Hunt for the Quark Gluon Plasma

QGP as a “Unicorn”. Experimentalists as hunters, so (in this field), “All theorists are...”
Finally do something!

http://www.anti-powerpoint-party.com
Theory vs Experiment

Theory:

Effective theory for deconfinement, near $T_c$.
Today: only the "pure" glue theory (no dynamical quarks)
Based upon detailed results from the lattice

Experiment:

Soon: including quarks
Will provide competition to AdS/CFT
SU(3) gauge theory without quarks, temperature $T$

(Weakly) first order transition at $T_c \sim 290$ MeV

The standard plot

![Plot of standard model parameters with SU(3) gauge theory and quark transition at $T_c$.](image)

- $\varepsilon/T^4$ (energy density)
- $3p/T^4$ (pressure)

$e(T) =$ energy density
$p(T) =$ pressure

WHOT: Umeda, Ejiri, Aoki, Hatsuda, Kanaya, Maezawa, Ohno, 0809.2842

$\beta=6.0, \xi=1, N_s=16$ (i1)
$\beta=6.0, \xi=1, N_s=24$ (i2)
$\beta=6.2, \xi=1, N_s=22$ (i3)
$\beta=6.1, \xi=4, N_s=20$ (a2)

continuum S.B. limit
fixed $N_l=8$ result [9]
Another plot of the same

Plot conformal anomaly, \( (e-3p)/T^4 \): large peak above \( T_c \). \( \sim g^4 \) as \( T \to \infty \)

\[
\frac{e - 3p}{T^4}
\]

\( \uparrow \)

\( T \to \infty \)

\( T_c \)

\( \beta = 6.1, \xi = 4, N_s = 20 \) (a2)

\( \beta = 6.0, \xi = 1, N_s = 24 \) (i2)

fixed \( N_t = 8 \) result [9]

WHOT: Umeda, Ejiri, Aoki, Hatsuda, Kanaya, Maezawa, Ohno, 0809.2842
Strong vs weak coupling at $T_c$?

Resummed perturbation theory at 3-loop order works down to $\sim 3$ $T_c$. *Intermediate* coupling: $\alpha_s(T_c) \sim 0.3$. *Not* so big... So what happens below $\sim 3$ $T_c$?

Want effective theory; e.g.: chiral pert. theory: expand in $m_\pi/f_\pi$, exact as $m_\pi \to 0$.

But there is *no* small mass scale for SU(3) in “semi”-QGP, $T_c \to 3$ $T_c$. 

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Andersen, Su, & Strickland, 1005.1603
What to expand in?

Consider SU(N) for different N. # perturbative gluons $\sim N^2 - 1$. Scaled by ideal gas values, $e$ and $p$ for $N = 3, 4$ and 6 look very similar. Implicitly, expand about infinite N. Explicitly, assume classical expansion ok.

$\frac{e}{e_{\text{ideal}}}$ vs $\frac{\epsilon}{\epsilon_{\text{SB}}}$ vs $\frac{p}{p_{\text{SB}}}$ vs $\frac{p}{p_{\text{ideal}}}$

$N = 3, 4, 6$

- N=3: Boyd et al, lat/9602007
- N = 4 & 6: Datta & Gupta, 1006.0938

Thursday, November 3, 11
Conformal anomaly $\approx N$ independent

For SU(N), “peak” in $e^{-3p/T^4}$ just above $T_c$. Approximately uniform in $N$.

Not near $T_c$: transition 2nd order for $N = 2$, 1st order for all $N \geq 3$

$N = 3$: weakly 1st order. $N = \infty$: strongly 1st order (even for latent heat/N$^2$)

Datta & Gupta, 1006.0938
To study the tail in \((e-3p)/T^4\), multiply by \(T^2/(N^2-1)T_c^2\):

\[
\frac{1}{N^2 - 1} \frac{e - 3p}{T^2 T_c^2}
\]

approximately constant, independent of \(N\)

Datta & Gupta, 1006.0938
Precise results for three colors

From WHOT:

\[ p(T) \approx \# \left( T^4 - c \frac{T^2 T^2_c}{2} \right), \quad T/T_c : 1.2 \rightarrow 2.0 \]

\[ c \approx 1.00 \pm 0.01 \]

\[ \frac{1}{8} \frac{e - 3p}{T^2 T^2_c} \]

WHOT: Umeda, Ejiri, Aoki, Hatsuda, Kanaya, Maezawa, Ohno, 0809.2842
How to get a term $\sim T^2$ in the pressure?

Expand pressure of ideal, massive gas in powers of mass $m$:

$$\int d^4 p \ tr \log(p^2 + m^2) = \# T^4 - \#' m^2 T^2 + \ldots$$

Quasi-particle models: choose $m(T)$ to fit pressure.

Need $m(T)$ to increase *sharply* as $T \to T_c$ to suppress pressure. Inelegant...

$$\frac{m(T)}{T_c} = \frac{a}{(t - 1)^b} + c t$$

$t = T/T_c$, $a = .47$, $b = .13$, $c = .39$

Above: Castorina, Miller, & Satz, 1101.1255

Originally: Peshier, Kampfer, Pavlenko & Soff, PRD 1996
A simple solution

Assume there is some potential, \( V(q) \).
The vacuum, \( q_0 \), is the minimum of \( V(q) \):

\[
\frac{dV(q)}{dq} \bigg|_{q=q_0} = 0
\]

Pressure is the value of the potential at the minimum:

\[
p(T) = -V(q_0)
\]

For \( T > 1.2 T_c \), a constant \( \sim T^2 \) in the pressure, is due to a constant \( \sim T^2 \) in \( V(q) \):

\[
V(q) = -\# \left( T^4 - T^2 T_c^2 + T^2 T_c^2 \tilde{V}(q) \right)
\]

Above \( 1.2 T_c \), \( \langle q \rangle = 0 \). Except near \( T_c \), for most of the semi-QGP, the non-perturbative part of the pressure, \( \sim T^2 \), is due just to a constant

Region where \( \langle q \rangle \neq 0 \), and \( V(q) \) matters, is very narrow: \( T: T_c \rightarrow 1.2 T_c \)

Unexpected consequence of precise lattice data.
Large N: makes sense to speak of classical \( \langle q \rangle \) instead of fluctuations.

Generalization of Meisinger, Miller, Ogilvie ph/0108009
Dumitru, Guo, Hidaka, Korthals-Altes, & RDP, arXiv:1011.3820 + 1112.?
Also: Y. Hidaka & RDP, 0803.0453, 0906.1751, 0907.4609, 0912.0940.
Hidden \(Z(2)\) spins in \(SU(2)\)

Consider \textit{constant} gauge transformation:

\[
U_c = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -1
\]

As \(U_c \sim 1\), locally gluons \textit{invariant}:

\[
A_\mu \rightarrow U_c^\dagger A_\mu U_c = + A_\mu
\]

Nonlocally, Wilson \textit{line} changes:

\[
L = \mathcal{P} e^{ig \int_0^1 A_0 d\tau} \rightarrow -L
\]

\(L\) \sim propagator for “test” quark.

\(SU(3): \det U_c = 1 \Rightarrow j = 0, 1, 2\)

\(SU(N): U_c = e^{2\pi i j/N} 1\): \(Z(N)\) symmetry.

\(Z(N)\) spins of \textit{‘t Hooft}, without \textit{quarks}\)

Quarks \sim background \(Z(N)\) field, \textit{break} \(Z(N)\) sym.

\[
\psi \rightarrow U_c \psi = - \psi
\]
Hidden Z(3) spins in SU(3)

Lattice, A. Kurkela, unpub.’d: 3 colors, loop $l$ complex. Distribution of loop shows Z(3) symmetry:

$\text{Im } l \uparrow$

$T \gg T_c$

$T \sim T_c$

$T < T_c$

Interface tension: box long in $z$. Each end: distinct but degenerate vacua. Interface forms, action $\sim$ interface tension:

$T > T_c$: order-order interface = ‘t Hooft loop: measures response to magnetic charge

Korthals-Altes, Kovner, & Stephanov, hep-ph/9909516

Also: if trans. 1st order, order-disorder interface at $T_c$. 

$Z \propto e^{-\sigma_{int} V_{tr}}$
Usual spins vs Polyakov Loop

\( L = \text{SU}(N) \text{ matrix} \), Polyakov loop \( l \sim \text{trace} \):

Confinement: \( F_{\text{test} \, qk} = \infty \Rightarrow \langle l \rangle = 0 \)

Above \( T_c \), \( F_{\text{test} \, qk} < \infty \Rightarrow \langle l \rangle \neq 0 \)

\( \langle l \rangle \) measures ionization of color:
partial ionization when \( 0 < \langle l \rangle < 1 \): “semi”-QGP

Svetitsky and Yaffe ’80:
SU(3) 1st order because \( Z(3) \) allows cubic terms:

\[
\mathcal{L}_{\text{eff}} \sim l^3 + (l^*)^3
\]

Does not apply for \( N > 3 \). So why is deconfinement 1st order for all \( N \geq 3 \)?
Polyakov Loop from Lattice: pure Glue, no Quarks

Lattice: *(renormalized)* Polyakov loop. Strict order parameter

Three colors: Gupta, Hubner, Kaczmarek, 0711.2251.

Suggests *wide* transition region, like pressure, to $\sim 4 T_c$.

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![Diagram showing Polyakov Loop from Lattice](image-url)

- Confined $\rightarrow$ *SemiQGP* $\rightarrow$ "Complete" QGP $\rightarrow$

- Direct renormalization

- QQ renormalization

- $L^r_3$

- $T/T_c$

- $T=0$ $\uparrow$ $T_c$ $\uparrow$ $\sim 4 T_c$ $\uparrow$ $T$ $\rightarrow$

- $\langle$loop$\rangle$ $\uparrow$

- $\sim 0.4$
Quarks ~ background Z(3) field. Lattice: Bazavov et al, 0903.4379.

3 quark flavors: weak Z(3) field, does not wash out approximate Z(3) symmetry.
Transition region \textit{narrow}: for pressure, < 1.2 $T_c$!
For interface tensions, < 4 $T_c$...

Above 1.2 $T_c$, pressure dominated by \textit{constant} term $\sim T^2$.

What does this term come from? Gluon mass $m(T)$? But inelegant...

SU(N) in 2+1 dimensions: ideal $\sim T^3$. Caselle + ...: \textit{also} $T^2$ term in pressure.
But mass would be $m^2 T$, \textit{not} $m T^2$.

$T^2$ term like free energy of massless fields in 2 dimensions: string? Above $T_c$?

\textit{Need to include quarks!}

Can then compute temperature dependence of:

shear viscosity, energy loss of light quarks, damping of quarkonia...
Lattice: SU(N) in 2+1 dimensions

SU(N) in 2+1 dim’s for N = 2, 3, 4, 5, & 6. Below plot of $T_c/T$, not $T/T_c$. Clear evidence for non-ideal terms $\sim T^2$, not $\sim T$

$$p(T) \approx \# \left( T^3 - c T^2 T_c \right), \; c \approx 1.$$

Caselle, Castagnini, Feo, Gliozzi, Gursoy, Panero, Schafer, 1111.0580.
With quarks: “$T_c$” moves down: *which* $T_c$?

Just glue: $T_c^{\text{deconf}} \sim 280$ MeV. Standard lore: with quarks, *one* “$T_c$”, decreases.

Matrix model: $T_c^{\text{deconf}}$ constant. With *light* quarks $T_c^{\text{chiral}} < T_c^{\text{deconf}}$.

*Not* two trans.’s, just ⟨loop⟩ small when $T << T_c^{\text{deconf}}$.

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**Plot Description:**

- **Graph Title:** $L_{\text{ren}}(T)$ vs. $T$ in MeV
- **Axes:**
  - $L_{\text{ren}}(T)$ on the y-axis
  - $T$ [MeV] on the x-axis

**Graph Details:**

- **Data Points:**
  - Lattice: Bazavov and Petreczky + HotQCD, 1110.2160
  - HISQ/tree: $N_t=6$
  - $N_t=8$
  - $N_t=12$
  - Stout, cont.
  - SU(3)
  - SU(2)

**Legend:**

- With quarks: $T_c^{\text{chiral}}$ increases.
- Without quarks: $T_c^{\text{chiral}}$ decreases.

**Annotations:**

- Ren’d loop $\uparrow$
- $T_c^{\text{chiral}}$, $T_c^{\text{deconf}}$

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Shear viscosity changes with $T$

In semi-QGP, $\eta$ suppressed from pert. value through function $R(q)$. Not like kinetic theory Log sensitivity, through constant $\kappa$

$$\eta = \frac{c_{\text{pert}} T^3}{g^4 \log(\kappa/g^2 N_c)} R(q)$$


$R(q)$: Y. Hidaka & RDP, 0912.0940.
“Bleaching” of color near $T_c$.

*Roughly* speaking, as $\langle \text{loop} \rangle \rightarrow 0$, *all* colored fields disappear.

Quarks, in fundamental rep. as $\langle \text{loop} \rangle$. Gluons, in adjoint rep., as $\langle \text{loop} \rangle^2$.

Bleaching of color as $T \rightarrow T_c$: *robust* consequence of the confinement of color.

**QGP:** quarks and gluons. **Semi-QGP:** dominated by quarks, by $\sim \langle \text{loop} \rangle$.

Why recombination works at RHIC but not at LHC? 

$\left( v_2 / \# \text{quarks} \text{ vs kinetic energy}/\# \text{quarks} \right)$

Suppression of color universal for all fields, *independent* of mass.

Why charm quarks flow the same as light quarks? (single charm vs pions)

An effective theory can provide a bridge from lattice simulations to experiment.
Matrix model: two colors

Simple approximation

Two colors: transition 2nd order, vs 1st for $N \geq 3$

Using large $N$ at $N = 2$
Matrix model: SU(2)

*Simple approximation: constant* $A_0 \sim \sigma_3$, *nonperturbative, $\sim 1/g$:*

$$A^{cl}_0 = \frac{\pi T}{g} q \sigma_3 \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad L(q) = \begin{pmatrix} e^{i \pi q} & 0 \\ 0 & e^{-i \pi q} \end{pmatrix}$$

*Single dynamical field, $q$*

Loop $l$ real. $Z(2)$ degenerate vacua $q = 0$ and 1:

$$
\begin{array}{c}
\llap{-1} \times \\
q = 1
\end{array} \quad \begin{array}{c}
\llap{0} \times \\
q = \frac{1}{2}
\end{array} \quad \begin{array}{c}
\llap{1} \times \\
q = 0
\end{array}
\quad \text{Re } l \rightarrow
$$

Point *halfway in between*: $q = \frac{1}{2}$, $l = 0$.

Confined vacuum, $L_c$,

$$L_c = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

Classically, $A^{cl}_0$ has zero action: *no* potential for $q$. 
Potential for $q$, interface tension

Computing to one loop order about $A_0^{cl}$ gives a potential for $q$: Gross, RDP, Yaffe, ‘81

\[ V_{pert}(q) = \frac{4\pi^2}{3} T^4 q^2 (1 - q)^2 \]

Use $V_{pert}(q)$ to compute $\sigma$: Bhattacharya, Gocksch, Korthals-Altes, RDP, ph/9205231.

\[ V_{tot}(q) = \frac{2\pi^2 T^2}{g^2} \left( \frac{dq}{dz} \right)^2 + V_{pert}(q) \quad \Rightarrow \sigma = \frac{4\pi^2}{3\sqrt{6}} \frac{T^2}{\sqrt{g^2}} \]

Balancing $S_{cl} \sim 1/g^2$ and $V_{pert} \sim 1$ gives $\sigma \sim 1/\sqrt{g^2}$ (not $1/g^2$).

Width interface $\sim 1/g$, justifies expansion about constant $A_0^{cl}$. GKA ‘04: $\sigma \sim ... + g^2$
Potentials for the q’s

Add *non*-perturbative terms, by *hand*, to generate \( \langle q \rangle \neq 0 \):

*By hand?* \( V_{\text{non}}(q) \) from: monopoles, vortices...  

Liao & Shuryak: ph/0611131, 0706.4465, 0804.0255, 0804.4890

\[
V_{\text{eff}}(q) = V_{\text{pert}}(q) + V_{\text{non}}(q)
\]

\[V_{\text{eff}}(q) \uparrow\]

\( T \gg T_c: \langle q \rangle = 0,1 \to\)

\( T < T_c: \langle q \rangle = \frac{1}{2} \to\)

\( V_{\text{eff}}(q) \uparrow\)
Three possible “phases”

Two phases are familiar:

\[ \langle q \rangle = 0, 1: \quad \langle l \rangle = \pm 1: \text{“Complete” QGP: usual perturbation theory. } T \gg T_c. \]

\[ \langle q \rangle = 1/2: \quad \langle l \rangle = 0: \text{confined phase. } T < T_c \]

Also a third phase, “partially” deconfined (adjoint Higgs phase)

\[ 0 < \langle q \rangle < 1/2: \quad \langle l \rangle < 1: \text{“semi”-QGP. } \quad \text{From some } x T_c > T > T_c \quad x? \]

Lattice: one transition, to confined phase, at \( T_c \). No other transition above \( T_c \). Still, there is an intermediate phase, the “semi”-QGP

\textit{Strongly constrains possible non-perturbative terms, } V_{\text{non}}(q).
Getting three “phases”, one transition

Simple guess: $V_{\text{non}} \sim \text{loop}^2,$

$$V_{\text{eff}} \sim \frac{a}{\pi^2} (\ell^2 - 1) + q^2(1 - q)^2$$

$$\sim q^2(1 - a) - 2q^3 + \ldots$$

1st order transition \textit{directly} from complete QGP to confined phase, \textit{not} 2nd

Generic if $V_{\text{non}}(q) \sim q^2$ at $q << 1$.

Easy to avoid, \textit{if} $V_{\text{non}}(q) \sim q$ for small $q$. Then $\langle q \rangle \neq 0$ at all $T > T_c$.

Imposing the symmetry of $q \leftrightarrow 1 - q$, $V_{\text{non}}(q)$ \textit{must} include

$$V_{\text{non}}(q) \sim q(1 - q)$$

Term $\sim q$ at small $q$ avoids transition from pert. QGP to adjoint Higgs phase
Cartoons of deconfinement

Consider:

\[ V_{eff} = q^2 (1 - q)^2 - a q(1 - q), \quad a \sim T_c^2 / T^2 \]

\( \downarrow a = 0: \) complete QGP

\( \downarrow a = \frac{1}{4}: \) semi QGP

\( a = \frac{1}{2}: \) Stable vacuum at \( q = \frac{1}{2} \)

Transition second order
0-parameter matrix model, $N = 2$

Meisinger, Miller, Ogilvie ph/0108009, MMO:
take $V_{\text{non}} \sim T^2$

$$V_{\text{non}}(q) = \frac{4\pi^2}{3} T^2 T_c^2 \left( -\frac{c_1}{5} q(1 - q) + \frac{c_3}{15} \right)$$

Two conditions: transition occurs at $T_c$, pressure($T_c$) = 0
Fixes $c_1$ and $c_3$, no free parameters. But not close to lattice data (from ’89!)

\[
\frac{(e - 3p)}{3T^4} \uparrow
\]

Lattice: Engels, Fingberg, Redlich, Satz, Weber ‘89
1-parameter matrix model, $N = 2$

Dumitru, Guo, Hidaka, Korthals-Altes, RDP ‘10: to usual perturbative potential,

$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \left( -\frac{1}{20} + q^2 (1 - q)^2 \right)$$

Add - by hand - a non-pert. potential $V_{non} \sim T^2 T_c^2$. Also add a term like $V_{pert}$:

$$V_{non}(q) = \frac{4\pi^2}{3} T^2 T_c^2 \left( -\frac{c_1}{5} q (1 - q) - c_2 q^2 (1 - q)^2 + \frac{c_3}{15} \right)$$

Now just like any other mean field theory. $\langle q \rangle$ given by minimum of $V_{eff}$:

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q)$$

$$\frac{d}{dq} V_{eff}(q) \bigg|_{q=\langle q \rangle} = 0$$

$\langle q \rangle$ depends nontrivially on temperature.

Pressure value of potential at minimum:

$$p(T) = -V_{eff}(\langle q \rangle)$$
Choose \( c_2 \) to fit \( e^{-3p/T^4} \): optimal choice

\[
c_1 = 0.23, \quad c_2 = 0.91, \quad c_3 = 1.11
\]

Reasonable fit to \( e^{-3p/T^4} \); also to \( p/T^4, e/T^4 \).

N.B.: \( c_2 \sim 1 \). At \( T_c \), terms \( \sim q^2(1-q)^2 \) *almost* cancel.
Width of transition region, 0- vs 1-parameter

1-parameter model: get sharper $e^{-3p/T^4}$ because $\langle q \rangle \rightarrow 0$ much quicker above $T_c$. Physically: sharp $e^{-3p/T^4}$ implies region where $\langle q \rangle$ is significant is narrow

N.B.: $\langle q \rangle \neq 0$ at all $T$, but numerically, negligible above $\sim 1.2 T_c$; $p \sim \langle q \rangle^2$. Above $\sim 1.2 T_c$, the $T^2$ term in the pressure is due entirely to the constant term, $c_3$!

Above $\sim 1.2 T_c$, the $T^2$ term in the pressure is due entirely to the constant term, $c_3$!
Polyakov loop: 1-parameter matrix model ≠ lattice

Lattice: *renormalized* Polyakov loop. 0-parameter model: close to lattice
1-parameter model: *sharp* disagreement. \( \langle l \rangle \) rises to ~ 1 *much* faster - ?

Sharp rise also found using Functional Renormalization Group (FRG): 
Braun, Gies, Pawlowski, 0708.2413; Marhauser, Pawlowski, 0812.1144

\[
\langle l \rangle \uparrow \quad \Leftrightarrow \quad 0\text{-parameter}
\]
\[
\downarrow \quad 1\text{-parameter}
\]

\[ Lattice: \quad \text{Cardoso, Cardoso, Bicudo, 1104.5432} \]

Can reconcile by (arbitrary) shift in zero point energy
\[ \langle l \rangle \rightarrow e^{-E_0 / T} \langle l \rangle \]
Interface tension, $N = 2$

$\sigma$ vanishes as $T \to T_c$, $\sigma \sim (t-1)^{2\nu}$.

Ising $2\nu \sim 1.26$; Lattice: $\sim 1.32$.

Matrix model: $\sim 1.5$: $c_2$ important.

$$\sigma(T) = \frac{4\pi^2 T^2}{3\sqrt{6} g^2} \frac{(t^2 - 1)^{3/2}}{t (t^2 - c_2)} , \quad t = \frac{T}{T_c}$$

Semi-class.: GKA ’04. Include corr.’s $\sim g^2$ in matrix $\sigma(T)$ (ok when $T > 1.2 T_c$)

N.B.: width of interface diverges as $T \to T_c$, $\sim \sqrt{(t^2 - c_2)/(t^2-1)}$.

Lattice: de Forcrand, D’Elia, Pepe, lat/0007034
Lattice: $A_0$ mass as $T \to T_c$ - up or down?

Gauge invariant: 2 pt function of loops:

$$\langle \text{tr } L^\dagger(x) \text{ tr } L(0) \rangle \sim e^{-\mu x / x^d}$$

$\mu/T$ goes down as $T \to T_c$

Kaczmarek, Karsch, Laermann, Lutgemeier lat/9908010

Gauge dependent: singlet potential

$$\langle \text{tr } (L^\dagger(x) L(0)) \rangle \sim e^{-m_D x / x}$$

$m_D/T$ goes up as $T \to T_c$

Cucchieri, Karsch, Petreczky lat/0103009, Kaczmarek, Zantow lat/0503017

Tuesday: Tereza Mendes

Which way do masses go as $T \to T_c$? Both are constant above $\sim 1.5 \ T_c$. 

Adjoint Higgs phase, $N = 2$

$A_0^{cl} \sim q \sigma_3$, so $\langle q \rangle \neq 0$ generates an (adjoint) Higgs phase:

RDP, ph/0608242; Unsal & Yaffe, 0803.0344, Simic & Unsal, 1010.5515

In background field, $A = A_0^{cl} + A^{qu}$: $D_0^{cl} A^{qu} = \partial_0 A^{qu} + i g [A_0^{cl}, A^{qu}]$

Fluctuations $\sim \sigma_3$ not Higgsed, $\sim \sigma_{1,2}$ Higgsed, get mass $\sim 2 \pi T \langle q \rangle$

Hence when $\langle q \rangle \neq 0$, for $T < 1.2 T_c$, splitting of masses:

At $T_c$: $m_{diag} = 0$, $m_{off} \sim 2 m_{pert}$.

$m_{pert} = \sqrt{2/3} g T$:

$m/m_{pert} \sim .56$ at $1.5 T_c$, from $V_{non}$. 

$T \uparrow$
Matrix model: $N \geq 3$

Why the transition is *always* 1st order

One parameter model
Path to $\mathbb{Z}(3)$, three colors

SU(3): *two* diagonal $\lambda$'s, so *two* $q$'s:

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$\mathbb{Z}(3)$ paths: move along $\lambda_8$, not $\lambda_3$: $q_8 \neq 0$, $q_3 = 0$.

$$A_0 = \frac{2\pi T}{3g} (q_3 \lambda_3 + q_8 \lambda_8)$$

$$L = e^{\frac{2\pi i q_8 \lambda_8}{3}}$$

$q_8 = 0 \quad L = 1$
$q_8 = 3/8 \quad L = e^{\frac{2\pi i}{3}}$
$q_8 = 1 \quad L = e^{\frac{2\pi i}{3}}$
Path to confinement, three colors

Now move along $\lambda_3$: \[ L = e^{2\pi i q_3 \lambda_3 / 3} \]

In particular, consider $q_3 = 1$:
- Elements of $e^{2\pi i/3} L_c$ same as those of $L_c$.
- Hence $\text{tr} \ L_c = \text{tr} \ L_c^2 = 0$: $L_c$ confining vacuum

Path to confinement: along $\lambda_3$, not $\lambda_8$, $q_3 \neq 0$, $q_8 = 0$. 

\[ q_3 = 0 \quad \ell = 1 \]
\[ q_3 = 3/8 \quad \ell \approx .8 \]
\[ q_3 = 1 \quad \ell = 0 \]
General potential for any SU(N)

Ansatz: constant, diagonal matrix
\[ A_{ij}^0 = \frac{2\pi T}{g} q_i \delta^{ij} \quad L_{ij} = e^{2\pi i q_j} \delta_{ij} \]

For SU(N), \(\sum_{j=1}^{N} q_j = 0\). Hence N-1 independent \(q_j\)'s, = # diagonal generators.

At 1-loop order, the perturbative potential for the \(q_j\)'s is

\[ V_{pert}(q) = \frac{2\pi^2}{3} T^4 \left( -\frac{4}{15} (N^2 + 1) + \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 \right), \quad q_{ij} = |q_i - q_j| \]

As before, assume a non-perturbative potential \(\sim T^2 T_c^2\):

\[ V_{non}(q) = \frac{2\pi^2}{3} T^2 T_c^2 \left( -\frac{c_1}{5} \sum_{i,j} q_{ij} (1 - q_{ij}) - c_2 \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 + \frac{4}{15} c_3 \right) \]
Path to confinement, four colors

Move to the confining vacuum along *one* direction, $q^c_j$: (For general interfaces, need *all* N-1 directions in $q_j$ space)

- Perturbative vacuum: $q = 0$.
- Confining vacuum: $q = 1$.
- Four colors:

\[ q^c_j = \left( \frac{2j - N - 1}{2N} \right) q, \ j = 1 \ldots N \]

\[ \begin{align*}
q &= 0 \\
\ell &= 1
\end{align*} \]

\[ \begin{align*}
q &= 1/2 \\
\ell &\approx .65
\end{align*} \]

\[ \begin{align*}
q &= 1 \\
\ell &= 0
\end{align*} \]

General N: confining vacuum = *uniform* distribution for eigenvalues of $L$

For infinite N, distribution flat.
Cubic term for all $N \geq 3$, so transition first order

Define $\phi = 1 - q$,
Confining point $\phi = 0$

\[
V_{tot} = \frac{\pi^2 (N^2 - 1)}{45} T_c^4 t^2 \left( t^2 - 1 \right) \tilde{V}(\phi, t), \quad t = \frac{T}{T_c}
\]

\[
\tilde{V}(\phi, t) = -m_{\phi}^2 \phi^2 - 2 \left( \frac{N^2 - 4}{N^2} \right) \phi^3 + \left( 2 - \frac{3}{N^2} \right) \phi^4
\]

No term linear in $\phi$. Cubic term in $\phi$ for all $N \geq 3$.
Along $q^c$, about $\phi = 0$ there is no symmetry of $\phi \to -\phi$ for any $N \geq 3$.

Hence terms $\sim \phi^3$, and so a first order transition, are ubiquitous.
Special to matrix model, with the $q_i$’s elements of Lie algebra.

Svetitsky and Yaffe ’80: $V_{\text{eff}}(\text{loop}) \Rightarrow$ 1st order only for $N=3$; loop in Lie group

Also 1st order for $N \geq 3$ with FRG: Braun, Eichhorn, Gies, Pawlowski, 1007.2619.
Cubic term for four colors

Construct $V_{\text{eff}}$ either from $q$’s, or equivalently, loops: $\text{tr } \mathbf{L}$, $\text{tr } \mathbf{L}^2$, $\text{tr } \mathbf{L}^3$...

$N = 4$: $|\text{tr } \mathbf{L}|^2$ and $|\text{tr } \mathbf{L}^3|^2$ not symmetric about $q = 1$, so cubic terms, $\sim (q - 1)^3$.

($|\text{tr } \mathbf{L}^2|^2$ symmetric, residual $Z(2)$ symmetry)

Cubic terms special to moving along $q_c$ in a matrix model. Not true in loop model

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Cubic term for four colors

Asymmetric in reflection about \( q = 1 \)

\(|\text{tr } L^3|^2 \Rightarrow \)

\( \Leftarrow|\text{tr } L|^2 \)

\( \uparrow 1.2 \quad q = 1 \quad \uparrow .8 \)

\( q^c \Rightarrow \)
Results for $N=3$ similar to $N=2$.
0-parameter model way off.
Good fit $e^{-3p/T^4}$ for 1-parameter model, \[ c_1 = 0.32, \quad c_2 = 0.83, \quad c_3 = 1.13 \]

Again, $c_2 \sim 1$, so at $T_c$, terms $\sim q^2(1-q)^2$ almost cancel.
Renormalized Polyakov loop from lattice does not agree with either matrix model \( \langle l \rangle - 1 \sim \langle q \rangle^2 \): By \( 1.2 T_c \), \( \langle q \rangle \sim 0.05 \), negligible.

Again, for \( T > 1.2 T_c \), the \( T^2 \) term in pressure due entirely to the constant term, \( c_3 \)!

Rapid rise of \( \langle l \rangle \) as with FRG: Braun, Gies, Pawlowski, 0708.2413

\[ \langle l \rangle \uparrow \] 
\[ 0 \text{-parameter} \] 
\[ 1 \text{-parameter} \downarrow \]

\[ T \rightarrow T_c \]
\[ T \rightarrow 2T_c \]

\[ \langle l \rangle \rightarrow e^{-E_0/T} \langle l \rangle \]

Lattice: Gupta, Hubner, and Kaczmarek, 0711.2251.

Cannot reconcile by shift in zero point energy.

Can not reconcile by shift in zero point energy

\[ \langle l \rangle \rightarrow e^{-E_0/T} \langle l \rangle \]
Interface tension, $N = 2$ and $3$

Order-order interface tension, $\sigma$, from matrix model close to lattice.
For $T > 1.2 \, T_c$, path along $\lambda_8$; for $T < 1.2 \, T_c$, along both $\lambda_8$ and $\lambda_3$.

$\sigma(T_c)/T_c^2$ nonzero but small, $\sim .02$. Results for $N = 2$ and $N = 3$ similar - ?
Adjoint Higgs phase, $N = 3$

For $SU(3)$, deconfinement along $A_0^{cl} \sim q \lambda_3$. Masses $\sim [\lambda_3, \lambda_i]$: two off-diagonal. Splitting of masses only for $T < 1.2 T_c$:

Measureable from singlet potential, $\langle tr L^\dagger(x) L(0) \rangle$, over all $x$.

At $T_c$: $m_{\text{diag}}$ small, but $\neq 0$

$m_{\text{pert}} = g T$, $m/m_{\text{pert}} \sim .8$ at $1.5 T_c$, from $V_{\text{non}}$. 4 off-diagonal, $K$’s

2 off-diagonal, $\pi$’s

2 diagonal modes
Matrix model: $N \geq 3$

To get the latent heat right, two parameter model.

Thermodynamics, interface tensions improve
Latent heat, and a 2-parameter model

Latent heat, $e(T_c)/T_c^4$: 1-parameter model too small:
1-para.: 0.33. BPK: 1.40 ± .1; DG: 1.67 ± .1.

$$c_3(T) = c_3(\infty) + \frac{c_3(1) - c_3(\infty)}{t^2}, \quad t = \frac{T}{T_c}$$

2-parameter model, $c_3(T)$. Like MIT bag constant
WHOT: $c_3(\infty) \sim 1$. *Fit* $c_3(1)$ to DG latent heat
Fits lattice for $T < 1.2 \ T_c$, overshoots above.

$$c_3(1) = 1.33, \ c_3(\infty) = .95$$

$$c_1 = .833, \ c_2 = .552$$

Bag const ~ (262 MeV)$^4$

$c_2$ *not* near 1, vs 1-para.
2-parameter model, N = 4

Assume $c_3(\infty) = 0.95$, like N=3. Fit $c_3(1)$ to latent heat, Datta & Gupta, 1006.0938
Order-disorder $\sigma(T_c)/T_c^2 \sim .08$, vs lattice, .12, Lucini, Teper, Wenger, lat/0502003

$$c_3(1) = 1.38, \quad c_3(\infty) = .95, \quad c_1 = 1.025, \quad c_2 = 0.39$$
2-parameter model, $N = 6$

Order-disorder $\sigma(T_c)/T_c^2 \sim 0.25$, vs lattice, 0.39, Lucini, Teper, Wenger, lat/0502003

$c_3(1) = 1.42$, $c_3(\infty) = 0.95$, $c_1 = 1.21$, $c_2 = 0.23$