-loop} \\ \text{gauge field } A(t) : \text{ moduli} \\ \text{integrated non-perturbatively by MC sim.} \end{cases} \]

\( \mathcal{N} = 4 \text{ SYM on } R \times S^3 \)

(Aharony et al. ’03)
Simulating Quantum Universe

Another interpretation of D0 brane quantum mechanics

(different large-N limit)

Matrix Theory
Banks-Fischler-Shenker-Susskind (’97)

microscopic description of M Theory

\( \text{c.f.) Matrix cosmology} \)
Freedman-Gibbons-Schnabl (hep-th/0411119)

How does our 4d space-time appear from 10d (11d) space-time?

\( \text{e.g.) SO(9) } \xrightarrow{\text{SSB}} \text{SO(3) ?} \)

Gaussian expansion method (GEM)
Aoyama-J.N.-Okubo-Takeuchi, in progress

application to matrix QM
Kabat-Lifschytz (‘99)

BH thermodynamics
Kabat-Lifschytz-Lowe(’00)
Gaussian Expansion Method

\[ F = -\log \left( \int d\phi \ e^{-S} \right) \]

\[ \tilde{S}(\epsilon, t) \equiv S_G(t) + \epsilon \{ S - S_G(t) \} \]

\[ \tilde{F}(\epsilon, t) \equiv -\log \left( \int d\phi \ e^{-\tilde{S}(\epsilon, t)} \right) \]

\[ \tilde{F}(\epsilon, t) = \sum_{k=0}^{\infty} \epsilon^k f_k(t) \]

\[ \tilde{F}_n(t) \equiv \sum_{k=0}^{n} f_k(t) \]

Truncate the series at \( k = n \) and set \( \epsilon = 1 \).

How to identify the plateau? if \( S_G \) contains many param.

self-consistency eq.:

\[ \frac{\partial}{\partial t} \tilde{F}_n(t) = 0 \]

Search for concentration of solutions

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Schwarzschild radius and black hole...
Dynamical generation of space-time in type IIB matrix model

\[ S = N \text{tr} \left\{ -\frac{1}{4}[X_\mu, X_\nu]^2 + \frac{1}{2} \psi_\alpha (\Gamma_\mu)_{\alpha\beta}[X_\mu, \psi_\beta] \right\} \]

Gaussian expansion method
J.N.-Sugino ('01), Kawai et al. ('01), ...

Analogous studies
in the D=6 model (less SUSY)
Aoyama-J.N.-Okubo, in prep.

more systematic studies of SSB patterns

\[ \text{SO}(6) \rightarrow \text{SSB} \rightarrow \text{SO}(3) \]

finite extra dimensions!

confirmation by MC sim.
Aoyama-Azuma-Hanada-J.N., in progress

Jun Nishimura (KEK)
Schwarzschild radius and black hole...
Results of GEM for the little IIB matrix model

Aoyama-J.N.-Okubo, in prep.

SO(5)  SO(4)  SO(3)  SO(5)  SO(4)  SO(3)  SO(5)  SO(4)  SO(3)

magnify this region

Krauth-Nicolai-Staudacher (‘98)
Results of GEM for the little IIB matrix model (cont’d)

concentration of solutions identified

suggesting:

$SO(6) \xrightarrow{SSB} SO(3)$
Results of GEM for the little IIB matrix model (cont’d)

extent of the eigenvalue distribution in the extended/shrunk direction

finite extra dimension ?!

SO(5), extended
SO(4), extended
SO(3), extended

shrunk directions
Future prospects

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