All-loop amplitudes of the Reggeon Field Theory via the stochastic approach.

Rodion Kolevatov[1,3] *Konstantin Boreskov*[2] *Larissa Bravina*[1]

[1]University of Oslo, Norway

[2]ITEP, Moscow, Russia

[3]Saint-Petersburg State University, Russia

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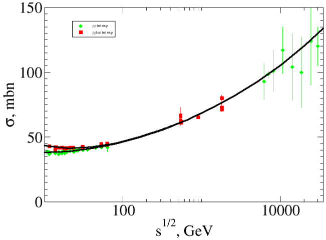
Outline

Power-like contributions to the amplitude



PDG fit:

$$\sigma_{tot}^{pp(\bar{p})} = 18.3 s^{0.095} + 60.1 s^{-0.34} \pm 32.8 s^{-0.55}$$

Optical theorem:

$$\sigma_{tot} = \frac{1}{s} 2 \mathrm{Im} A_{el}(q=0) \equiv 2 \mathrm{Im} T_{el}(q=0)$$

Indication: High energy elastic scattering goes via quasiparticle, “Reggeon”, exchanges with powerlike asymptotic in c.m.energy. Leading contirbution – Pomeron, $T_{\mathbb{P}} \sim s^{\Delta}$, $\Delta > 0$.

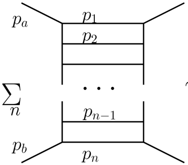
Caveat: Single Pomeron exchange violates Froissart bound ($\sigma_{tot} \lesssim C \ln^2 s$)

s-channel ($s \to \infty$, $t = Q^2$ small) dominant contributions

Analiticity&unitarity:

- Power-like terms come from poles in the complex L plane of the t-channel amplitude, Pomeron = the rightmost singularity

Field theories (φ^3, QCD):

- $\sum_n$ 

$\sim s^{\alpha(t)}$

$$p_1^+ \gg p_2^+ \gg \ldots \gg p_n^+$$
$$p_1^- \ll p_2^- \ll \ldots \ll p_n^-$$
$$p^\pm = p^0 \pm p^3$$

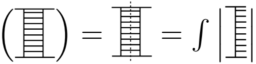
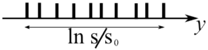
For phenomenological applications: $\mathbb{R}/\mathbb{P}$ = exchange of a "ladder" structure in the t-channel with ordering of the ladder rungs in rapidity $y = 1/2 \ln p_+/p_-$

Contributions to σ_{tot}

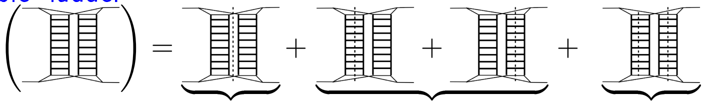
Contributions to imaginary part (Cutkosky rules):

- Cut the diagram for the elastic scattering amplitude
- Put cut lines on the mass shell, integrate over the phase space

Single “ladder” exchange – uniform rapidity distribution

$2\mathrm{Im}\,T_1 = 2\mathrm{Im}$  $d\tau_n \longrightarrow$ 

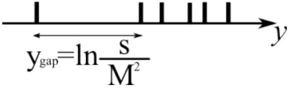
Double “ladder”

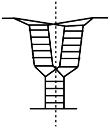
$2\mathrm{Im}$ 

Contributions to σ_{tot}

Rapidity gaps – splitting of the “ladder”:

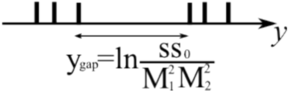
Single diffraction dissociation

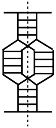




+ abs. corrections

Double diffraction dissociation





+ abs. corrections

Reggeon Field Theory = the theory of the Pomeron (Reggeon) exchanges and interactions. The underlying principles of the RFT are analyticity and t-channel unitarity of the elastic amplitude.

RFT

The theory of Pomeron and Reggeon exchanges is known to be very successfull phenomenologically:

- Gives reliable predictions of hadronic X-sections
  - The $\sigma_{tot} \lesssim C \ln^2 s$ comes out quite naturally (taking into account multiple Pomeron exchanges)
- Cuts of the RFT diagrams define X-sections of various inelastic processes via AGK rules (a special case of Cutkosky rules)
- Good description of the events with rapidity gaps (single and double diffraction). At higher energies the loop contributions become increasingly important.

Account of loop contirbution is an untrivial task and is under investigation by several groups (Ostapchenko, Khoze et al., Poghosyan; also Lund group non-RFT approach).

RFT

The elastic amplitude $T = A/(8\pi s)$ is written as (Regge factorization):

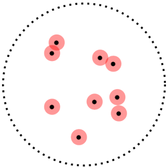
$$T = \sum_{n,m} V_n \otimes G_{nm} \otimes V_m$$

Green functions G_{mn} are obtained within the effective field theory, process independent

$$\mathcal{L} = \frac{1}{2}\phi^\dagger(\overleftarrow{\partial_y} - \overrightarrow{\partial_y})\phi - \alpha'(\nabla_{\mathbf{b}}\phi^\dagger)(\nabla_{\mathbf{b}}\phi) + \Delta\phi^\dagger\phi + \mathcal{L}_{int}.$$

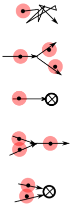
For $\mathcal{L}_{int} = i\, r_{3P}\phi^\dagger\phi(\phi^\dagger + \phi) + \chi{\phi^\dagger}^2\phi^2$
it is possible to use reaction-diffusion (or “stochastic”) models for obtaining the Green functions with account of all loops.
[Grassberger&Sundermeyer’78; Boreskov’01]

The stochastic model.



Consider a system of classic "partons" in the transverse plane with:

- Diffusion (chaotical movement) D;
- Splitting (λ – prob. per unit time)
- Death (m_1)
- Fusion ($\sigma_\nu \equiv \int d^2b\, p_\nu(b)$)
- Annihilation ($\sigma_{m_2} \equiv \int d^2b\, p_{m_2}(b)$)



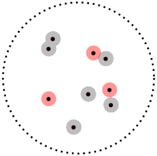
Parton number and positions are described in terms of

probability densities $\rho_N(y, \mathcal{B}_N)$ ($N = 0, 1, ...; \mathcal{B}_N \equiv \{b_1, \ldots, b_N\}$)

with normalization $p_N(y) \equiv \frac{1}{N!} \int \rho_N(y, \mathcal{B}_N) \prod d\mathcal{B}_N$; $\sum_0^\infty p_N = 1$.

Inclusive distributions

S-parton inclusive distributions:



$$f_s(y;\mathcal{Z}_s) = \sum_N \frac{1}{(N-s)!}\int d\mathcal{B}_N\ \rho_N(y;\mathcal{B}_N)\prod_{i=1}^{s}\delta(\mathbf{z}_i-\mathbf{b}_i);$$

$\int d\mathcal{Z}_s f_s(y;\mathcal{Z}_s) = \sum \frac{N!}{(N-s)!}\, p_N(y) \equiv \mu_s(y)$. – factorial moments.

Example: Start with a single parton with only diffusion and splitting allowed.

$$f_1^{1\ \text{parton}}(y,b) = \frac{\exp(\lambda y)\exp(-b^2/4Dy)}{4\pi D y}.$$

– the bare Pomeron propagator.

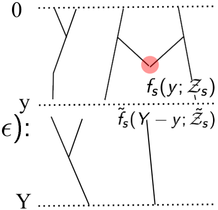
The set of evolution equations for $f_s(\mathcal{Z}_s)$, $(s=1,\ldots)$ coincides with the set of equations for the Green functions of the RFT.

The amplitude.

Green functions:

$f_s(y;\mathcal{Z}_s) \propto \sum_m \int d\mathcal{X}_m\, V_m(\mathcal{X}_m) G_{mn}(0;\mathcal{X}_m|y;\mathcal{Z}_n)$;

$f_m(y = 0, \mathcal{X}_m) \propto V_m(\mathcal{X}_m)$ – particle–mPomeron vertices



The amplitude ($g(b)$ assumed narrow; $\int g(b)d^2b \equiv \epsilon$):
$T(Y) = \langle A|T|\tilde{A}\rangle =$

$$= \sum_{s=1}^{\infty} \frac{(-1)^{s-1}}{s!} \int d\mathcal{Z}_s d\tilde{\mathcal{Z}}_s f_s(y;\mathcal{Z}_s)\tilde{f}_s(Y-y;\tilde{\mathcal{Z}}_s) \prod_{i=1}^{s} g(\mathbf{z}_i - \tilde{\mathbf{z}}_i - \mathbf{b}).$$

It does not depend on the linkage point *y* ("boost invariance") **if**

$$\lambda \int g(b)d^2b = \int p_{m_2}(b)d^2b + \frac{1}{2}\int p_\nu(b)d^2b\ ,$$

Correspondence RFT–Stochastic model

We use the simplest form of $g(b)$, $p_{m_2}(b)$ and $p_\nu(b)$:

$$p_{m_2}(\mathbf{b}) = m_2\ \theta(a - |\mathbf{b}|); \quad p_\nu(\mathbf{b}) = \nu\ \theta(a - |\mathbf{b}|);$$
$$g(\mathbf{b}) = \theta(a - |\mathbf{b}|);.$$

with a – some small scale; $\epsilon \equiv \pi a^2$.

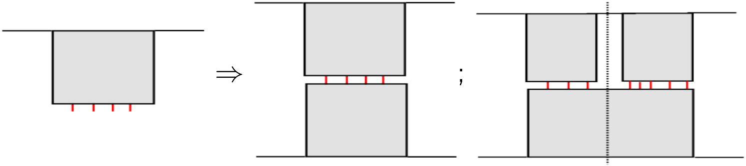
RFT	stochastic model
Rapidity y	Evolution time y
Slope α'	Diffusion coefficient D
$\Delta = \alpha(0) - 1$	$\lambda - m_1$
Splitting vertex r_{3P}	$\lambda\sqrt{\epsilon}$
Fusion vertex r_{3P}	$(m_2 + \frac{1}{2}\nu)\sqrt{\epsilon}$
Quartic coupling χ	$\frac{1}{2}(m_2 + \nu)\epsilon$

Boost invariance $(\lambda = m_2 + \frac{\nu}{2}) \Leftrightarrow$ equality of fusion and splitting vertices.

Summary of the stochastic approach

The approach allows to compute numerically (via the explicit evolution of the stochastic system) the RFT Green functions in their convolutions which correspond to

- the elastic scattering amplitude
- the single diffractive cut of the amplitude.



Peculiarities of the stochastic approach to the RFT:

- Presence of the triple and $2 \rightarrow 2$ couplings
- Regularization scale (equivalient to the cutoff or the Pomeron size in RFT) enters via functions $g(b)$, $p_{m_2}(b)$ and p_ν.
- Neglect of the real part of the $\mathbb{P}$ exchange amplitude.

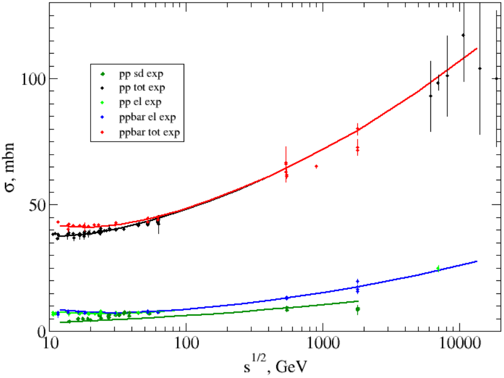
Fitting the cross sections

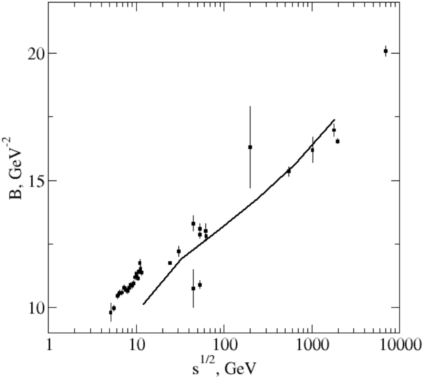
The calculation method is described in detail in R.K., K.Boreskov and L.Bravina, Eur. Phys. J. C **71** (2011) 1757 [arXiv:1105.3673 [hep-ph]]. In addition to that in the ongoing calculations we

- Implement two-channel eikonal p–$n\mathbb{P}$ vertices to incorporate low-M^2 diffraction ($|p\rangle = \alpha|1\rangle + \beta|2\rangle$)
- Account the secondary Reggeons contribution in the lowest order
- Neglect the real part of the Pomeron exchange amplitude (keeping it for the secondary Reggeons)
- Neglect central diffraction in calculation of SD cross sections.

Cross sections

Prelliminary results on X-sections and slope ($B = \frac{d}{dt} \ln \frac{d\sigma_{el}}{dt}\Big|_{t=0}$):
fit with $\Delta = 0.255$ (compare with 0.095 of the PDG fit), reg. scale $a = 0.018\text{fm}{=}0.09\text{GeV}^{-1}$, $\alpha' = 0.0035\text{fm}^2{=}0.09\text{GeV}^{-2}$, $r_{3\mathbb{P}} = 0.087\text{GeV}^{-1}$ [Kaidalov'79].





Conclusions

Results of an academic interest (from the paper in EPJC**71**):

- The full account of loop corrections doesn't turn the Pomeron into the subcritical as in 0D RFT ($T \sim s^{\Delta}$ with $\Delta < 0$) though effectively reduces the intercept value.
- The role of $2 \to 2$ coupling is minor in 2D compared to 0D RFT.

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Phenomenological outcome:

- We are able to compute all-loop total, elastic, high-M^2 SD X-sections and elastic scattering slope within a single approach.

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Further challenges:

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Further challenges:

- Complete the fitting, obtain predictions for the 14TeV LHC run;
- dN/dy energy dependence;
- Study of the high-density regime Green Functions.

Backup – cross sections definitions

$$\sigma^{\rm tot}(Y) = 2\,{\rm Im} M(Y,\mathbf{q}=0), \quad \sigma^{\rm el} = \int \frac{d^2q}{(2\pi)^2}\,|M(Y,\mathbf{q})|^2\ ,$$

$$f(Y,\mathbf{b}) = \frac{1}{(2\pi)^2}\int d^2q\, e^{-i\mathbf{qb}} M(Y,\mathbf{q})\ .$$

$$\sigma^{\rm tot}(Y) = 2\int d^2b\,{\rm Im} f(Y,\mathbf{b})\ , \quad \sigma^{\rm el} = \int d^2b\,|f(Y,\mathbf{b})|^2.$$

$$f(Y,\mathbf{b}) \simeq iT(Y,\mathbf{b}), \quad T \equiv {\rm Im} f$$

Backup - calculation method

Taking an explicit note of the initial parton distributions

$$T = \sum_{n,k} P_n(\mathcal{X}) \otimes \underbrace{\sum_s \frac{(-1)^{s-1}}{s!} f_{ns}(\mathcal{X}|\mathcal{Z}) \otimes \prod g(\mathcal{Z} - \tilde{\mathcal{Z}}) \otimes \tilde{f}_{ks}(\tilde{\mathcal{X}}|\tilde{\mathcal{Z}})}_{T_{sample}} \otimes \tilde{P}_k(\tilde{\mathcal{X}}).$$

Main idea: simulate a sample of 2 parton sets which correspond to f_s and $\tilde{f}_s$ on the average, compute T_{sample} and make its MC average. For N partons with fixed positions

$$f_s(\mathcal{Z}_s) = \sum_{\{\hat{\mathbf{x}}_{i_1},\ldots,\hat{\mathbf{x}}_{i_s}\} \in \hat{\mathcal{X}}_N} \delta(\mathbf{z}_1 - \hat{\mathbf{x}}_{i_1}) \ldots \delta(\mathbf{z}_s - \hat{\mathbf{x}}_{i_s})$$

$$T_{\mathrm{sample}} = \sum_{s=1}^{N_{min}} (-1)^{s-1} \sum_{i_1 < i_2 \ldots < i_s} \sum_{j_1 < \ldots < j_s} g_{i_1 j_1} \cdots g_{i_s j_s}.$$

- expansion of T_{sample} in the number of **P** exchanges s;
- works for any position of the linkage point y.

Backup – calculation method 2

Setting the linkage point to full rapidity interval $y = Y$ simplifies the calculation: $\tilde{f}_s(y = 0, \mathcal{Z}_s) = N_s(\mathcal{Z}_s)/\epsilon^{s/2}$ and the MC average involves evolution from only one side:

$$T = \sum_n P_n(\mathcal{X}) \otimes \underbrace{\sum_s \frac{(-1)^{s-1}}{s!} f_{ns}(\mathcal{X}|\mathcal{Z}) \otimes \prod g(\mathcal{Z} - \tilde{\mathcal{X}}) \otimes \tilde{P}_s(\tilde{\mathcal{X}}).}_{T_{sample}}$$