Hand-in exam

for the course in Particle Physics Phenomenology

Below is a list of exercises, adding up to a maximum of 40 points. At least half of that, 20 points, is required to pass the exam. Partial answers give points in rough proportion to how close they are to a correct solution. To allow an individual assessment, collaboration should be held at a minimum. (Discussing interpretation of the exercises is acceptable, copying solutions is not.)

Unless otherwise specified, you are free to do sensible approximations (but motivate them!). If you find ambiguities in how to address a problem, then notice this and explain the choices you make. When computer programs are used to obtain results, they should be appended to the solution. The code should contain enough comments to be understandable, generally be well structured, and preferably in C, C++, C#, Java or Fortran.

Last date to hand in the exam is Tuesday 15 November 2011. Either send it
by e-mail to torbjorn@thep.lu.se (scanned handwritten is acceptable), or
by snail mail to Torbjörn Sjöstrand, Department of Astronomy and Theoretical Physics, Sölvegatan 14A, SE-223 62 Lund, Sweden.

Note that, in addition to the exercises, you are also expected to rehearse/digest the course contents, making use of the slides and the literature list.

1. (2 p) What is the expected maximal spatial and temporal extent of the hadronization region for events at the LHC at 7 TeV?

2. (2 p) Show that, for a $2 \to 2$ process with massive incoming and outgoing particles,

$$s + t + u = \sum m_i^2 ,$$

   where the sum runs over all of them.

3. (2 p) Show by explicit addition of two massless four-vectors with fixed $E_{\perp 1}$ and $E_{\perp 2}$ that, for small separations, their invariant mass only depends on R, rather than on $\Delta\eta$ and $\Delta\varphi$ separately.

4. (3 p) A Higgs particle H^0 may be produced by several different subprocesses at the LHC. Two of the most important ones are $\mathrm{gg} \to \mathrm{H}^0$ and $\mathrm{W}^+\mathrm{W}^- \to \mathrm{H}^0$ (where the W's are generated by branchings $\mathrm{q} \to \mathrm{q}'\mathrm{W}$, with the q' retaining most of the energy and thus moving close to the original q direction). Compare the colour structure of the events in the two processes and discuss what differences this could lead to in particle production patterns. Initially neglect multiparton interactions and thereafter comment what changes MPI's could lead to.

5. (3 p) Assume that particles are produced flat in rapidity, $\mathrm{d}n/\mathrm{d}y = 1$, between $-5 < y < 5$, and with a fixed $p_\perp = 0.5$ GeV. Show the pseudorapidity distribution $\mathrm{d}n/\mathrm{d}\eta$ for the three cases that $m = 0.14$, 0.50 and 0.94 GeV (roughly π, K and p, respectively). Specifically, provide $(\mathrm{d}n/\mathrm{d}\eta)/(\mathrm{d}n/\mathrm{d}y)$ at $y = 0$ and the η range in the three cases.

6. (5 p) Consider a part of an event, consisting of three massless particles, all at $\varphi = 0$, but with (pseudo)rapidities y and transverse momenta as follows:

   $y_1 = -0.5, \quad p_{\perp 1} = 10;$
   $y_2 = 0.0, \quad p_{\perp 2} = 80;$
   $y_3 = 0.4, \quad p_{\perp 3} = 60.$

   In which order would these be joined, and resulting in which final jet y and $p_\perp$, for each of the $k_\perp$, Cambridge/Aachen and anti-$k_\perp$ clustering algorithms? The jet size parameter is $R = 0.6$, and the standard E-scheme should be used for recombinations.

7. (5 p) A stable massless quark radiates soft gluons according to the approximate expression

$$\mathrm{d}\mathcal{P} \approx C_\mathrm{F} \frac{\alpha_\mathrm{s}}{2\pi} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{2\mathrm{d}\omega}{\omega} \,, \tag{1}$$

   where θ is the gluon emission angle and $\omega = E_g$. If the quark is massive, this expression is changed to

$$\mathrm{d}\mathcal{P} \approx C_\mathrm{F} \frac{\alpha_\mathrm{s}}{2\pi} \frac{\theta^2\,\mathrm{d}\theta^2}{(\theta^2 + 1/\gamma^2)^2} \frac{2\mathrm{d}\omega}{\omega} \,, \tag{2}$$

   where $\gamma = E/m$ is the ordinary Lorentz boost factor of the quark. Finally, if the quark additionally is unstable and has a width Γ, the expression further generalizes to

$$\mathrm{d}\mathcal{P} \approx C_\mathrm{F} \frac{\alpha_\mathrm{s}}{2\pi} \frac{\theta^2\,\mathrm{d}\theta^2}{\left(\theta^2 + \frac{1}{\gamma^2}\right)^2 + \left(\frac{\Gamma}{\gamma\omega}\right)^2} \frac{2\mathrm{d}\omega}{\omega} \,, \tag{3}$$

   a) Compare qualitatively the shape (including peak positions and relative normalizations) of the three alternatives above for the radiation pattern in squared angle $\mathrm{d}\mathcal{P}/\mathrm{d}\theta^2$ for gluon energies $\omega \ll \Gamma$.

   b) Ditto for $\omega \approx \Gamma$.

   c) Ditto for $\omega \gg \Gamma$.

   d) A top quark of mass $m_t \approx 175$ GeV has a width of $\Gamma_t \approx 1.5$ GeV. Consider a top kicked out with roughly 500 GeV energy (in the subcollision frame). At what angles is the radiation pattern peaked for the emission of 1 GeV and 10 GeV gluons, respectively?

8. (6 p) Compare the jet broadening induced in ordinary $e^+e^- \to \mathrm{q\overline{q}}$ events by perturbative gluon emission and by nonperturbative fragmentation. You are allowed to simplify as follows.

   - For the perturbative emission, calculate the $p_\perp$ spectrum of gluons emitted from the primary $\mathrm{q\overline{q}}$ dipole only, e.g. using the ARIADNE-style

     $$\frac{\mathrm{d}N}{\mathrm{d}p_\perp^2\,\mathrm{d}y} \approx \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{4}{3}\,\frac{2}{p_\perp^2}$$

     emission rate and an approximate dipole phase space $p_{\perp\mathrm{max}} \approx \sqrt{s}/2$, $y_\mathrm{max} \approx \ln(\sqrt{s}/2p_\perp)$. (Thus neglect the extra emission off gluons, as well as Sudakov factors, energy–momentum conservation and coherence effects.) Use a fixed but sensible α_s.
   - For the nonperturbative hadronization, consider the Gaussian $p_\perp$ spectrum of primary hadrons from a simple $\mathrm{q\overline{q}}$ string, where particles can be assumed produced with a flat distribution $\mathrm{d}N/\mathrm{d}y \approx 4/3$ out to a $y_\mathrm{max} \approx \ln(\sqrt{s}/m_\rho)$. (Thus neglect the more complicated string topology caused by the shower, and also neglect secondary decays.)

   Calculate the average summed $p_\perp$ per event of either source and compare as a function of energy. At about which energy do the two curves cross, i.e. when is perturbative and nonperturbative jet broadening equally important?

9. (6 p) A photon partly behaves like a hadron, and it is therefore possible to define a parton distribution function for it. This will consist of both a perturbative and a nonperturbative component. To simplify, consider a perturbative branching $\gamma \to \mathrm{u\overline{u}}$ with splitting function $P_{\gamma\to\mathrm{q\overline{q}}}(z) \propto z^2 + (1-z)^2$ (like $\mathrm{g} \to \mathrm{q\overline{q}}$). Let this specify the $u(x, Q_0)$ valence quark content of a photon of some initial scale $Q_0 = 1$ GeV. Study how this component evolves in x as Q is increased. Specifically, show the shape of $xu(x, Q)$ for $Q = 100$ GeV. (For instance in 100 bins linearly between 0 and 1.) How does the mean value $\langle x\rangle$ of this distribution change from Q_0 to Q. You do not need to study the kinematics of branchings, are allowed to neglect the effect of branchings with $1 - z < 0.0001$, and can use a constant α_s value.

10. (6 p) Write and study a semi-realistic final-state parton-shower generator, as follows. Assume that the evolution variable is transverse momentum $p_\perp$ in branchings $a \to bc$,

$$d\mathcal{P}_{a\to bc} = \frac{\mathrm{d}p_\perp^2}{p_\perp^2} \frac{\alpha_\mathrm{s}}{2\pi} P_{a\to bc}(z)\,\mathrm{d}z \ ,$$

times a Sudakov factor. Thus the $p_\perp$ of one branching defines the $p_{\perp\mathrm{max}}$ for the subsequent evolution of the two daughters (when kinematically allowed, see below).

Two possible branching types, $\mathrm{q} \to \mathrm{qg}$ and $\mathrm{g} \to \mathrm{gg}$, must be included in the evolution equations, while $\mathrm{g} \to \mathrm{q\overline{q}}$ can be neglected. The quark is assumed (almost) massless, like for u and d, and a fixed value $\alpha_\mathrm{s} = 0.2$ can be used as a typical average over the value at different scales.

The z variable in a branching provides a sharing of energy E between the two daughter partons. Each parton must have $E \geq p_\perp$ (where $p_\perp$ is the scale at which it is being produced), but the kinematics of branchings need not be constructed further than that. A lower cutoff $p_{\perp\mathrm{min}} = 1$ GeV sets the termination of the shower evolution, and is also used to set the minimal energy of partons.

The shower is supposed to start from a quark with a fixed energy E_0, and with a $p_{\perp\mathrm{max}} = E_0$ for the first branching. Do the simulations for four energies, $E_0 = 50$, 100, 200 and 400 GeV. In each case study the mean value and root-mean-square width of the number of gluons emitted by $\mathrm{q} \to \mathrm{qg}$ branchings, and of the final number of gluons in an event.