An $SU(3)$ parton shower and orthogonal color bases

- First results from an SU(3), rather than SU(∞) parton shower
  In collaboration with Simon Plätzer (DESY), on the arXiv tomorrow!
- Minimal orthogonal multiplet based color bases for treating the SU(3) color space
  In collaboration with Stefan Keppeler (Tübingen University), work in progress

Spåtind
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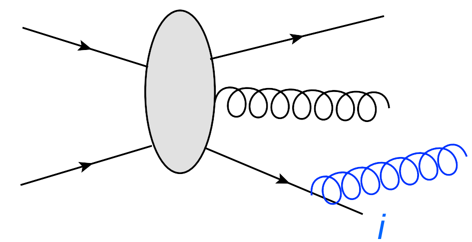
Malin Sjödahl

Parton showers

- Parton showers are indispensable tools for predicting and analyzing LHC data
- Perturbative part describes the emission of collinear and soft radiation
- To enable this parton showers work with: leading $\log(k_\perp^2)$, leading order (but resummed), leading N_c, and approximated momentum space
- Leading N_c: treat the 3 colors of nature as infinitely many (in soft part), most often corrections are $\mathcal{O}(\frac{1}{N_c^2})$, but sometimes $\mathcal{O}(\frac{1}{N_c})$

- Collinear singularity: A parton can be seen as being emitted from one other parton using

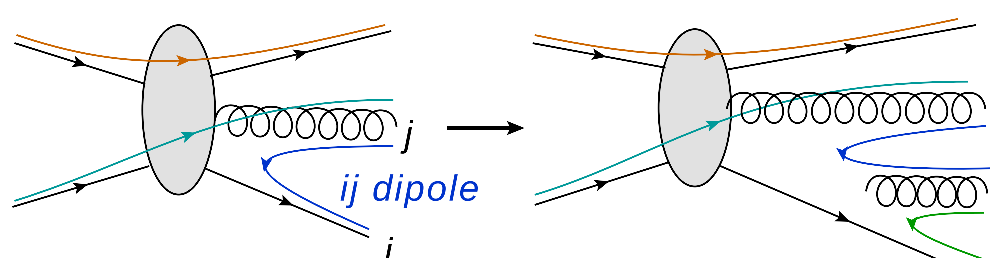
$$\Delta_k(t) = \exp(-\sum_{i} \int_{t_0}^{t} \frac{dt'}{t'} \alpha_s(t') \int \frac{dz}{2\pi} P_{ik})$$



Color structure treated as if $N_c = 3$, $C_F = 4/3$ (not $3/2$)

- In the soft limit the next parton is emitted coherently from a pair of color connected partons, "dipole shower"

$$\Delta_k(t) = \exp(-\frac{2}{\pi} \sum_{\text{dipoles i,j(i)}} \int_{t_0}^{t} \frac{dt'}{t'} \alpha_s(t') \int \frac{dy d\phi}{2\pi} \frac{k_T^2 p_i.p_j}{2\, p_i.k\; p_j.k})$$



An $SU(3)$ improved parton shower

In collaboration with Simon Plätzer

- For $N_c = 3$ *all* pairs can potentially radiate

$$\Delta_k(t) = \exp(-\frac{2}{\pi} \sum_{\text{pairs i,j} \neq \text{i}} \sum_{mn} \int_{t_0}^{t} \frac{dt'}{t'} \alpha_s(t') \int \frac{dy\, d\phi}{2\pi} \dots$$

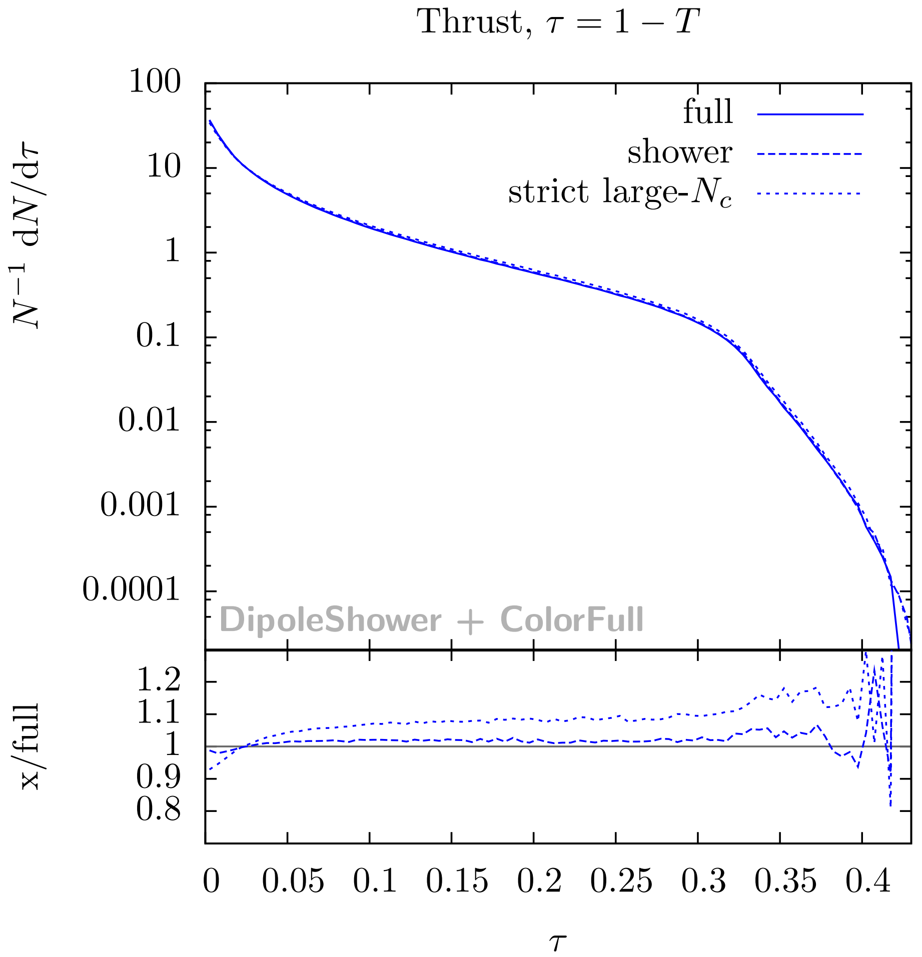
$$\frac{1}{\text{Norm}} c_n c_m^* < C^n | \frac{k_T^2 p_i \cdot p_j}{2\, p_i \cdot k\; p_j \cdot k} T^{(i)} \cdot T^{(j)} | C^m >)$$

- For N_p partons $N_p(N_p - 1)/2$ pairs $\longrightarrow$ many more color suppressed terms
- Large sum, but doable if we have a basis for color space
- In the standard basis the number of basis vectors grows roughly like $(N_g + N_{q\bar{q}})!$ and it's non-orthogonal $\longrightarrow$ $(N_g + N_{q\bar{q}})!^2$ terms

- Also like to keep the virtual color rearranging terms $\longrightarrow$ rotate state in color space as well $\longrightarrow$ matrix exponentiation needed $\longrightarrow$ future goal

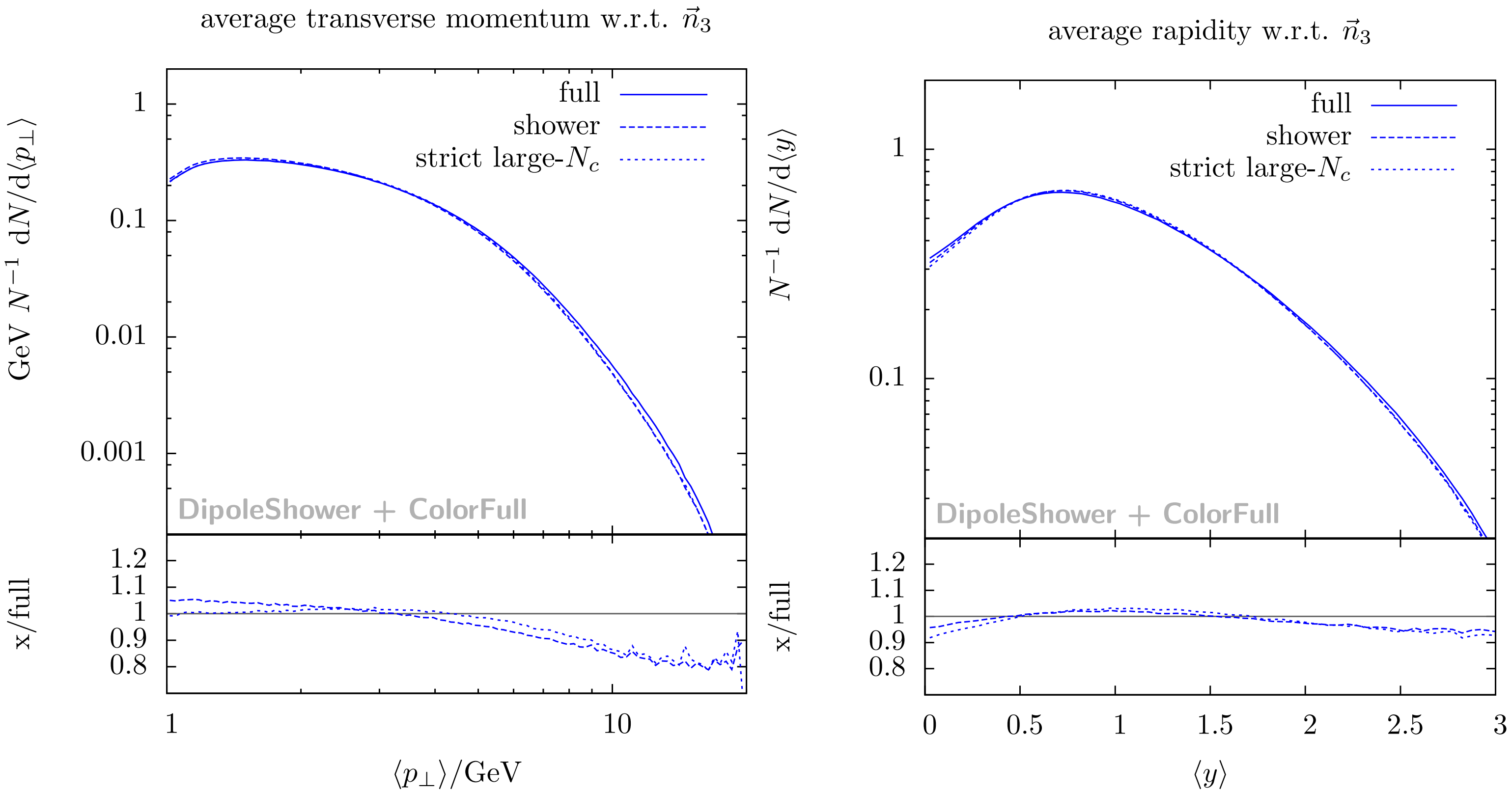
Preliminary results for LEP

For standard observables, small deviations of order few percent



Preliminary results for LEP

For tailored observables we find much larger differences



Average transverse momentum and rapidity of softer particles with respect to the thrust axis defined by the three hardest partons

Conclusion and outlook

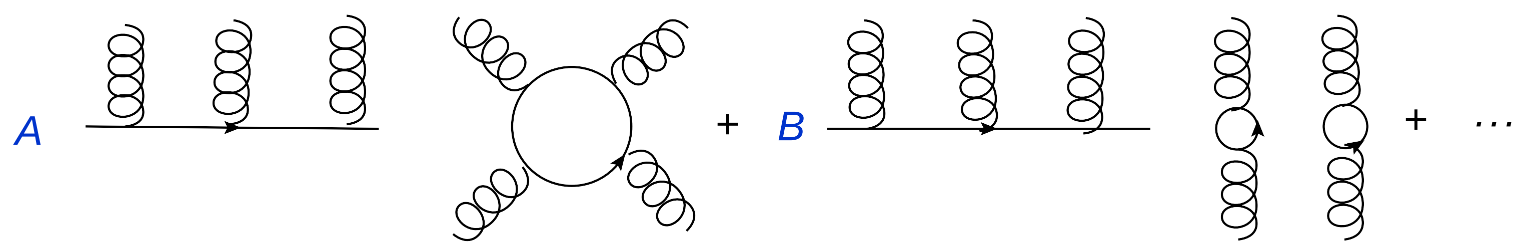
- For standard observables we find small deviations for LEP, of order a few percent
- For tailored observables we find much larger differences
- Keeping C_F to its $N_c = 3$ value 4/3 (as is done in standard showers), rather than 3/2, tends to improve the approximation
- Leading N_c was probably a very good approximation for standard observables at LEP
- At the LHC we have many more colored particles, so (many more)^2 possible color suppressed interference terms

A basis for the color space

- The color space is a finite dimensional vector space equipped with a (real) scalar product

$$< A, B >= \sum_{a,b,c,\ldots} A_{a,b,c,\ldots} (B_{a,b,c,\ldots})^*$$

- In general an amplitude can be written as linear combination of different color structures, like

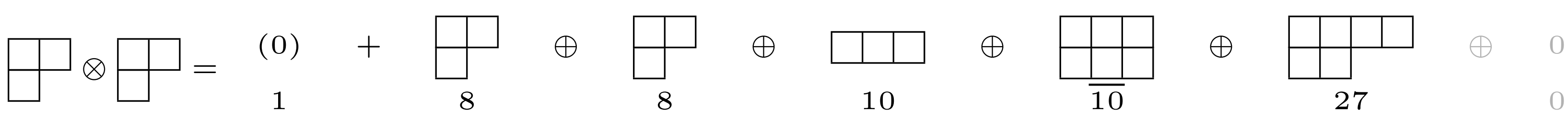


- This is the kind of (trace) bases used in our current parton shower, and most NLO calculations, but it is non-orthogonal and overcomplete $\rightarrow$ major bottleneck when many partons

A minimal orthonormal basis for the color space

In collaboration with Stefan Keppeler

- Want orthonormal minimal basis for color space
- Basis vectors can be enumerated using Young tableaux multiplication



and constructed if projection operators are known

- The problem is the construction of the corresponding projection operators; the Young-tableaux operate with "quark-units" but the physical particles include anti-quarks and gluons

- New idea: Successively build up projection operators using quark and anti-quark projection operators and project out already constructed projection operators
- Prove that this strategy works for all projection operators for any N_g and any N_c
- Prove that the gluon projection operators can be used for constructing minimal, complete, orthonormal bases for any $SU(N_c)$, and any number and kind of particles (and any order in perturbation theory)
- We are almost there, in particular the $3g \to 3g$ projection operators have been constructed, and the $6g$ basis written down

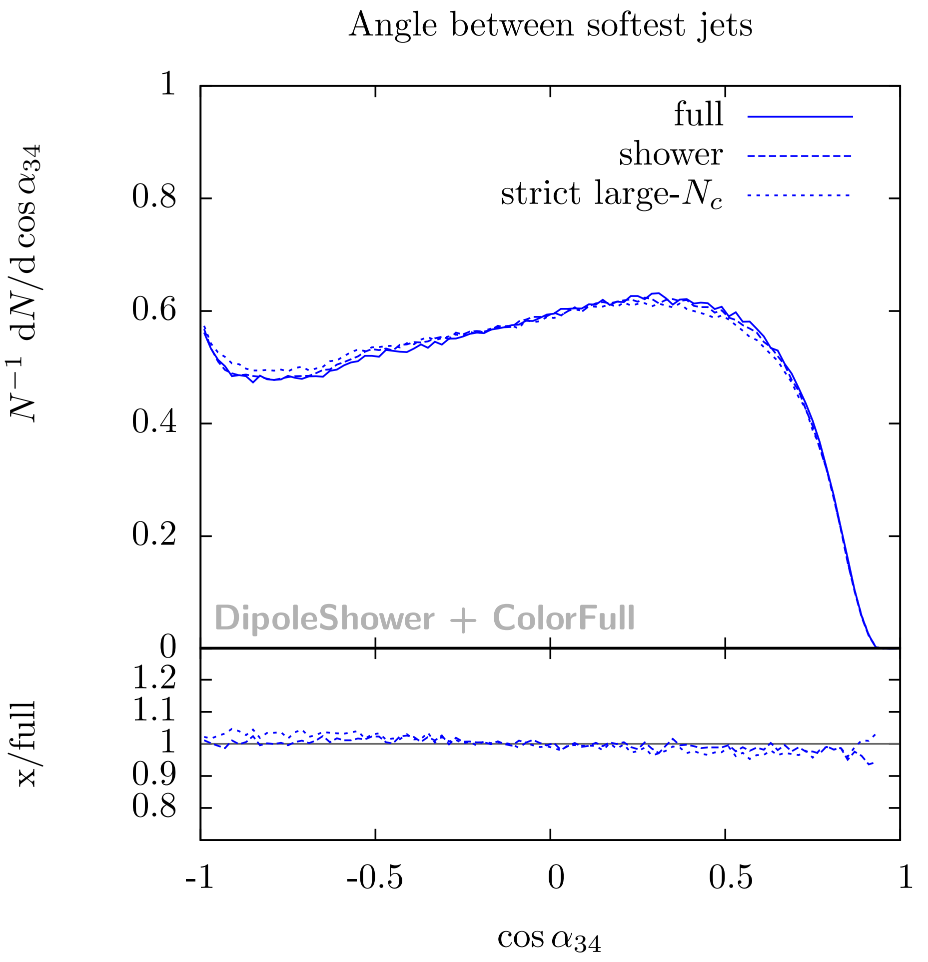
Conclusion and outlook

- A general recipe for construction of minimal orthogonal multiplet based bases for any QCD process it on its way
- Number of basis vectors grows only exponentially for $N_c = 3$
- This has the potential to very significantly speed up calculations in the color space of $SU(N_c)$
- ...but before that a lot of implementational work remains

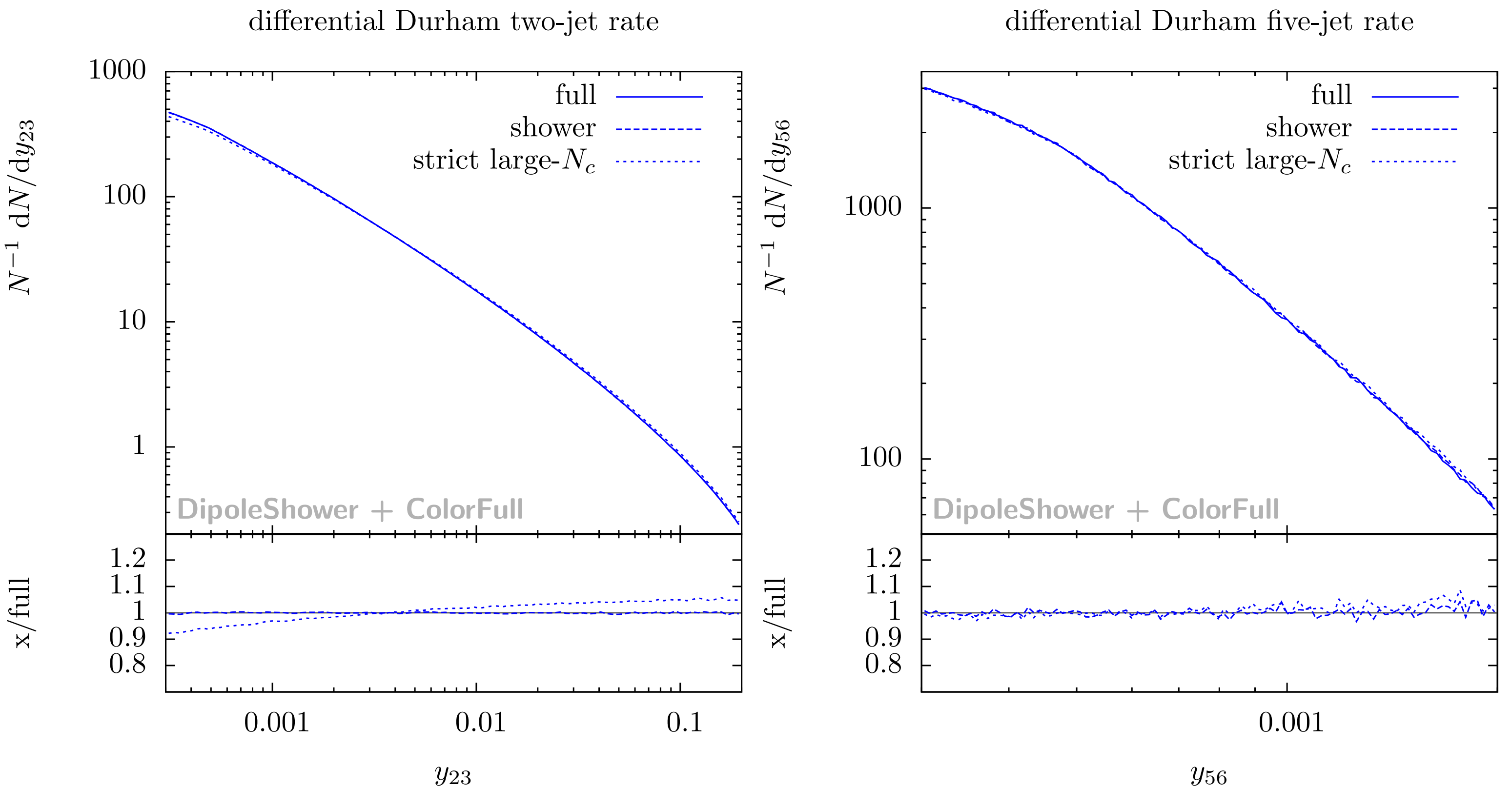
Thank you for your attention

Backup: More results for LEP

Cos(angle) between third and fourth jet



Backup: More results for LEP



Jet separation between 2nd and 3rd jet, and 5th and 6th jet

$y = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})/s$

Backup: Loop/resummation induced N_c-suppressed terms

- At loop level (resummed level), another source of suppressed terms comes from virtual gluon exchanges which rearrange the color structure $\longrightarrow$ **exponentiation** has to be done at the **amplitude level**

$$\exp(-\int_{t_0}^{t}\frac{dt'}{t'}\alpha_{\mathrm{s}}(t)\int ...\mathbf{Matrix})|\text{state}>$$

The color rearranging terms tend to be suppressed, but in

$$\exp(-\int_{t_0}^{t}\frac{dt'}{t'}(\text{moderate}+\text{small}))$$

the small number is not irrelevant when $\int_{t_0}^{t}\frac{dt'}{t'}$ is large!
$\longrightarrow$ different source of N_c-suppressed terms which should be included

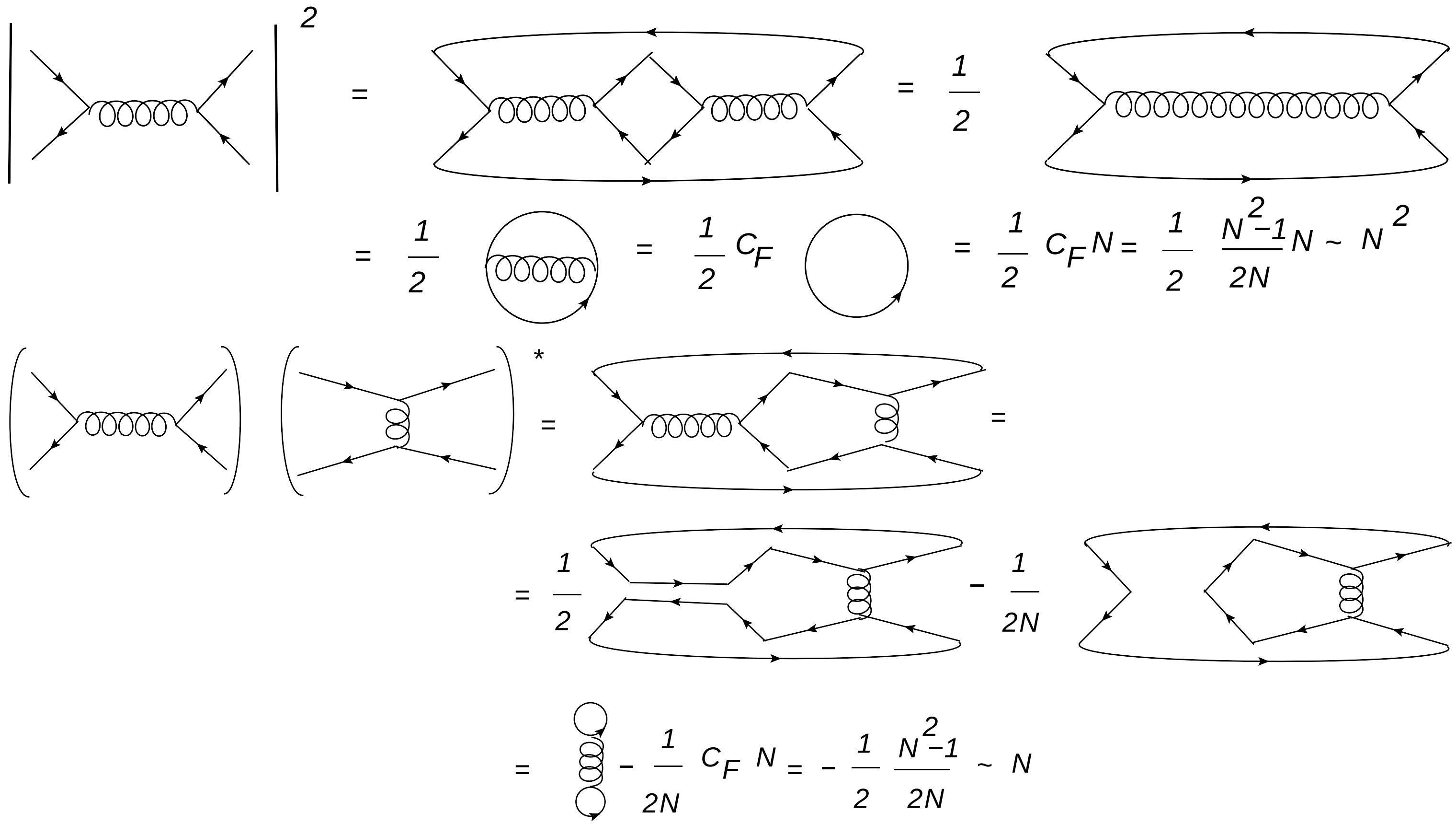
Backup: Number of projection operators and basis vectors

Case	Projectors $N_c = 3$	Projectors $N_c = \infty$	Vectors $N_c = 3$	Vectors $N_c = \infty$
2g $\rightarrow$ 2g	6	7	8	9
3g $\rightarrow$ 3g	29	51	145	265
4g $\rightarrow$ 4g	166	513	3 598	14 833
5g $\rightarrow$ 5g	1 002	6 345	107 160	1 334 961

Table 1: Number of projection operators and basis vectors for $N_g \rightarrow N_g$ gluons *before* imposing projection operators and vectors to appeare in selfadjoint combinations

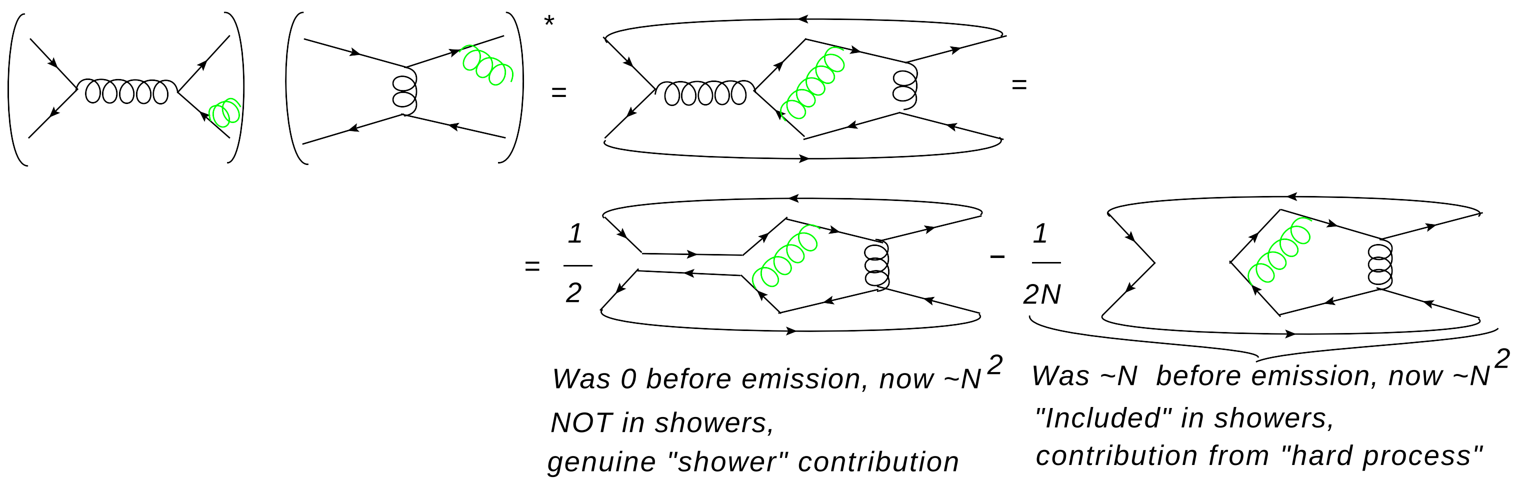
Backup: N_c suppressed terms

That non-leading color terms are suppressed by $1/N_c^2$, is guaranteed only for same order α_s diagrams with only gluons ('t Hooft 1973)



Backup: N_c suppressed terms

For a parton shower there may also be terms which only are suppressed by one power of N_c

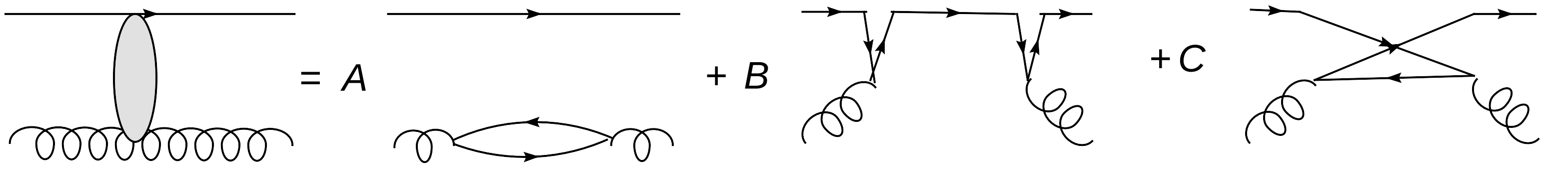


The leading N_c contribution scales as N_c^2 before emission and N_c^3 after

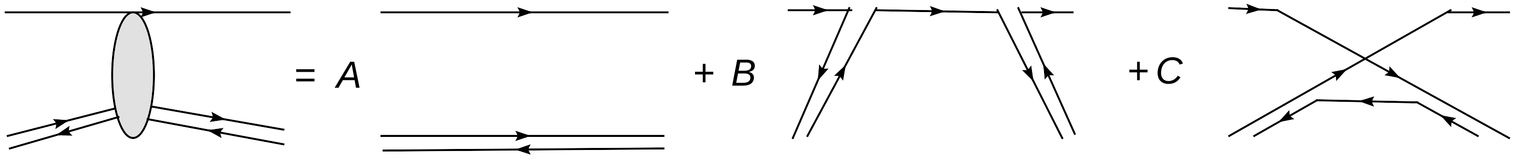
Backup: $N_c \to \infty$ limit of trace basis

Example: $\mathrm{qg} \to \mathrm{qg}$

- Split gluons into $\mathrm{q\bar{q}}$ pairs and connect lines in all possible ways

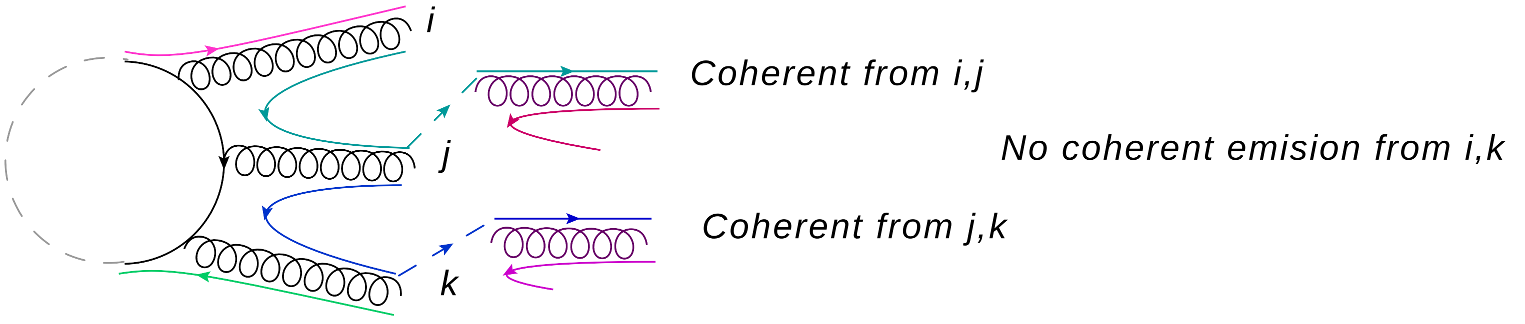


- In the $N_c \to \infty$ limit



Backup: A dipole shower in the "trace" basis

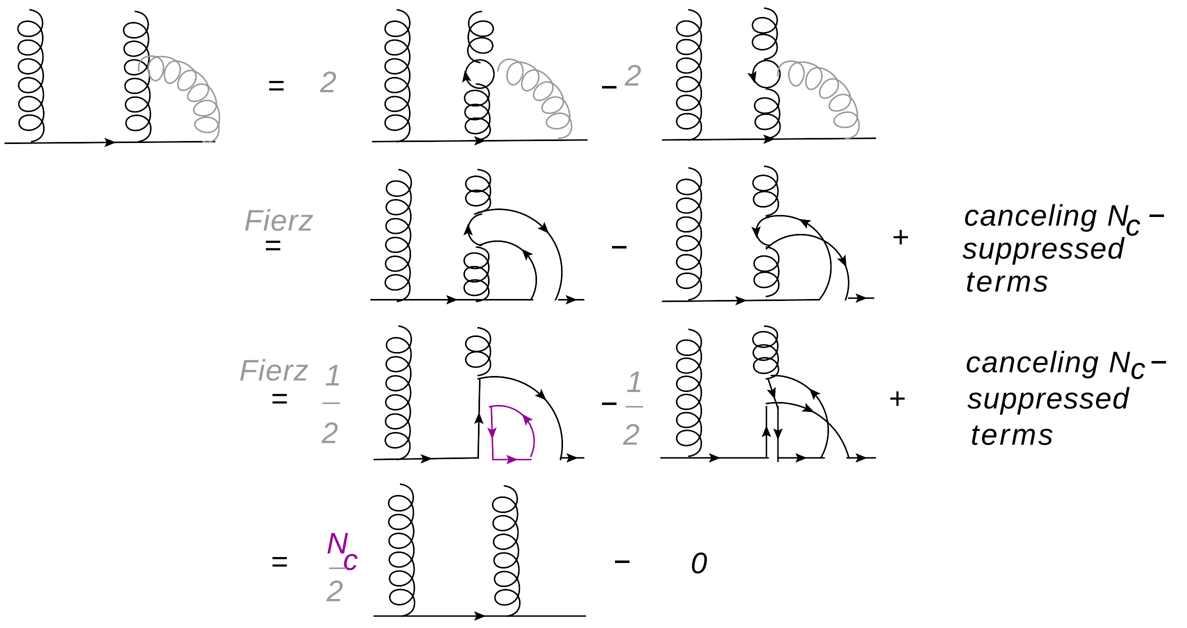
- A dipole shower can easily be thought of in the language of the $N_c \to \infty$ limit of the "trace" basis



- Also, it is easy to see that in this limit only "color neighbors" radiate, i.e. only neighboring partons on the quark-lines in the basis radiate $\longrightarrow$ trace basis well suited for comparing to parton showers

Backup: Gluon exchange

A gluon exchange in this basis "directly" i.e. without using scalar products gives back a linear combination of (at most 4) basis tensors



- N_c-enhancement possible only for near by partons
  $\longrightarrow$ only "color neighbors" radiate in the $N_c \to \infty$ limit