

# **Parton Distribution Functions for the LHC**

***Present Status & Uncertainty determination***

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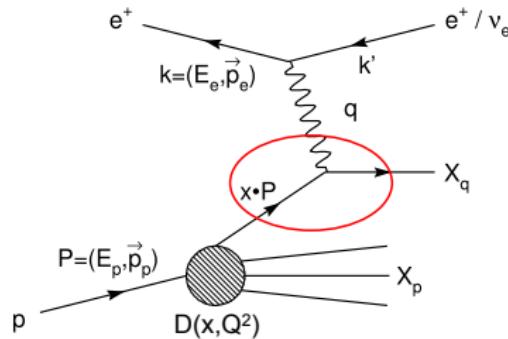
***Ph. D. course on Particle Physics Phenomenology***

NBI Copenhagen  
October 4, 2011

# What are Parton Distribution Functions?

## Definition - Factorization

- Consider a process with one hadron in the initial state



- According to the **Factorization Theorem** we can write the cross section as

$$d\sigma = \sum_a \int_0^1 \frac{d\xi}{\xi} D_a(\xi, \mu^2) d\hat{\sigma}_a \left( \frac{x}{\xi}, \frac{\hat{s}}{\mu^2}, \alpha_s(\mu^2) \right) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$



# What are Parton Distribution Functions?

Definition - DGLAP equations

- The initial condition cannot be computed in Perturbation Theory  
(Lattice? In principle yes, but ...)
- ... but the energy scale dependence is governed by DGLAP evolution equations

$$\frac{\partial}{\ln Q^2} q^{NS}(\xi, Q^2) = P^{NS}(\xi, \alpha_s) \otimes q^{NS}(\xi, Q^2)$$
$$\frac{\partial}{\ln Q^2} \left( \begin{array}{c} \Sigma \\ g \end{array} \right) (\xi, Q^2) = \left( \begin{array}{cc} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{array} \right) (\xi, \alpha_s) \otimes \left( \begin{array}{c} \Sigma \\ g \end{array} \right) (\xi, Q^2)$$

- ... and the **splitting functions**  $P$  can be computed in PT and are known up to **NNLO**

(**LO** - Dokshitzer; Gribov, Lipatov; Altarelli, Parisi, 1977)

(**NLO** - Floratos, Ross, Sachrajda; Gonzalez-Arroyo, Lopez, Yndurain; Curci, Furmanski, Petronzio, 1981)

(**NNLO** - Moch, Vermaseren, Vogt, 2004)



# Problem

Faithful estimation of errors on PDFs

- Single quantity: **1- $\sigma$  error**
- Multiple quantities: **1- $\sigma$  contours**
- Function: need an **"error band" in the space of functions**  
(i.e. the probability density  $\mathcal{P}[f]$  in the space of functions  $f(x)$ )

**Expectation values** are **Functional integrals**

$$\langle \mathcal{F}[f(x)] \rangle = \int \mathcal{D}f \mathcal{F}[f(x)] \mathcal{P}[f(x)]$$



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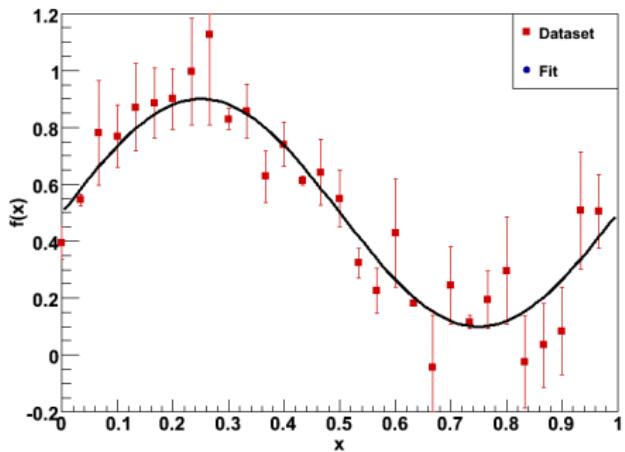
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$$\langle \mathcal{F}[f(x)] \rangle = \int \mathcal{D}f \mathcal{F}[f(x)] \mathcal{P}[f(x)]$$

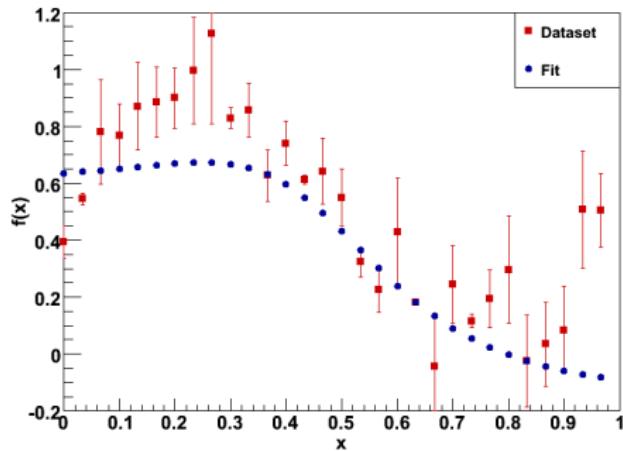
**Determine a function from a finite set of data points**



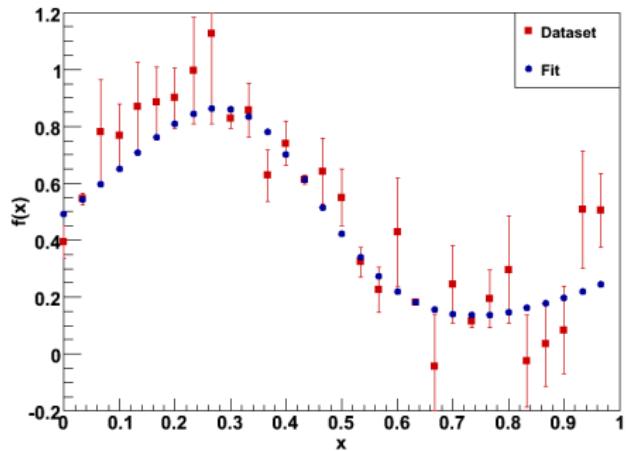
# Proper Fitting avoiding Overlearning



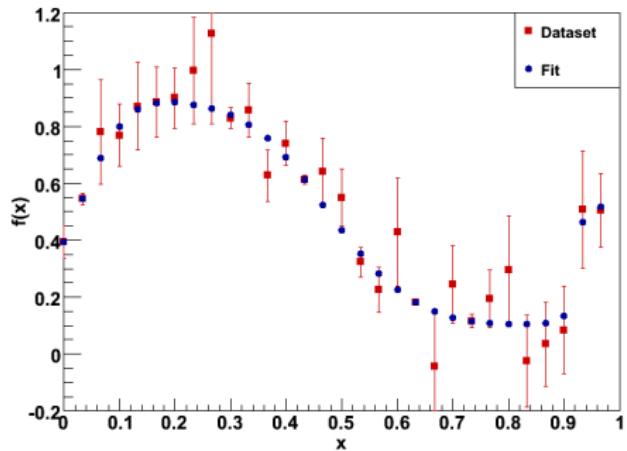
# *Proper Fitting avoiding Overlearning*



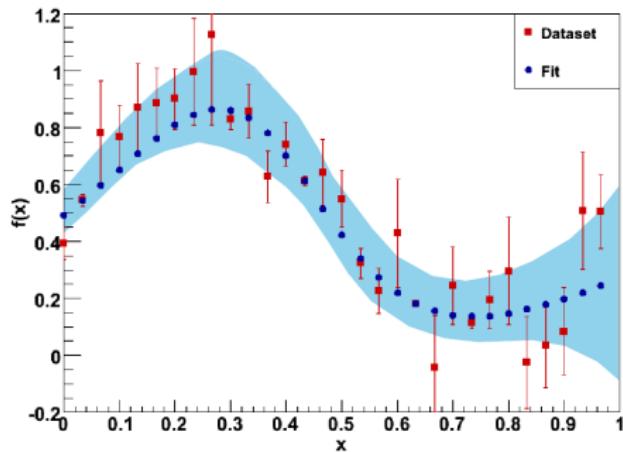
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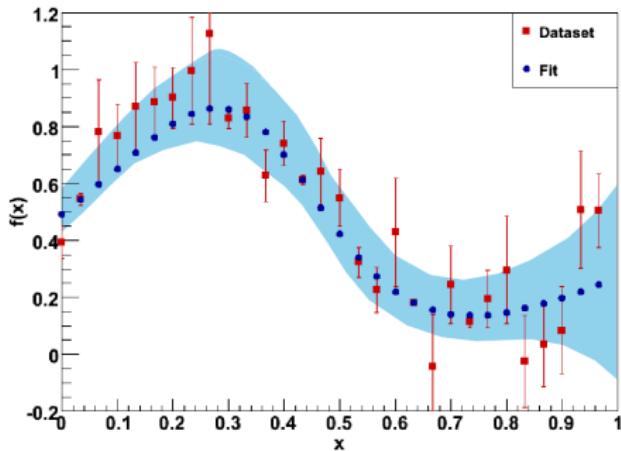
# *Proper Fitting avoiding Overlearning*



# *Proper Fitting avoiding Overlearning*



# *Proper Fitting avoiding Overlearning*



- Need a **redundant parametrization** to avoid parametrization bias.
- Need a way of **stopping the fit before overlearning** sets in to avoid fitting statistical noise.
- Need a **reliable error estimation**.



# DATA SET SELECTION



# *Choice of Dataset*

Global vs. Restricted dataset

## **Restricted set Analyses**

- **HERAPDF**: use only HERA DIS data
- **AB(K)M/JR**: use Fixed target DIS, HERA and Drell-Yan data
  - Focus on the most precise dataset(s)
  - Avoid possible incompatibilities
  - Limited flavour separation
  - Neglect important constraints (gluon at medium/large-x)

## **Global Analyses**

- **CTEQ-TEA/MSTW/NNPDF**: HERA DIS, Fixed target DIS and Drell-Yan, Vector Boson and Inclusive jet production at colliders
  - Focus on completeness
  - Reliable flavour separation
  - Possible data incompatibilities



# Choice of Dataset

Which data constrain which PDFs

**H1, ZEUS:**  $F_2^{e^\pm p}(x, Q^2)$

**BCDMS:**  $F_2^{\mu p}(x, Q^2), F_2^{\mu d}(x, Q^2)$

**NMC:**  $F_2^{\mu p}(x, Q^2), F_2^{\mu d}(x, Q^2), \frac{F_2^{\mu n}(x, Q^2)}{F_2^{\mu p}(x, Q^2)}$

**SLAC:**  $F_2^{\mu p}(x, Q^2), F_2^{\mu d}(x, Q^2)$

**E665:**  $F_2^{\mu p}(x, Q^2), F_2^{\mu d}(x, Q^2)$

**CCFR, NuTeV, CHORUS:**  $F_{2,3}^{\nu(\bar{\nu})p}(x, Q^2)$

$\Rightarrow q, \bar{q}$  at all  $x$   
 $\Rightarrow g$  at moderate and small  $x$

**E605, E702, E866:**  $pN \rightarrow \mu\bar{\mu} + X$

**E605:** Drell-Yan  $p, n$  asymmetry

**CDF:**  $W$  rapidity asymmetry

**CDF, D0:** Inclusive jet data

**CCFR, NuTeV:** Dimuon data

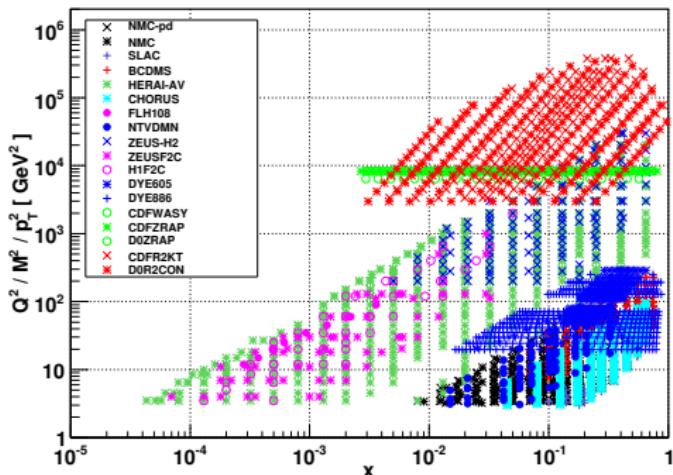
$\Rightarrow \bar{q}, (g)$   
 $\Rightarrow \bar{u}, \bar{d}$   
 $\Rightarrow u/d$  ratio at high- $x$   
 $\Rightarrow g$  at high- $x$   
 $\Rightarrow s, \bar{s}$  sea



# NNPDF 2.1

## Dataset

NNPDF2.1 dataset



3338 data points

OBS	Data set
<b>Deep Inelastic Scattering</b>	
$F_2^d/F_2^p$	NMC-pd
$F_2^p$	NMC, SLAC, BCDMS
$F_2^d$	SLAC, BCDMS
$\sigma_{NC}^\pm$	HERA-I, ZEUS (HERA-II)
$\sigma_{CC}^\pm$	HERA-I, ZEUS (HERA-II)
$F_L$	H1
$\sigma_\nu, \sigma_{\bar{\nu}}$	CHORUS
dimuon prod.	NuTeV
$F_2^c$	ZEUS (99,03,08,09)
$F_2^c$	H1 (01,09,10)
<b>Drell-Yan &amp; Vector Boson prod.</b>	
$d\sigma^{DY}/dM^2 dy$	E605
$d\sigma^{DY}/dM^2 dx_F$	E866
W asymm.	CDF
Z rap. distr.	D0/CDF
<b>Inclusive jet prod.</b>	
Incl. $\sigma^{(jet)}$	CDF ( $k_T$ ) - Run II
Incl. $\sigma^{(jet)}$	D0 (cone) - Run II



# PDF PARAMETRIZATION



# PDF parametrization

## Theoretical Requirements - QCD Sum Rules

- Valence Sum Rules

$$\int_0^1 [u(x, Q^2) - \bar{u}(x, Q^2)] dx = 2, \quad \int_0^1 [d(x, Q^2) - \bar{d}(x, Q^2)] dx = 1$$

$$\int_0^1 [s(x, Q^2) - \bar{s}(x, Q^2)] dx = 0$$

A proton has net quantum numbers of 2 up and 1 down quarks.

- Momentum Sum Rule

$$\sum_{a=q,\bar{q},g} \int_0^1 x f_a(x, Q^2) dx = 1$$

Momenta of all partons must add up to the proton momentum.

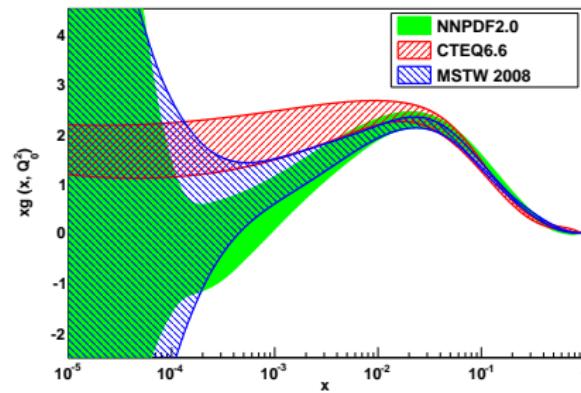


# PDF parametrization

## Theoretical Requirements - Positivity of Observables

- Cross-sections must be positive

- Typically realized imposing PDF positivity
- ... but remember: PDFs are not Probability Density Functions (nor are they observables)
- No probabilistic interpretation beyond Leading Order
- Might result in excessive constraints on PDFs.



# PDF parametrization

Standard Approach (CTEQ-TEA/MSTW/ABKM/HERAPDF)

- Introduce a simple functional form with enough free parameters

$$q(x, Q^2) = x^\alpha (1 - x)^\beta P(x; \lambda_1, \dots, \lambda_n).$$

- "Theoretically motivated"-form
  - $x \rightarrow 0 : q \propto x^{a_1}$  - Regge-like behaviour
  - $x \rightarrow 1 : q \propto (1 - x)^{a_2}$  - quark counting rules
  - $P(x; \lambda_1, \dots, \lambda_n)$ : affects medium- $x$ , just a convenient functional form

# PDF parametrization

## Standard Parametrization

### Parton Distributions Combination

$$xu_v(x, Q^2) = A_u x^{\eta_1} (1 - x)^{\eta_2} (1 + \epsilon_u \sqrt{x} + \gamma_u x)$$

$$xd_v(x, Q^2) = A_d x^{\eta_3} (1 - x)^{\eta_4} (1 + \epsilon_d \sqrt{x} + \gamma_d x)$$

$$xS(x, Q^2) = A_S x^{\delta_S} (1 - x)^{\eta_S} (1 + \epsilon_S \sqrt{x} + \gamma_S x)$$

$$x\Delta(x, Q^2) = A_\Delta x^{\eta_\Delta} (1 - x)^{\eta_S+2} (1 + \gamma_\Delta + \delta_\Delta x^2)$$

$$xg(x, Q^2) = A_g x^{\delta_g} (1 - x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g x) + A_{g'} x^{\delta_{g'}} (1 - x)^{\eta_{g'}}$$

$$x(s + \bar{s})(x, Q^2) = A_+ x^{\delta_S} (1 - x)^{\eta_+} (1 + \epsilon_S \sqrt{x} + \gamma_S x)$$

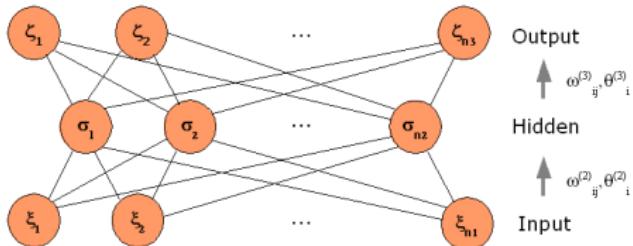
$$x(s - \bar{s})(x, Q^2) = A_- x^{\delta_-} (1 - x)^{\eta_-} (1 + x/x_0)$$

**29** parameters



# PDF parametrization

## The NNPDF Approach - Neural Networks



- Neural Networks are **non-linear** statistical tools.
- Any continuous function can be approximated with neural network with one internal layer and non-linear neuron activation function.
- **Efficient minimization algorithms** for complex parameter spaces.
- They provide a parametrization which is **redundant** and **robust** against variations.



# PDF parametrization

Neural Networks ... just another basis of functions

## Multilayer feed-forward networks

- Each neuron receives input from neurons in preceding layer and feeds output to neurons in subsequent layer
- Activation determined by **weights** and **thresholds**

$$\xi_i = g \left( \sum_j \omega_{ij} \xi_j - \theta_i \right)$$

- Sigmoid activation function

$$g(x) = \frac{1}{1 + e^{-\beta x}}$$



# PDF parametrization

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A 1-2-1 NN:

$$\xi_1^{(3)}(\xi_1^{(1)}) = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - \xi_1^{(1)} \omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - \xi_1^{(1)} \omega_{21}^{(1)}}}}}$$



# PDF parametrization

## NNPDF 2.1 Parametrization

### Parton Distributions Combination

### NN architecture

Singlet ( $\Sigma(x)$ )	$\Rightarrow$	2-5-3-1 (37 pars)
Gluon ( $g(x)$ )	$\Rightarrow$	2-5-3-1 (37 pars)
Total valence ( $V(x) \equiv u_V(x) + d_V(x)$ )	$\Rightarrow$	2-5-3-1 (37 pars)
Non-singlet triplet ( $T_3(x)$ )	$\Rightarrow$	2-5-3-1 (37 pars)
Sea asymmetry ( $\Delta_s(x) \equiv \bar{d}(x) - \bar{u}(x)$ )	$\Rightarrow$	2-5-3-1 (37 pars)
Total Strangeness ( $s^+(x) \equiv (s(x) + \bar{s}(x))/2$ )	$\Rightarrow$	2-5-3-1 (37 pars)
Strange valence ( $s^-(x) \equiv (s(x) - \bar{s}(x))/2$ )	$\Rightarrow$	2-5-3-1 (37 pars)

259 parameters



# PDF UNCERTAINTIES



# PDF Uncertainties

## The Hessian Method

- The figure of merit to be minimized in the fit is the fully correlated  $\chi^2$

$$\chi^2 = \sum_{I,J=1}^{N_{dat}} (D_I - T_I(\{a\})) [(\text{cov})^{-1}]_{IJ} (D_J - T_J(\{a\}))$$

where  $D_k$  and  $T_k$  are the **data** and **theory** values for each data point

- Important to properly **account** for **correlated uncertainties**.
- Special care needed for **normalization uncertainties**  
(in general multiplicative uncertainties).

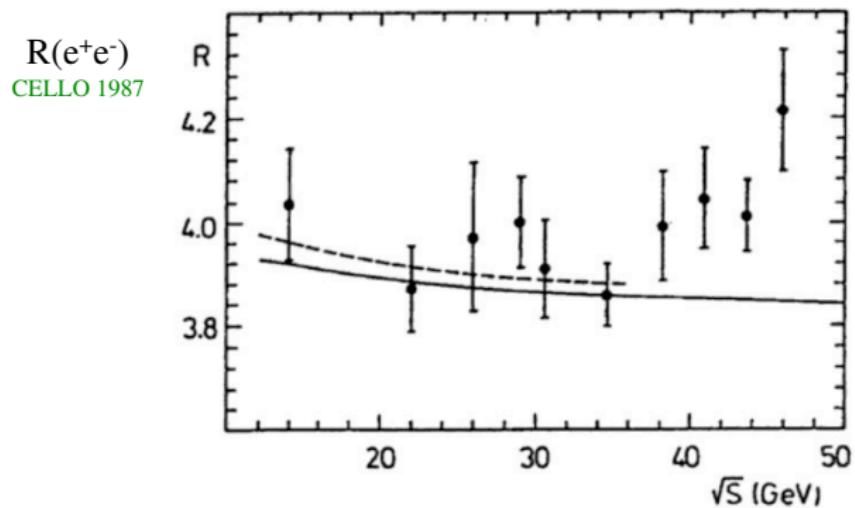
[R. D. Ball et al., arXiv:0912.2276]



# PDF Uncertainties

## Normalization Uncertainties

m-cov



Dashed line: data below 36 GeV

Solid line: all data

D'Agostini 1994



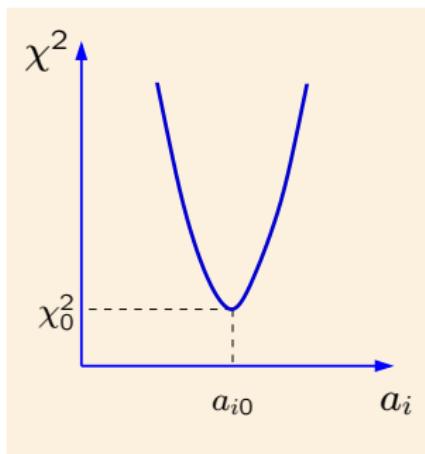
# PDF Uncertainties

## Multidimensional error Analysis

[P. Nadolsky, CTEQ School 2009]

Experimental observables   Theoretical cross sections   PDF parametrizations   Statistical aspects   Practical applications

### Multi-dimensional error analysis



- Minimization of a likelihood function ( $\chi^2$ ) with respect to  $\sim 30$  theoretical (mostly PDF) parameters  $\{a_i\}$  and  $> 100$  experimental systematical parameters



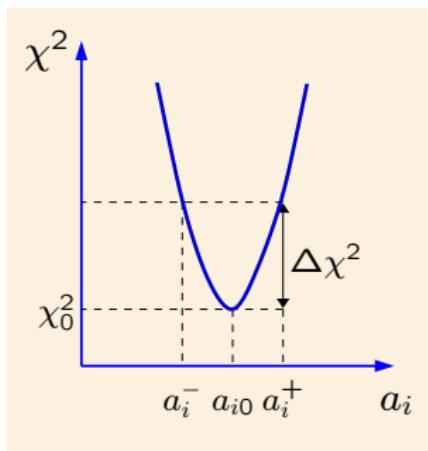
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### Multi-dimensional error analysis



- Establish a confidence region for  $\{a_i\}$  for a given tolerated increase in  $\chi^2$
- In the ideal case of perfectly compatible Gaussian errors, 68% c.l. on a physical observable  $X$  corresponds to  $\Delta\chi^2 = 1$  independently of the number  $N$  of PDF parameters

See, e.g., P. Bevington, K. Robinson, Data analysis and error reduction for the physical sciences



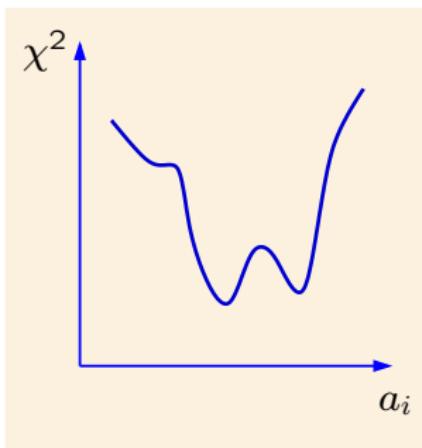
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### Multi-dimensional error analysis



#### Pitfalls to avoid

##### ■ "Landscape"

- disagreements between the experiments

In the worst situation, significant disagreements between  $M$  experimental data sets can produce up to  $N \sim M!$  possible solutions for PDF's, with  $N \sim 10^{500}$  reached for "only" about 200 data sets



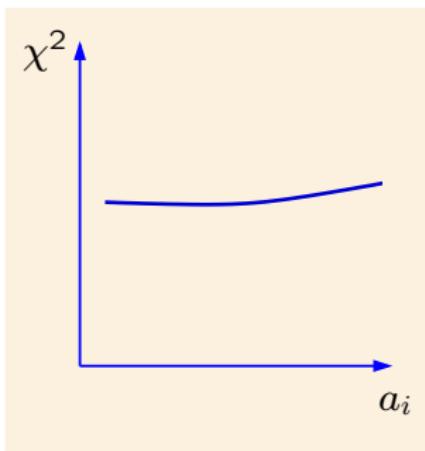
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### Multi-dimensional error analysis



#### Pitfalls to avoid

- Flat directions
  - ▶ unconstrained combinations of PDF parameters
  - ▶ dependence on free theoretical parameters, especially in the PDF parametrization
  - ▶ impossible to derive reliable PDF error sets



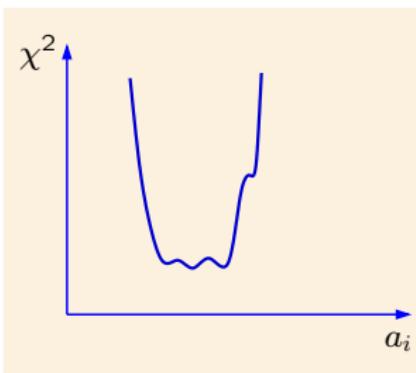
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### Multi-dimensional error analysis



The actual  $\chi^2$  function shows

- a well pronounced global minimum  $\chi_0^2$
- weak tensions between data sets in the vicinity of  $\chi_0^2$  (mini-landscape)
- some dependence on assumptions about flat directions



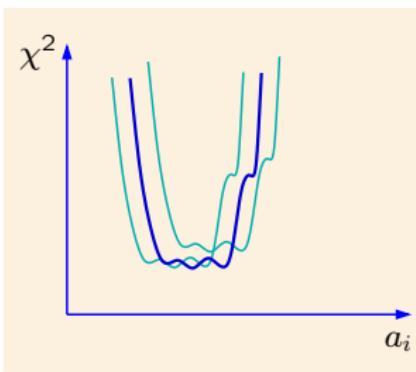
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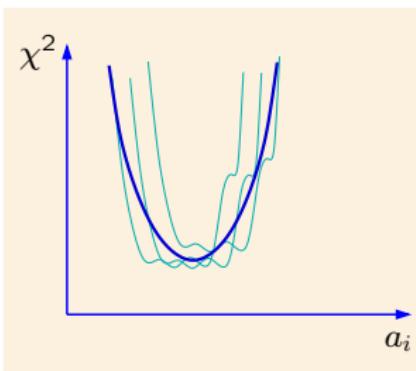
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The likelihood is approximately described by a quadratic  $\chi^2$  with a revised tolerance condition  $\Delta\chi^2 \leq T^2$



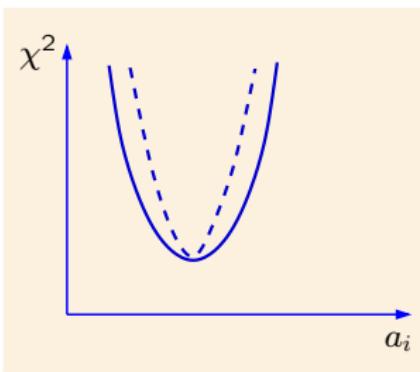
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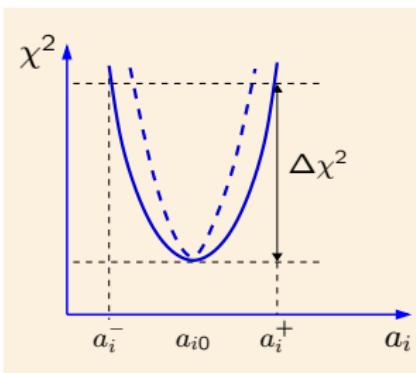
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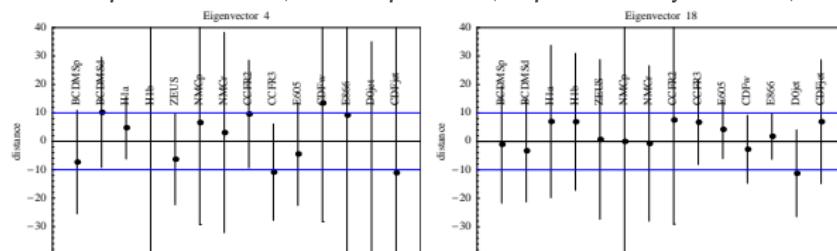
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### CTEQ6 tolerance criterion (2001)

Acceptable values of PDF parameters must agree at  $\approx 90\%$  c.l. with all experiments included in the fit, *for a plausible range of assumptions about the PDF parametrization, scale dependence, experimental systematics, ...*



Can be crudely approximated (but does not have to) by assuming  $T \approx 10$  for all PDF parameters

A somewhat stricter variant of this criterion is applied in the MSTW'08 analysis



# PDF Uncertainties

## The Hessian Method

- Used by most PDF fitters (CTEQ, MSTW, ABKM, HERAPDF).
- Given an observable which depends on a set of parameters  $\{z\}$

$$X(\{z\}) = X_0 + z_i \partial_i X(\{z\})$$

the variance is given by

$$\sigma_X^2 = \sigma_{ij} \partial_i X \partial_j X$$

with  $\sigma_{ij}$  being the covariance matrix in parameter space.

- **Diagonalization:** choose the  $z_i$  as eigenvectors of  $\sigma_{ij}$  with unit eigenvalues so that

$$\sigma_X^2 = |\vec{\nabla} X|^2$$

- Relies on **linear error propagation**, i.e. **Gaussian approximation**



# PDF Uncertainties

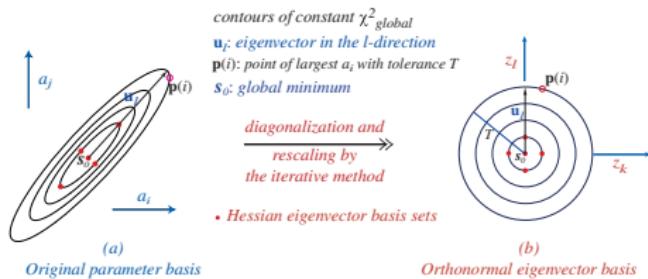
## The Hessian Method

[P. Nadolsky, CTEQ School 2009]

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### Tolerance hypersphere in the PDF space

2-dim  $(i,j)$  rendition of  $N$ -dim (22) PDF parameter space



A hyperellipse  $\Delta\chi^2 \leq T^2$  in space of  $N$  physical PDF parameters  $\{a_i\}$  is mapped onto a filled hypersphere of radius  $T$  in space of  $N$  orthonormal PDF parameters  $\{z_i\}$



# PDF Uncertainties

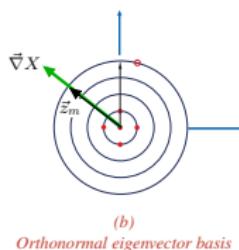
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PDF error for a physical observable  $X$  is given by

$$\Delta X = \vec{\nabla}X \cdot \vec{z}_m = \left| \vec{\nabla}X \right| = \frac{1}{2} \sqrt{\sum_{i=1}^N (X_i^{(+)} - X_i^{(-)})^2}$$



# PDF Uncertainties

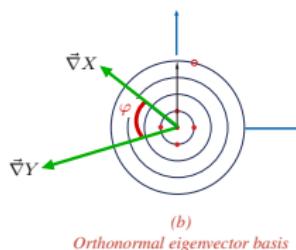
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Correlation cosine for observables  $X$  and  $Y$ :

$$\cos \varphi = \frac{\vec{v}X \cdot \vec{v}Y}{\Delta X \Delta Y} = \frac{1}{4 \Delta X \Delta Y} \sum_{i=1}^N \left( X_i^{(+)} - X_i^{(-)} \right) \left( Y_i^{(+)} - Y_i^{(-)} \right)$$



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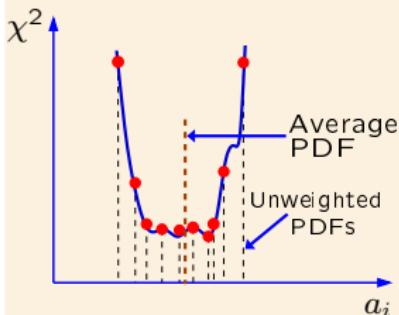
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[P. Nadolsky, CTEQ School 2009]

Experimental observables   Theoretical cross sections   PDF parametrizations   Statistical aspects   Practical applications

### Confidence intervals in global PDF analyses

#### Monte-Carlo sampling of the PDF parameter space



A very general approach that

- realizes stochastic sampling of the probability distribution  
(Alekhin; Giele, Keller, Kosower; NNPDF)
- can parametrize PDF's by flexible neural networks (NNPDF)
- does not rely on smoothness of  $\chi^2$  or Gaussian approximations



# PDF Uncertainties

## The Monte Carlo Method

- Used by NNPDF (and old Fermi and Alekhin sets).
- Generate  $N_{rep}$  Monte Carlo **replicas of the data**  
(Monte Carlo in the space of parameters is not a smart idea, because of flat directions)
- Fit a set of PDFs to each replica, the **ensemble of replicas** gives the **probability density** in the space of PDFs
- You get a set of  $N_{rep}$  replicas, compute central values, standard deviations, correlations as you would do for any Monte Carlo ensemble:

$$\langle \mathcal{F}[\{q\}] \rangle = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} \mathcal{F}[\{q^{(k)}\}]$$

$$\sigma_{\mathcal{F}} = \sqrt{\frac{N_{rep}}{N_{rep}-1} (\langle \mathcal{F}[\{q\}]^2 \rangle - \langle \mathcal{F}[\{q\}] \rangle^2)}$$



# PDF Uncertainties

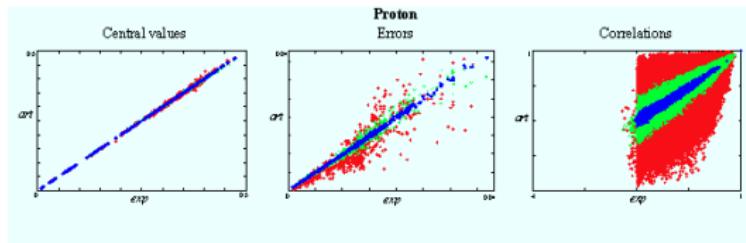
The Monte Carlo Method in practice

- Generate artificial data according to distribution

$$O_i^{(art)(k)} = (1 + r_N^{(k)} \sigma_N) \left[ O_i^{(exp)} + \sum_{p=1}^{N_{sys}} r_p^{(k)} \sigma_{i,p} + r_{i,s}^{(k)} \sigma_s^i \right]$$

where  $r_i$  are univariate (gaussian) random numbers

- Validate Monte Carlo **replicas** against experimental data  
(statistical estimators, faithful representation of errors, convergence rate increasing  $N_{rep}$ )



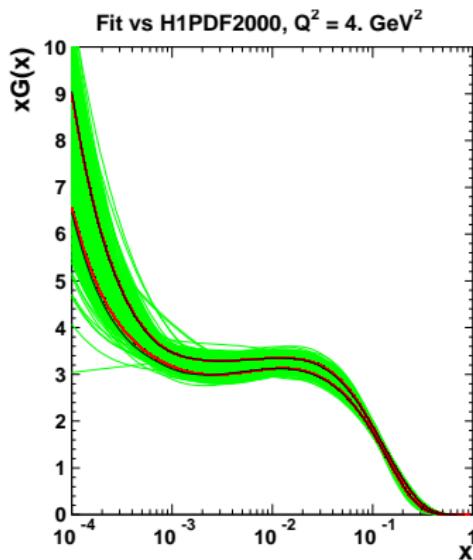
- $\mathcal{O}(1000)$  replicas needed to reproduce correlations to percent accuracy



# PDF Uncertainties

## Hessian vs. Monte Carlo

- **Q:** Are the two methods equivalent?  
**A:** Hessian and MC method give the same results in the linear error propagation approximation
- **Q:** Nice, but what about in the real world?  
**A:** [arXiv:0901.2504](#), pg. 41
- **Q:** Wait a minute, you assumed gaussian errors ...  
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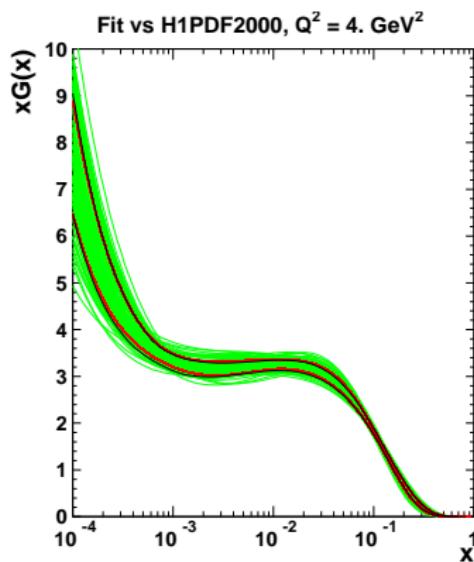
[S. Glazov and V. Radescu]



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[S. Glazov and V. Radescu]



# PDF fits status



# The present status of PDF fits

## The PDF sets Matrix

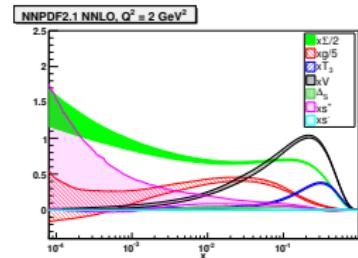
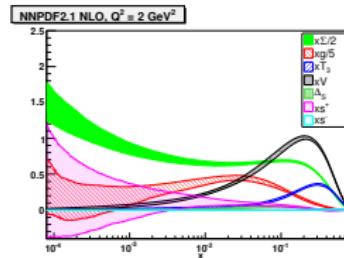
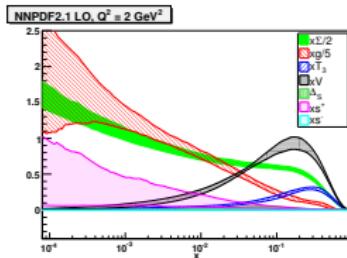
	Dataset	Pert. Order	Heavy Flavours	$\alpha_S$	Param.	Uncert.
<b>ABKM09</b>	DIS (FT, HERA) Drell-Yan (FT)	NLO NNLO	FFN (BMSN)	fitted	6 indep. PDF Polynomial (25 param.)	Hessian ( $\Delta\chi^2 = 1$ )
<b>CT10</b>	DIS (FT, HERA) Drell-Yan (FT, Tev) Jets (Tevatron)	LO NLO	GM-VFNS (S-ACOT)	external	6 indep. PDF Polynomial (26 param.)	Hessian ( $\Delta\chi^2 = 100$ )
<b>JR09</b>	DIS (FT, HERA) Drell-Yan (FT) Jets (Tevatron)	NLO NNLO	FFN VFN	fitted	5 indep. PDF Polyinom. (15 param.)	Hessian ( $\Delta\chi^2 = 1$ )
<b>HERAPDF1.5</b>	DIS (HERA)	NLO NNLO	GM-VFNS (TR)	external	5 indep. PDF Polnom. (14 param.)	Hessian ( $\Delta\chi^2 = 1$ )
<b>MSTW08</b>	DIS (FT, HERA) Drell-Yan (FT, Tev) Jets (HERA, Tev)	LO NLO NNLO	GM-VFNS (TR)	fitted	7 indep. PDF Polynom. (20 param.)	Hessian ( $\Delta\chi^2 \sim 25$ )
<b>NNPDF2.1</b>	DIS (FT, HERA) Drell-Yan (FT, Tev) Jets (Tevatron)	LO NLO NNLO	GM-VFNS (FONLL)	external	7 indep. PDF Neural Netw. (259 param.)	Monte Carlo



# The present status of PDF fits

PDFs ... a family portrait

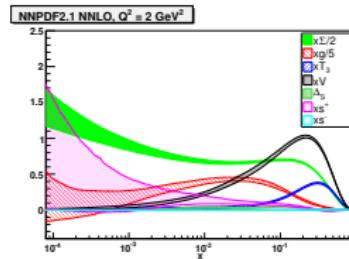
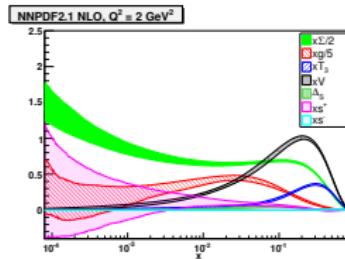
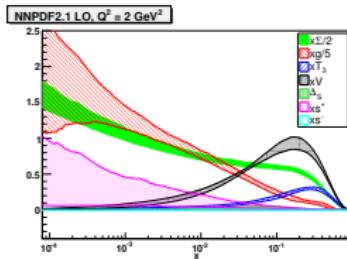
- At the starting scale ( $2 \text{ GeV}^2$ ) ...



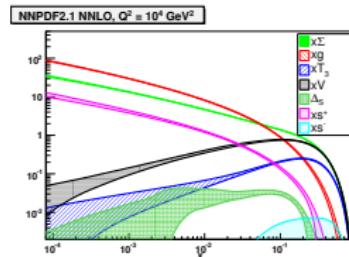
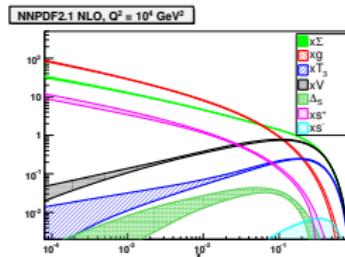
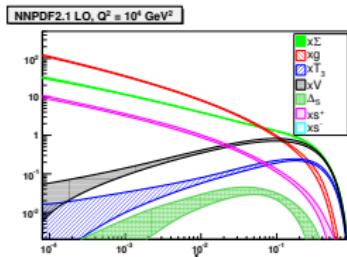
# The present status of PDF fits

PDFs ... a family portrait

- At the starting scale ( $2 \text{ GeV}^2$ ) ...



- ... and at the typical EW scale ( $100 \text{ GeV}^2$ )



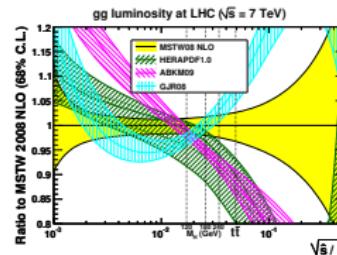
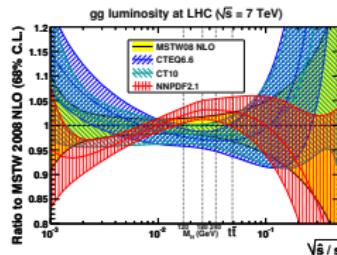
# The present status of PDF fits

## Comparison between Parton Luminosities

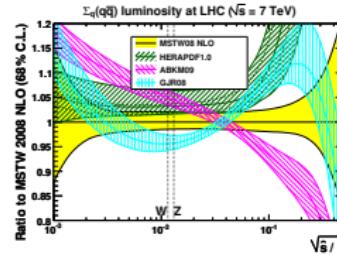
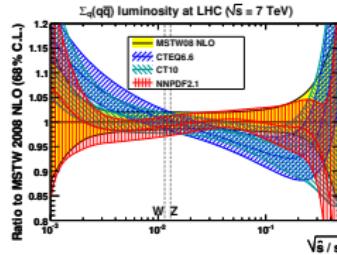
- When trying to understand **differences between PDF sets** it is useful to look at **parton luminosities**

$$\Phi_{ij}(M_X^2) = \frac{1}{s} \int_{\tau}^1 \frac{x_1}{x_1} f_i(x_1, M_X^2) f_j(\tau/x_1, M_X^2)$$

gg:



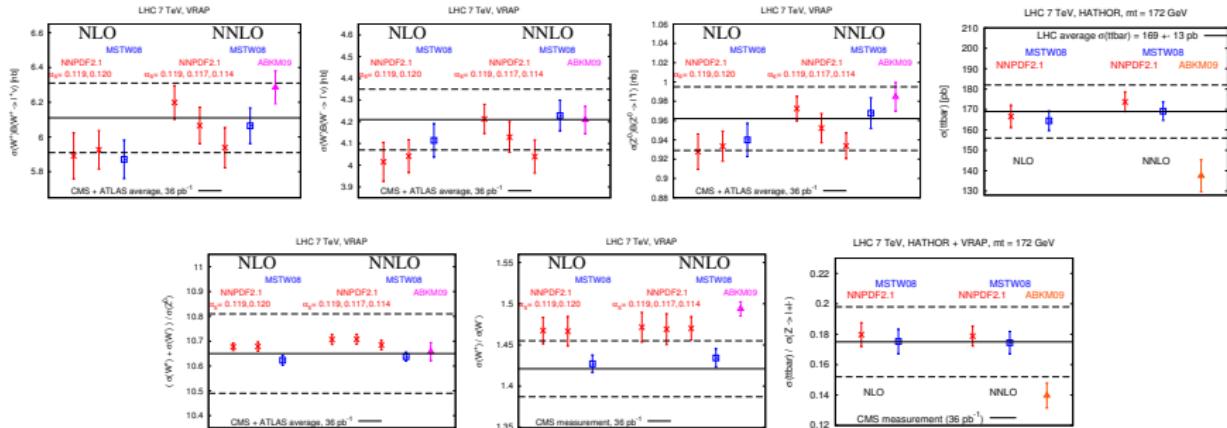
q-q-bar:



# The present status of PDF fits

Comparisons to LHC data

- Predictions for **LHC Standard Candles** compared to **LHC data**



- LHC data will soon be precise enough to distinguish between different predictions.

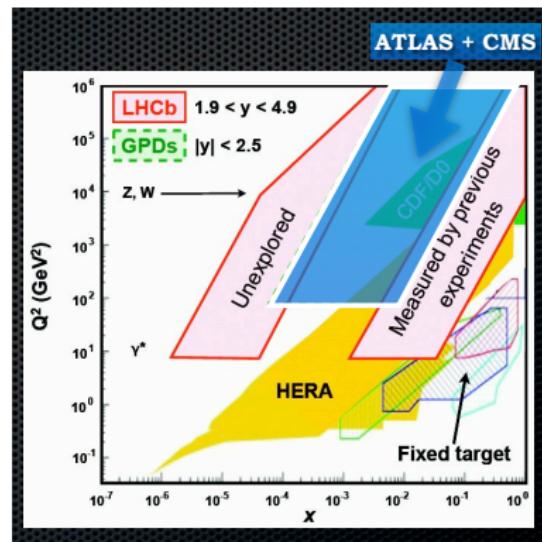


# The present status of PDF fits

W lepton asymmetry data at the LHC

$$A_W^l = \frac{\sigma(pp \rightarrow W^+ \rightarrow l^+ \nu_l) - \sigma(pp \rightarrow W^- \rightarrow l^- \bar{\nu}_l)}{\sigma(pp \rightarrow W^+ \rightarrow l^+ \nu_l) + \sigma(pp \rightarrow W^- \rightarrow l^- \bar{\nu}_l)}$$

- **ATLAS**: muon charge asymmetry ( $31\text{pb}^{-1}$ ) [[ArXiv:1103:2929](#)]
- **CMS**: muon charge asymmetry ( $36\text{pb}^{-1}$ ) [[ArXiv:1103:3470](#)]

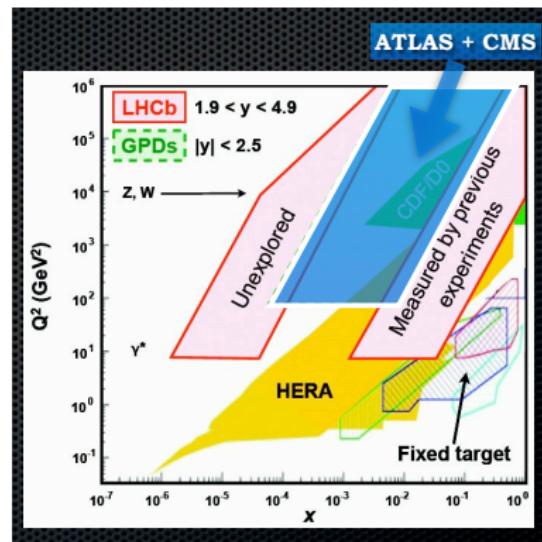


# The present status of PDF fits

W lepton asymmetry data at the LHC

$$A_W^l \sim \frac{u(x_1, M_W^2) \bar{d}(x_2, M_W^2) - d(x_1, M_W^2) \bar{u}(x_2, M_W^2)}{u(x_1, M_W^2) \bar{d}(x_2, M_W^2) + d(x_1, M_W^2) \bar{u}(x_2, M_W^2)}$$

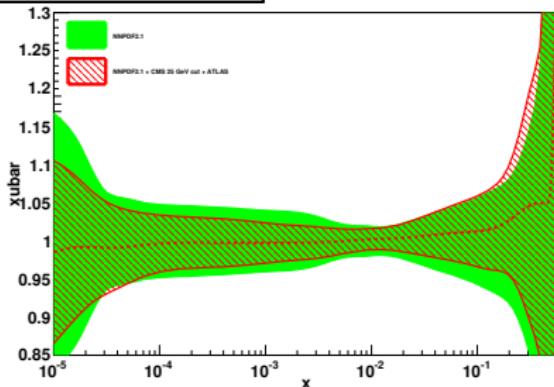
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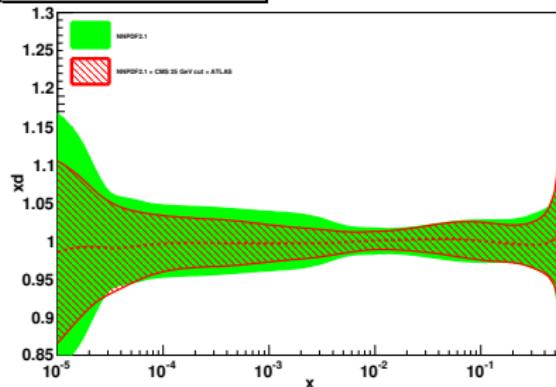
# The present status of PDF fits

First constraints on PDFs from LHC

$Q^2 = M_W^2$ , ratio to NNPDF2.1



$Q^2 = M_W^2$ , ratio to NNPDF2.1



- ATLAS and CMS data compatible with data included in global analysis
- The provide important constraint to PDFs in the small medium-x region
- Significant uncertainty reduction for light (anti-)flavour distributions



# *The present status of PDF fits*

... the data we would love to have from the LHC

- Medium- and large- $x$  **gluon**
  - Prompt photons
  - Inclusive Jets
  - $t$ -quark distributions ( $p_{\perp}$ ,  $y$ ) (?)
- **Light flavour separation** at medium- & small- $x$ 
  - Low-mass Drell-Yan
  - High-mass  $W$  production
  - $Z$  rapidity distribution
  - $W(+\text{jets})$  asymmetry
- **Strangeness & Heavy Flavours**
  - $W + c$
  - $Z + c, \gamma + c$
  - $Z + b$

