Selected Topics in the Theory of Heavy Ion Collisions

Lecture 2

Urs Achim Wiedemann
CERN Physics Department
TH Division

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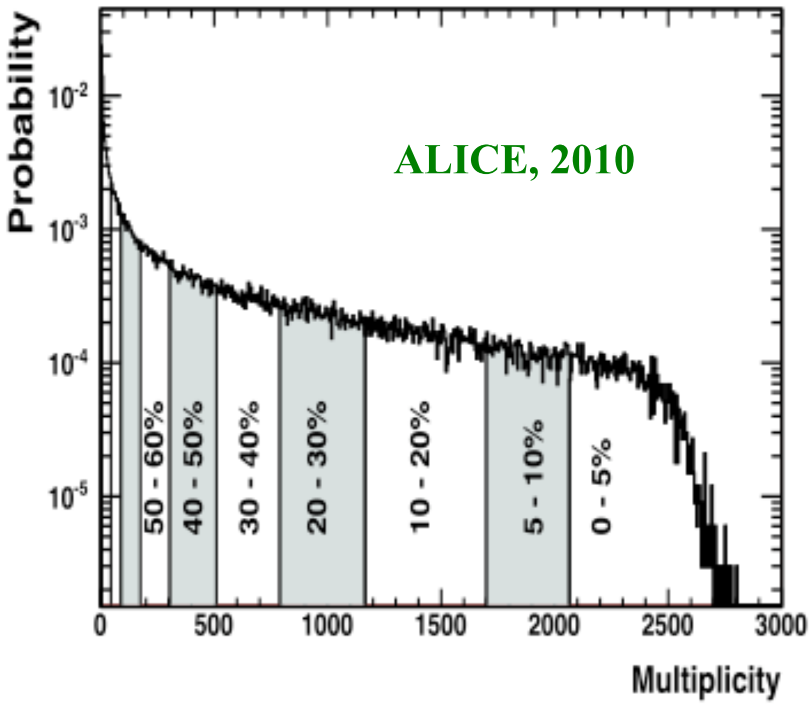
Recall lecture 1: we discussed how to

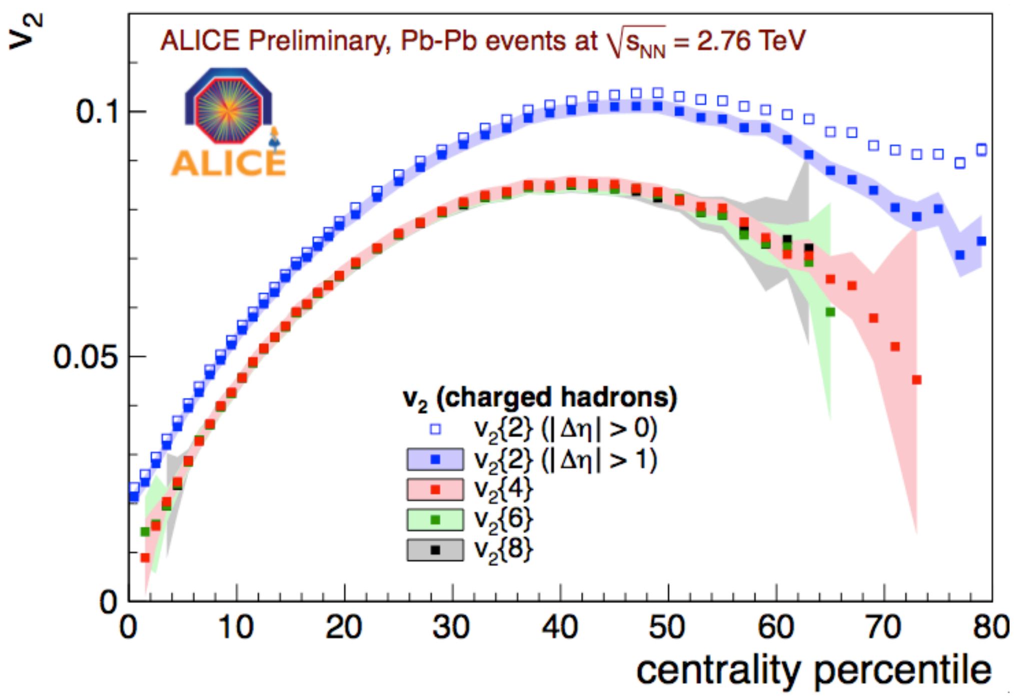
- associate an impact parameter range $b \in [b_{min}, b_{max}]$ to an event class in A+A.

  (namely by selecting multiplicity classes via Glauber theory)

- analyze azimuthal asymmetries and disentangle collective flow from fluctuations

  (namely by a cumulant analysis of flow harmonics)





Lecture 2 continues here …

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II.8. Alternative flow measurements: Q-cumulants

Construction of 'standard' cumulants involves sum over M(M-1) terms to 2nd order

(2.14) ~ M^4 terms to 4th order,
~ M^6 to 6th order, etc

$$\langle 2 \rangle \equiv \left\langle e^{i\,n\,(\phi_1 - \phi_2)} \right\rangle = \frac{1}{M(M-1)} \sum_{\substack{i,j=1 \\ (i \neq j)}}^{M} e^{i\,n\,(\phi_i - \phi_j)}$$

Problem: For typical event multiplicity M this becomes computationally expensive

Solution: Use Q-vector of harmonic N (sum over M terms only!)

(2.15)

$$Q_n \equiv \sum_{i=1}^{M} e^{i\,n\,\phi_i}$$

to construct cumulants

Problem: check this!
Bilandzic, Snellings, Voloshin, arXiv:1010.0233 [nucl-ex]

(2.16)

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

(2.17)

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2\,\mathrm{Re}\left[Q_{2n} Q_n^* Q_n^*\right] - 4(M-2)|Q_n|^2}{M(M-1)(M-2)(M-3)} + \frac{2}{(M-1)(M-2)}$$

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II.9.Yet another method: EP

For each event, one estimates directly the orientation of the event plane (**EP**)

(2.18)
$$\psi_n \equiv \tan^{-1} \frac{\sum_{i=1}^{M} w_i \sin(n\phi_i)}{\sum_{i=1}^{M} w_i \cos(n\phi_i)} \Big/ n$$

(2.19) One then measures
$$\frac{dN}{d(\phi-\psi_n)} = \frac{\langle N \rangle}{2\pi}\left[1 + \sum_{k=1}^{\infty} 2\, v_{kn}^{obs} \cos\left(kn(\phi-\psi_n)\right)\right]$$

(2.20) But we want to measure w.r.t. true reaction plane orientation ψ_r
$$E\frac{dN}{d^3p} = \frac{1}{2\pi}\frac{dN}{p_t dp_t dy}\left[1 + \sum_{n=1}^{\infty} 2\, v_n\{EP\} \cos\left(n(\phi-\psi_r)\right)\right]$$

(2.21) Correction needed
$$v_n\{EP\} = v_n^{obs} \Big/ \langle \cos n(\psi_n - \psi_r) \rangle \equiv v_n^{obs} / R$$

(2.22) Event-plane resolution R estimated e.g. from sub-event method (A,B,C indep. sub-events)
$$R = \sqrt{\frac{\langle \cos n(\psi_n^A - \psi_n^B) \rangle \langle \cos n(\psi_n^A - \psi_n^C) \rangle}{\langle \cos n(\psi_n^B - \psi_n^C) \rangle}}$$

Poskanzer, Voloshin, PRC58 (1998) 1671

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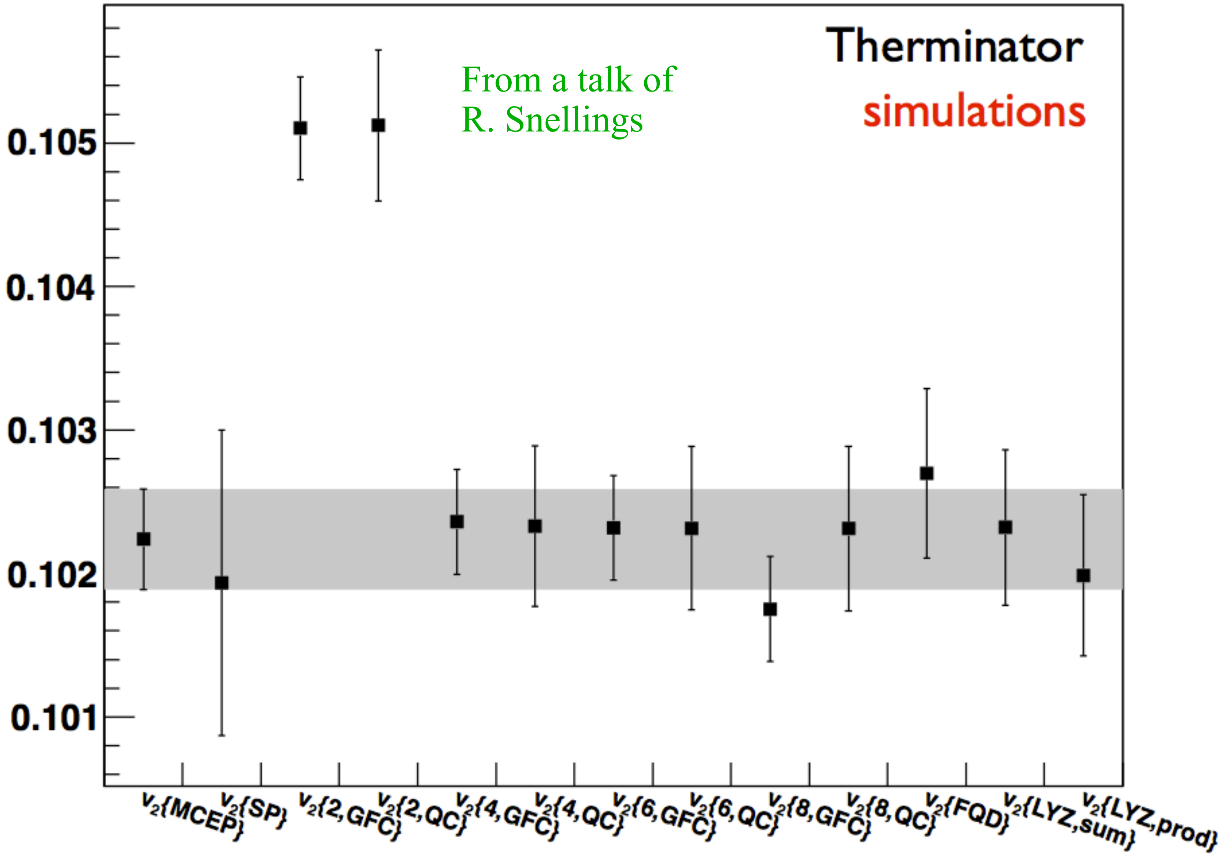
II.10.Consistency of flow analysis methods



Many important technical issues not touched here.

Take home message:

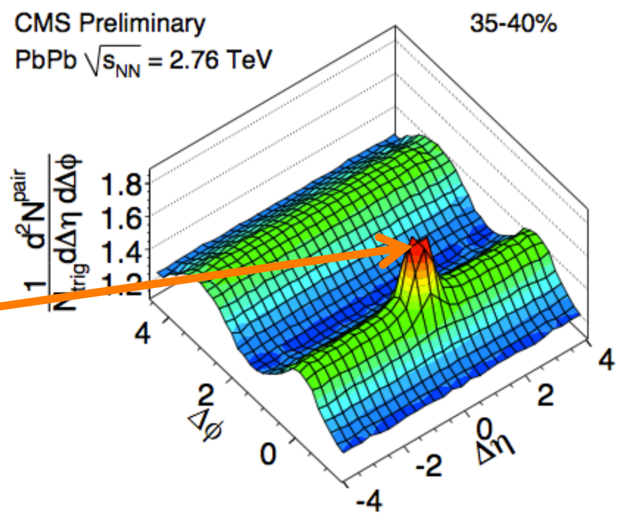
- There are many flow analysis methods with different systematic uncertainties.
- They are "generally" consistent, deviations are "relatively well" understood.

II.11.Flow in measured two-particle correlations

Flow harmonics measured via particle correlations.
Here: look directly at correlations of 'trigger' with 'associate' particle
(often pt-cuts on 'trig' and 'assoc')

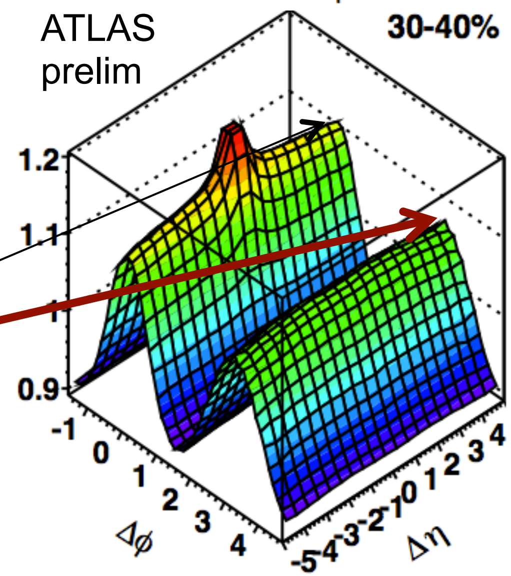
If flow dominated, then

$$\text{(2.23)}\quad \frac{2\pi}{N_{pairs}}\left\langle \frac{dN_{pairs}}{d\Delta\phi}\right\rangle = 1 + \sum_{n=1}^{\infty} 2\left\langle v_n^{(trig)} v_n^{(assoc)}\right\rangle \cos\left(n\Delta\phi\right)$$



Characteristic features:

1. Small-angle jet-like correlations around $\Delta\phi \approx \Delta\eta \approx 0$ (this is a non-flow effect)
2. Long-range rapidity correlation (almost rapidity-independent)
3. Elliptic flow v_2 seems to dominate (for the semi-peripheral collisions shown here)
4. **Away-side peak** at $\Delta\phi \approx \pi$ is smaller (implies ***non-vanishing odd harmonics v1, v3,*** …)



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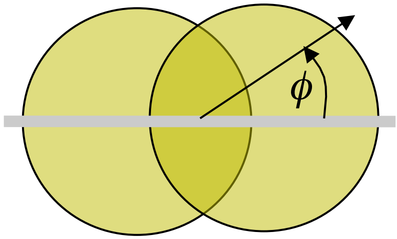
II.12.Non-vanishing odd flow harmonics



Event-averaged (non-fluctuating) **initial conditions** have nuclear overlap with

$\phi \to \phi + \pi$ **symmetry**

Dynamics cannot break this symmetry of the initial conditions

$$\Rightarrow v_{2n+1} = 0 \quad \forall n$$

Conclusion: Non-vanishing odd harmonics are unambiguous signal for Event-by-Event fluctuations in **initial conditions.**

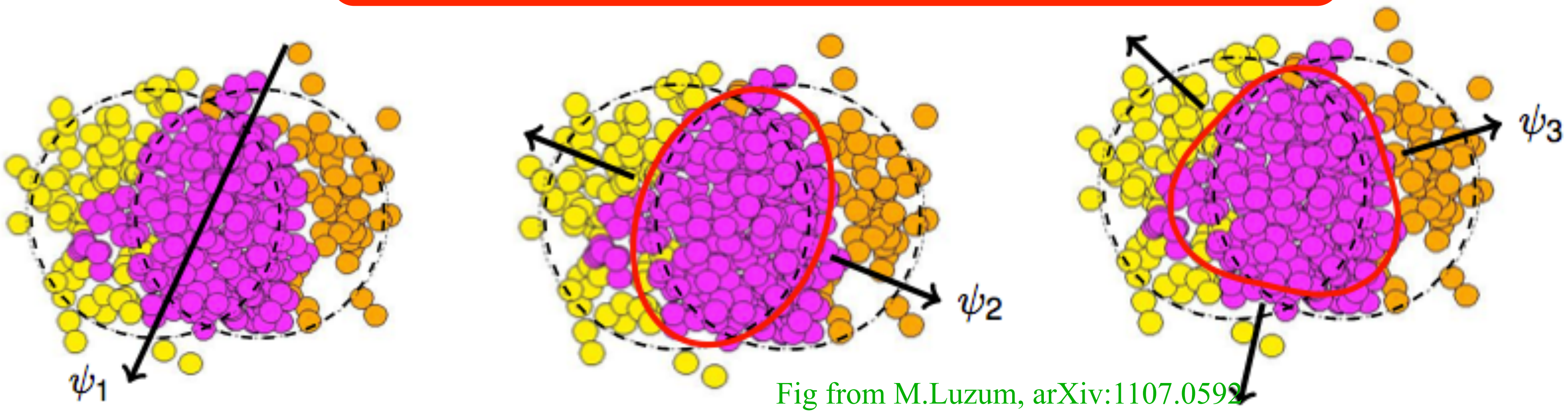


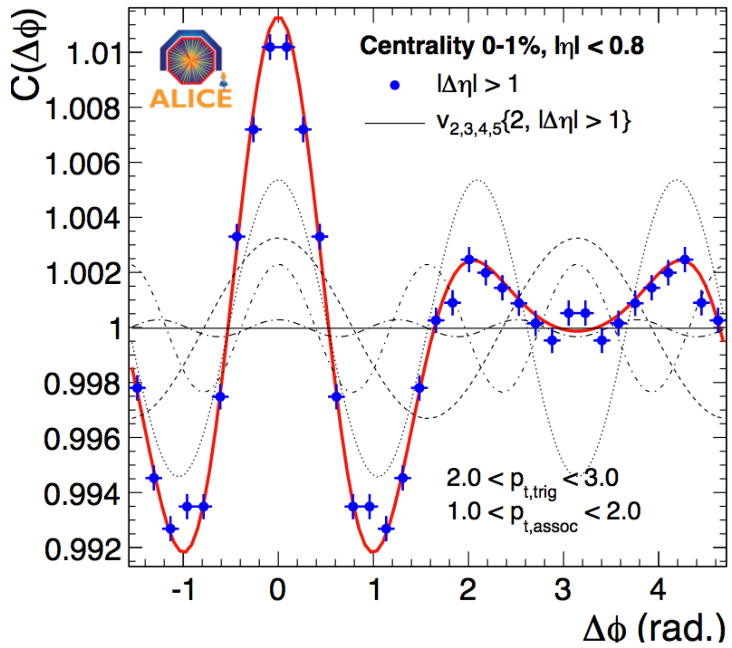
Fig from M.Luzum, arXiv:1107.0592

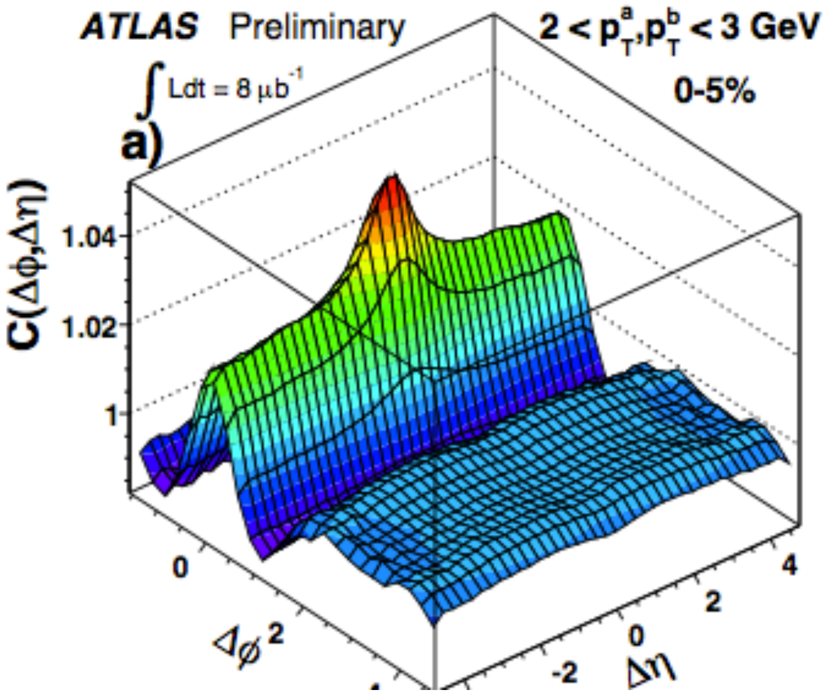
II.13. Odd harmonics dominate central collisions

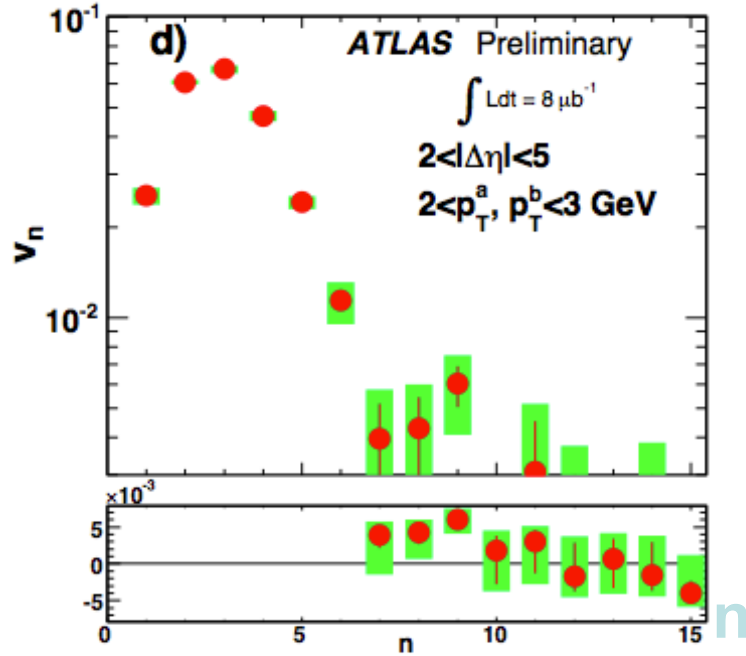
In the most central 0-5% events,

(2.24) $$v_3 \geq v_2$$

Fluctuations in initial conditions dominate flow measurements







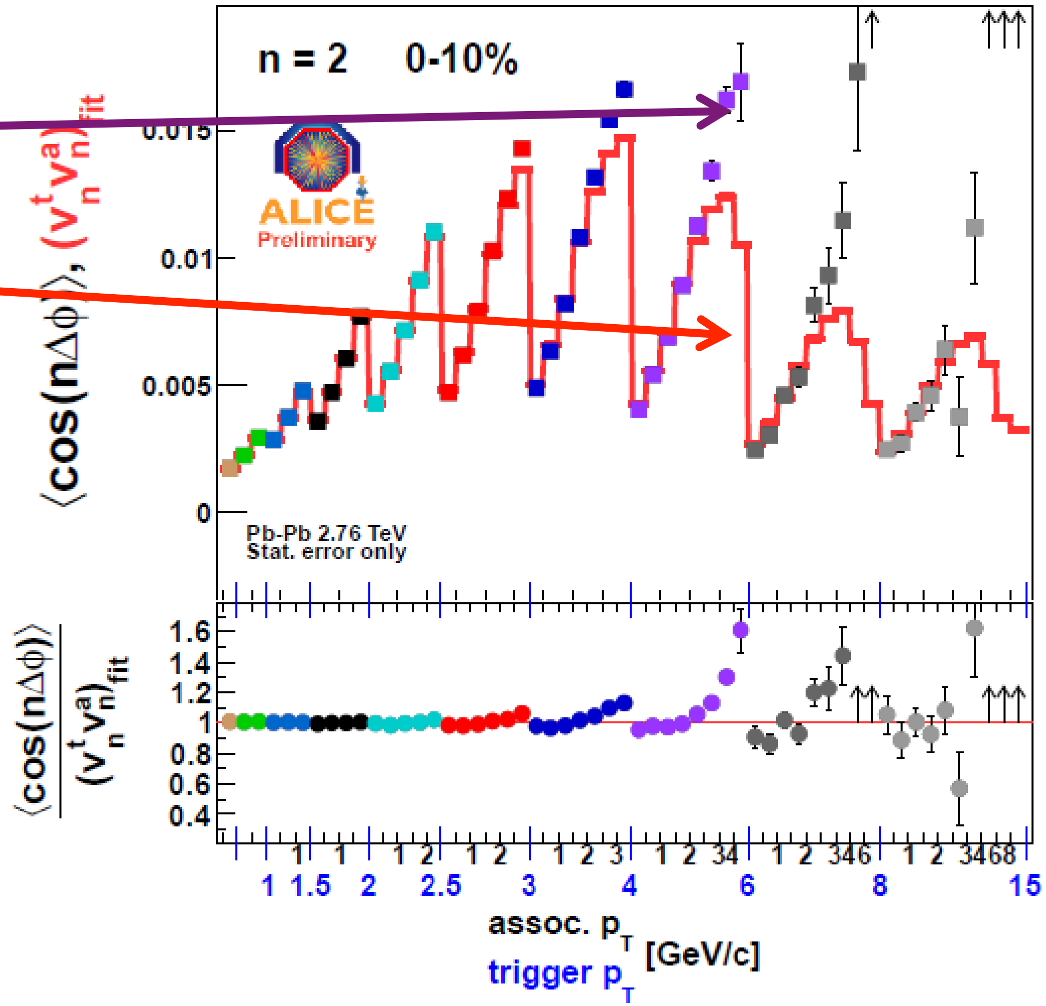
II.14. Factorization of 2-particle correlations

If these fluctuations in the initial conditions *propagate collectively* to the measured flow harmonics,
then 2-particle-correlations must factorize.

Do they? Check (2.23)

$$\frac{2\pi}{N_{pair}}\frac{dN_{pair}}{d\Delta\phi} = 1 + 2\sum_{n=1}^{\infty} \langle v_n^{(t)} v_n^{(a)} \rangle \cos(n\,\Delta\phi)$$

At sufficiently low p_T, data consistent with assumption of collective propagation.



II.15.Characterizing spatial asymmetries

To discuss propagation of fluctuations in initial conditions, need to quantify them.
Characterize **spatial eccentricities**, e.g., via moments of transverse density

(2.24) $\varepsilon_{m,n} e^{in\phi_{m,n}} \equiv -\frac{\left\{ r^m e^{in\phi} \right\}}{\left\{ r^m \right\}}, \quad \varepsilon_n \equiv \varepsilon_{n,n}$ $\qquad \left\{ \ldots \right\} \equiv \frac{\int d^2x\, \rho(x) \ldots}{\int d^2x\, \rho(x)}$

Simplifying working hypothesis (commonly used)
- EbyE asymmetry of initial condition is a **purely spatial eccentricity**
- **spatial eccentricity** is related to (**momentum**) **flow** by ***linear response***

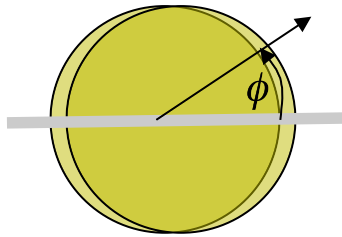
(2.25) $v_n \exp\left[in\psi_n \right] = k\, \varepsilon_n \exp\left[in\phi_n \right] + corr$

For tests, see e.g.
F. Gardim et al, arXiv:1111.6538

Final aim: to understand the dynamical mechanism that maps
fluctuating initial conditions onto flow harmonics

Aside: In most central collision, event-averaged (non-fluctuating) initial conditions would lead to

$$\varepsilon_n \approx 0 \Rightarrow v_n \approx 0$$



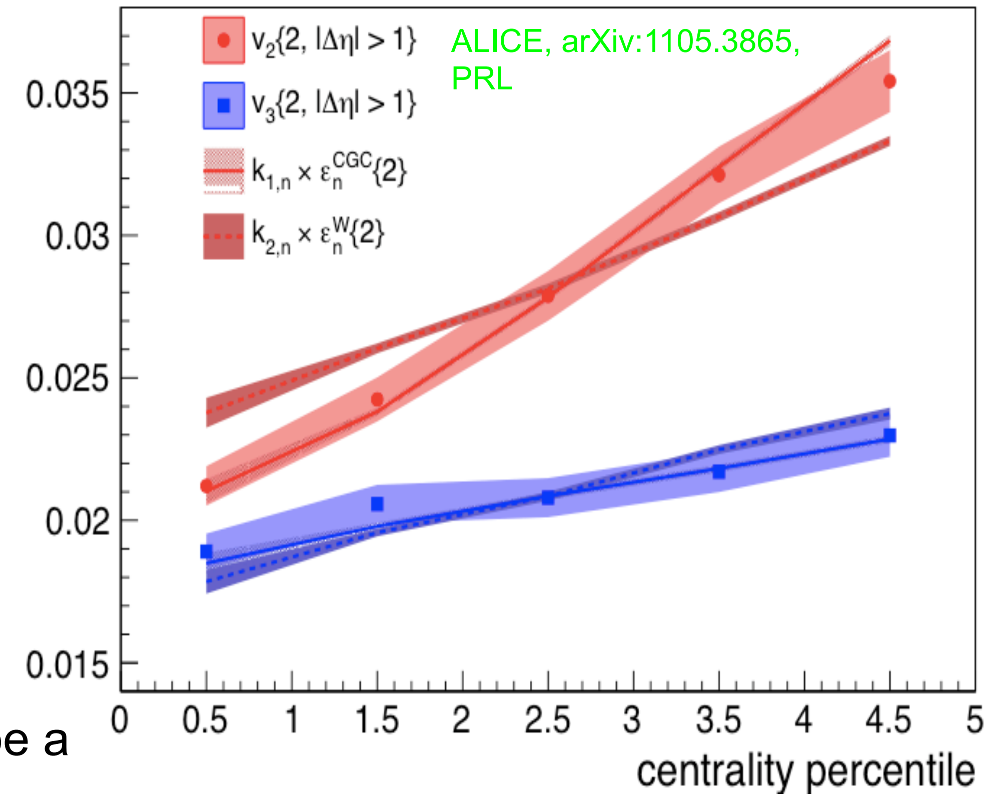
Thus, no geometric reason for 2nd harmonics to dominate fluctuating initial conditions (see II.13).

II.16. Comparing spatial eccentricies with flow

Simple models for initial spatial eccentricities and their centrality dependence can be based on supplementing e.g. Glauber model with notion of energy density:

(2.26)

$$\varepsilon\left(\underline{x}\right) \equiv \sum_{i=1}^{N_{part}} \varepsilon_{NN}\left(\underline{x} - \underline{x}_i\right) = \frac{K}{2\pi\sigma^2} \sum_{i=1}^{N_{part}} \exp\left[-\frac{\left(\underline{x} - \underline{x}_i\right)^2}{2\sigma^2} \right]$$



Spatial eccentricities

- details model-dependent
- for some models

(2.27)

$$v_n \propto \varepsilon_n$$

- Linear response (2.27) seems to be a fair first approximation
- But deviations from linear response (2.27) do not disprove a model of eccentricity in initial conditions. They could be accounted for by non-linear dynamics. (**to which we turn now**).

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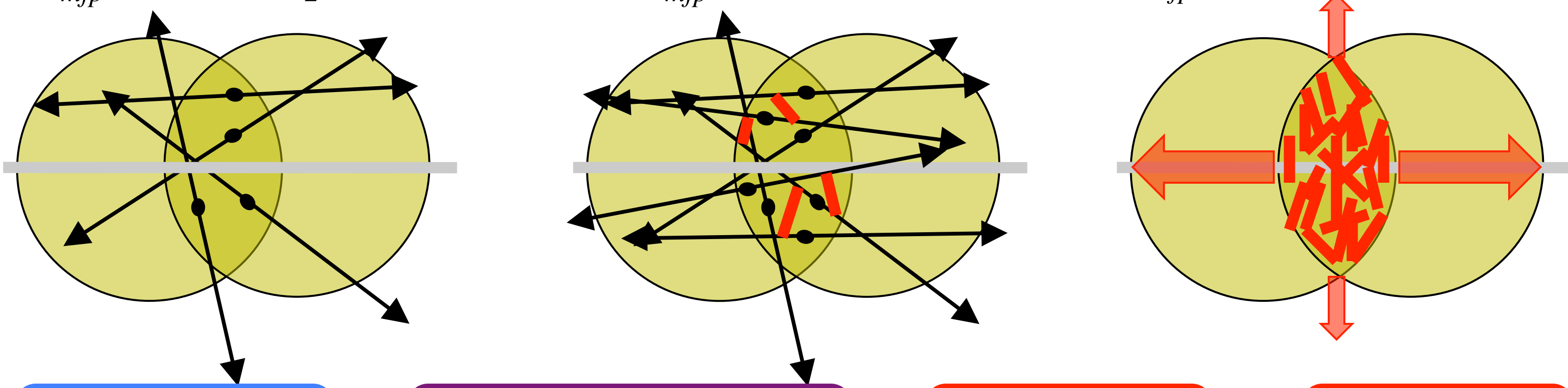
III. Dynamical framework for collective flow

We seek a dynamical framework that maps

initial conditions	→	**particle spectra**
- their average eccentricities - their EbyE fluctuations		- their p_T - and η - dependence - their flow harmonics

Mean free path vs. collectivity: $\lambda_{mfp} \approx \infty \Rightarrow v_2 = 0$ | $\lambda_{mfp} \approx finite$ | $\lambda_{mfp} \approx 0 \Rightarrow v_2 = \max$



Theory tools: Free streaming → Particle cascade (QCD transport theory) → Dissipative fluid dynamics → Perfect fluid dynamics

System p+p ?? … A+A … ??

Study **fluid dynamics** as relevant theoretical baseline for discussing collective effects …

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III.1. Fluid dynamics - the basics

Consider matter in local equilibrium, characterized locally by its energy momentum tensor, the density of n charges, and a flow field:

- energy momentum tensor $T^{\mu\nu}$ .... 10 indep. components
- conserved charges N_i^{μ} .... 4n indep. components

Tensor decomposition w.r.t. flow field $u_\mu(x)$ projector $\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$

(3.1) $$N_i^{\mu} = n_i\, u^{\mu} + \bar{n}_i$$

(3.2) $$T^{\mu\nu} = \varepsilon\, u^{\mu} u^{\nu} - p\Delta^{\mu\nu} + q^{\mu} u^{\nu} + q^{\nu} u^{\mu} + \Pi^{\mu\nu}$$

(3.3) (1 comp.) $\varepsilon \equiv u_\mu T^{\mu\nu} u_\nu$ energy density

(3.4) (1 comp.) $p \equiv -T^{\mu\nu}\Delta_{\mu\nu}/3$ isotropic pressure

(3.5) (3 comp.) $q^{\mu} \equiv \Delta^{\mu\alpha} T_{\alpha\beta} u^{\beta}$ heat flow

(3.6) (5 comp.) $\Pi^{\mu\nu} \equiv \left[\left(\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\mu}_{\beta}\Delta^{\nu}_{\alpha}\right)/2 - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3\right] T^{\alpha\beta}$ shear viscosity

In Local Rest Frame (LRF) $u_\mu = (1,0,0,0)$

Convenient choice of frame: Landau frame: $u = u_L \Rightarrow q^{\mu} = 0$

Eckard frame: …

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III.2. Equations of motion for a perfect fluid

A fluid is perfect if it is locally isotropic at all space-time points. This implies

(3.7) $$N_i^\mu = n_i u^\mu + \bar{n}_i$$ (n comp.)

(3.8) $$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu}$$ (5 comp.)

The equations of motion are then determined by conservation laws

(3.9) $$\partial_\mu N_i^\mu \equiv 0$$ (n constraints)

(3.10) $$\partial_\mu T^{\mu\nu} \equiv 0$$ (4 constraints)

and the equation of state

(3.11) $$p = p(\varepsilon, n)$$ (1 constraint)

Here, information from ab initio calculations (lattice) or models enters.

Hydrodynamic simulations are numerical solutions of (3.7),(3.8).
'Systematic' model uncertainties arise from

- specifying initial conditions
- specifying the decoupling of particles ('freeze-out')
- assuming that non-perfect terms in (3.7),(3.8) can be dropped
- specifying (3.11)

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III.3. Two-dimensional Bjorken fluid dynamics

Main assumption: initial conditions for thermodynamic fields do not depend on space-time rapidity

(3.12) $$\eta = \frac{1}{2} \ln \left[\frac{t+z}{t-z} \right]$$

Longitudinal flow has 'Hubble form':

(3.13) $$v_z = z/t$$

Bjorken scaling means that hydrodynamic equations preserve Hubble form

(3.14) $$u^\mu = \cosh y_T \left(\cosh \eta, v_x, v_y, \sinh \eta \right)$$ **Longitudinally boost-invariant flow profile**

(3.15) at mid-rapidity $$v_r(\tau, r, \eta = 0) \equiv \tanh y_T(\tau, r)$$

(3.16) at forward rapidity $$v_r(\tau, r, \eta) \equiv \frac{v_r(\tau, r, \eta = 0)}{\cosh \eta}$$

Problem: show that e.o.m. (3.10) preserve longitudinal boost-invariance of initial conditions.
solution see e.g. Kolb+Heinz, PRC62 (2000) 054909

III.4. 2-dim "perfect" Hydro Simulations: Input…

Initialization: thermo-dynamic fields $\varepsilon(\tau, r, \eta = 0)$ have to be initialized, e.g. by

(3.17) $$\varepsilon_{init}(\underline{r}) = \varepsilon(\tau_0, \underline{r}, \eta = 0) \propto \left(\frac{1-x}{2} \overline{N}^{AB}_{part}(\underline{b}, \underline{r}) + x \overline{N}^{AB}_{coll}(\underline{b}, \underline{r}) \right)$$

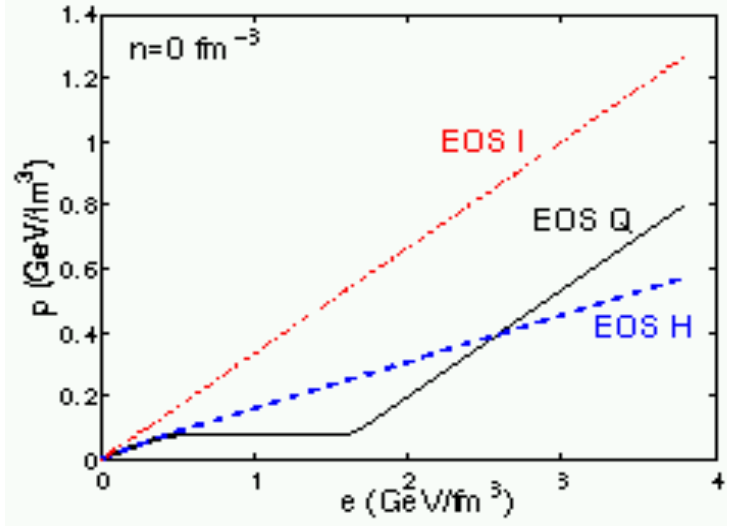
Equation of state: $p(\varepsilon, n)$

(3.18) Velocity of sound: $c_s^2 = \frac{\partial p}{\partial \varepsilon}$

(3.19) Expectations: $c_s^2 \approx 0.15$ **Soft EOS**

$c_s^2 = 1/3$ **Hard EOS**

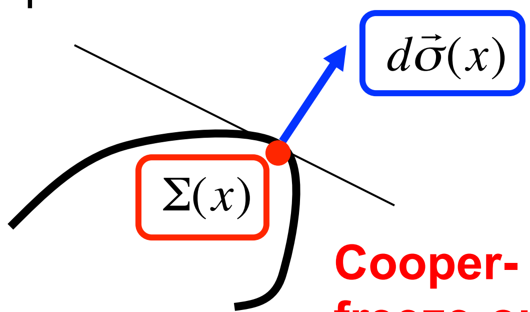
Input from (many) models and from lattice QCD.



Freeze-out: local temperature $T(x) = T_{fo}$ defines space-time hypersurface $\Sigma(x)$, from which particles decouple with spectrum

(3.20) $$E \frac{dN_i}{d\vec{p}} = \frac{g_i}{(2\pi)^3} \int_{\Sigma} \vec{p}.d\vec{\sigma}(x)\, f_i(p.u(x), x)$$

(3.21) $$f_i(E, x) = \frac{1}{\exp[(E - \mu_i(x))/T(x)] \pm 1}$$

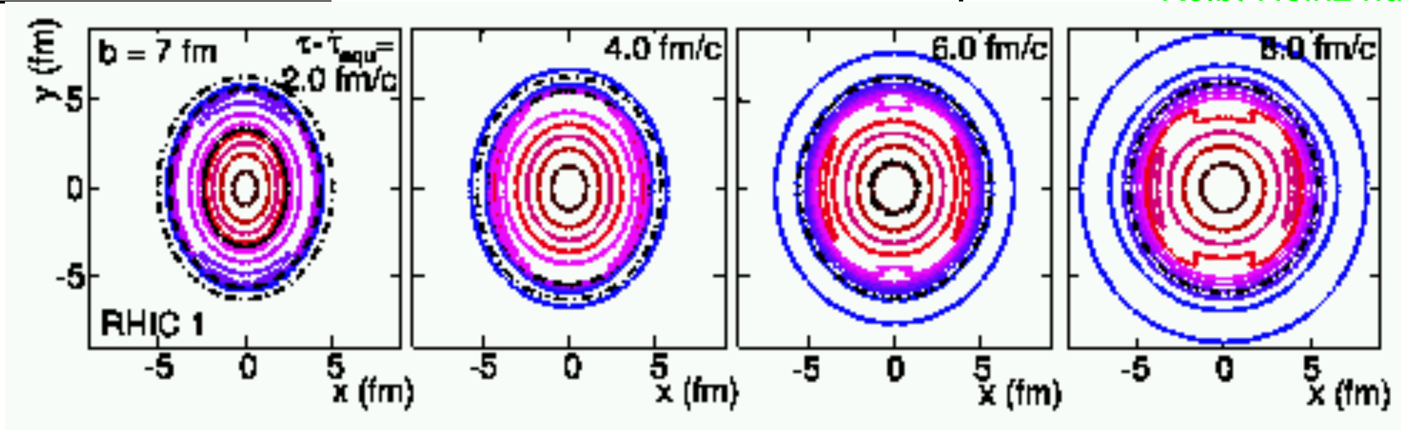


Cooper- Frye freeze-out

III.5. 2D-simulations with event-averaged IC

Results of simulations: time evolution in transverse plane

Kolb, Heinz nucl-th/0305084



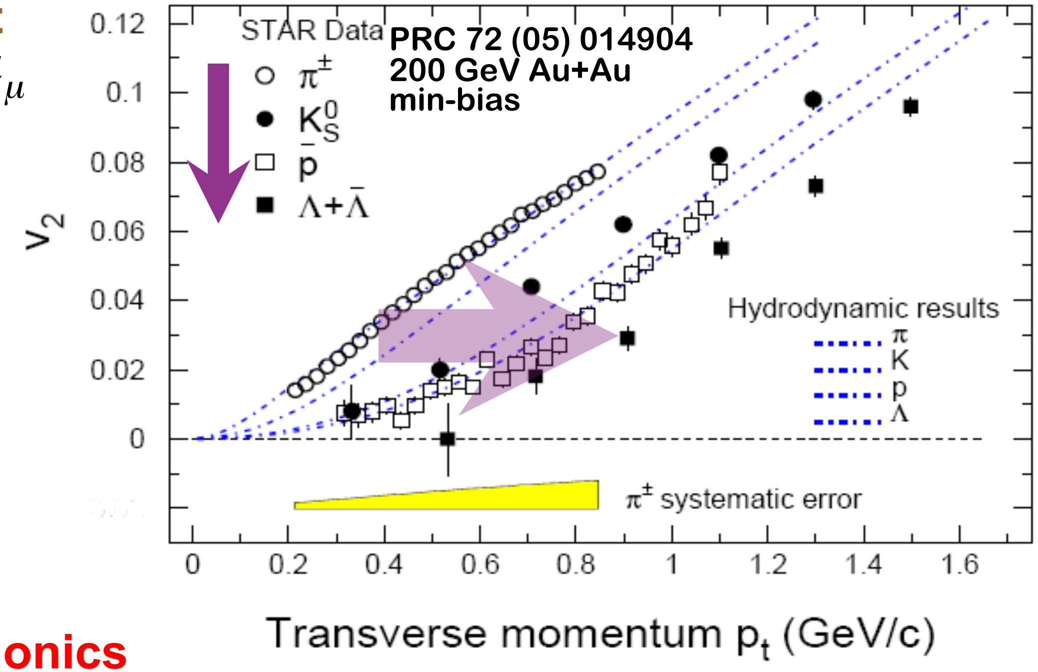
Conclusions from such studies:

- initial **transverse pressure gradient**
  ⇒ ϕ - dependence of flow field u_μ
  ⇒ elliptic flow $v_2(p_T)$

- size and pt-dependence of v_2 data accounted for by hydro ('maximal')

- characteristic **mass dependence**, since all particle species emerge from common flow field u_μ

- **BUT: no fluctuations, no odd harmonics**



III.6. Dissipative corrections to a perfect fluid

Small deviations from a locally isotropic fluid can be accounted for by restoring

(3.7) $$N_i^\mu = n_i\, u^\mu + \bar{n}_i^\mu$$ (4n comp.)

(3.8) $$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu}$$ (10 comp.)

When does perfect fluid assumption fail? Consider conserved current:

(3.22) $$\partial_\mu j^\mu = \partial_\mu\left(\rho\, u^\mu\right) = \rho \underbrace{\partial_\mu u^\mu}_{\text{expansion scalar}} + \underbrace{u^\mu \partial_\mu}_{\text{comoving } t-\text{derivative}} \rho = 0$$

Spatio-temporal variations of macroscopic fluid should be small if compared to microscopic reaction rates

(3.23) $$\Gamma \cong n\sigma >> \theta = \partial_\mu u^\mu$$

Dissipative corrections characterized by gradient expansion!

Now, the conservation laws and equation of state

$$\partial_\mu N_i^\mu \equiv 0 \qquad \text{(n constraints)}$$

$$\partial_\mu T^{\mu\nu} \equiv 0 \qquad \text{(4 constraints)}$$

$$p = p(\varepsilon, n) \qquad \text{(1 constraint)}$$

are not sufficient to constrain all independent thermo-dynamic fields in (3.7),(3.8).

How do we obtain additional constraints?

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III.7. 1st order dissipative fluid dynamics

Since conservation laws + eos do not close equations of motion, one seeks additional constraints from expanding 2nd law of thermodynamics to 1st order

(3.24) $S^\mu = s\,u^\mu + \beta q^\mu$ Entropy to first order

Use $\varepsilon + p = \mu n + Ts$ and $u_\nu \partial_\mu T^{\mu\nu} \equiv 0$ to write:

(3.25) $T\partial_\mu S^\mu = (T\beta - 1)\partial q + q(\dot{u} + T\partial\beta) + \Pi^{\mu\nu}\partial_\nu u_\mu + \Pi\theta \geq 0$

To warrant that entropy increases, require:

$\beta \equiv 1/T$

Navier-Stokes
1st order hydro

(3.26) bulk viscosity $\Pi \equiv \varsigma\,\theta$

(3.27) heat conductivity $q^\mu \equiv \kappa T \Delta^{\mu\nu}\left(\partial_\nu \ln T - \dot{u}_\nu\right)$

(3.28) shear viscosity $\Pi^{\mu\nu} \equiv 2\eta\left[\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right)/2 - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3\right]\partial^\alpha u^\beta$

Determines $\Pi, q^\mu, \Pi^{\mu\nu}$ in terms of flow, energy density and dissipative coeff.

(3.29) $$\partial_\mu S^\mu = \frac{\Pi^2}{\varsigma T} - \frac{q\,q}{\kappa T^2} + \frac{\Pi^{\mu\nu}\Pi_{\mu\nu}}{2\eta T} \geq 0$$

Problem: instantaneous acausal propagation.

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III.8. 2nd order viscous hydro – entropy derivation

Expand entropy to 2nd order in dissipative gradients

(3.30) $S^\mu = s u^\mu + \beta q^\mu + \alpha_0 \Pi q^\mu + \alpha_1 \Pi^{\mu\nu} q_\nu + u^\mu \left(\beta_0 \Pi^2 + \beta_1 q q + \beta_2 \Pi^{\mu\nu} \Pi_{\mu\nu} \right)$

Now, need 9 eqs. to determine $\Pi, q^\mu, \Pi^{\mu\nu}$

$\partial_\mu S^\mu \geq 0$ leads to differential equations for $\Pi, q^\mu, \Pi^{\mu\nu}$

which involve $\alpha_0, \alpha_1, \beta, \beta_0, \beta_1, \beta_2, \varsigma, \kappa, \eta$

Entropy increase determined by **shear viscosity** (if vorticity neglected)

(3.31) $$T \partial_\mu S^\mu = \Pi_{\mu\nu} \left[-\beta_2 D \Pi^{\mu\nu} + \tfrac{1}{2} \left\langle \nabla^\mu u^\nu \right\rangle \right] \equiv \frac{1}{2\eta} \Pi_{\mu\nu} \Pi^{\mu\nu}$$ $\beta_2 = \tau_\Pi / 2\eta$

Equations of motion involve **relaxation time** and **viscosity**.

Notations: covariant derivative $d_\mu u^\nu \equiv \partial_\mu u^\nu + \Gamma^\nu_{\alpha\mu} u^\alpha$

Convective derivative $D \equiv u^\mu d_\mu$

Nabla operator $\nabla^\mu \equiv \Delta^{\mu\nu} d_\nu = d^\mu - u^\mu D$

Angular bracket $\left\langle A^{\mu\nu} \right\rangle \equiv \left[\tfrac{1}{2} \left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha \right) - \tfrac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \right] A^{\alpha\beta}$

III.9. Fluid dynamics from transport theory

Dissipative fluid dynamics can also be derived as the long wave-length limit of transport theory.

Consider Boltzmann equation with relaxation time approximation

(3.32) $$p^\mu d_\mu f(x,p) = C \approx -\left(u^\mu p_\mu\right)\frac{f - f_{eq}}{\tau_\pi}$$

Consider small departures from local thermal equilibrium, quadratic ansatz

(3.33) $$f = f_{eq}\left[1 + \varepsilon_{\mu\nu}(x,p)p^\mu p^\nu\right] \qquad \varepsilon_{\mu\nu} = \frac{1}{2T^2(\varepsilon + p)}\Pi_{\mu\nu}$$

With this ansatz, we write momentum moments from the Boltzmann eq.

… long journey …

(3.34)
$$(\varepsilon + p)Du^\mu = \nabla^\mu p - \Delta^\mu_\nu \nabla^\sigma \Pi^{\nu\sigma} + \Pi^{\mu\nu} Du_\nu$$
$$D\varepsilon = -(\varepsilon + p)\nabla_\mu u^\mu + \tfrac{1}{2}\Pi^{\mu\nu}\left\langle \nabla_\nu u_\mu \right\rangle$$
$$\tau_\pi \Delta^\mu_\alpha \Delta^\nu_\beta D\Pi^{\alpha\beta} + \Pi^{\mu\nu} = \eta\left\langle \nabla^\mu u^\nu \right\rangle - 2\tau_\pi \Pi^{\alpha(\mu}\omega^{\nu)}_\alpha$$

2nd order **Israel-Stewart** fluid dynamic equations of motion.

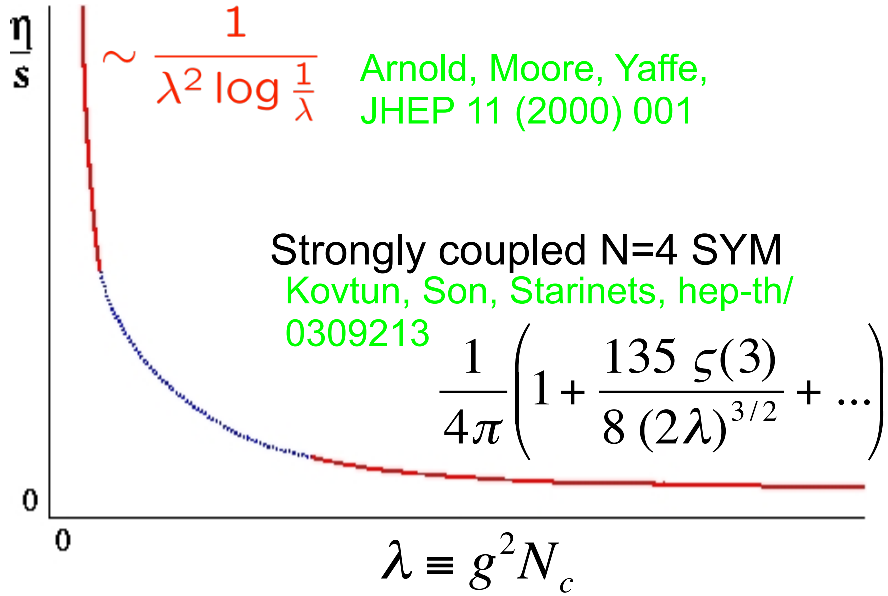
III.10. Input: transport coefficients are fundamental properties of hot QCD matter

The Green-Kubo formula defines transport coefficient as long wave-length limit of retarded Green’s function of energy-momentum tensor

(3.35)
$$G^R_{xy,xy}(\omega,0) \equiv \int dt\, dx\, e^{i\omega t}\, \Theta(t) \left\langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \right\rangle_{eq}$$

$$\eta \equiv - \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} G^R_{xy,xy}(\omega,0)$$

Calculable from first principles in quantum field theory (QCD)



First attempts in finite temperature lattice QCD: H. Meyer, PRD76 (2007) 101701

Motivates the scanning of

$$\eta / s$$

in units of $1/4\pi$

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III.11. Input: relaxation times

Also relaxation times are calculable from first principles in QFT …
In some theories with gravity dual, e.g. N=4 SYM, ***all*** relaxation times and transport coefficients are known

Bhattacharyya, Hubeny, Minwalla,Rangamani 2008
Kanitschneider, Skenderis (2009)
Buchel, Myers (2009)
Romatschke (2009)

in the **weak coupling limit**,

(3.36)
$$\tau_\pi\big|_{\lambda<<1} \sim 5.9 \frac{\eta}{\varepsilon + p}$$

and in the **strong coupling limit**

(3.37)
$$\tau_\pi\big|_{\lambda>>1} \sim \left(4 - 2\ln 2 + \frac{375}{8}\varsigma(3)\lambda^{-3/2}\right)\frac{\eta}{\varepsilon + p} \approx \frac{0.2}{T}$$

Relaxation time is very short

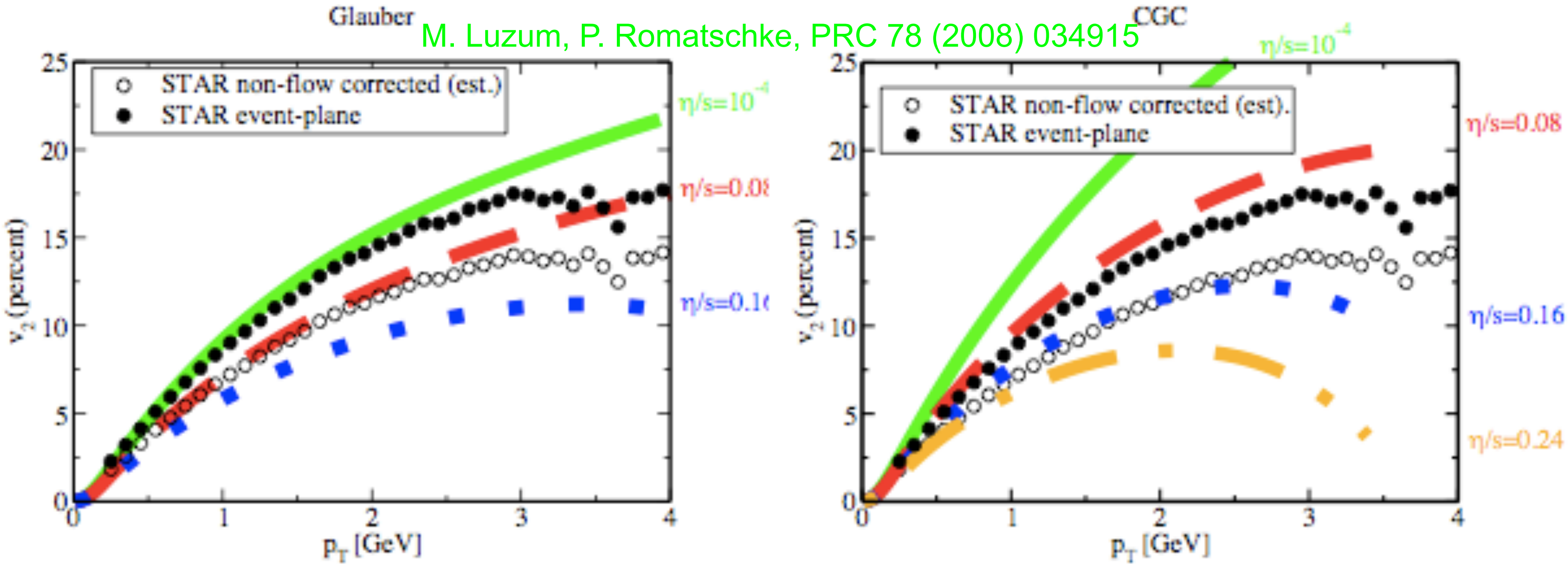
Remarkable curiosity: all modes propagate causal
(need not be the case since hydro holds in long wavelength limit only)

Numerical simulations show very weak dependence on value of relaxation time (see following slides).

III.12. Sensitivity of flow on shear viscosity

Elliptic flow decreases strongly even for close to minimal values of η / s

M. Luzum, P. Romatschke, PRC 78 (2008) 034915



(3.38) To understand order of magnitude, consider 1st order Navier-Stokes dissipative hydrodynamics

$$\frac{d(\tau s)}{d\tau} = \frac{\frac{4}{3}\eta}{\tau T}$$

'Perfect liquid' description applicable, if change of entropy small compared to s

$$\frac{\eta}{\tau T}\frac{1}{s} << 1$$

(3.39) Put in numbers $\tau \sim 1\, fm/c, \quad T \sim 200\, MeV$ $\Rightarrow$ $\frac{\eta}{s} << 1$

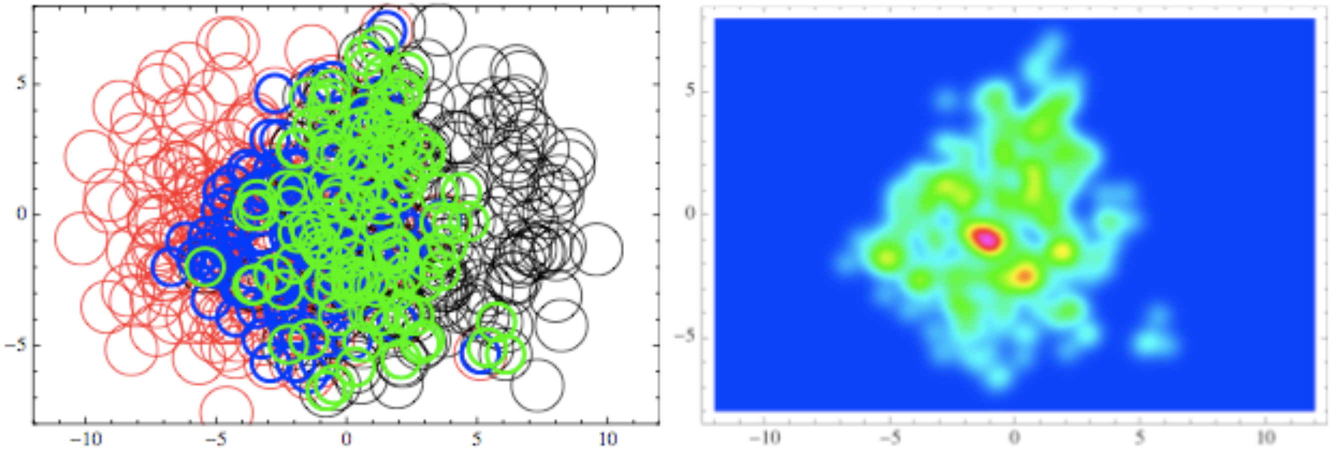
U.A.Wiedemann

III.13. Input with EbyE fluctuations

EbyE fluctuations needed to account for odd harmonic flow coefficients.

- Typical transverse energy density distribution from Glauber model

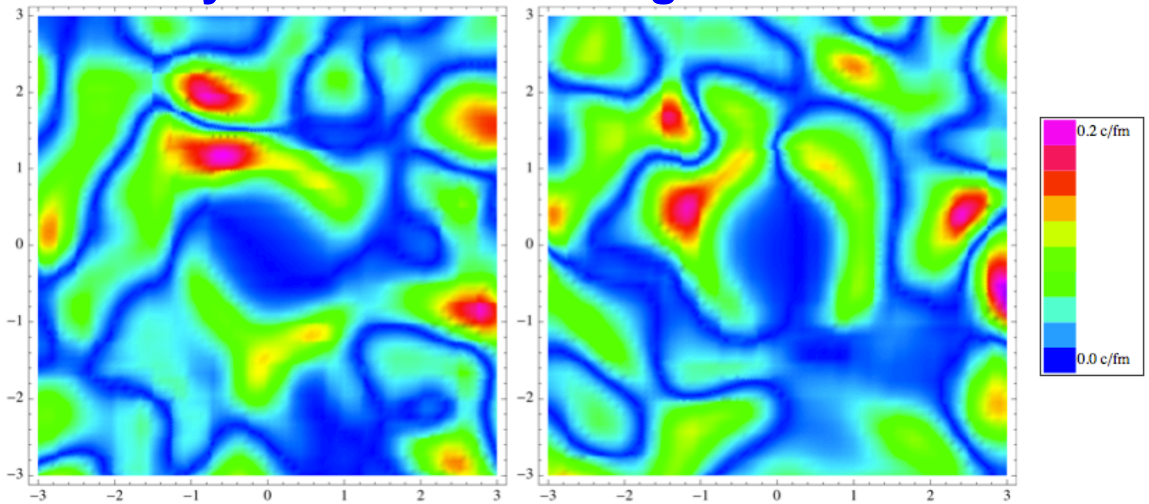
Relevance for v3 first pointed out by B. Alver and G. Roland, PRC81 (2010) 054905



S. Flörchinger, UAW, arXiv:1108.5535, JHEP in press

- Fluctuations in initial velocity fields (normally not included)

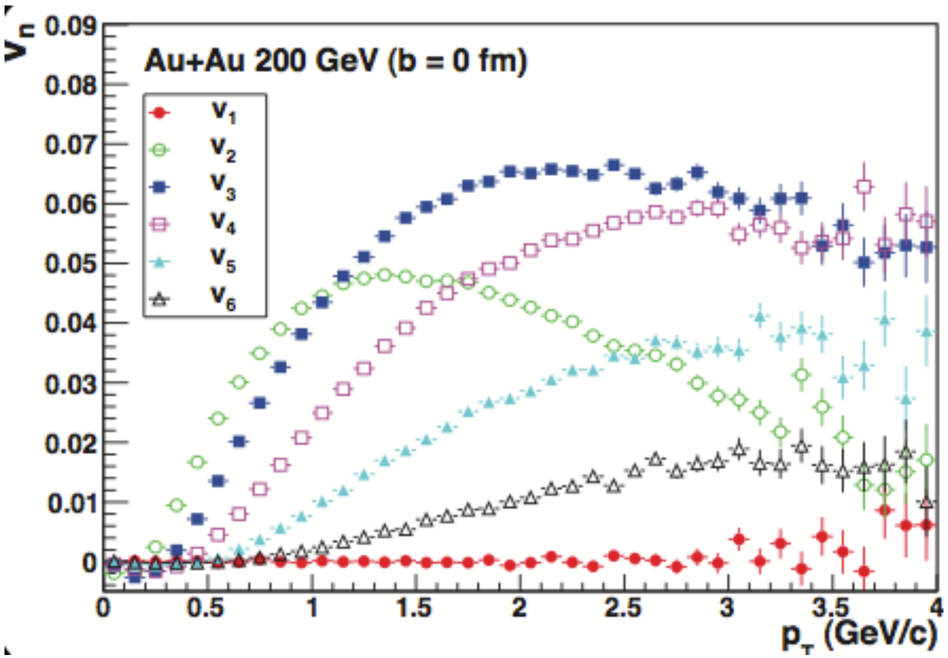
Vorticity of flow field **Divergence of flow field**

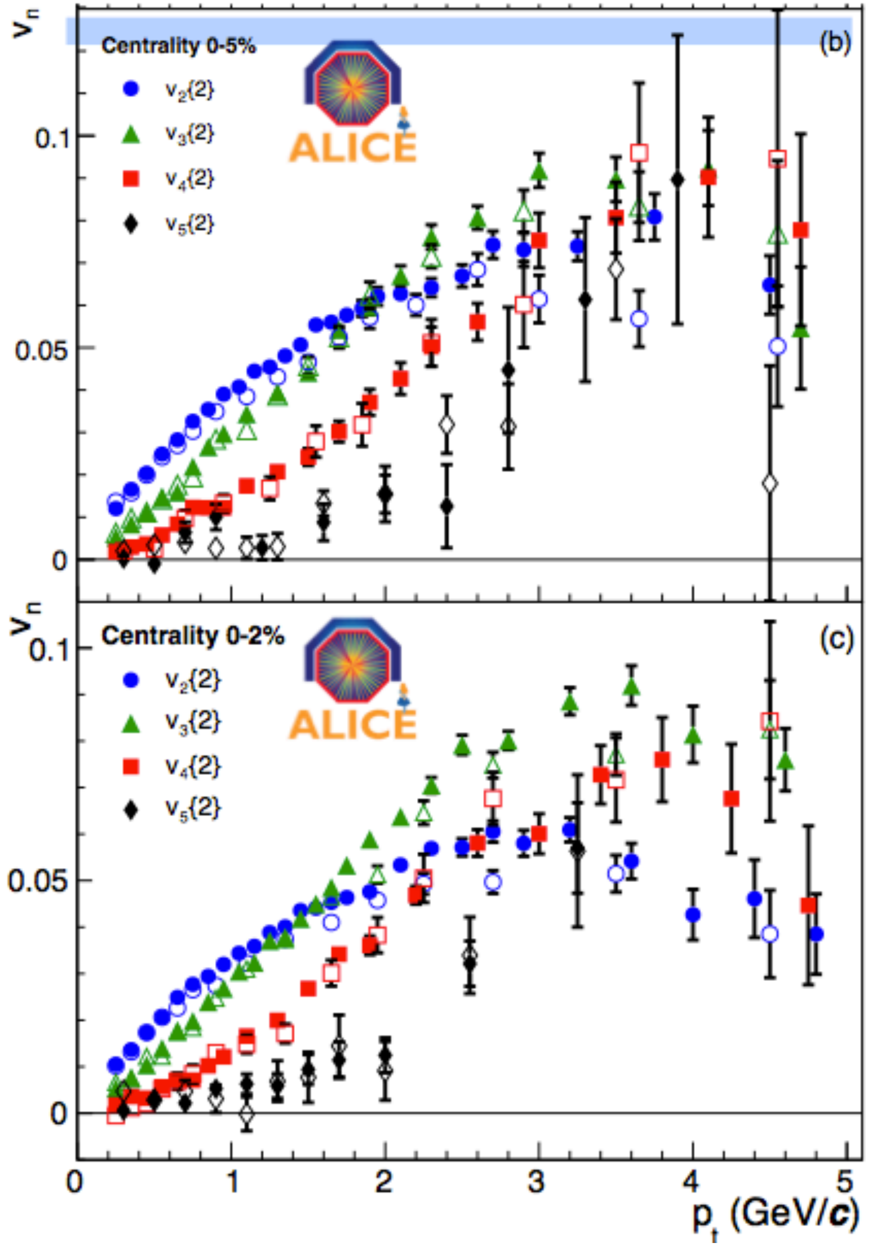


III.14. Odd harmonics in transport models…

- AMPT: includes fluctuations in the initial state …

G-L Ma & X.N. Wang, arXiv:1011.5249v2



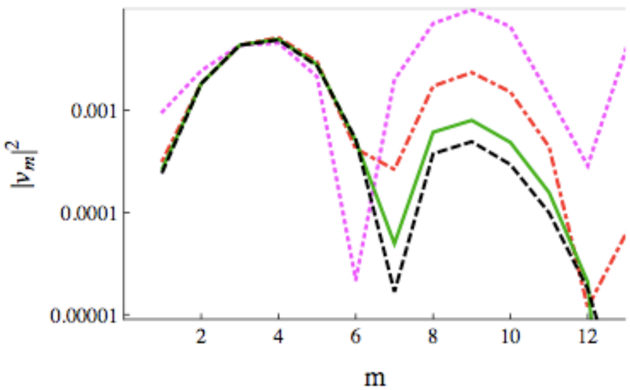


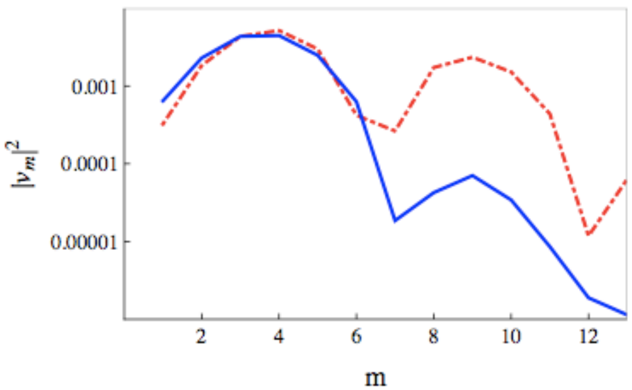
- This is not a fluid dynamic simulation but the AMPT transport model has very small m.f.p's

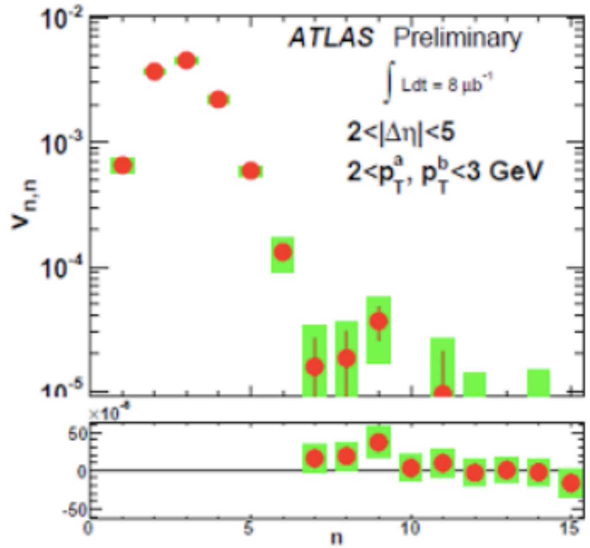
III.15. How does fluid dynamics propagate fluctuations in heavy ion collisions?

P. Staig and E. Shuryak, arXiv:1109.6633

- Consider <u>linear</u> fluid dynamic perturbations on top of analytically known event-averaged fluid dynamic solution (Gubser's model)
- Find that higher Fourier modes of fluid dynamic perturbations dissipate faster
- Emphasize analogy with CMB radiation spectrum







III.16. How does fluid dynamics propagate fluctuations in heavy ion collisions?

S. Flörchinger, UAW, arXiv:1108.5535, JHEP in press

- If fluid dynamic description holds, Reynold's number is

$$\mathrm{Re} \propto 1/(\eta/s) \cong 1-10$$

- consider linear and non-linear propagation of fluid dynamic perturbations on top of analytically known Bjorken model:

  late time dynamics governed (after coord. trafo) by 2-dim Navier-Stokes equation

Heavy Ions

- Bjorken expansion (1-dim)
- time-scale sufficient for fluid dynamic description? (exp support but no deep th understanding)
- expansion delays onset of non-linearities only in longitudinal dimension
- **dynamics of fluctuations gives access to material properties** (viscosities, relaxation times, calculable from 1st principles of QFT)

CMB

- Hubbel expansion (3-dim)
- time scale clearly sufficient for fluid dynamic description
- expansion delays onset of non-linearities
- dynamics of fluctuations gives access to matter content of Universe

Much more to come …