Model for Color suppressed mode $\overline{B_d^0} \to \pi^0\pi^0$

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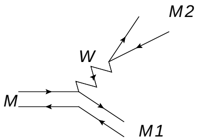
Extended chiral quark model for light energetic quarks

$B \to \pi\pi$ decays

- Non-leptonic decays always difficult due to QCD corrections
- Use lattice, quark models, sum rules, eg
- From 1999, Beneke et. al.: *QCD factorization* For $B \to \pi\pi, \pi K, \ldots$
  Corrections to factorization:
  $\frac{\alpha_s}{\pi}$ (calculable), $\frac{\Lambda_{QCD}}{m_b}$ (not calculable).
- $\overline{B_d^0} \to \pi^0\pi^0$ is not factorizable
- For $\overline{B_d^0} \to \pi^0\pi^0$, calculated previously (QCD fact, SCET, QCD sum rules) but amplitude factor 2 off.
- We will use new LEχQM which includes hard pions and soft gluons

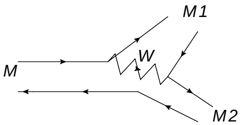
Quark Diagrams for Non-Leptonic Decays

$\overline{B_d^0} \to \pi^0\pi^0$ and $\overline{B_d^0} \to \pi^+\pi^-$ are dynamically different.



color allowed

$\overline{B_d^0} \to \pi^+\pi^-$



color suppressed

$\overline{B_d^0} \to \pi^0\pi^0$

quark process $b \to u d \bar{u}$

Effective Lagrangian Non-Leptonic Decays

Effective non-leptonic Lagrangian at quark level:

$$\mathcal{L}_{eff} = -G\left[c_A(\mu)Q_A + c_B(\mu)Q_B\right]$$

$\mu =$ renormalization scale. C_i = Wilson coefficient

$$Q_A = 4(\bar{u}_L\gamma_\mu b_L)(\bar{d}_L\gamma^\mu u_L),\ Q_B = 4(\bar{u}_L\gamma_\mu u_L)(\bar{d}_L\gamma^\mu b_L)$$

At tree level $c_A = 1$ and $c_B = 0$
For “flavor mismatch”, use Fierz transformation:

$$\hat{Q}_i \rightarrow \hat{Q}_i^{Fierz} = \frac{1}{N_c}\, j_L^\alpha(q_1 \rightarrow q_4)\, j_\alpha^L(q_3 \rightarrow q_2) + 2\,\hat{Q}_{color}$$

$$\hat{Q}_{color} \ = \ j_L^\alpha(q_1 \rightarrow q_4)^a\, j_\alpha^L(q_3 \rightarrow q_2)^a \quad ; \ \ j_L^\alpha(q_i \rightarrow q_j)^a = \overline{(q_j)_L}\,\gamma^\alpha\, t^a\,(q_i)_L$$

Colored operators might dominate

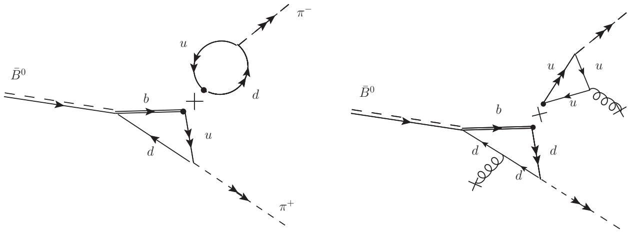
$$\mathcal{M}_{\pi^0\pi^0} = 4G\Big[(c_B + \frac{c_A}{N_c})\left\langle\pi^0|\bar{u}_L\gamma_\mu u_L|0\right\rangle\left\langle\pi^0|\bar{d}_L\gamma^\mu b_L|\bar{B}^0\right\rangle$$
$$+2c_A\left\langle\pi^0\pi^0|(\bar{u}_L\gamma_\mu t^a u_L)(\bar{d}_L\gamma^\mu t^a b_L)|\bar{B}^0\right\rangle\Big]$$

$\left(c_B + \frac{c_A}{N_c}\right) \approx 0$ Color suppressed

$$\mathcal{M}_{\pi^+\pi^-} = 4G\Big[(c_A + \frac{c_B}{N_c})\left\langle\pi^-|\bar{d}_L\gamma_\mu u_L|0\right\rangle\left\langle\pi^+|\bar{u}_L\gamma^\mu b_L|\bar{B}^0\right\rangle$$
$$+2c_B\left\langle\pi^+\pi^-|(\bar{d}_L\gamma_\mu t^a u_L)(\bar{u}_L\gamma^\mu t^a b_L)|\bar{B}^0\right\rangle\Big]$$

$\left(c_A + \frac{c_B}{N_c}\right) \approx 1$ color allowed

$\overline{B^0} \to \pi^+\pi^-$, $\overline{B^0} \to \pi^0\pi^0$ in LEχQM



$$\mathcal{M}_{\pi^+\pi^-} = 4G\,(c_A + \frac{c_B}{N_c})\left\langle \pi^- | \bar{d}_L \gamma_\mu u_L | 0 \right\rangle \left\langle \pi^+ | \bar{u}_L \gamma^\mu b_L | \bar{B}^0 \right\rangle$$

$$\mathcal{M}_{\pi^0\pi^0} = 4G\,2c_A \left\langle \pi^0\pi^0 | (\bar{u}_L \gamma_\mu t^a u_L)(\bar{d}_L \gamma^\mu t^a b_L) | \bar{B}^0 \right\rangle$$

How to calculate $\langle M_1\, M_2 |\, \hat{Q}^{color}\, | M \rangle$?

The Chiral Quark Model (χQM)

To be used for colored operators! Light $q = u, d, s$ sector:

$$\mathcal{L}_{\chi QM} = \mathcal{L}_{QCD} + \mathcal{L}_{\chi}$$

$$\mathcal{L}_{\chi} = -m \left(\overline{q}_R \, \Sigma \, q_L + \overline{q}_L \, \Sigma^{\dagger} q_R \right)$$

$\Rightarrow$ Meson-quark couplings.

$m =$ *constituent* light quark mass, due to chiral symmetry breaking.

“Rotated version” ; flavour rotated “constituent quark fields”

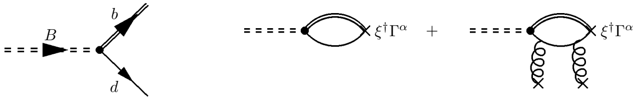
$$\chi_L = \xi q_L \quad , \quad \chi_R = \xi^{\dagger} q_R \quad ; \qquad \xi \cdot \xi = \Sigma$$

$$\mathcal{L}_{\chi QM} = \overline{\chi} \left[\gamma^{\mu} (iD_{\mu} + \mathcal{V}_{\mu} + \gamma_5 \mathcal{A}_{\mu}) - m \right] \chi + \ldots$$

The $HL\chi QM$

$$\mathcal{L}_{HL\chi QM} = \mathcal{L}_{HQET} + \mathcal{L}_{\chi QM} + \mathcal{L}_{Int}$$

$$\mathcal{L}_{Int} = -G_H \left[\overline{\chi}_f \, \overline{H_v^f} \, Q_v + \overline{Q_v} \, H_v^f \, \chi_f \right]$$



Ingtegrating out quarks, should give the known HLχPT terms
$\Rightarrow$ Physical and model dependent parameters, $f_\pi, \langle \overline{q}q \rangle, f_H, g_A$ linked to (divergent) loop integrals in HLχQM!

Fit in strong sector: $m \sim 220$ MeV, $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4} \sim 315$ MeV, $G_H{}^2 = \frac{2m}{f^2}\,\rho$
where $\rho \sim 1$ and $\rho = \rho(f_\pi, \langle \frac{\alpha_s}{\pi} G^2 \rangle, m, g_A)$

(A.Hiorth and J.O.Eeg, PRD 66 (2002))

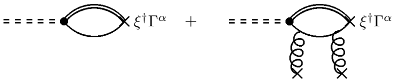
Soft gluons

Hard gluons are integrated out

Soft gluons couple to quarks in the loops

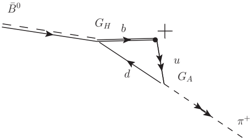
Introduce gluon condensates (model dep.) by (Novikov et al.):

$$g_s^2 G^a_{\mu\nu} G^a_{\alpha\beta} \rightarrow 4\pi^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \, ,$$



The $LE\chi QM$

$$\mathcal{L}_{intq} = G_A\,(\bar{q}\,\gamma_\mu\gamma_5(\partial^\mu M)\,q_n) + h.c \qquad , \; M = \text{meson fields}$$



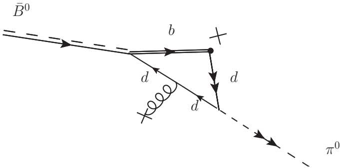
Coupling G_A determined by loop diagram for $\zeta^{(v)}$
(Orsay group (Charles et al 1999)):

$$\langle P|V^\mu|H\rangle = 2E\left[\zeta^{(v)}(M_H,E)\,n^\mu + \zeta_1^{(v)}(M_H,E)\,v^\mu\right] \;,$$

$$G_A = \frac{4\zeta^{(v)}}{m^2\,G_H\,F}\sqrt{\frac{E}{M_H}} = \left(\frac{4\hat{c}f_\pi}{m\,F\,\sqrt{2\rho}}\right)\frac{1}{E^{\frac{3}{2}}} \;,$$

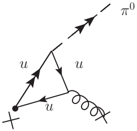
Coupling $G_A \sim N_c^{-1/2}$ fixed. $F = N_c/(16\pi) + ... \sim 10^{-1}$
(J.O.Eeg and L.E. Leganger, PRD 82 (2010)) for $B \to \pi D$

Calculating the color suppressed mode



The $B \to \pi_n$ current with color

$$J^{\mu}_{1G}(B \to \pi^0)^a \;=\; g_s\, G^a_{\rho\nu}\, \frac{G_H\, G_A}{128\pi} \epsilon^{\sigma\rho\nu\lambda}\, n_\sigma \mathrm{Tr}\left(\gamma^\mu L H_v \gamma_\lambda\, \xi^\dagger M_n\right) \;\;,$$



The colored current for outgoing hard $\pi_{\tilde{n}}$:

$$J^{\mu}(\pi_{\tilde{n}})^a \;=\; g_s G^a_{\alpha\beta}\; 2 \left(-\frac{G_A E}{4}\right)\; Y\; \tilde{n}_\sigma \epsilon^{\sigma\alpha\beta\mu}\, \mathrm{Tr}\left[\lambda^X\, M_{\tilde{n}}\right] \;\;,$$

λ^X = SU(3) flavor matrix, Y is a Loop factor:

$$Y \;=\; \frac{f_\pi^2}{4m^2 N_c}\left(1 \;-\; \frac{1}{24 m^2 f_\pi^2}\langle \frac{\alpha_s}{\pi} G^2\rangle\right)$$

Use Novikov procedure to form gluon condensate.

Result: ratio of amplitudes

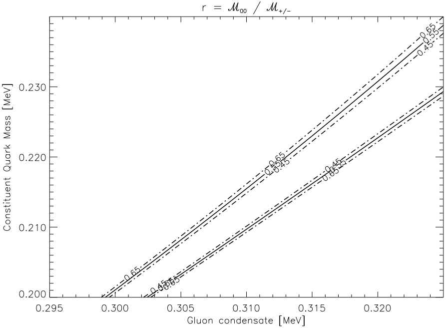
$$r \;\equiv\; \frac{\mathcal{M}(\overline{B^0_d} \to \pi^0\pi^0)_{\text{Non-Fact}}}{\mathcal{M}(\overline{B^0_d} \to \pi^+\pi^-)_{\text{Fact}}} \;=\; \frac{c_A}{c_f}\,\frac{\kappa}{N_c}\,\frac{E\,\zeta^{(v)}}{\sqrt{m\,M_B}}\;,$$

κ = model-dependent hadronic factor, dimensionless and $\sim (N_c)^0$:

$$\kappa \;=\; \left(\frac{\pi\,N_c\,\langle\frac{\alpha_s}{\pi}G^2\rangle}{2\,F^2\,m^4\sqrt{2\rho}}\right)\;Y.$$

From scaling behaviour of $\zeta^{(v)}$ with $C = \hat{c}\,m^{\frac{3}{2}}$

$$r \;\simeq\; \left(\frac{c_A}{c_f}\,\kappa\,\hat{c}\right)\frac{1}{N_c}\,\frac{m}{E}\;\sim\;\frac{\Lambda_{QCD}}{m_b}$$



Experimental value of $r = 0.5$ can be accommodated for $m \sim 220$ Mev and $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4} \sim 315$ MeV. - as in previous work.

BUT: Result very sensitive to variations of m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$

Conclusions

- Have constructed LEχQM in accordance with $\langle \pi_n | j_V^\mu | B \rangle$
- Good news: Color suppressed ($\sim 1/N_c$) ampl. $\overline{B_d^0} \to \pi^0\pi^0$ can be accomodated numerically! And: amp. $\sim m/2E \simeq \Lambda_{QCD}/m_b$
- Bad news: Obtained amplitude very sensitive to m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$

Conclusions

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- Bad news: Obtained amplitude very sensitive to m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$
- Thank you

Meson loop contributions to $\overline{B_d^0} \to \pi^0\pi^0$

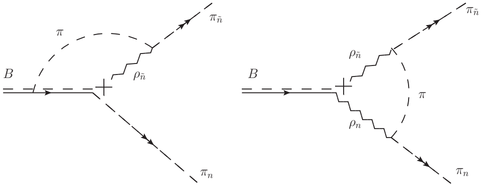

Figure: Meson loops for $\overline{B_d^0} \to \pi\pi$.

Heavy Quark Effective Theory (*HQET*)

Project out movement of heavy quark ($p_Q = m_Q\, v + k\ ,\ v^2 = 1$)

$$Q_v^{(\pm)}(x) = e^{\pm i m_Q v\cdot x} P_\pm(v) Q(x)\ \ ,\ P_\pm(v) = \frac{1}{2}(1 \pm \gamma \cdot v)$$

$$\mathcal{L}_{HQET} = \pm \overline{Q_v^{(\pm)}} i v \cdot D Q_v^{(\pm)} + \mathcal{O}(m_Q^{-1})$$

Heavy quark propagator:

$$S(p_Q) \ \rightarrow\ \frac{P_+(v)}{k \cdot v}$$

Replacements in quark operators

$$b \rightarrow Q_{v_b}^{(+)} \quad , \ c \rightarrow Q_{v_c}^{(+)} \quad , \ \overline{c} \rightarrow Q_{\bar{v}}^{(-)}$$

Large Energy Eff. Th. (*LEET* $\rightarrow$ *SCET*)

Project out movement of light energetic quark: $p_q^\mu = E\, n^\mu + k^\mu$,

$$q_\pm(x) = e^{iEn\cdot x}\,\mathcal{P}_\pm\, q(x) \;\; ; \; n(\text{ or } \tilde{n}) \; = \; (1,0,0,\pm\eta)$$

$$\mathcal{P}_+ = \frac{1}{4}\,\gamma\cdot n\,(\gamma\cdot\tilde{n} \,+\, \delta) \quad , \; \mathcal{P}_- = \frac{1}{4}\,(\gamma\cdot\tilde{n} \,-\, \delta)\,\gamma\cdot n$$

$$\eta = \sqrt{1-\delta^2}\,, \;\; n^2 = \tilde{n}^2 = \delta^2\,, \, v\cdot n = v\cdot\tilde{n} = 1.$$

$$\delta \sim \frac{\Lambda_{QCD}}{E} \;\; ; \quad S(p_q) \rightarrow \frac{\gamma\cdot n}{2n\cdot k}$$

Project out q_- gives eff. Lagr. for $q_+ \; = \; q_n$ ($\sim$ as Orsay group):

$$\mathcal{L}_{LEET\delta} \; = \; \bar{q}_n \left(\frac{1}{2}(\gamma\cdot\tilde{n} + \delta) \right) (in\cdot D)\, q_n + \mathcal{O}(E^{-1}) \; ,$$

In the formal limits $M_H \to \infty$ and $E \to \infty$, $\langle P \mid V^\mu \mid H \rangle$ of the form (Orsay group (Charles et al 1999)):

$$\langle P|V^\mu|H\rangle = 2E\left[\zeta^{(v)}(M_H, E)\, n^\mu + \zeta_1^{(v)}(M_H, E)\, v^\mu\right] \ ,$$

where

$$\zeta^{(v)} = C\frac{\sqrt{M_H}}{E^2} \quad , \quad C \sim (\Lambda_{QCD})^{3/2} \qquad , \qquad \frac{\zeta_1^{(v)}}{\zeta^{(v)}} \sim \delta \ \sim \ \frac{1}{E}$$

Behavior constistent with the energetic quark having x close to one, where x = quark momentum fraction of the outgoing pion.