CHIRAL SYMMETRY AT HIGH ENERGIES

Johan Bijnens

Lund University

bijnens@thep.lu.se
http://www.thep.lu.se/∼bijnens

Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- Effective Field Theory
- Chiral Perturbation Theory
- Hard Pion Chiral Perturbation Theory:
  idea and applications

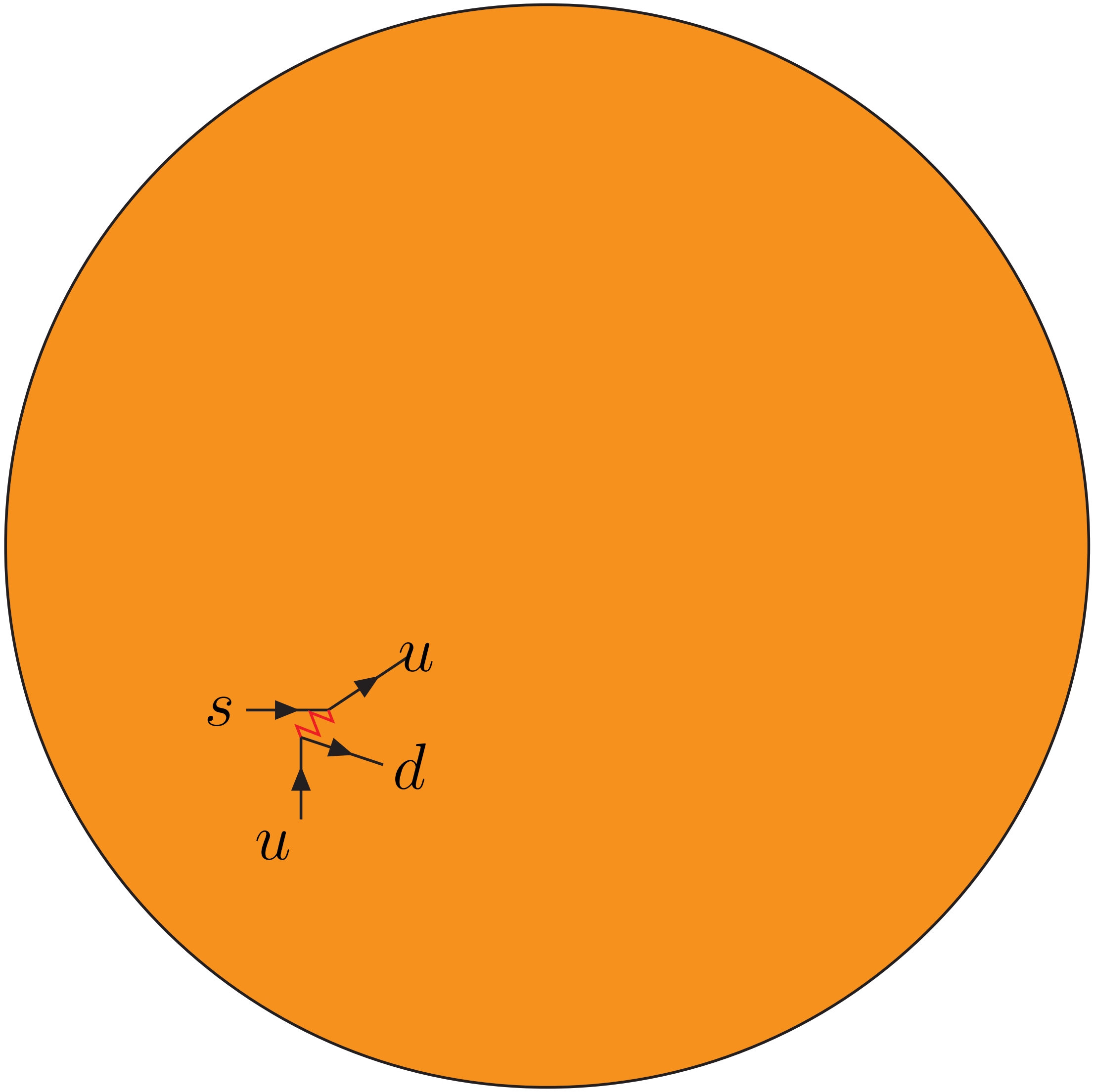
Overview

- Effective Field Theory
- Chiral Perturbation Theory
- Hard Pion Chiral Perturbation Theory: idea and applications
  - $K_{\ell 3}$ Flynn-Sachrajda, arXiv:0809.1229
  - $K \to \pi\pi$ JB+ Alejandro Celis, arXiv:0906.0302
  - F_π^S and F_π^V JB + Ilaria Jemos, arXiv:1011.6531 a two-loop check
  - $B, D \to \pi$ JB + Ilaria Jemos, arXiv:1006.1197
  - $B, D \to \pi, K, \eta$ JB + Ilaria Jemos, arXiv:1011.6531
  - $\chi_c(J = 0, 2) \to \pi\pi, KK, \eta\eta$ JB+Ilaria Jemos, arxiv:1109.5033

Why is low-energy stuff needed?

A weak decay:

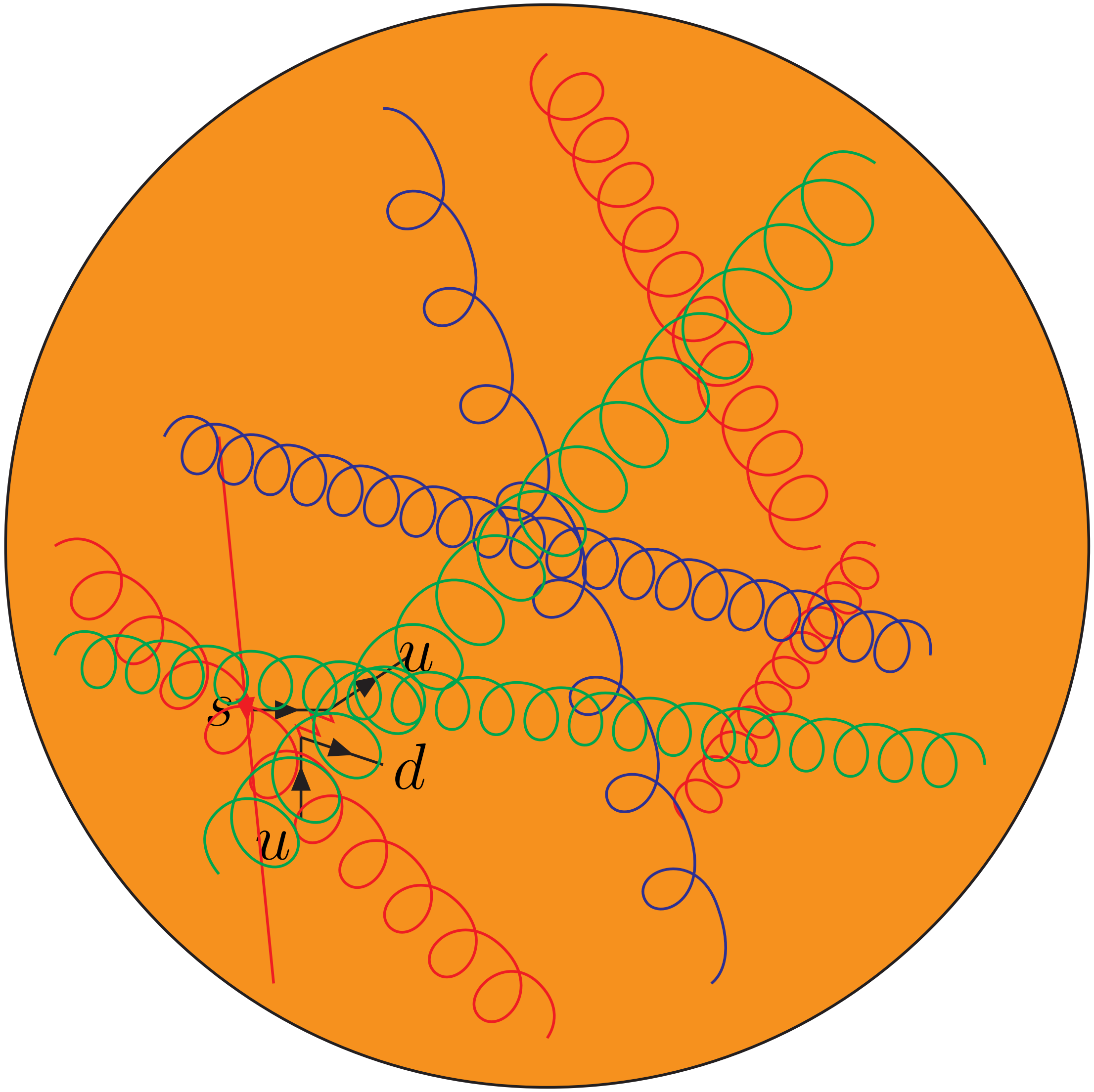
Hadron: 1 fm
W-boson: 10^{-3} fm
new physics:
at $\lesssim 10^{-4}$ fm



Why is low-energy stuff needed?

A weak decay:

Hadron: 1 fm
W-boson: 10^{-3} fm
new physics:
at $\lesssim 10^{-4}$ fm



Effective Field Theory

- gap in the spectrum $\Longrightarrow$ separation of scales
- with the lower degrees of freedom, build the most general effective Lagrangian

Effective Field Theory

- gap in the spectrum $\Longrightarrow$ separation of scales
- with the lower degrees of freedom, build the most general effective Lagrangian

  - $\infty\#$ parameters
  - Where did my predictivity go ?

Effective Field Theory

- gap in the spectrum $\Longrightarrow$ separation of scales
- with the lower degrees of freedom, build the most general effective Lagrangian

  - $\infty\#$ parameters
  - Where did my predictivity go ?

$\Longrightarrow$ Need some ordering principle: power counting
Higher orders suppressed by powers of $1/\Lambda$

Effective Field Theory

- gap in the spectrum $\Longrightarrow$ separation of scales
- with the lower degrees of freedom, build the most general effective Lagrangian

  - $\infty\#$ parameters
  - Where did my predictivity go ?

$\Longrightarrow$ Need some ordering principle: power counting
Higher orders suppressed by powers of $1/\Lambda$

$\Longrightarrow$ Or some other way to handle an infinite number of parameters

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Perturbation Theory

- Chiral Symmetry:

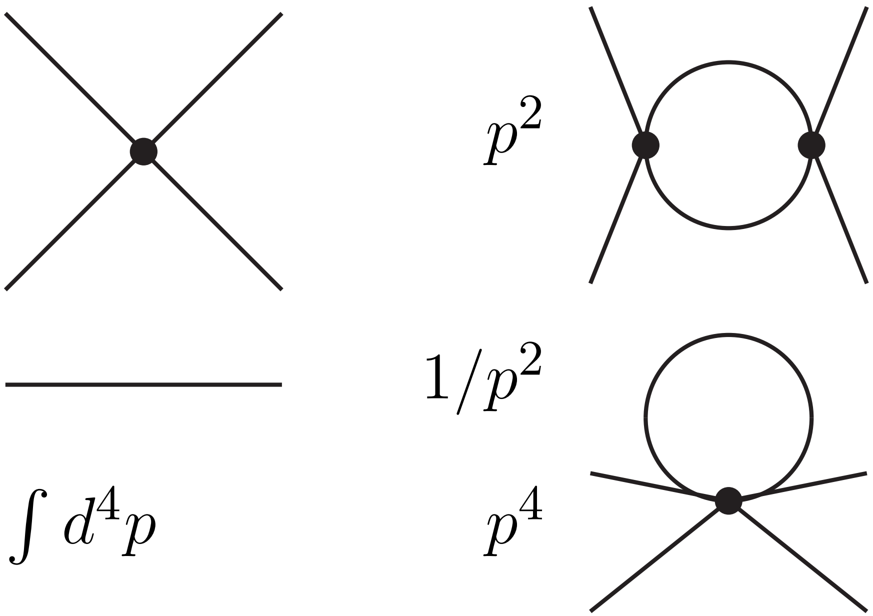
  $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

  So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.
- Spontaneously broken: $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
- $SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$
- 8 generators broken $\Longrightarrow$ 8 (pseudo)-Goldstone bosons and interaction vanishes at zero momentum
- 8 candidates light compared to other hadrons: $\pi^0, \pi^+, \pi^-, K^+, K^-, K^0, \overline{K^0}, \eta$

Hard pion ChPT?

- In Meson ChPT: the powercounting is from all lines in Feynman diagrams having soft momenta
- thus powercounting = (naive) dimensional counting

Power counting in momenta (all lines soft):

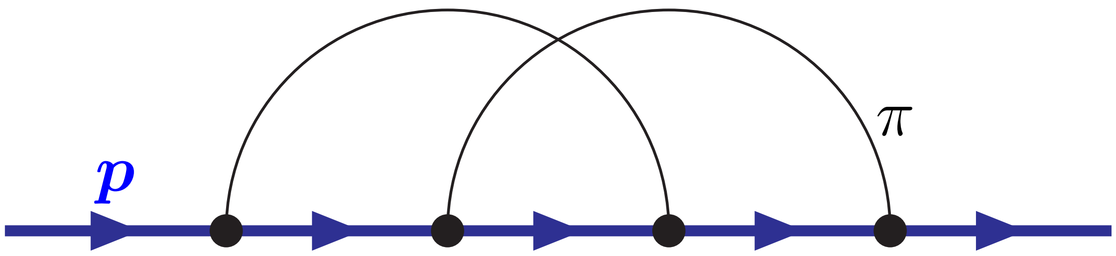


$$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$$

$$(p^2)\,(1/p^2)\,p^4 = p^4$$

Hard pion ChPT?

- Baryon and Heavy Meson ChPT: $p, n, \ldots\ B, B^*$ or D, D^*
  - $p = M_B v + k$
  - Everything else soft



- Works because baryon or b or c number conserved so the non soft line is continuous
- General idea: All M_B dependence can be absorbed in the parameters of the higher order Lagrangians (LECs), since it is analytic in the other parts k.

Hard pion ChPT?

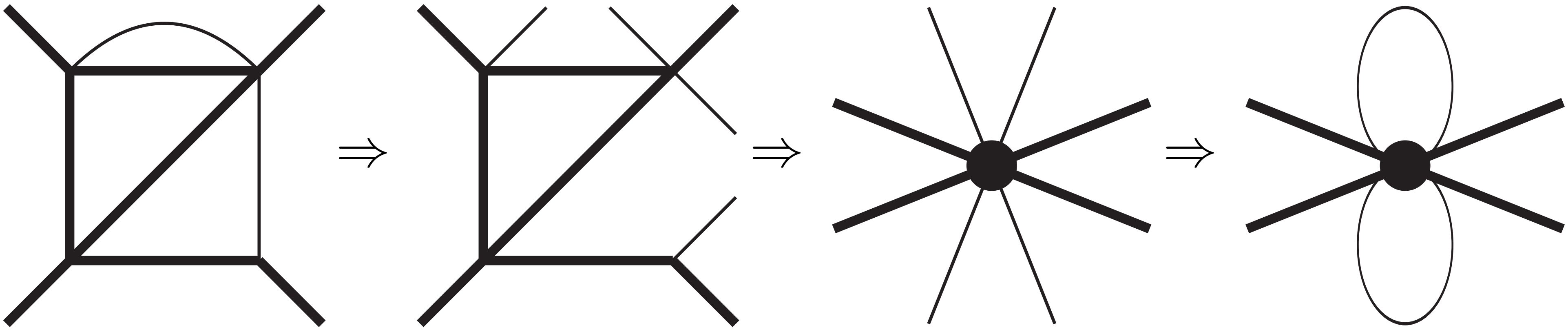
- Heavy Kaon ChPT: Flynn-Sachrajda argued $K_{\ell 3} : K \to \pi \ell \nu$ also for q^2 away from q^2_{max}: ie fast pion
- JB-Celis Argument generalizes to other processes with hard/fast pions and applied to $K \to \pi\pi$
- JB Jemos $B, D \to D, \pi, K, \eta$ vector formfactors, charmonium decays and a two-loop check
- General idea: heavy/fast dependence again reabsorbed in LECs, since it is analytic in the other parts k.

Hard pion ChPT?

Field Theory: a process at given external momenta

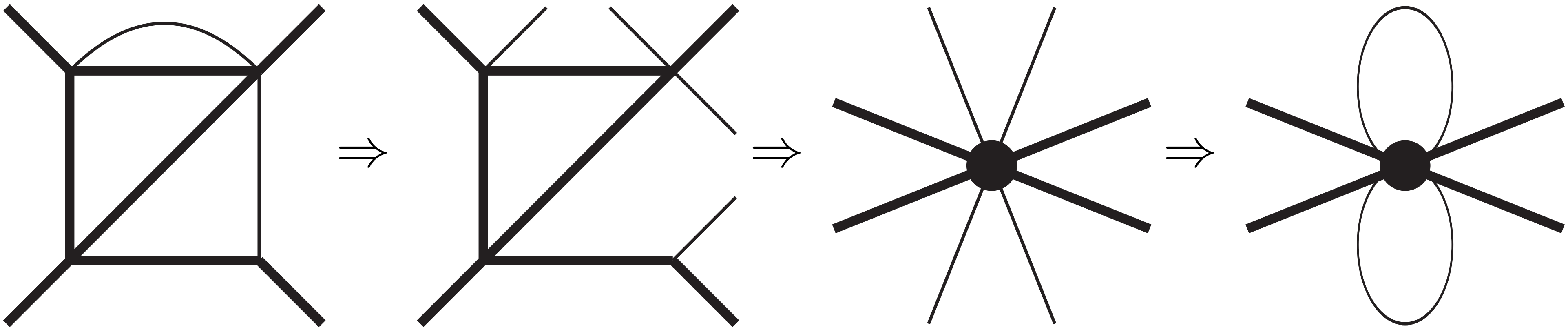
- nonanalyticities in the light masses come from soft lines
- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta (Weinberg's folklore theorem)
- Loops with this Lagrangian reproduce the original nonanalicities in the light masses

Hard pion ChPT?



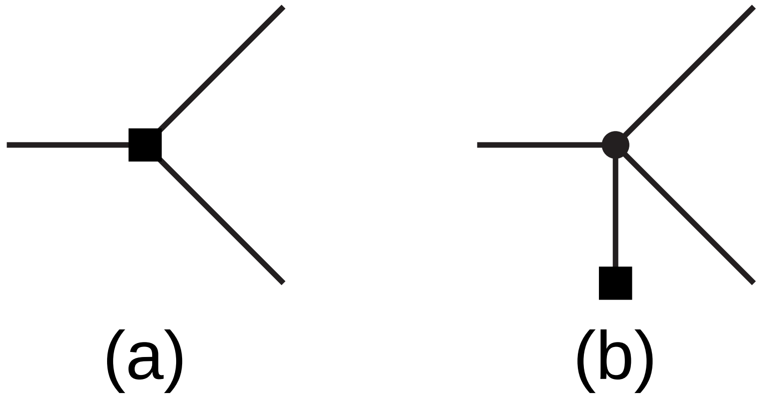
- The effective tree Lagrangian is in hadron fields but all possible orders of the momenta included: possibly an infinite number of terms
- If symmetries present, Lagrangian should respect them
- but my powercounting is gone

Hard pion ChPT?



- The effective tree Lagrangian is in hadron fields but all possible orders of the momenta included: possibly an infinite number of terms
- If symmetries present, Lagrangian should respect them
- In some cases we can prove that up to a certain order in the expansion in light masses, not momenta, matrix elements of higher order operators are reducible to those of lowest order.

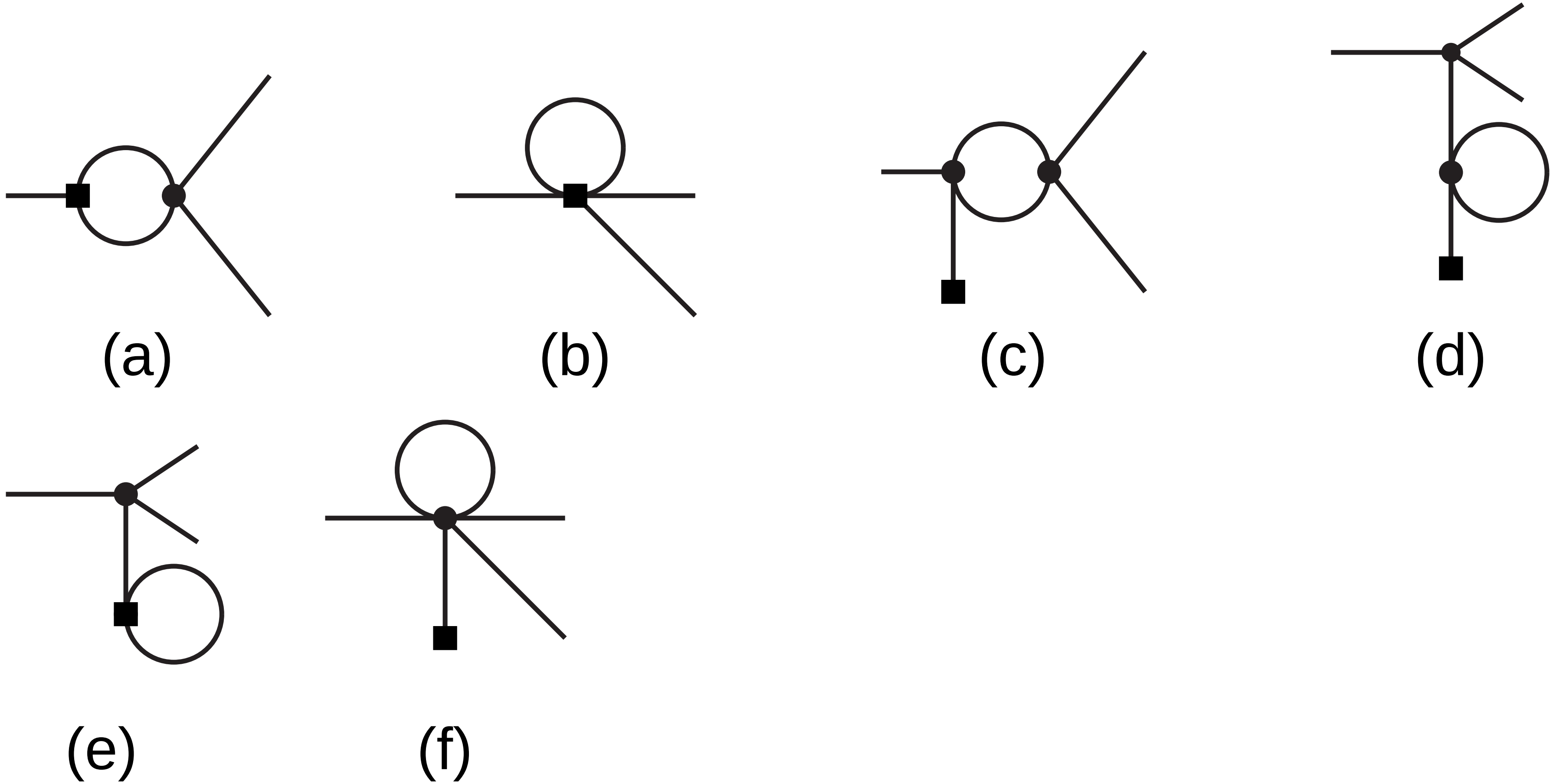
$K \to \pi\pi$: Tree level



$$A_0^{LO} = \frac{\sqrt{3}i}{2F^2}\left[-\frac{1}{2}E_1 + (E_2 - 4E_3)\,\overline{M}_K^2 + 2E_8\overline{M}_K^4 + A_1 E_1\right]$$

$$A_2^{LO} = \sqrt{\frac{3}{2}}\frac{i}{F^2}\left[(-2D_1 + D_2)\,\overline{M}_K^2\right]$$

$K \to \pi\pi$: One loop



$K \to \pi\pi$: One loop

Diagram	A_0	A_2
Z	$-\frac{2F^2}{3}A_0^{LO}$	$-\frac{2F^2}{3}A_2^{LO}$
(a)	$\sqrt{3}i\left(-\frac{1}{3}E_1+\frac{2}{3}E_2\overline{M}_K^2\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{2}{3}D_2\overline{M}_K^2\right)$
(b)	$\sqrt{3}i\left(-\frac{5}{96}E_1-\left(\frac{7}{48}E_2+\frac{25}{12}E_3\right)\overline{M}_K^2+\frac{25}{24}E_8\overline{M}_K^4\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{61}{12}D_1+\frac{77}{24}D_2\right)\overline{M}_K^2$
(e)	$\sqrt{3}i\frac{3}{16}A_1E_1$	
(f)	$\sqrt{3}i\left(\frac{1}{8}E_1+\frac{1}{3}A_1E_1\right)$	

The coefficients of $\overline{A}(M_\pi^2)/F^4$ to A_0 and A_2. $\overline{A}(M_\pi^2) = -\frac{M_\pi^2}{16\pi^2}\log\frac{M_\pi^2}{\mu^2}$

- $$A_0^{NLO} = A_0^{LO}\left(1+\frac{3}{8F^2}\overline{A}(M_\pi^2)\right)+\lambda_0 M_\pi^2+\mathcal{O}(M_\pi^4)$$
- $$A_2^{NLO} = A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M_\pi^2)\right)+\lambda_2 M_\pi^2+\mathcal{O}(M_\pi^4)$$

Hard Pion ChPT: A two-loop check

- Similar arguments to JB-Celis, Flynn-Sachrajda work for the pion vector and scalar formfactor JB-Jemos
- Predicts:

$$F_V(t,M_\pi^2) = F_V(t,0)\left(1-\frac{M_\pi^2}{16\pi^2F^2}\ln\frac{M_\pi^2}{\mu^2}+\mathcal{O}(M_\pi^2)\right)$$
$$F_S(t,M_\pi^2) = F_S(t,0)\left(1-\frac{5}{2}\frac{M_\pi^2}{16\pi^2F^2}\ln\frac{M_\pi^2}{\mu^2}+\mathcal{O}(M_\pi^2)\right)$$

- Note that $F_{V,S}(t,0)$ is now a coupling constant and can be complex

Hard Pion ChPT: A two-loop check

- Take the full two-loop ChPT calculation JB,Colangelo,Talavera, valid for $t, m_\pi^2 \ll \Lambda_\chi^2$
- Expand this for $t \gg m_\pi^2$
- $t^2 \ln t, \ldots\ldots$ terms go in $F_{S,V}(t,0)$
- HPChPT predicts how the chiral log $M_\pi^2 \log M_\pi^2$ appears at two loops
- The two loop precisely satisfies it with:

$$F_V(t,0) = 1 + \frac{t}{16\pi^2 F^2}\left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6}\ln\frac{t}{\mu^2}\right)$$
$$F_S(t,0) = 1 + \frac{t}{16\pi^2 F^2}\left(1 + 16\pi^2 l_4^r + i\pi - \ln\frac{t}{\mu^2}\right)$$

- Note that the needed coupling constants in HPChPT are indeed complex

$B, D \to \pi, K, \eta$

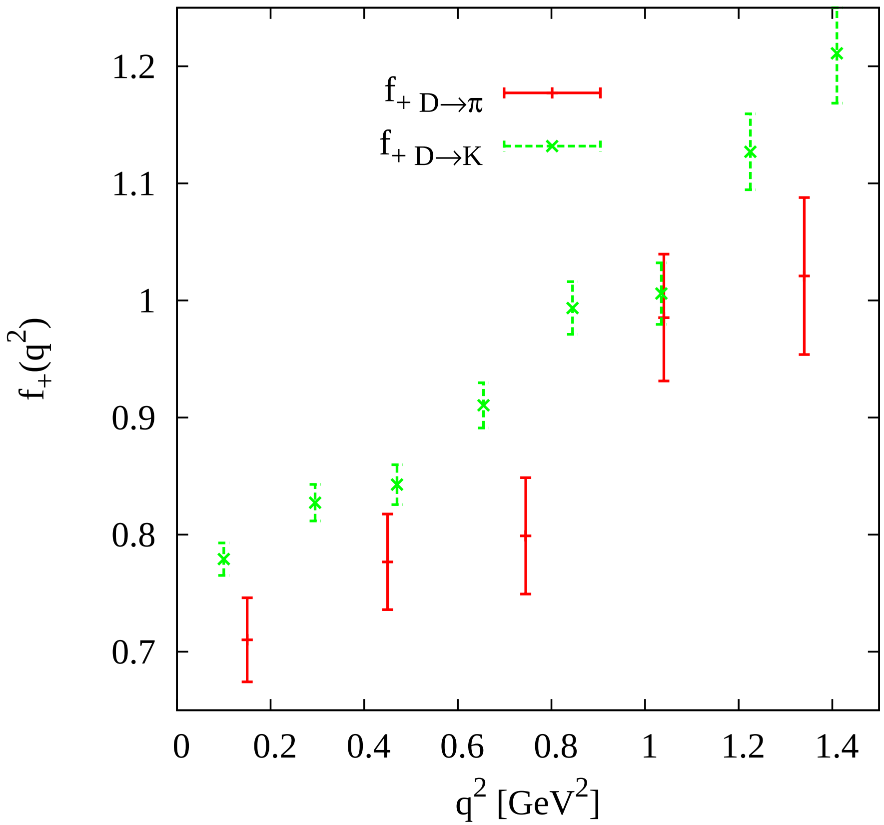
$$\left\langle P_f(p_f) \left| \overline{q}_i \gamma_\mu q_f \right| P_i(p_i) \right\rangle = (p_i + p_f)_\mu f_+(q^2) + (p_i - p_f)_\mu f_-(q^2)$$

$$f_{+B\to M}(t) = f^\chi_{+B\to M}(t) F_{B\to M}$$
$$f_{-B\to M}(t) = f^\chi_{-B\to M}(t) F_{B\to M}$$

- t away from endpoint
- $F_{B\to M}$ is always the same for f_+, f_- and f_0
- LEET: in this limit the two formfactors are related
  J. Charles et al, hep-ph/9812358

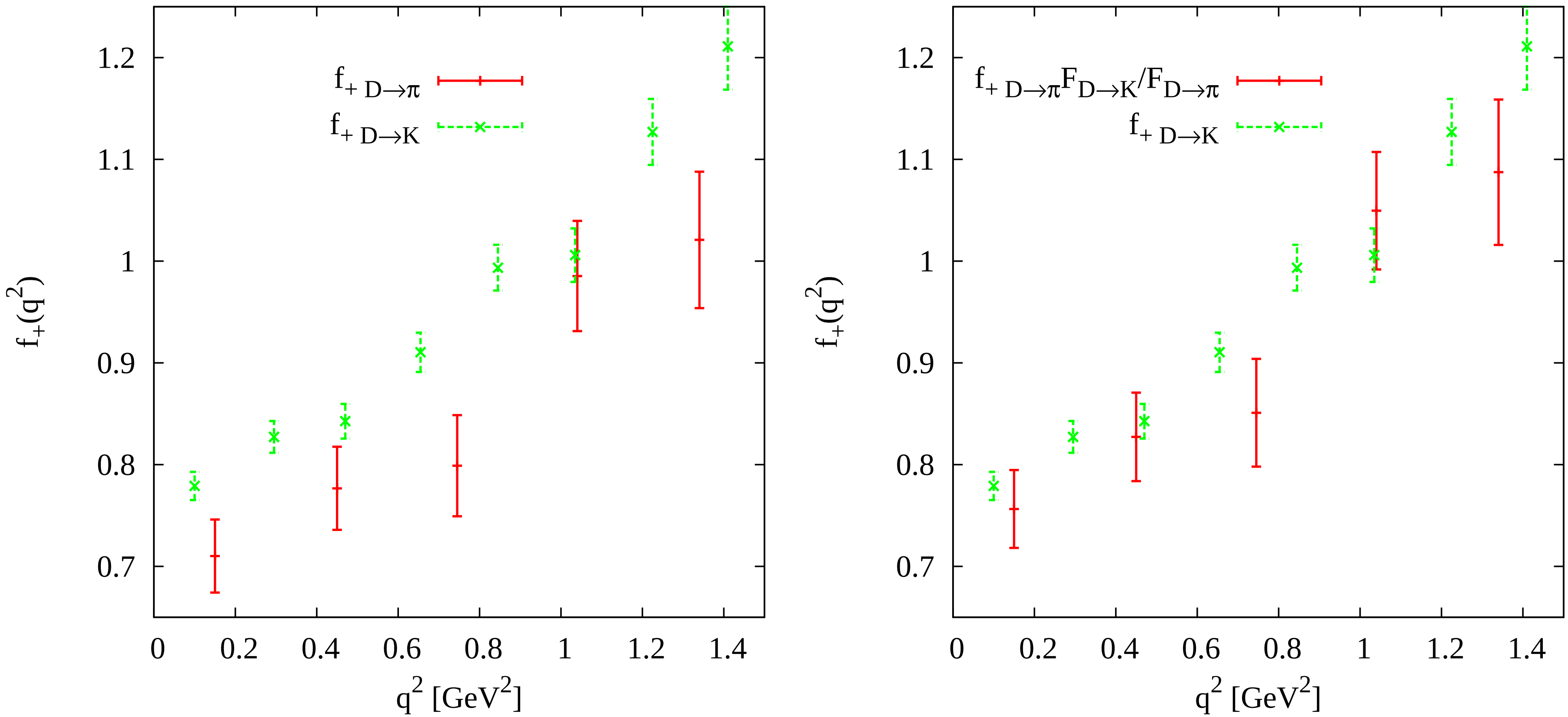
Experimental check

CLEO data on $f_+(q^2)|V_{cq}|$ for $D \to \pi$ and $D \to K$ with $|V_{cd}| = 0.2253$, $|V_{cs}| = 0.9743$



Experimental check

CLEO data on $f_+(q^2)|V_{cq}|$ for $D \to \pi$ and $D \to K$ with $|V_{cd}| = 0.2253$, $|V_{cs}| = 0.9743$



$$f_{+D\to\pi} = f_{+D\to K} F_{D\to\pi}/F_{D\to K}$$

Applications to charmonium

- We look at decays $\chi_{c0}, \chi_{c2} \to \pi\pi, KK, \eta\eta$
- $J/\psi, \psi(nS), \chi_{c1}$ decays to the same final state break isospin or U-spin or V-spin, they thus proceed via electromagnetism or quark mass differences: more difficult.

Applications to charmonium

- We look at decays $\chi_{c0}, \chi_{c2} \to \pi\pi, KK, \eta\eta$
- $J/\psi, \psi(nS), \chi_{c1}$ decays to the same final state break isospin or U-spin or V-spin, they thus proceed via electromagnetism or quark mass differences: more difficult.
- No chiral logarithm corrections
- These decays should have small $SU(3)_V$ breaking

Charmonium

	χ_{c0}		χ_{c2}	
Mass	3414.75 ± 0.31 MeV		3556.20 ± 0.09 MeV	
Width	10.4 ± 0.6 MeV		1.97 ± 0.11 MeV	
Final state	10^3 BR	$10^{10}\, G_0[\mathrm{MeV}^{-5/2}]$	10^3 BR	$10^{10} G_2[\mathrm{MeV}^{-5/2}]$
$\pi\pi$	8.5 ± 0.4	3.15 ± 0.07	2.42 ± 0.13	3.04 ± 0.08
K^+K^-	6.06 ± 0.35	3.45 ± 0.10	1.09 ± 0.08	2.74 ± 0.10
$K_S^0 K_S^0$	3.15 ± 0.18	3.52 ± 0.10	0.58 ± 0.05	2.83 ± 0.12
$\eta\eta$	3.03 ± 0.21	2.48 ± 0.09	0.59 ± 0.05	2.06 ± 0.09
$\eta'\eta'$	2.02 ± 0.22	2.43 ± 0.13	< 0.11	< 1.2

Experimental results for $\chi_{c0}, \chi_{c2} \to PP$ and the factors corrected for the known m^2 effects.

- $\pi\pi$ and KK are good to 10% (Note: 20% for F_K/F_π)
- $\eta\eta$ OK

Summary

Why is this useful:

- Lattice works actually around the strange quark mass
- need only extrapolate in m_u and m_d.
- Applicable in momentum regimes where usual ChPT might not work
- Three flavour case useful for B, D, χ_c decays
- tells us something nontrivial about otherwise very difficult quantities