Schwarzschild Radius and Black Hole Thermodynamics with α' Corrections from Simulations of SUSY Matrix Quantum Mechanics

Jun Nishimura (KEK)

"Lattice Supersymmetry and Beyond"
at The Niels Bohr International Academy, Nov.24, '08

Ref.) Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 ('08) 021601
Hanada-Miwa-J.N.-Takeuchi, arXiv:0811.2081[hep-th]
Hanada-Hyakutake-J.N.-Takeuchi,arXiv:0811.3102[hep-th]

- based on collaborations with

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Masanori Hanada (Weizmann Inst., Israel)

Yoshifumi Hyakutake (Osaka Univ.)

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→ Harish-Chandra Research Inst., India)

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0. Introduction

Supersymmetric large-N gauge theories

suitable formulation for describing superstrings non-perturbatively

- non-pert. formulation of superstring/M theories

  e.g.) Matrix Theory (Banks-Fischler-Shenker-Susskind '97)

  IIB matrix model (Ishibashi-Kawai-Kitazawa-Tsuchiya '97)

  ➢ dynamical origin of space-time dimensionality, gauge group, matters, etc.

- gauge/string duality

  e.g.) AdS/CFT Maldacena ('97)

  ➢ quantum/stringy description of black holes in terms of gauge theories

Difficulty : Strongly coupled dynamics should be investigated !

Monte Carlo simulation

can be a powerful approach as in lattice QCD, but...

SUSY : broken on the lattice

$\{Q, \bar{Q}\} \propto P_\mu$ → translational symmetries broken to discrete ones

- Lattice formulation preserving a part of supersymmetry by using various ideas such as twisting, orbifolding, ...

  (Cohen-Kaplan-Katz-Unsal,Catterall,Sugino,,Kanamori-Suzuki, Arianos-D'Adda-Feo-Kawamoto-Saito,Nagata,Damgaard-Matsuura,...)

- Non-lattice approach respecting SUSY maximally

  systems with 16 supercharges can be studied !

1d gauge theory (SUSY matrix QM) (This talk)

non-perturbative gauge fixing + Fourier mode cutoff

$\mathcal{N} = 4$ SYM on $R \times S^2$ and $R \times S^3$

fuzzy sphere (S^2)

Supersymmetric matrix quantum mechanics

1d U(N) gauge theory with 16 supercharges

$$S = \frac{N}{\lambda} \int_0^\beta dt \; \mathrm{tr} \left\{ \frac{1}{2} (D X_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 + (\text{fermionic part}) \right\}$$

non-perturbative formulation of M theory

BFSS conjecture — Banks-Fischler-Shenker-Susskind ('97)

This talk

low energy effective theory of N D0 branes

gauge/gravity correspondence (non-conformal ver.)

Itzhaki-Maldacena-Sonnenschein-Yankielowicz ('98)

SUSY mass deformation ⟹ plane-wave matrix model

Berenstein-Maldacena-Nastase ('02)

Expanding around a (multi-)fuzzy-sphere background

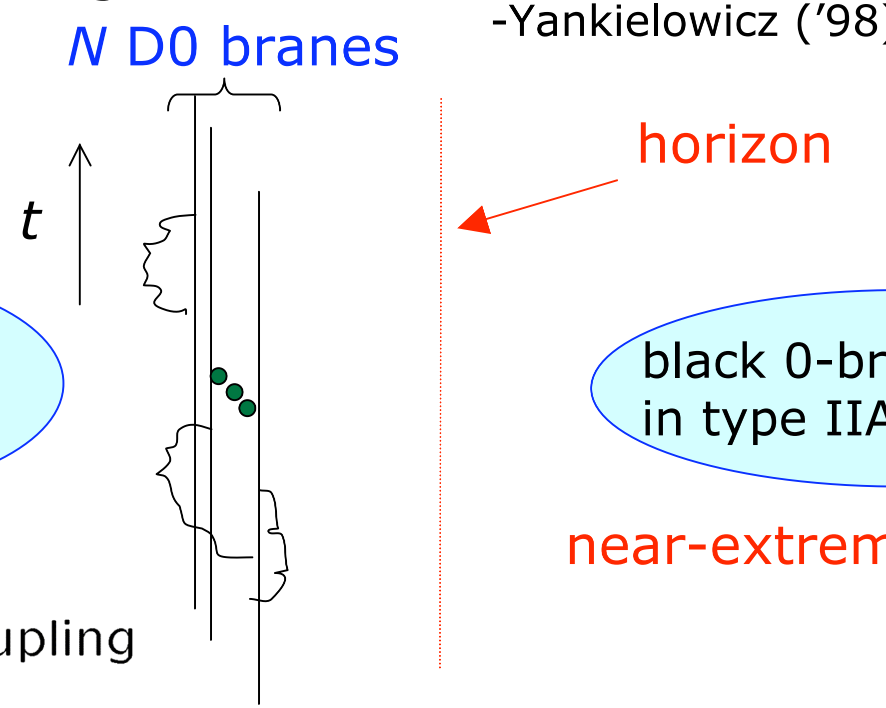
⟹ $\mathcal{N} = 4$ SYM on $R \times S^2$ and $R \times S^3$

Ishii-Ishiki-Shimasaki-Tsuchiya ('08)

Gauge-gravity duality for D0-brane system

type IIA superstring

Itzhaki-Maldacena-Sonnenschein -Yankielowicz ('98)



1d U(N) SUSY gauge theory

at finite T

λ : 't Hooft coupling

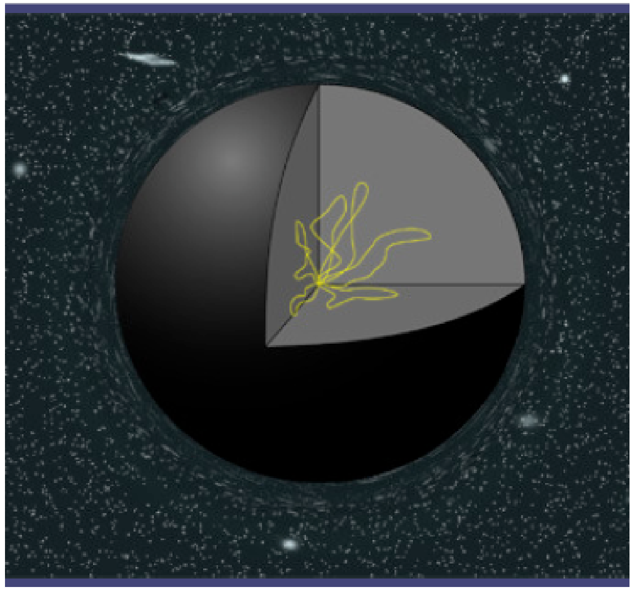
black 0-brane solution in type IIA SUGRA

near-extremal black hole

In the decoupling limit, the D0 brane system describes the black hole microscopically.

large N and large λ ⇒ SUGRA description : valid

Simulating superstrings inside a black hole



- **black hole thermodynamics**

Anagnostopoulos-Hanada-J.N.-Takeuchi ('08)

$$\frac{1}{N^2}\left(\frac{E}{\lambda^{1/3}}\right) = \underbrace{\frac{9}{14}\left\{4^{13}15^2\left(\frac{\pi}{7}\right)^{14}\right\}^{1/5}}_{7.41}\left(\frac{T}{\lambda^{1/3}}\right)^{14/5}$$

including α' corrections

Hanada-Hyakutake-J.N.-Takeuchi,arXiv:0811.3102

- **Schwarzschild radius** from **Wilson loop**

Hanada-Miwa-J.N.-Takeuchi, arXiv:0811.2081[hep-th]

$$W \equiv \mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^\beta dt\{A(t)+iX_9(t)\}\right] \sim \exp\left(\frac{\beta R_{\mathrm{Sch}}}{2\pi\alpha'}\right)$$

$$\ln W = \frac{\beta R_{\mathrm{Sch}}}{2\pi\alpha'} = \underbrace{\frac{1}{2\pi}\left\{\frac{16\sqrt{15}\pi^{7/2}}{7}\right\}^{2/5}}_{1.89}\left(\frac{T}{\lambda^{1/3}}\right)^{-3/5}$$

Plan of the talk

0. Introduction

1. Simulating SUSY matrix QM with 16 supercharges

2. Dual gravity description and black hole thermodynamics

3. Higher derivative corrections to black hole thermodynamics
   from SUSY QM

4. Schwarzschild radius from Wilson loop

5. Summary

1. Simulating SUSY QM with 16 supercharges

SUSY matrix QM with 16 supercharges

$$S_{\rm b} = \frac{N}{\lambda} \int_0^\beta dt \ {\rm tr} \left\{ \frac{1}{2}(DX_i(t))^2 - \frac{1}{4}[X_i(t), X_j(t)]^2 \right\}$$

$$S_{\rm f} = \frac{N}{\lambda} \int_0^\beta dt \ {\rm tr} \left\{ \frac{1}{2}\psi_\alpha D\psi_\alpha - \frac{1}{2}\psi_\alpha(\gamma_i)_{\alpha\beta}[X_i, \psi_\beta] \right\}$$

1d gauge theory $\quad D = \partial_t - i\,[A(t), \ \cdot \ ]$

$$\begin{cases} X_j(t) & (j = 1, \cdots, 9) \\ \psi_\alpha(t) & (\alpha = 1, \cdots, 16) \end{cases}$$

- $X_j(t)$: p.b.c.
- $\psi_\alpha(t)$: anti p.b.c.

$T = \beta^{-1}$ temperature

$\lambda = g^2 N$ 't Hooft coupling

$$\lambda_{\rm eff} = \frac{\lambda}{T^3}$$

$\lambda = 1$ (without loss of generality)

- low T ⇨ strongly coupled — dual gravity description
- high T ⇨ non-zero modes : weakly coupled (high T exp.)
  (zero modes : integrated non-perturbatively)

Kawahara-J.N.-Takeuchi,
JHEP 0712 (2007) 103, arXiv:0710.2188[hep-th]

Fourier-mode simulation respecting SUSY maximally

Hanada-J.N.-Takeuchi, PRL 99 (07) 161602 [arXiv:0706.1647]

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_{i,n} \mathrm{e}^{i\omega n t} \qquad \omega = \frac{2\pi}{\beta}$$

Note: Gauge symmetry can be fixed non-perturbatively in 1d.

- static diagonal gauge :

$$A(t) = \frac{1}{\beta}\mathrm{diag}(\alpha_1, \cdots, \alpha_N)$$

$$S_{\mathrm{FP}} = -\sum_{a<b} 2\ln\left|\sin\frac{\alpha_a - \alpha_b}{2}\right|$$

- residual gauge symmetry : $g(t) = \mathrm{diag}(\mathrm{e}^{i\omega\nu_1 t}, \cdots, \mathrm{e}^{i\omega\nu_N t})$

$$\begin{cases} \tilde{X}^{ab}_{i,n} \mapsto \tilde{X}^{ab}_{i,n-\nu_a+\nu_b} \\ \alpha_a \mapsto \alpha_a + 2\pi\nu_a \end{cases} \qquad \begin{aligned} X_i &\mapsto g X_i g^\dagger \\ A &\mapsto g A g^\dagger + i g \partial_t g^\dagger \end{aligned}$$

should be fixed by imposing $-\pi < \alpha_a \leq \pi$

c.f.) lattice approach : Catterall-Wiseman, PRD78 (08) 041502

2. Dual gravity description and black hole thermodynamics

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 ('08) 021601 [arXiv:0707.4454]

Dual gravity description

After taking the decoupling limit : $\alpha' \to 0$

$$U \equiv \frac{r}{\alpha'} \quad , \quad \lambda \equiv g_s N \alpha'^{-3/2} \quad \text{(fixed)}$$

$$f(U) \equiv \frac{U^{7/2}}{\sqrt{d_0 \lambda}} \left\{ 1 - \left(\frac{U_0}{U} \right)^7 \right\}$$

$$ds^2 = \alpha' \left\{ f(U) dt^2 + \frac{1}{f(U)} dU^2 + \sqrt{d_0 \lambda} U^{-3/2} d\Omega^2_{(8)} \right\}$$

range of validity: $N^{-10/21} \ll \frac{T}{\lambda^{1/3}} \ll 1$

Black hole thermodynamics

- Hawking temperature : $\frac{T}{\lambda^{1/3}} = \frac{7}{16\sqrt{15}\pi^{7/2}} \left(\frac{U_0}{\lambda^{1/3}} \right)^{5/2}$
- Bekenstein-Hawking entropy : $S = \frac{1}{28\sqrt{15}\pi^{7/2}} N^2 \left(\frac{U_0}{\lambda^{1/3}} \right)^{9/2}$

$$\Rightarrow \quad \frac{1}{N^2} \frac{E}{\lambda^{1/3}} = \underbrace{\frac{9}{14} \left\{ 4^{13} 15^2 \left(\frac{\pi}{7} \right)^{14} \right\}^{1/5}}_{7.41} \left(\frac{T}{\lambda^{1/3}} \right)^{14/5}$$

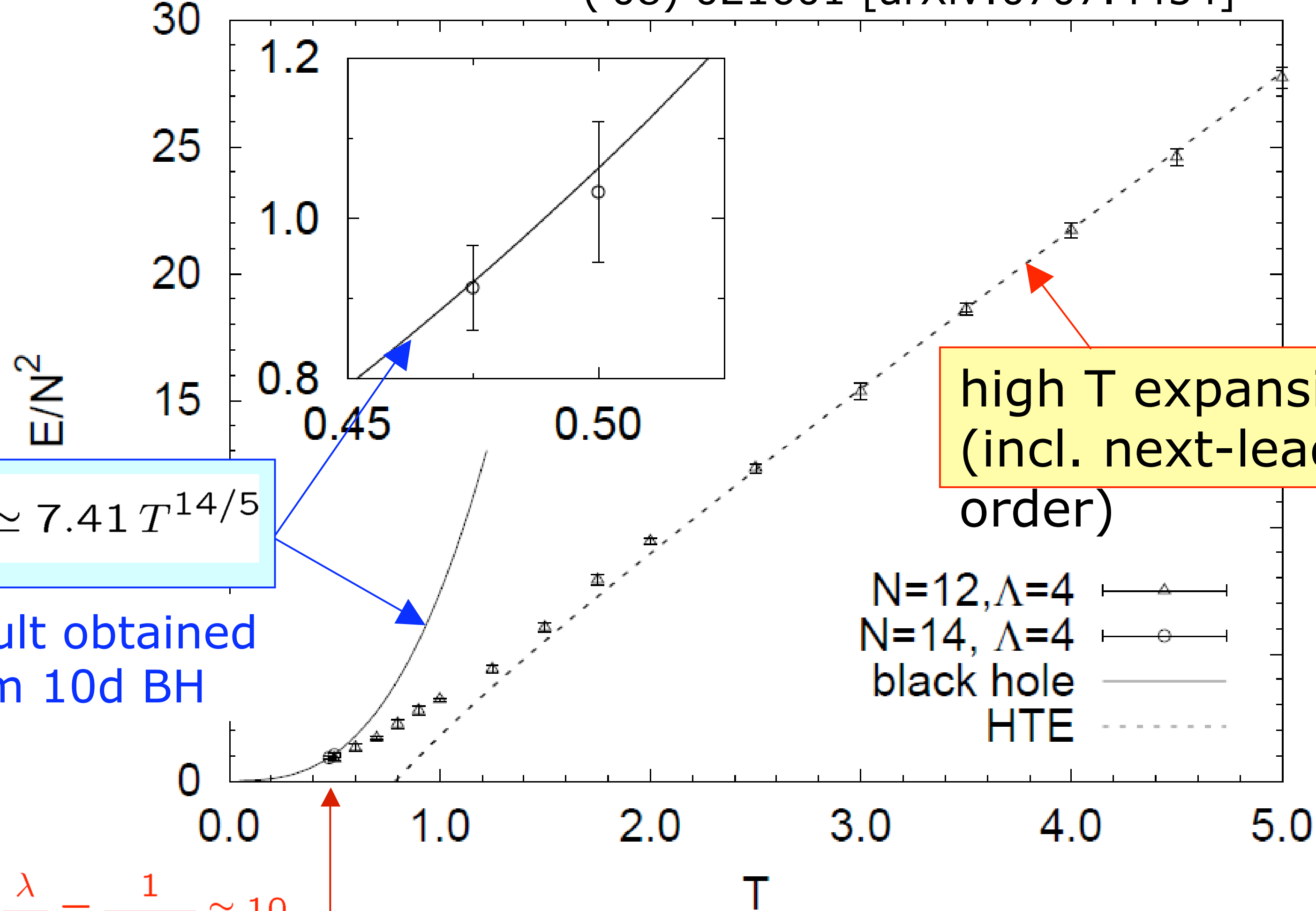
Klebanov-Tseytlin ('96)

Result: Internal energy

$$E = \frac{\partial}{\partial \beta}(\beta \mathcal{F})$$

free energy

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 ('08) 021601 [arXiv:0707.4454]



high T expansion (incl. next-leading order)

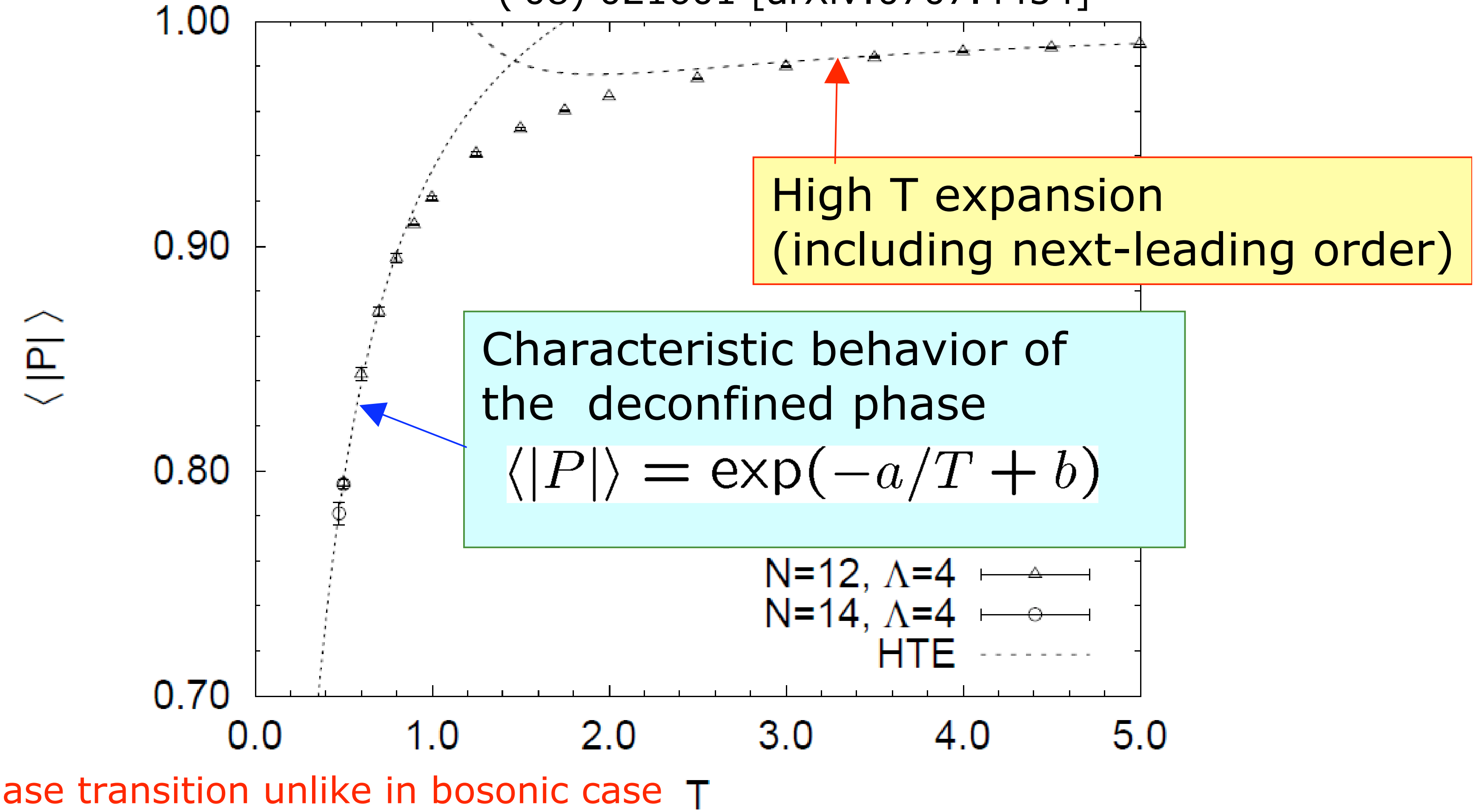
$$\frac{E}{N^2} \simeq 7.41\, T^{14/5}$$

result obtained from 10d BH

$$\lambda_{\rm eff} = \frac{\lambda}{T^3} = \frac{1}{0.46^3} \approx 10$$

Result: Polyakov line

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 ('08) 021601 [arXiv:0707.4454]



no phase transition unlike in bosonic case

⇨ consistent with analyses on the gravity size (Barbon et al., Aharony et al.)

3. Higher derivative corrections to black hole thermodynamics from SUSY QM

Hanada-Hyakutake-J.N.-Takeuchi, arXiv:0811.3102[hep-th]

α' corrections to type IIA SUGRA action

low energy effective action of type IIA superstring theory

⇦ tree-level scattering amplitudes of the massless modes

leading term : type IIA SUGRA action

$$\mathcal{S}_{(0)} = \frac{1}{16\pi G_{\mathrm{N}}} \int d^{10}x \sqrt{-g} \left\{ \mathrm{e}^{-2\phi}(R + 4\partial_\mu \phi \partial^\mu \phi) - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} \right\}$$

$$G_{\mathrm{N}} \sim \alpha'^4 g_s^2$$

explicit calculations of 2-pt and 3-pt amplitudes

⇨ $\mathcal{S}_{(1)} = \mathcal{S}_{(2)} = 0$

4-pt amplitudes

⇨ $$\mathcal{S}_{(3)} = \frac{\alpha'^3}{16\pi G_{\mathrm{N}}} \int d^{10}x \sqrt{-g} \left\{ \mathrm{e}^{-2\phi} \mathcal{R}^4 + \cdots \right\}$$

Complete form is yet to be determined,
but we can still make a dimensional analysis.

Black hole thermodynamics with α' corrections

curvature radius of the dual geometry

$$\rho^2 \sim \left(\frac{\lambda^{1/3}}{U_0}\right)^{3/2} \alpha'$$

α' corrections

$$\Rightarrow \quad \frac{\alpha'}{\rho^2} \sim \left(\frac{U_0}{\lambda^{1/3}}\right)^{3/2} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{3/5} \qquad \frac{T}{\lambda^{1/3}} \sim \left(\frac{U_0}{\lambda^{1/3}}\right)^{5/2}$$

corrections at α'^3 order gives

$$\frac{1}{N^2}\frac{E}{\lambda^{1/3}} = 7.41 \left(\frac{T}{\lambda^{1/3}}\right)^{14/5} \left\{1 + a\left(\frac{T}{\lambda^{1/3}}\right)^{9/5}\right\}$$

More careful treatment leads to the same conclusion.
(Hanada-Hyakutake-J.N.-Takeuchi,arXiv:0811.3102)

Setting $\lambda = 1$,

$$\frac{E}{N^2} = 7.41\, T^{14/5} - C\, T^{23/5}$$

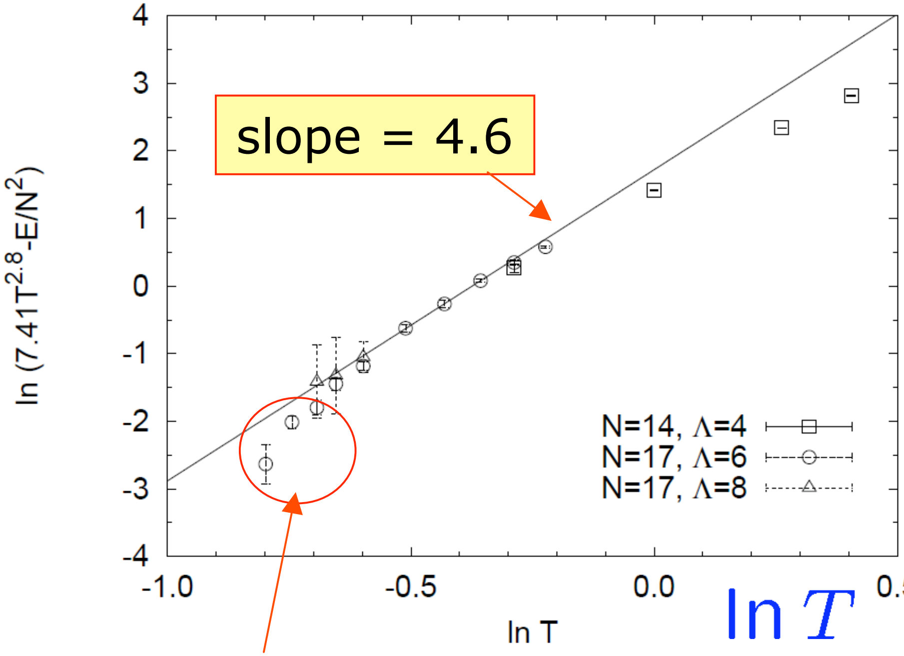
Higher derivative corrections to black hole thermodynamics from SUSY QM

Hanada-Hyakutake-J.N.-Takeuchi, arXiv:0811.3102[hep-th]

$$\frac{E}{N^2} = 7.41\, T^{14/5} - C\, T^{23/5}$$

higher derivative corrections

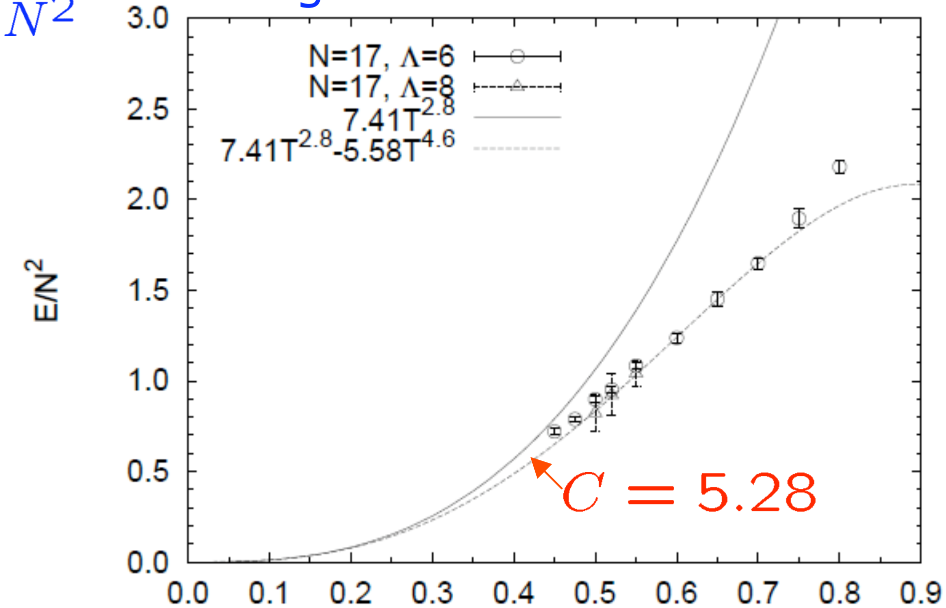
$\ln\left(7.41\, T^{14/5} - \frac{E}{N^2}\right)$



$\ln T$

finite cutoff effects

$\frac{E}{N^2}$



$C = 5.28$

T

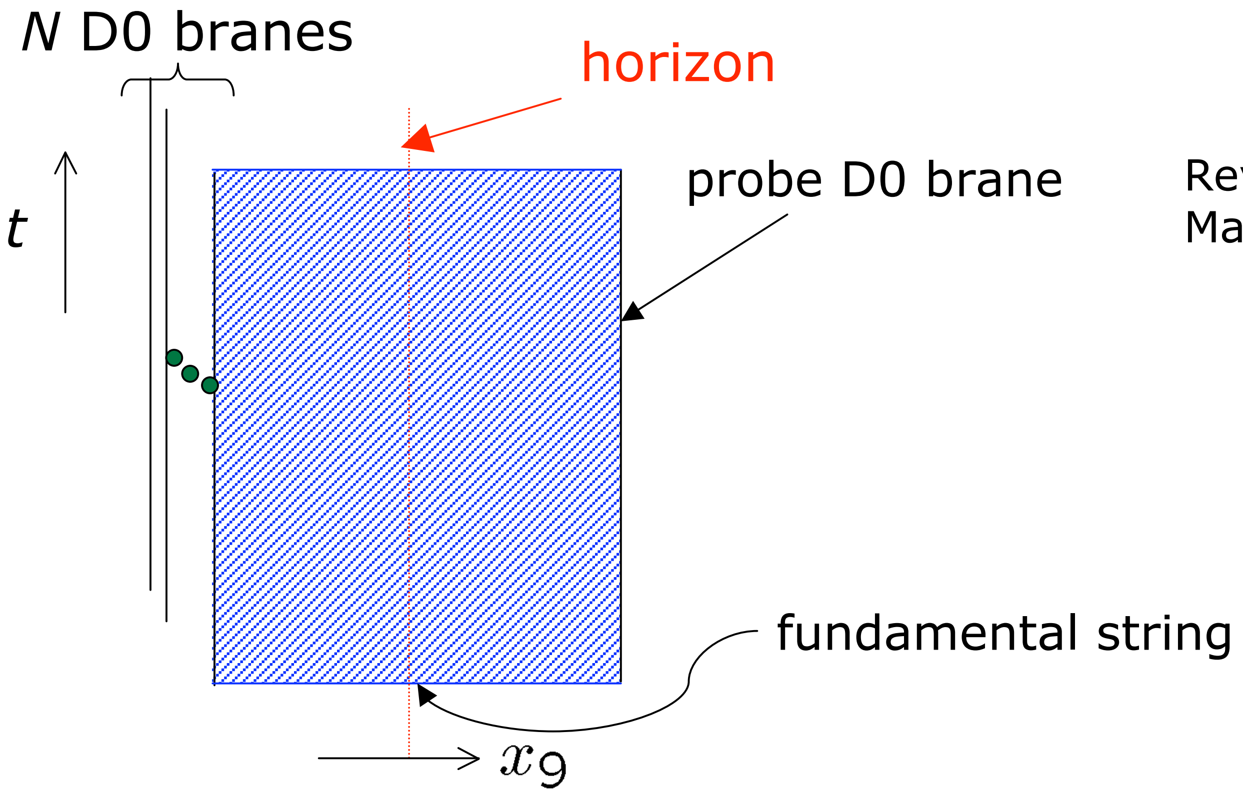
MC data at $T \lesssim 0.7$ can be nicely fitted with $C = 5.28$

3. Schwarzschild radius from Wilson loop

Hanada-Miwa-J.N.-Takeuchi , arXiv:0811.2081[hep-th]

Calculation of Wilson loop

Hanada-Miwa-J.N.-Takeuchi, arXiv:0811.2081[hep-th]



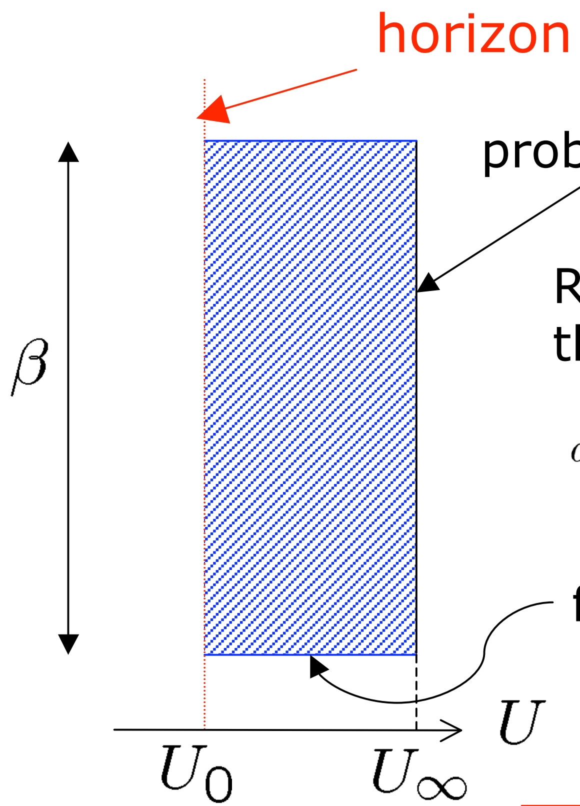
Rey-Yee ('98),
Maldacena ('98)

gauge theory side :

propagation of a test particle coupled to $A(t)$ and $X_9(t)$

$$W = \mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^\beta dt\{A(t) + iX_9(t)\}\right]$$

Calculation of Wilson loop (cont'd)



Replace N D0 branes by the black 0-brane background

$$ds^2 = \alpha' \left\{ f(U)dt^2 + \frac{1}{f(U)}dU^2 + (S^8 \text{ part}) \right\}$$

<u>gravity theory side :</u>

propagation of the string in the b.g. geometry

string action for the minimal surface :

$$S_{\text{string}} = \frac{1}{2\pi}\beta\,(U_\infty - U_0)$$

Calculation of Wilson loop (cont'd)

$$W\, e^{-M\beta} = e^{-S_{\text{string}}}$$ at large N and large λ

perimeter-law suppression factor
due to propagation of a particle with mass *M*

$$S_{\text{string}} = \frac{1}{2\pi}\beta\left(U_\infty - U_0\right)$$

$$\log W - \beta M = \frac{\beta U_0}{2\pi} - \frac{\beta U_\infty}{2\pi}$$

natural to identify

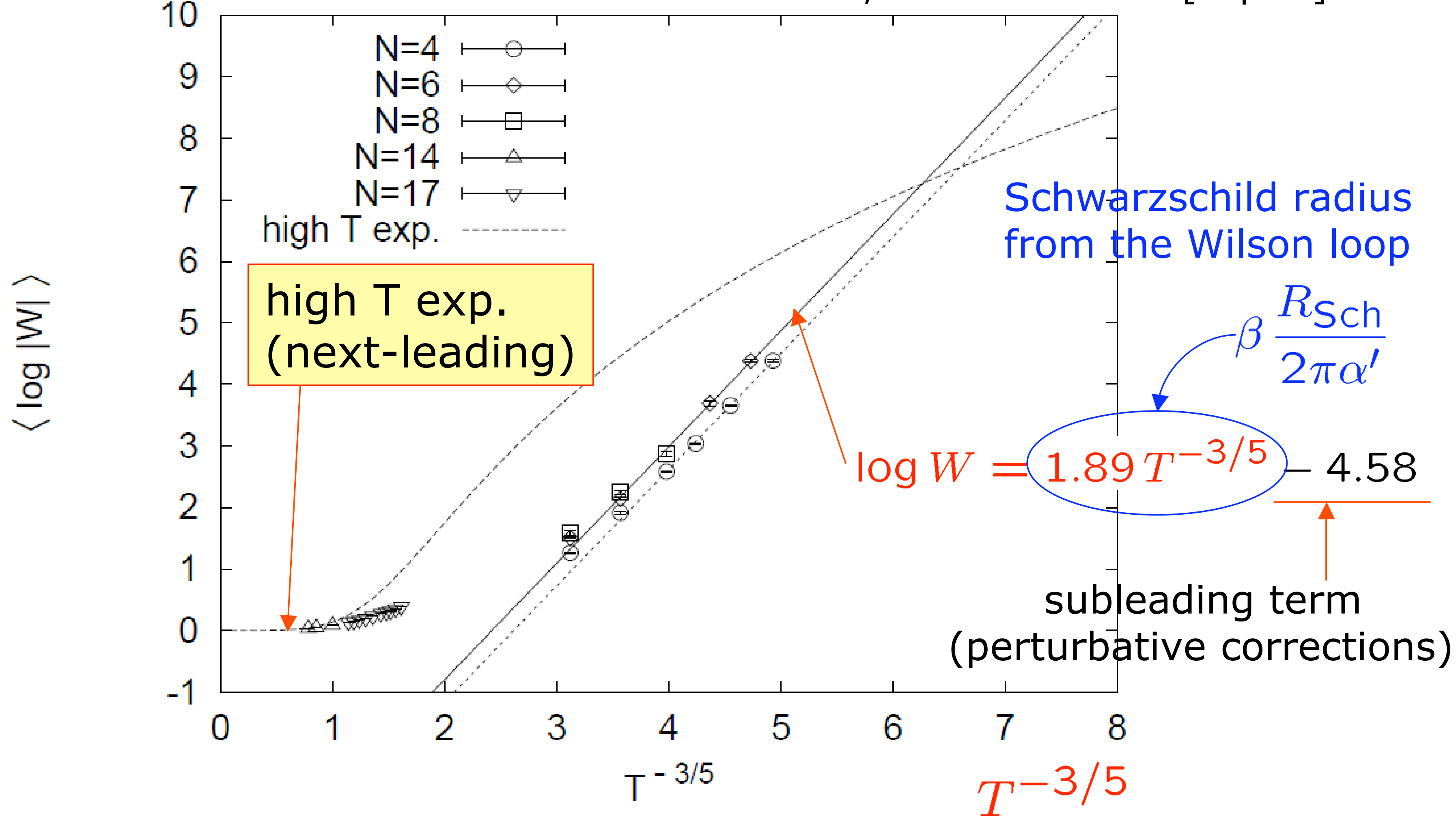
more sophisticated justification
a la Drukker-Gross-Ooguri ('99)

$$\log W = \frac{\beta U_0}{2\pi} = \frac{\beta R_{\text{Sch}}}{2\pi\alpha'} = \underbrace{\frac{1}{2\pi}\left\{\frac{16\sqrt{15}\pi^{7/2}}{7}\right\}^{2/5}}_{1.89}\left(\frac{T}{\lambda^{1/3}}\right)^{-3/5}$$

Results: Wilson loop

$$W = \mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^\beta dt\{A(t)+iX_9(t)\}\right]$$

Hanada-Miwa-J.N.-Takeuchi, arXiv:0811.2081[hep-th]



5. Summary

Summary

- Monte Carlo studies of supersymmetric large N gauge theories

  ⇨ powerful method for superstring theory

- simulating superstrings inside a black hole

  based on gauge/string duality

  { Black hole thermodynamics (E v.s. T relation)
    Schwarzschild radius reproduced from Wilson loop

- a highly nontrivial check of the duality

  microscopic origin of the black hole thermodynamics

  including higher derivative corrections !

Future prospects

- extension to lower SUSY case

  (Hanada-Matsuura-J.N., in progress)

  easier, and many things to explore

- higher dimensional case

  possible using mass deformation

  (Ishiki-Kim-J.N.-Tsuchiya, in progress)

- so far, planar limit

  important next step is to study non-planar limit

  Matrix theory, IIB matrix model

  sign problem has to be treated carefully

  toy model (Anagnostopoulos-Azuma-J.N., in prep.)

  6d IKKT model (Aoyama-Azuma-Hanada-J.N., in progress)

SO(6) ⇨ SO(3) from Gaussian expansion (Aoyama-J.N.-Okubo, in prep.)

6. Related on-going projects

SYM on $R \times S^2$, $R \times S^3$ from SUSY matrix QM

4d universe from 10d(11d) space-time ?

SYM on $R\times S^2,\ R\times S^3$ from SUSY matrix QM

respect SUSY maximally   c.f.) lattice approach

1) U(k) SYM on $R \times S^2$ in the planar limit

(dual geometry Lin-Maldacena ('05))

Fourier mode cut off

fuzzy sphere (matrix regularization)

plane wave matrix model

(matrix QM with mass terms & CS terms)

around k copies of the fuzzy sphere solution

$$X_a = L_a^{(n)} \otimes \mathbf{1}_k \quad (a=1,2,3)$$

$k\to\infty$ limit removes fuzzyness

SYM on $R\times S^2$, $R\times S^3$ from SUSY matrix QM (cont'd)

2) U(k) SYM on $R \times S^3$ in the planar limit

Fourier mode cut off

S^1 fibered to S^2

thermodynamics at strong coupling from the gravity side (Witten '98)

plane wave matrix model
around k copies of the multi-fuzzy-sphere solution

$$X_a = L_a \otimes \mathbf{1}_k \quad (a=1,2,3)$$ Ishii-Ishiki-Shimasaki-Tsuchiya ('08)

$$L_a = \begin{pmatrix} L_a^{(n)} & & & \\ & L_a^{(n+1)} & & \\ & & \ddots & \\ & & & L_a^{(n+s)} \end{pmatrix}$$

$k \to \infty$ limit removes fuzzyness

Agreement at weak coupling

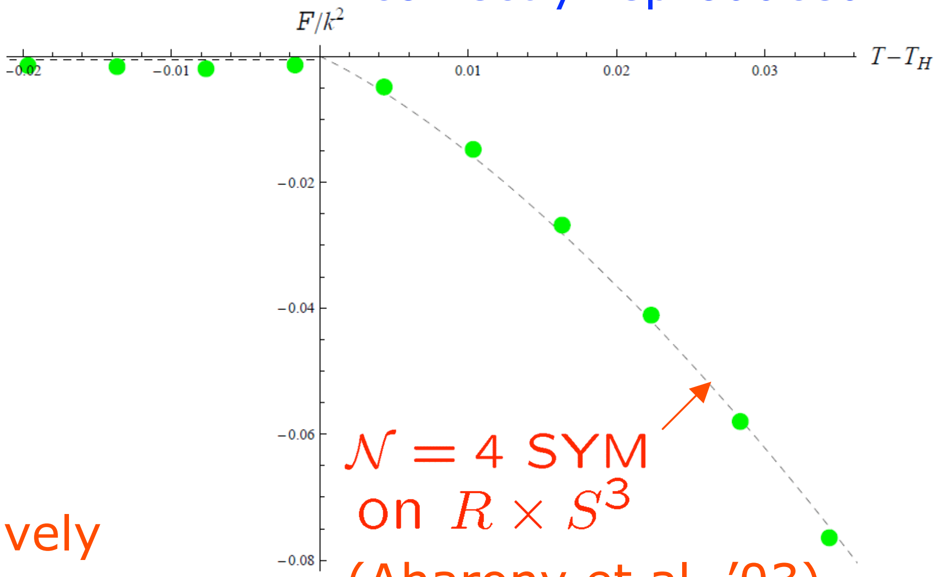
Ishiki-Kim-J.N.-Tsuchiya, arXiv:0810.2884[hep-th]

$$S_{\rm PWMM} = N\int_0^\beta dt\,{\rm tr}\Big\{\frac{1}{2}(DX_i)^2 + \frac{1}{2}\mu^2(X_a)^2 + \frac{1}{8}\mu^2(X_m)^2 + \frac{1}{2}i\mu\epsilon_{abc}X_a[X_b,X_c] - \frac{1}{4}[X_i,X_j]^2 + \frac{1}{2}\psi^\dagger D\psi + \frac{3}{8}i\mu\psi^\dagger\gamma^{123}\psi - \frac{1}{2}\psi^\dagger\gamma^i[X_i,\psi]\Big\}$$

$$X_a = L_a \otimes \mathbf{1}_k \quad (a=1,2,3)$$

$$L_a = \begin{pmatrix} L_a^{(n)} & & & \\ & L_a^{(n+1)} & & \\ & & \ddots & \\ & & & L_a^{(n+s)} \end{pmatrix}$$

deconfinement transition correctly reproduced!



$\mathcal{N}=4$ SYM on $R\times S^3$ (Aharony et al. '03)

For $\mu \gg 1$ (weak coupling),

- fluctuations of X_i and ψ integrated out at 1-loop
- gauge field $A(t)$: moduli integrated non-perturbatively by MC sim.

Simulating Quantum Universe

Another interpretation of D0 brane quantum mechanics

(different large-N limit)

Matrix Theory

Banks-Fischler-Shenker-Susskind ('97)

microscopic description of M Theory

> c.f.) Matrix cosmology
> Freedman-Gibbons-Schnabl (hep-th/0411119)

How does our 4d space-time appear from 10d (11d) space-time ?

e.g.) SO(9) $\xrightarrow{\text{SSB}}$ SO(3) ?

Gaussian expansion method (GEM) → application to matrix QM

Kabat-Lifschytz ('99)

BH thermodynamics

Kabat-Lifschytz-Lowe('00)

Aoyama-J.N.-Okubo-Takeuchi, in progress

Gaussian Expansion Method

$$F = -\log\left(\int d\phi\, \mathrm{e}^{-S}\right) \qquad S = \frac{1}{4} N\, \mathrm{tr}\phi^4$$

$$\tilde{S}(\epsilon, t) \equiv S_{\mathrm{G}}(t) + \epsilon\{S - S_{\mathrm{G}}(t)\} \qquad S_{\mathrm{G}}(t) = \frac{1}{2} N t\, \mathrm{tr}\phi^2$$

$$\tilde{F}(\epsilon, t) \equiv -\log\left(\int d\phi\, \mathrm{e}^{-\tilde{S}(\epsilon,t)}\right) \qquad F = \tilde{F}(1, t)$$

indep. of t

$$\tilde{F}(\epsilon, t) = \sum_{k=0}^{\infty} \epsilon^k f_k(t)$$

Truncate the series at $k = n$ and set $\epsilon = 1$.

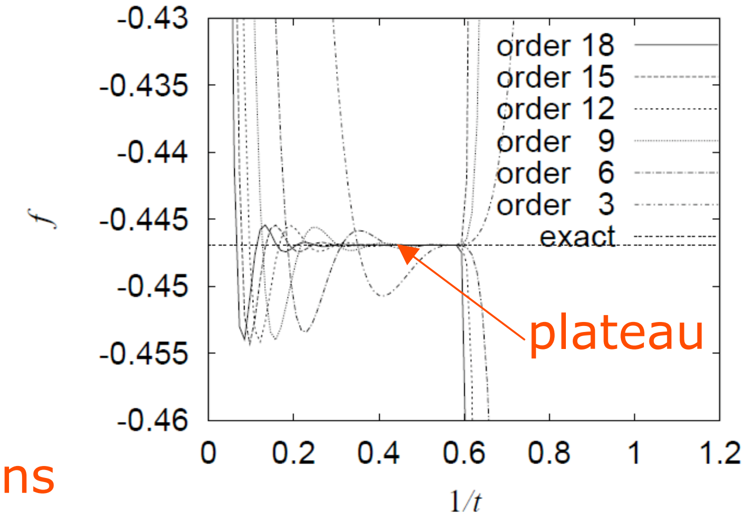
$$\tilde{F}_n(t) \equiv \sum_{k=0}^{n} f_k(t)$$

How to identify the plateau ?
if S_{G} contains many param.

self-consistency eq.:

$$\frac{\partial}{\partial t} \tilde{F}_n(t) = 0$$

Search for concentration of solutions



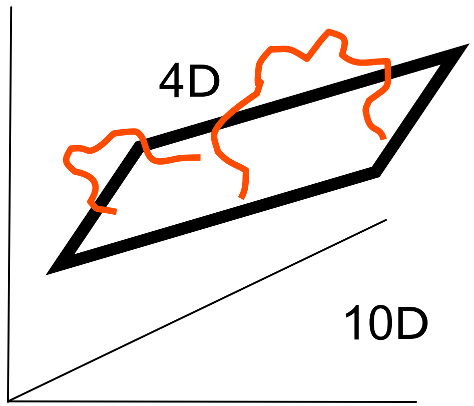
Dynamical generation of space-time in type IIB matrix model

$$S = N\mathrm{tr}\left\{-\frac{1}{4}[X_\mu, X_\nu]^2 + \frac{1}{2}\psi_\alpha(\Gamma_\mu)_{\alpha\beta}[X_\mu, \psi_\beta]\right\}$$

Gaussian expansion method

J.N.-Sugino ('01),
Kawai et al. ('01),...

Eigenvalue distribution of X_μ



Analogous studies
in the D=6 model (less SUSY)
Aoyama-J.N.-Okubo, in prep.

more systematic studies of SSB patterns

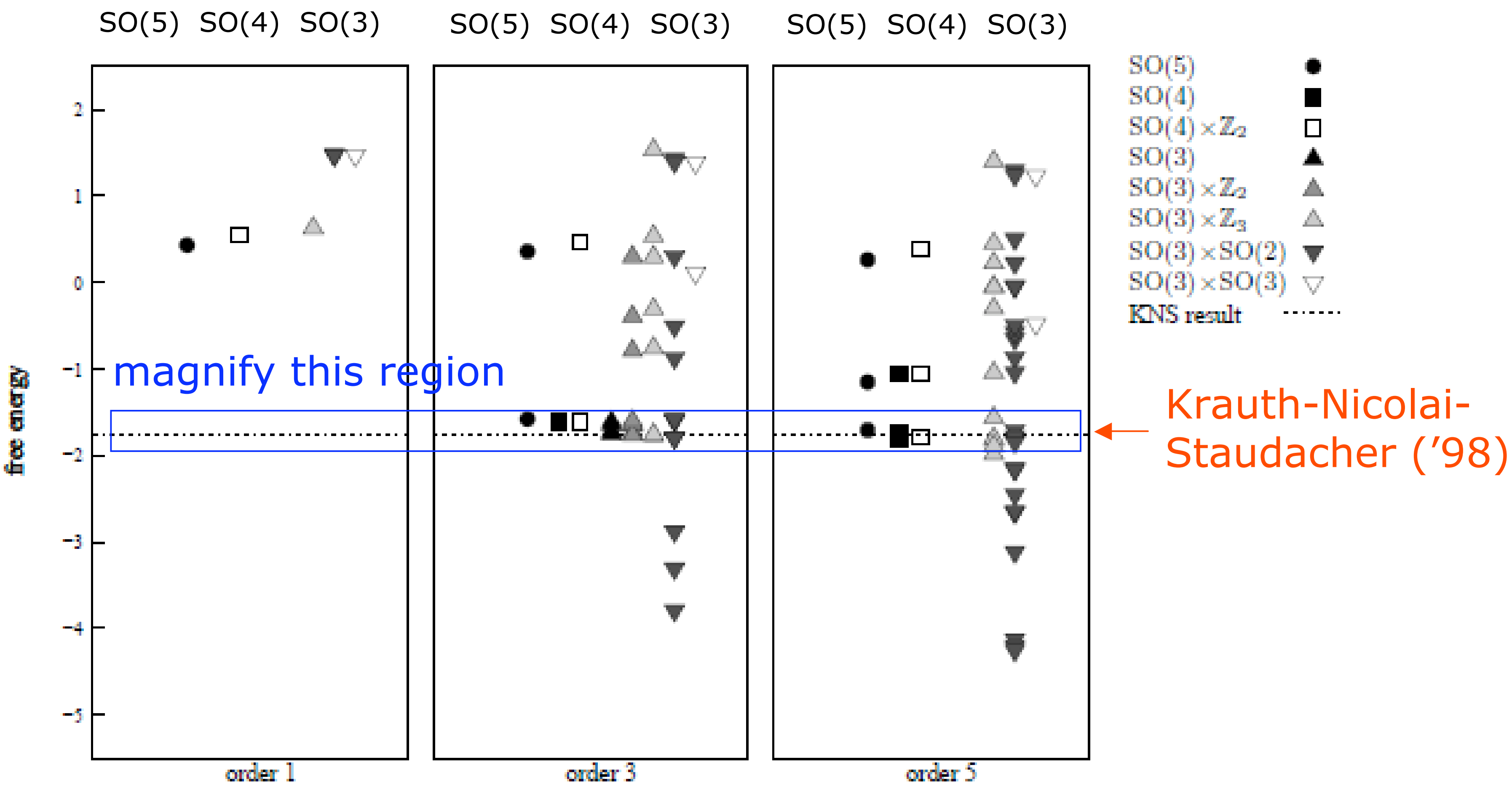
SO(6) $\xrightarrow{\text{SSB}}$ SO(3)

finite extra dimensions !

confirmation by MC sim.
Aoyama-Azuma-Hanada-J.N.,
in progress

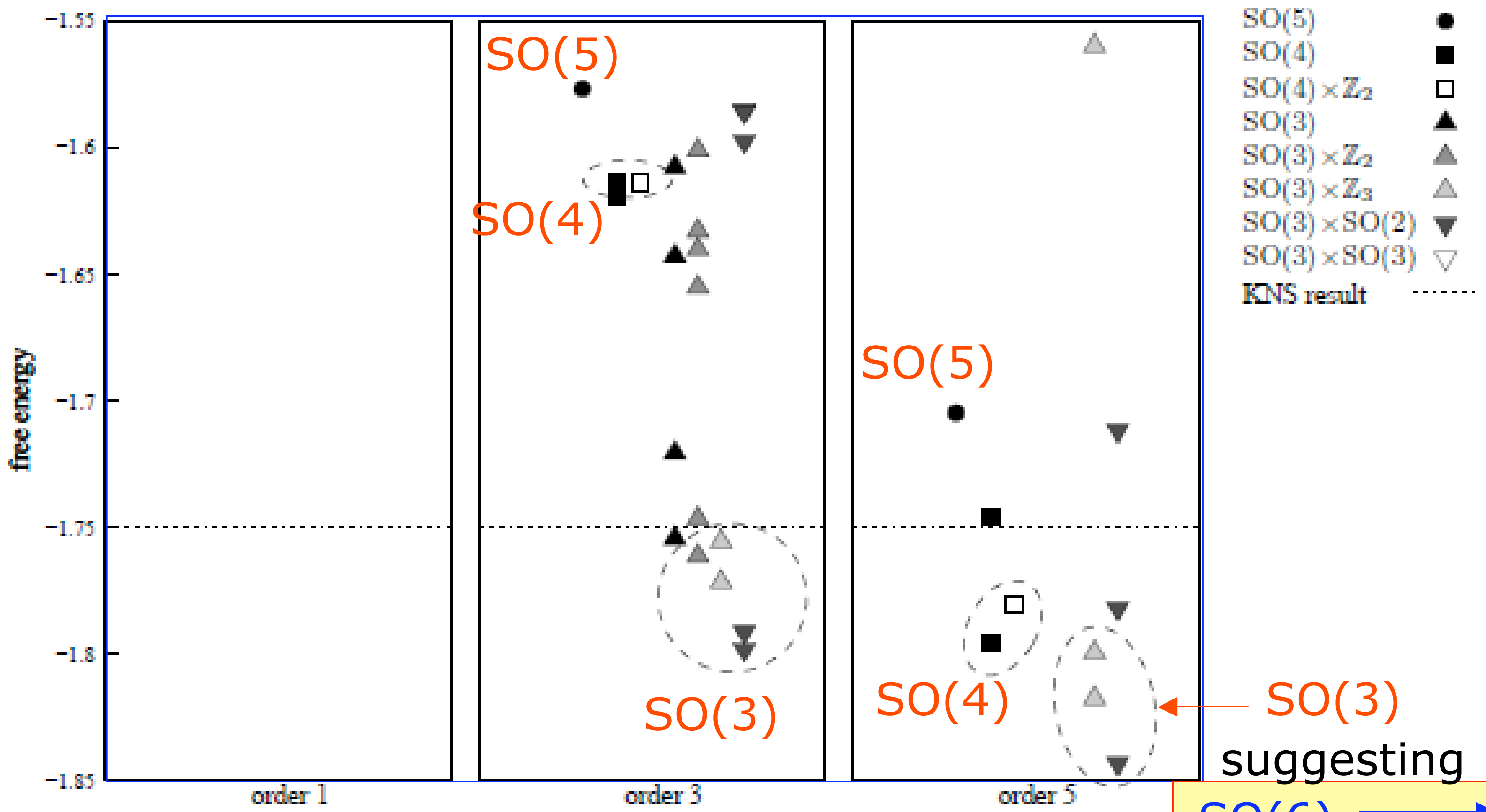
Results of GEM for the little IIB matrix model

Aoyama-J.N.-Okubo, in prep.



Results of GEM for the little IIB matrix model (cont'd)

concentration of solutions identified

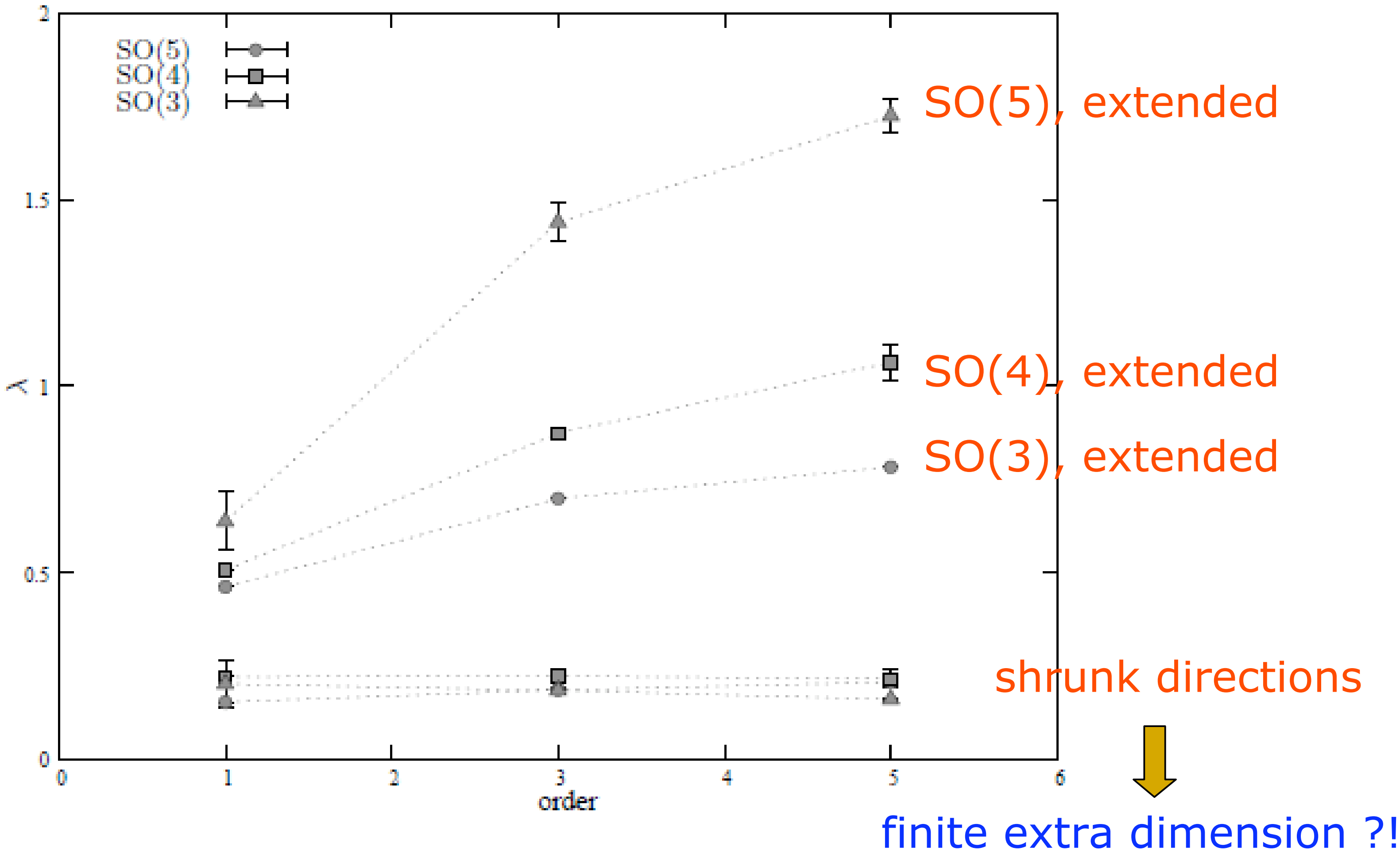


suggesting :

SO(6) $\xrightarrow{\text{SSB}}$ SO(3)

Results of GEM for the little IIB matrix model (cont'd)

extent of the eigenvalue distribution
in the extended/shrunk direction



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  toy model (Anagnostopoulos-Azuma-J.N., in prep.)

  6d IKKT model (Aoyama-Azuma-Hanada-J.N., in progress)

SO(6) ⇒ SO(3) from Gaussian expansion (Aoyama-J.N.-Okubo, in prep.)