Hunt for the Quark Gluon Plasma

QGP as a "Unicorn". Experimentalists as hunters,
so (in *this* field), "All theorists are..."

http://www.anti-powerpoint-party.com

Theory vs Experiment

Theory:

Effective theory for deconfinement, near T_c.
Today: only the "pure" glue theory (no dynamical quarks)
Based upon *detailed* results from the lattice

Experiment:

Soon: including quarks
Will provide competition to AdS/CFT

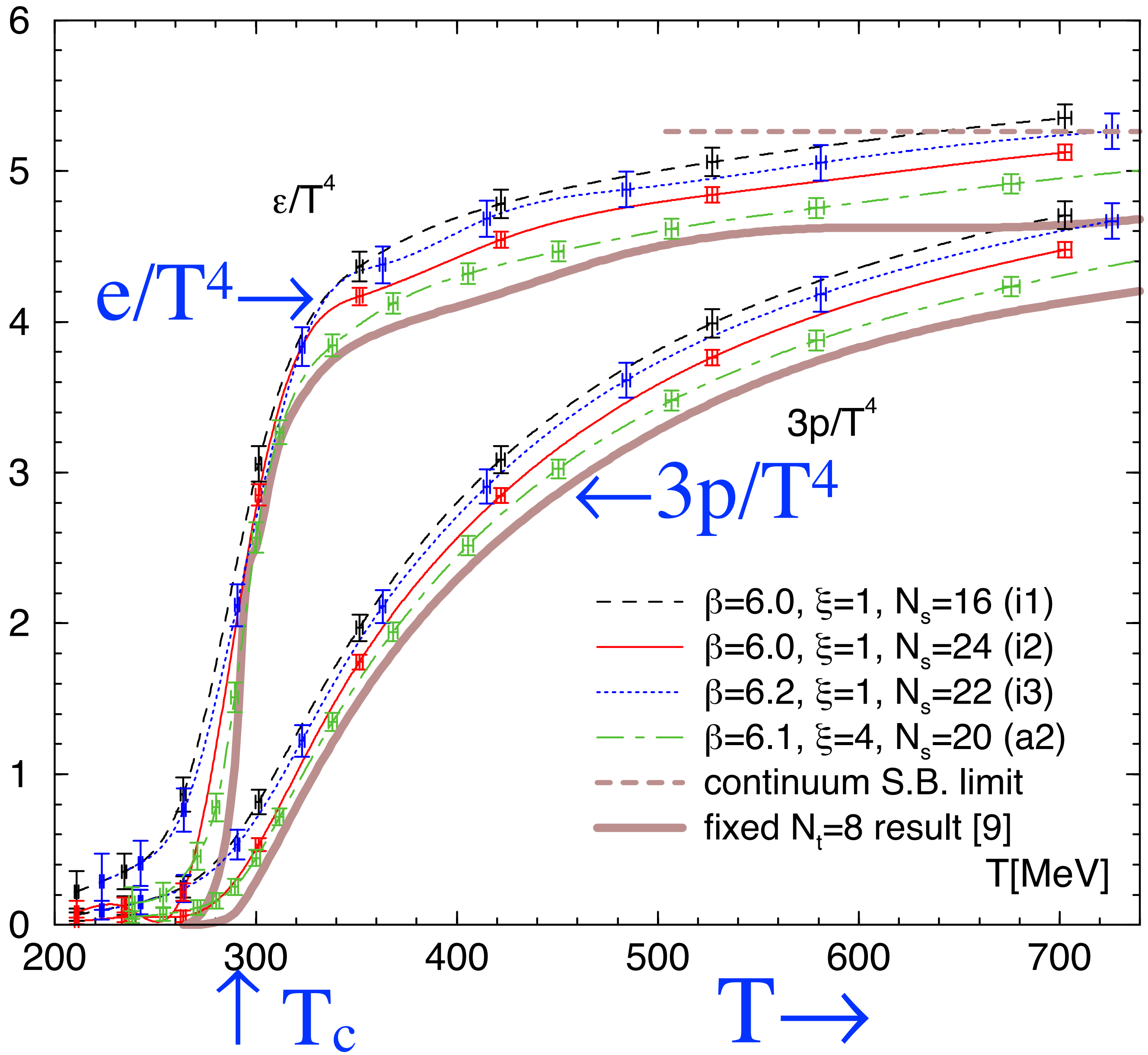
The standard plot

SU(3) gauge theory *without* quarks, temperature T

(Weakly) first order transition at $T_c \sim 290$ MeV

e(T)= energy density

p(T)= pressure

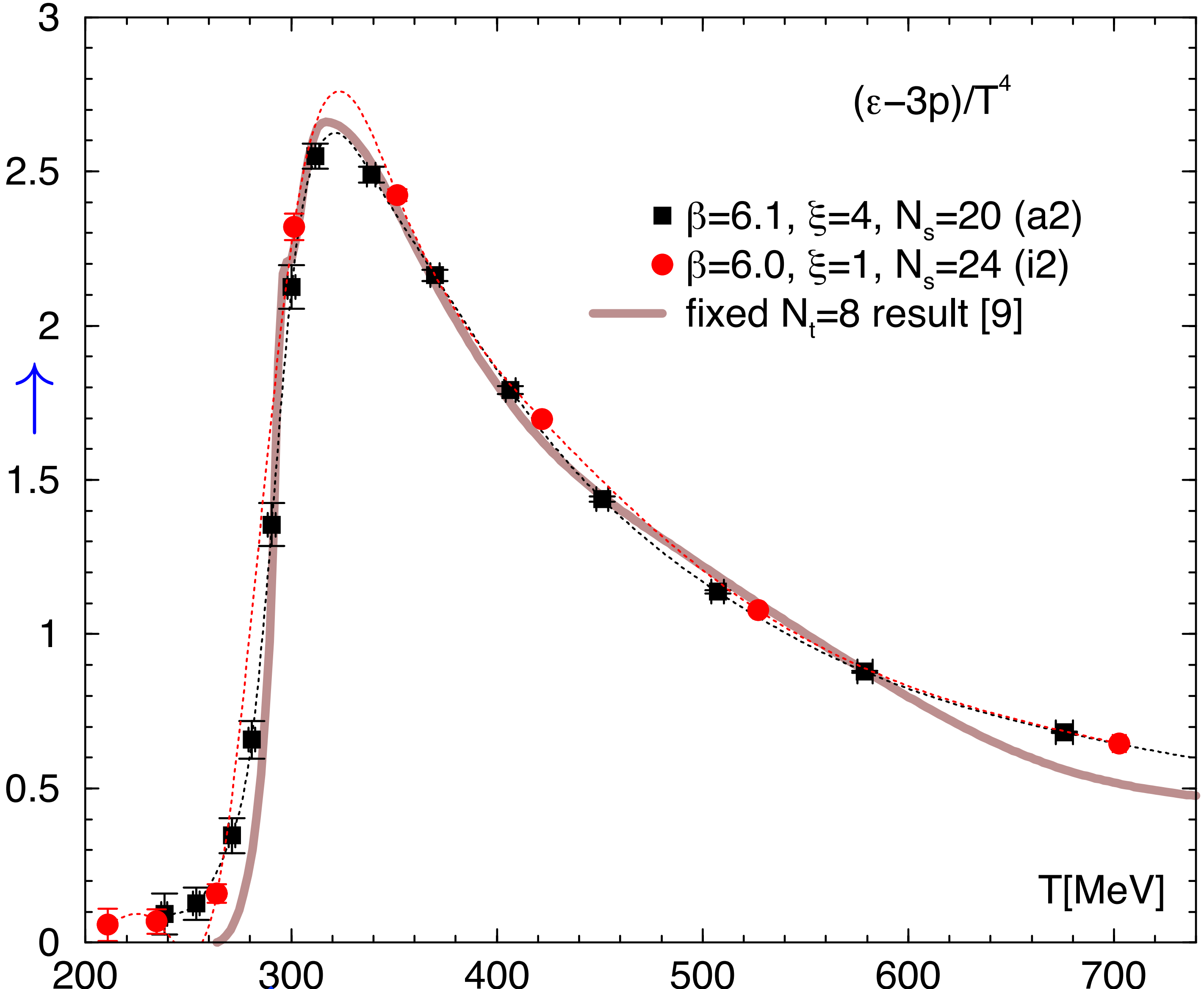


WHOT: Umeda, Ejiri, Aoki, Hatsuda, Kanaya, Maezawa, Ohno, 0809.2842

Another plot of the same

Plot conformal anomaly, $(e\text{-}3p)/T^4$: large peak above T_c. $\sim g^4$ as $T \to \infty$

$$\frac{e-3p}{T^4}\uparrow$$



WHOT: Umeda, Ejiri, Aoki, Hatsuda, Kanaya, Maezawa, Ohno, 0809.2842

$\uparrow T_c$ $\qquad T\rightarrow$

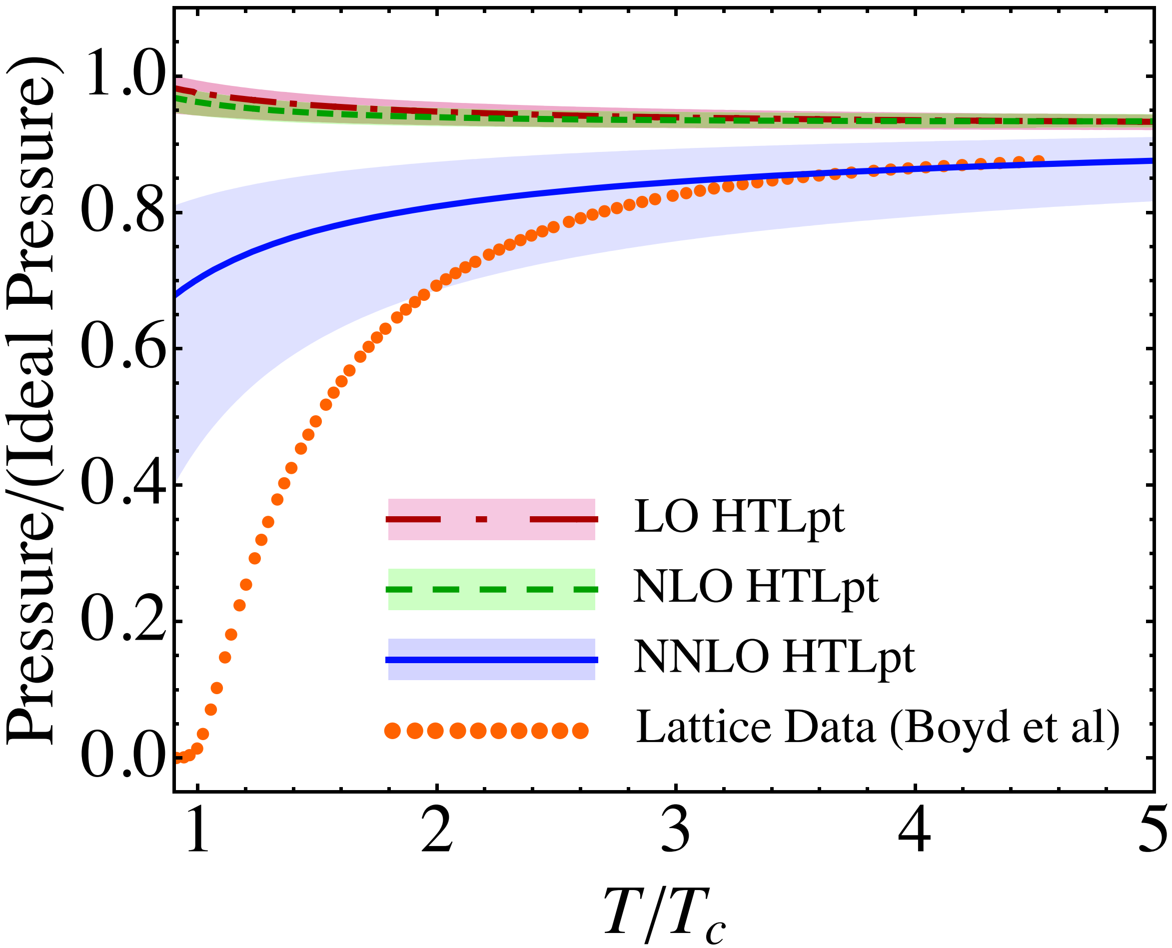
Strong vs weak coupling at T_c?

Resummed perturbation theory at 3-loop order works down to ~ 3 T_c.

Intermediate coupling: $\alpha_s(T_c) \sim 0.3$. *Not* so big... So what happens below ~ 3 T_c?

Want effective theory; e.g.: chiral pert. theory: expand in m_π/f_π, exact as $m_\pi \to 0$.

But there is *no* small mass scale for SU(3) in "semi"-QGP, $T_c \to 3\ T_c$.



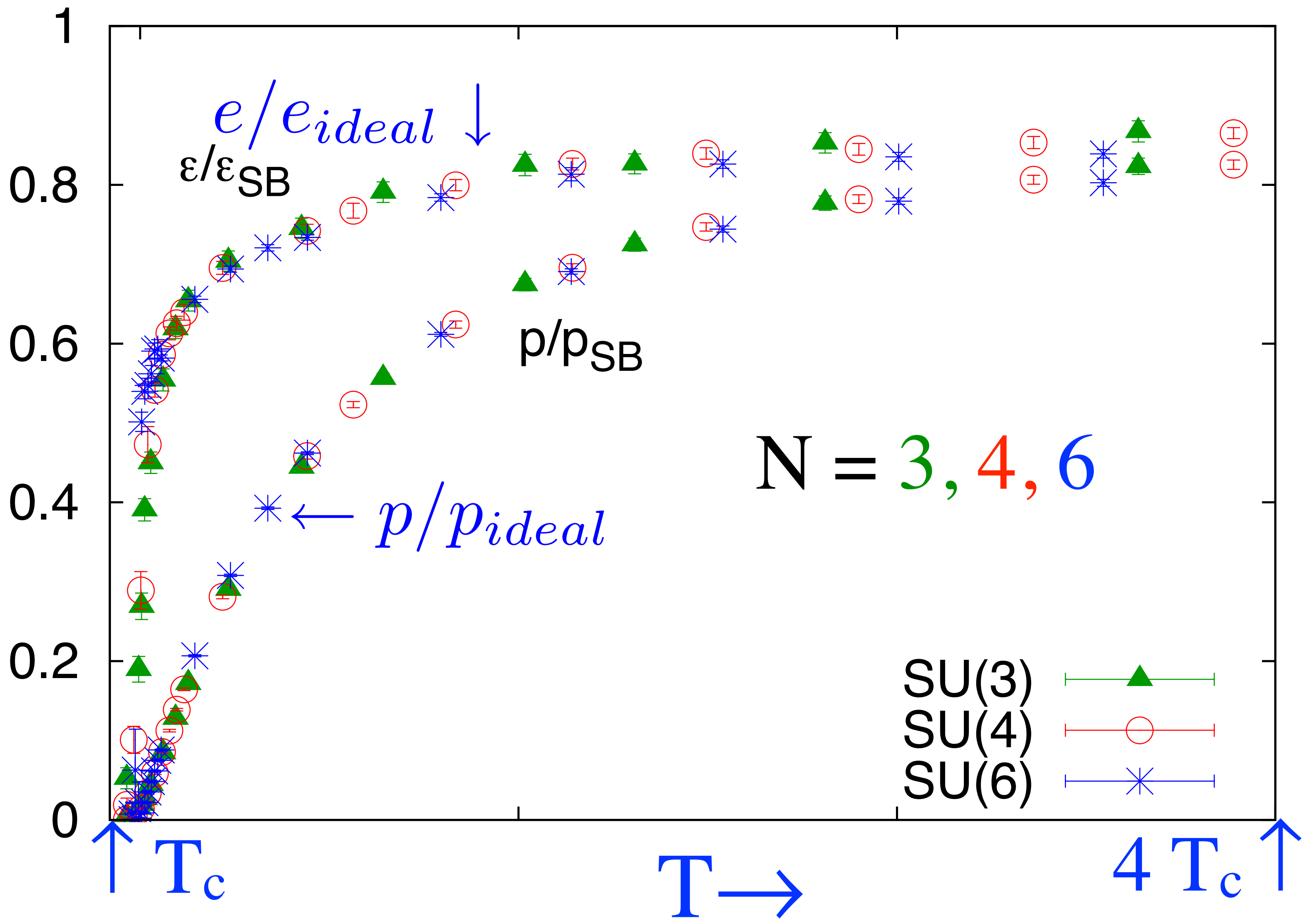
Andersen, Su, & Strickland, 1005.1603

What to expand in?

Consider SU(N) for *different* N. # perturbative gluons ~ N^2 - 1.

Scaled by ideal gas values, e and p for N = 3, 4 and 6 look *very* similar

Implicitly, expand about infinite N. Explicitly, assume classical expansion ok



N=3:
Boyd et al,
lat/9602007

N = 4 & 6:
Datta & Gupta,
1006.0938

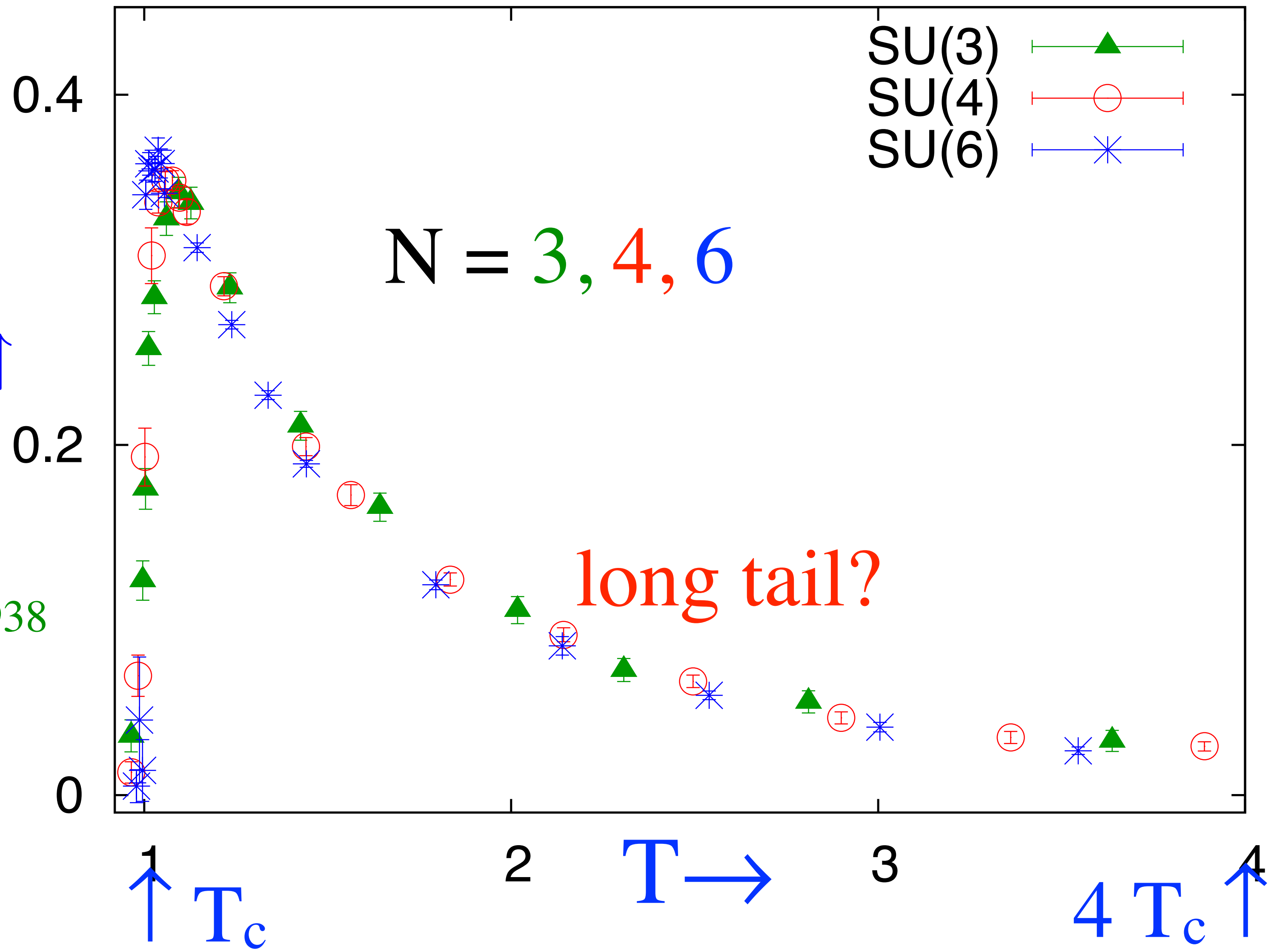
Conformal anomaly ≈ N independent

For SU(N), "peak" in e-3p/T^4 just above T_c. *Approximately* uniform in N.

Not near T_c: transition *2nd* order for N = 2, *1st* order for *all* N ≥ 3

N=3: *weakly* 1st order. N = ∞: *strongly* 1st order (even for latent heat/N^2)

$$\frac{1}{N^2-1}\,\frac{e-3p}{T^4}\uparrow$$



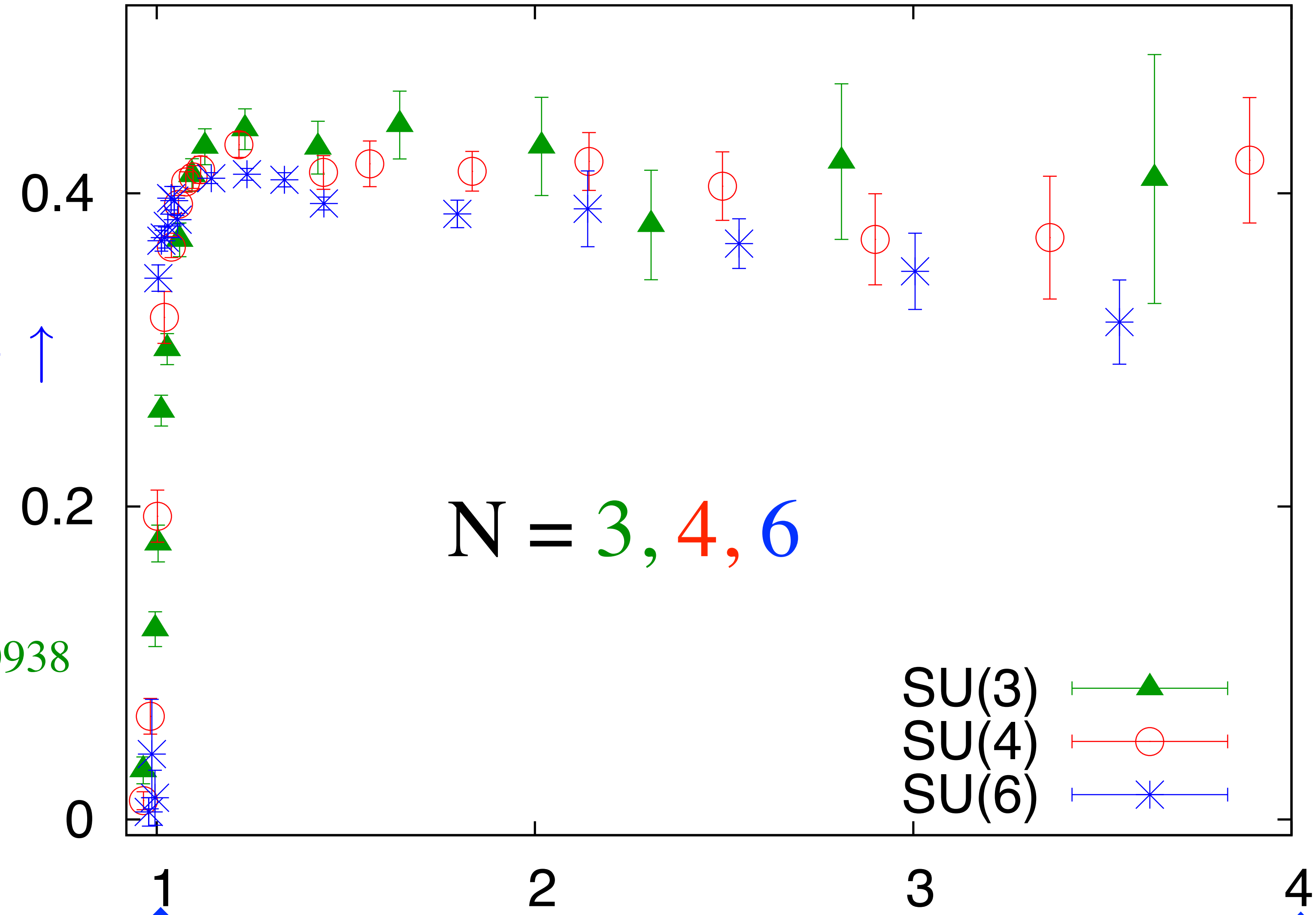
Datta & Gupta, 1006.0938

Tail in the conformal anomaly

To study the tail in $(e-3p)/T^4$, multiply by $T^2/(N^2-1)\,T_c^2$:

$(e-3p)/((N^2-1)T^2\,T_c^2)$ *approximately* constant, independent of N

$$\frac{1}{N^2-1}\;\frac{e-3p}{T^2\,T_c^2}\uparrow$$



Datta & Gupta, 1006.0938

$\uparrow T_c$ $\quad T\rightarrow$ $\quad 4\,T_c\uparrow$

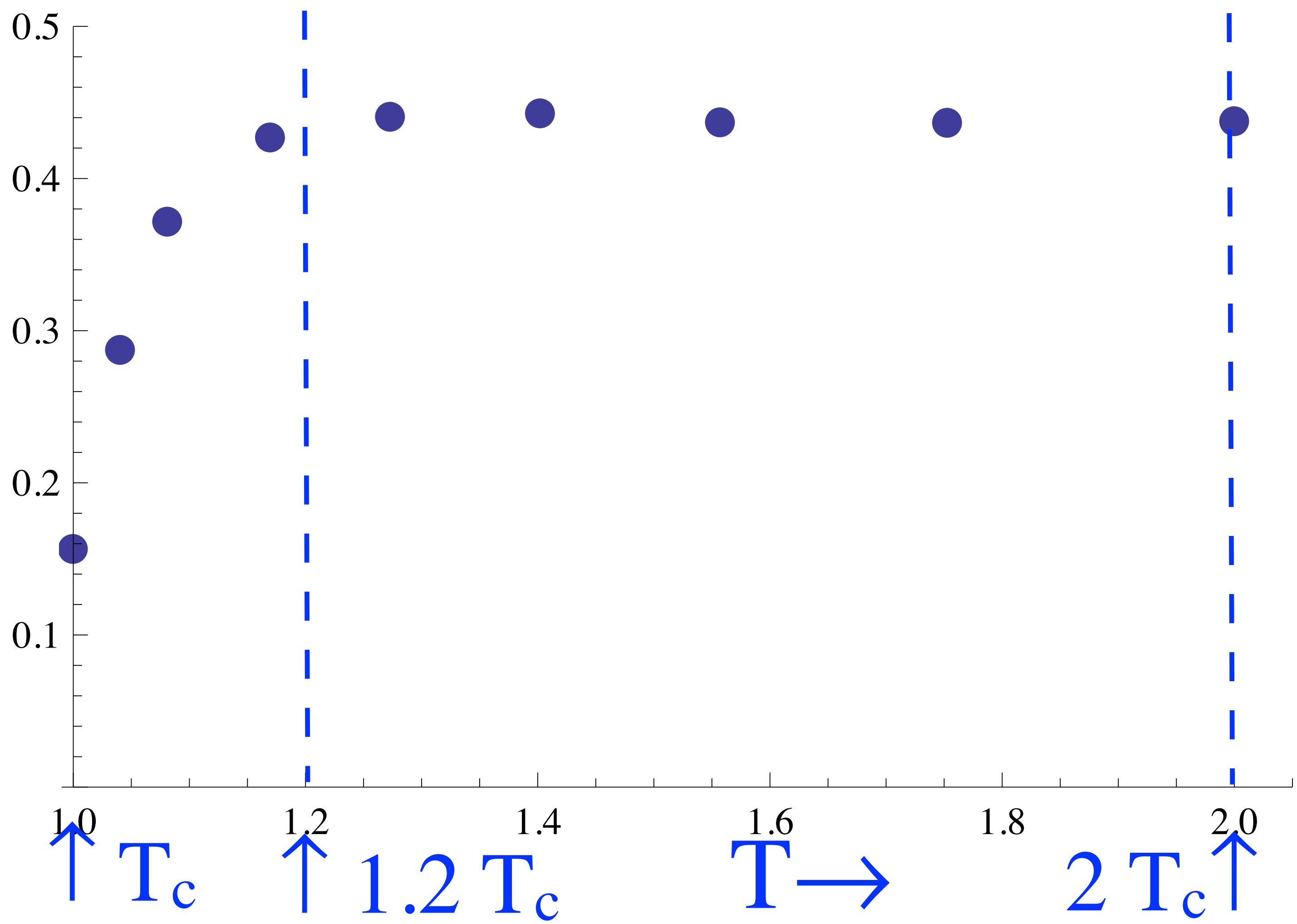
Precise results for three colors

From WHOT:

$$p(T) \approx \#\,(T^4 - c\,T^2 T_c^2)\,,\; T/T_c : 1.2 \to 2.0$$

$$c \approx 1.00 \pm 0.01$$

$$\frac{1}{8}\,\frac{e-3p}{T^2\,T_c^2}\uparrow$$



WHOT: Umeda, Ejiri, Aoki, Hatsuda, Kanaya, Maezawa, Ohno, 0809.2842

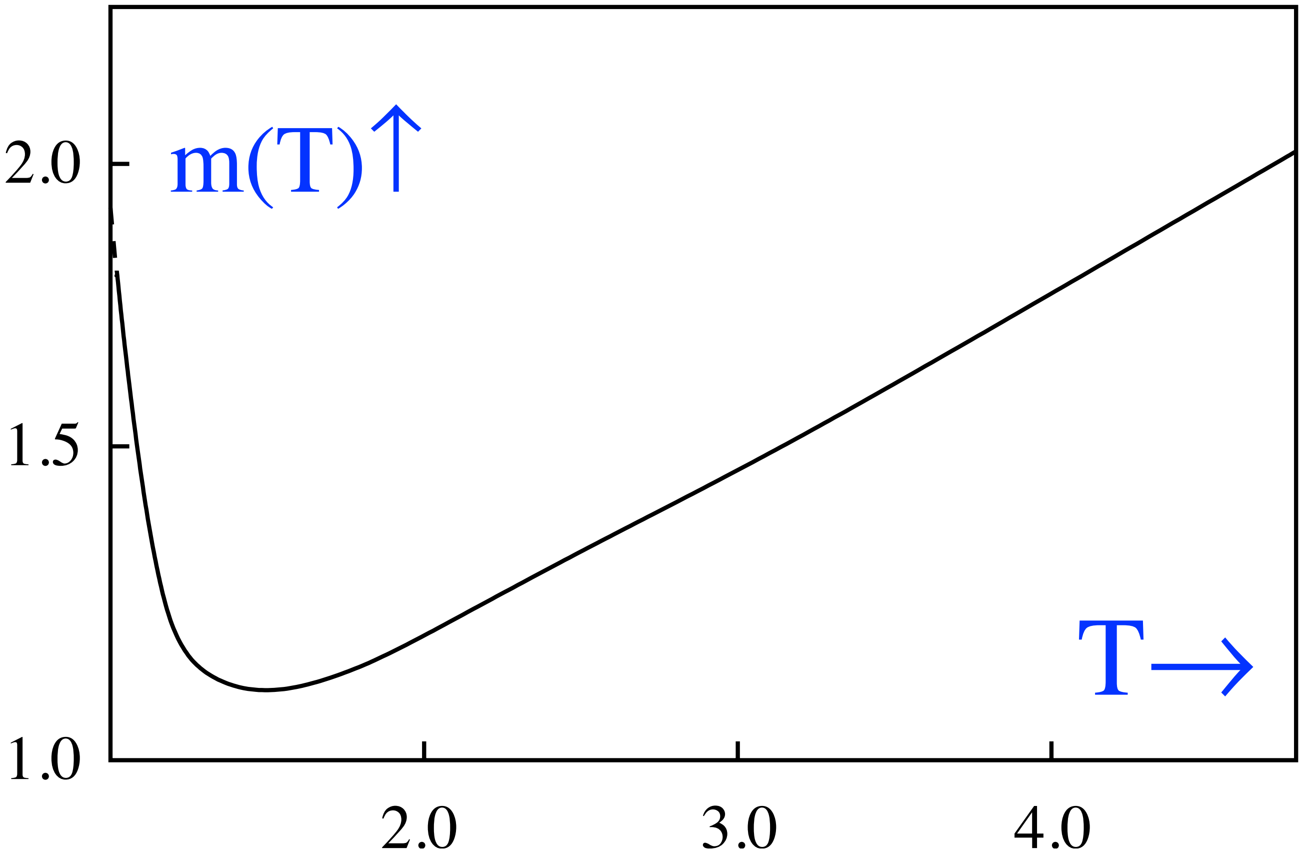
How to get a term ~ T^2 in the pressure?

Expand pressure of ideal, massive gas in powers of mass m:

$$\int d^4p \; \mathrm{tr}\log(p^2+m^2) = \#\, T^4 - \#'\, m^2\, T^2 + \ldots$$

Quasi-particle models: *choose* m(T) to *fit* pressure.

Need m(T) to increase *sharply* as T $\to$ T_c to suppress pressure. Inelegant...



$$\frac{m(T)}{T_c} = \frac{a}{(t-1)^b} + c\,t$$

$t = T/T_c$, $a = .47$,
$b = .13$, $c = .39$

Above: Castorina, Miller, & Satz, 1101.1255

Originally: Peshier, Kampfer, Pavlenko & Soff, PRD 1996

A simple solution

Assume there is some potential, V(q).
The vacuum, q_0, is the minimum of V(q):

$$\left.\frac{dV(q)}{dq}\right|_{q=q_0} = 0$$

Pressure is the value of the potential at the minimum:

$$p(T) = -V(q_0)$$

For T > 1.2 T_c, a *constant* ~T^2 in the pressure, is due to a *constant* ~T^2 in V(q):

$$V(q) = -\#\,(T^4 - T^2\,T_c^2 + T^2\,T_c^2\,\widetilde{V}(q))$$

Above 1.2 T_c , $\langle q \rangle = 0$. Except near T_c, for *most* of the semi-QGP,
the non-perturbative part of the pressure, ~T^2, is due *just* to a constant
Region where $\langle q \rangle \neq 0$, and V(q) matters, is *very* narrow: T: $T_c \rightarrow 1.2\ T_c$

Unexpected consequence of *precise* lattice data.
Large N: makes sense to speak of classical $\langle q \rangle$ instead of fluctuations.

Generalization of Meisinger, Miller, Ogilvie ph/0108009
Dumitru, Guo, Hidaka, Korthals-Altes, & RDP, arXiv:1011.3820 + 1112.?
Also: Y. Hidaka & RDP, 0803.0453, 0906.1751, 0907.4609, 0912.0940.

Hidden Z(2) spins in SU(2)

Consider *constant* gauge transformation:

$$U_c = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -\mathbf{1}$$

As $U_c \sim \mathbf{1}$, locally gluons *invariant*:

$$A_\mu \to U_c^\dagger\, A_\mu\, U_c = +\, A_\mu$$

Nonlocally, Wilson *line* changes:

$$\mathbf{L} = \mathcal{P}\, \mathrm{e}^{ig\int_0^{1/T} A_0\, d\tau} \to -\,\mathbf{L}$$

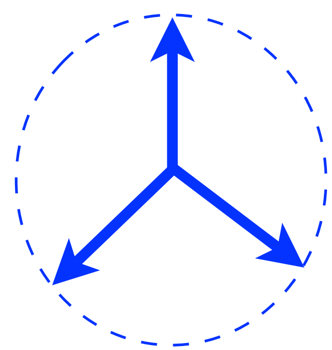
L ~ propagator for "test" quark.

SU(3): det U_c = 1 ⇒

$$U_c = \mathrm{e}^{2\pi i j/3}\,\mathbf{1}$$

j = 0, 1, 2

SU(N): $U_c = e^{2\pi i j/N}\,\mathbf{1}$: Z(N) symmetry.



Z(N) spins of 't Hooft, with*out* quarks

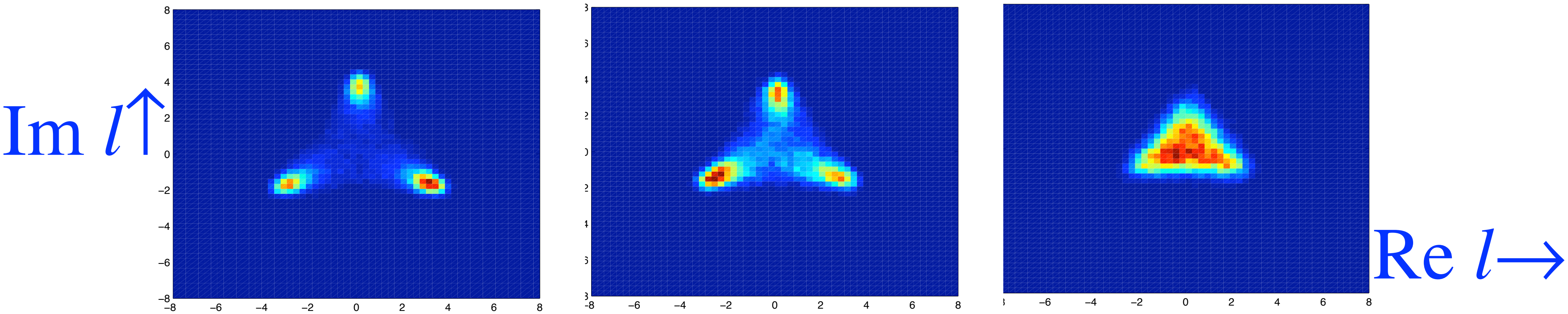
Quarks ~ background Z(N) field, *break* Z(N) sym.

$$\psi \to U_c \psi = -\,\psi$$

Hidden Z(3) spins in SU(3)

Lattice, A. Kurkela, unpub.'d: 3 colors, loop l complex.

Distribution of loop shows Z(3) symmetry:

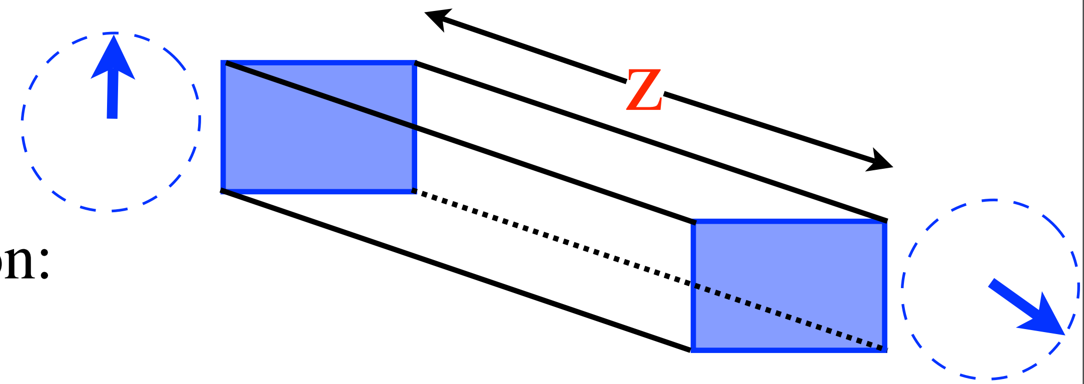


$T >> T_c$ $\quad$ $T \sim T_c$ $\quad$ $T < T_c$

Interface tension: box long in z.

Each end: distinct but *degenerate* vacua.

Interface forms, action ~ interface tension:



$T > T_c$: order-order interface = '*t Hooft loop*:

measures response to *magnetic* charge

Korthals-Altes, Kovner, & Stephanov, hep-ph/9909516

$$Z \sim \mathrm{e}^{-\sigma_{int} V_{tr}}$$

Also: *if* trans. 1st order, order-*dis*order interface *at* T_c .

Usual spins vs Polyakov Loop

L = SU(N) *matrix*, Polyakov loop l ~ trace:

$$\ell = \frac{1}{N}\,\mathrm{tr}\,\mathbf{L}$$

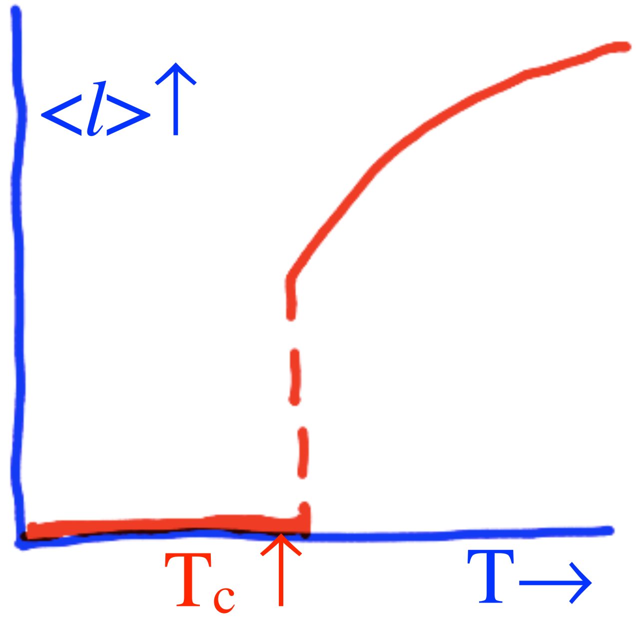
Confinement: $F_{\text{test qk}} = \infty \Rightarrow \langle\, l \,\rangle = 0$

$$< \ell > \sim \, \mathrm{e}^{-F_{\text{test qk}}/T}$$

Above T_c, $F_{\text{test qk}} < \infty \Rightarrow \langle\, l \,\rangle \neq 0$

$\langle\, l \,\rangle$ measures ionization of color:

partial ionization when $0 < \langle\, l \,\rangle < 1$: “semi”-QGP



Svetitsky and Yaffe ’80:

SU(3) 1st order because Z(3) allows *cubic* terms:

$$\mathcal{L}_{\text{eff}} \sim \ell^3 + (\ell^*)^3$$

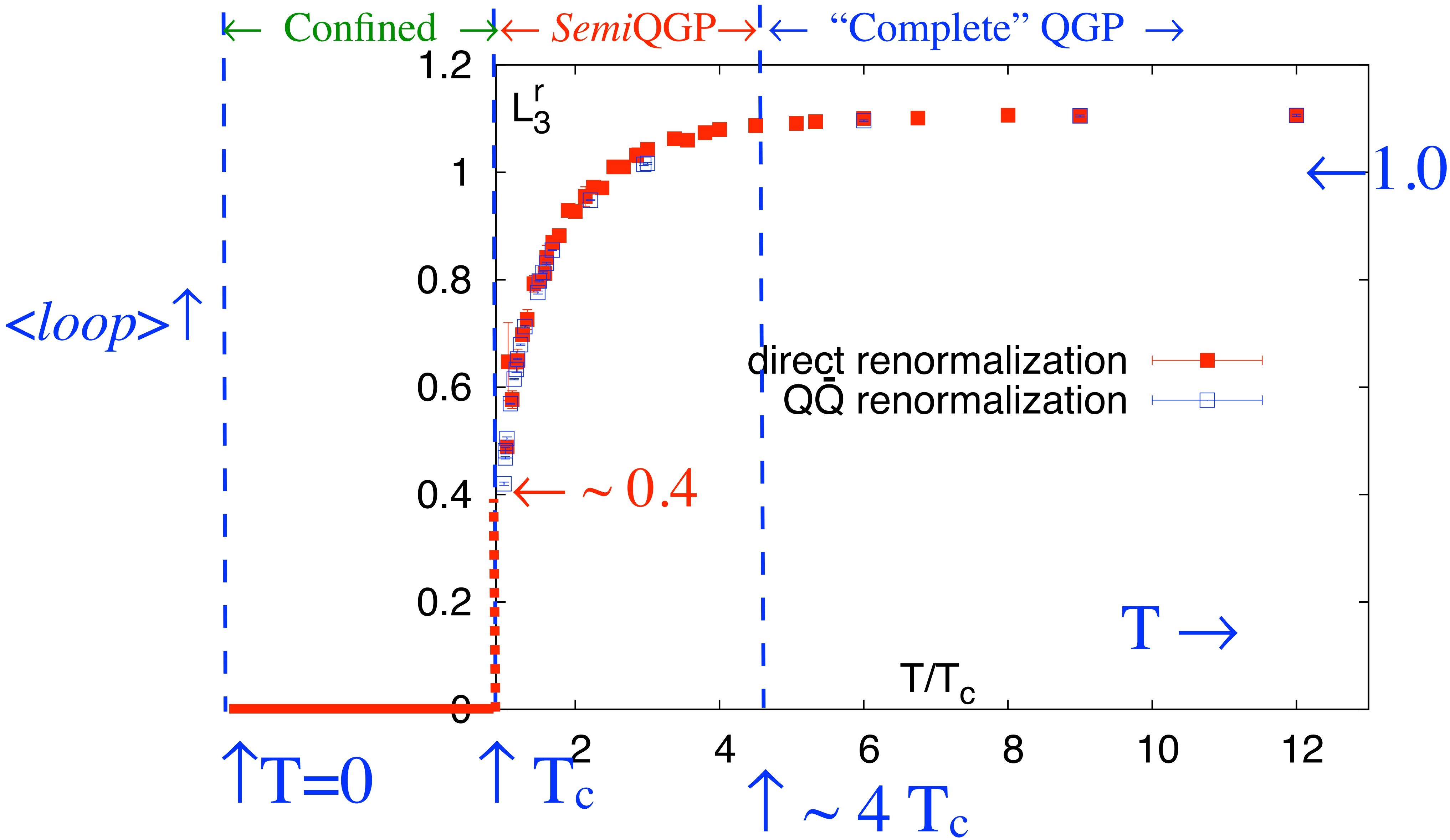
Does *not* apply for N > 3. *So why is deconfinement 1st order for all N ≥ 3?*

Polyakov Loop from Lattice: pure Glue, no Quarks

Lattice: (*renormalized*) Polyakov loop. Strict order parameter

Three colors: Gupta, Hubner, Kaczmarek, 0711.2251.

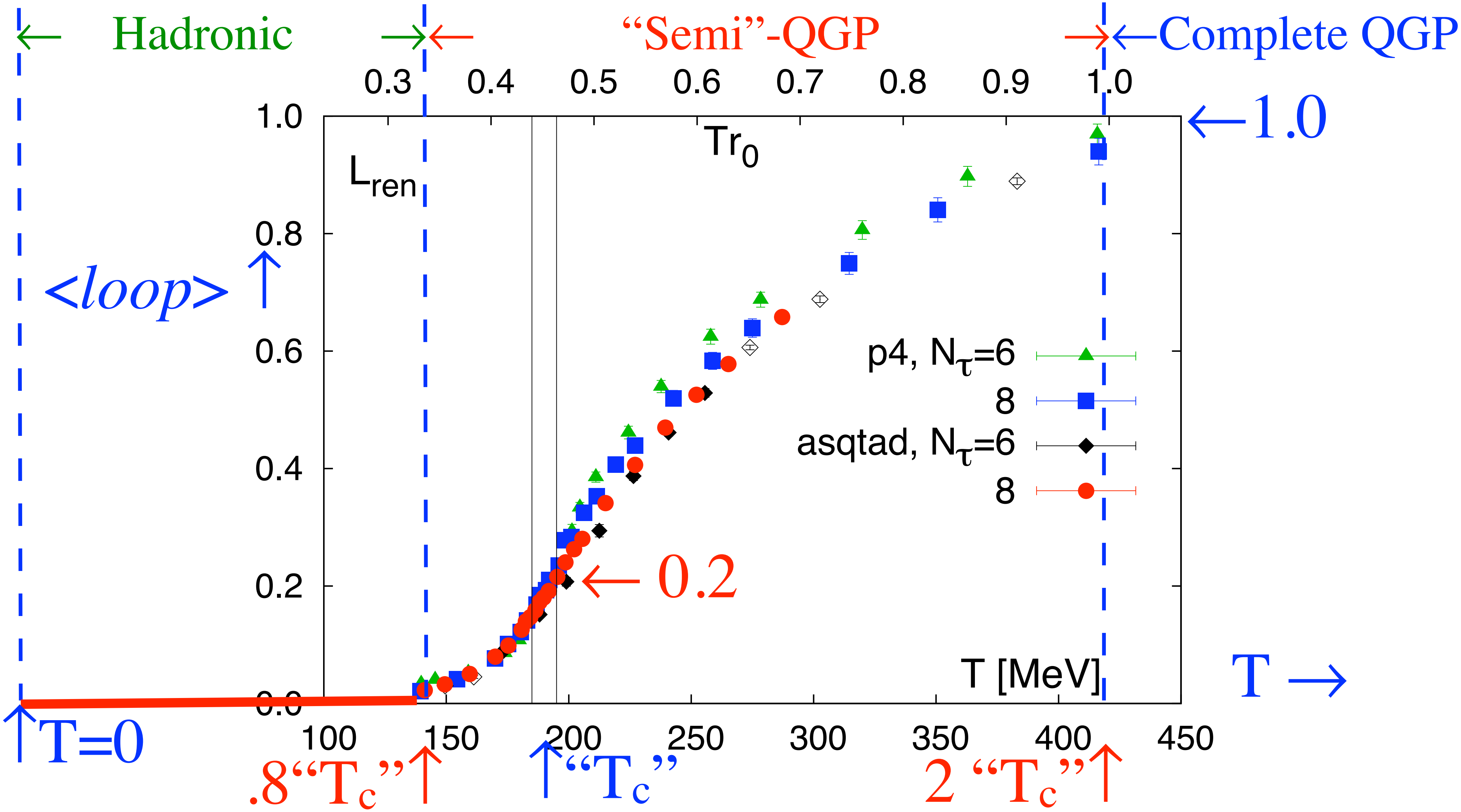
Suggests *wide* transition region, like pressure, to ~ 4 T_c.



Polyakov Loop from Lattice: Glue plus Quarks, “T_c”

Quarks ~ background Z(3) field. Lattice: Bazavov et al, 0903.4379.

3 quark flavors: *weak* Z(3) field, does *not* wash out approximate Z(3) symmetry.



Skipping to the punchline

Transition region *narrow*: for pressure, $< 1.2\ T_c$!

For interface tensions, $< 4\ T_c$...

Above $1.2\ T_c$, pressure dominated by *constant* term $\sim T^2$.

What does this term come from? Gluon mass $m(T)$? But inelegant...

SU(N) in 2+1 dimensions: ideal $\sim T^3$. Caselle + ...: *also* T^2 term in pressure.

But mass would be $m^2\ T$, *not* $m\ T^2$.

T^2 term like free energy of massless fields in 2 dimensions: string? Above T_c?

Need to include quarks!

Can then compute temperature dependence of:

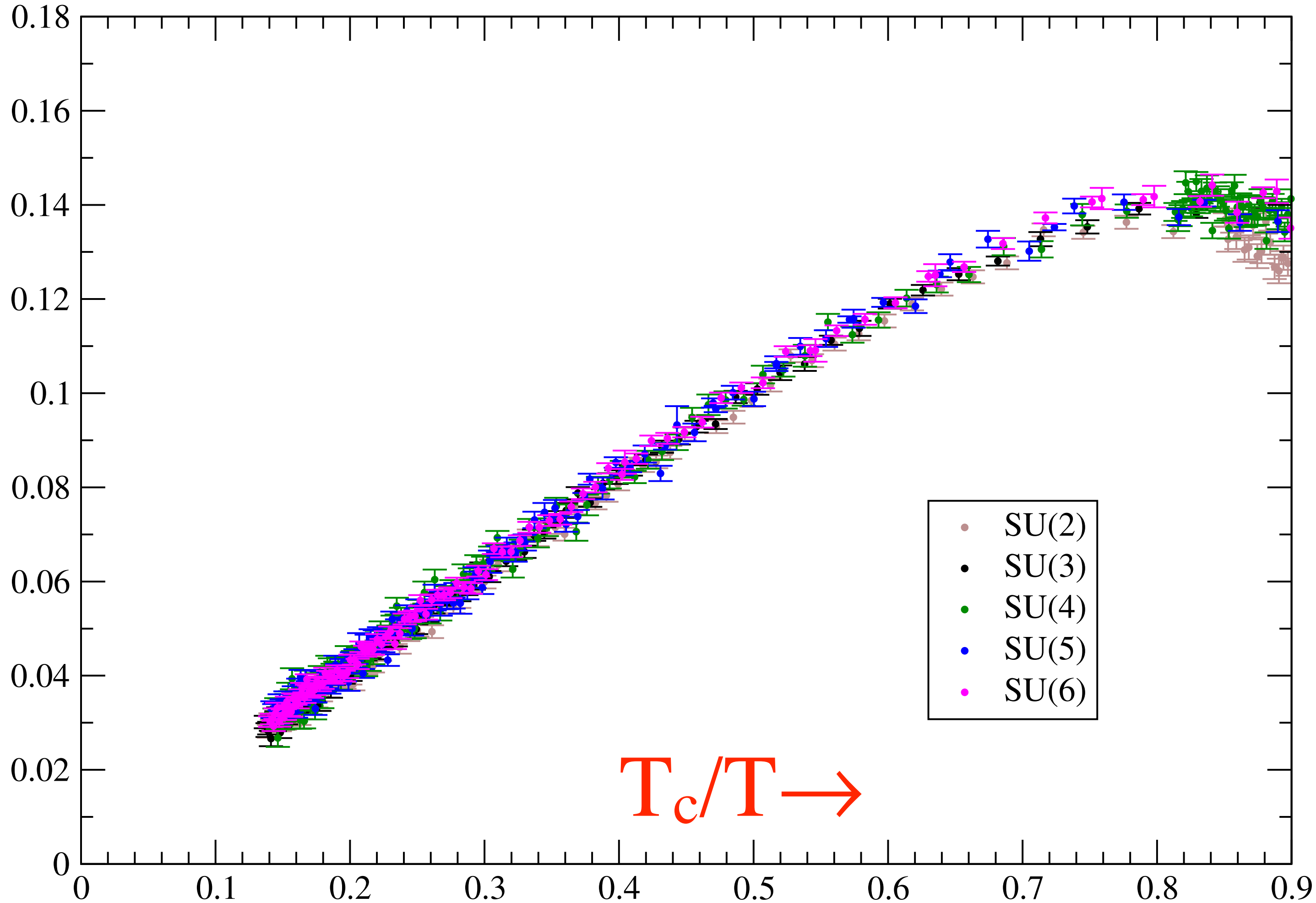
shear viscosity, energy loss of light quarks, damping of quarkonia...

Lattice: SU(N) in 2+1 dimensions

SU(N) in 2+1 dim's for N = 2, 3, 4, 5, & 6. Below plot of T_c/T, not T/T_c.
Clear evidence for non-ideal terms ~ T^2, not ~ T

$$p(T) \approx \# \left(T^3 - c\, T^2\, T_c\right),\ c \approx 1.$$

$$\frac{1}{N^2-1}\,\frac{e-2p}{T^3} \uparrow$$



$T_c/T \rightarrow$

$\uparrow 10\ T_c$ $\uparrow 2T_c$ $\uparrow 1.1T_c$

Caselle, Castagnini, Feo, Gliozzi, Gursoy, Panero, Schafer, 1111.0580.

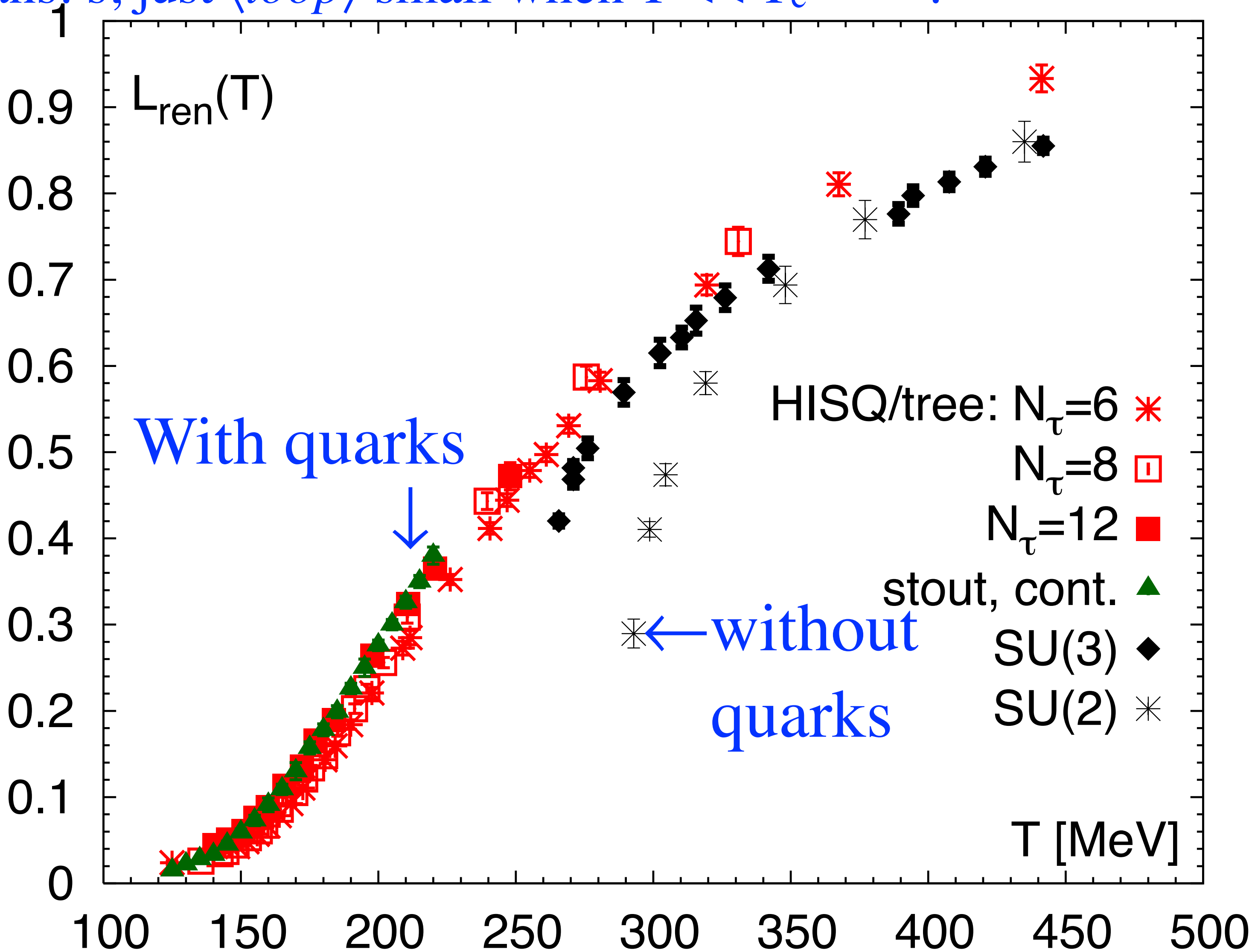
With quarks: "T_c" moves down: *which* T_c?

Just glue: T_c^{deconf} ~ 280 MeV. Standard lore: with quarks, *one* "T_c", decreases.

Matrix model: T_c^{deconf} constant. With *light* quarks T_c^{chiral} < T_c^{deconf}.

Not two trans.'s, just ⟨*loop*⟩ small when T << T_c^{deconf} .

Ren'd loop ↑



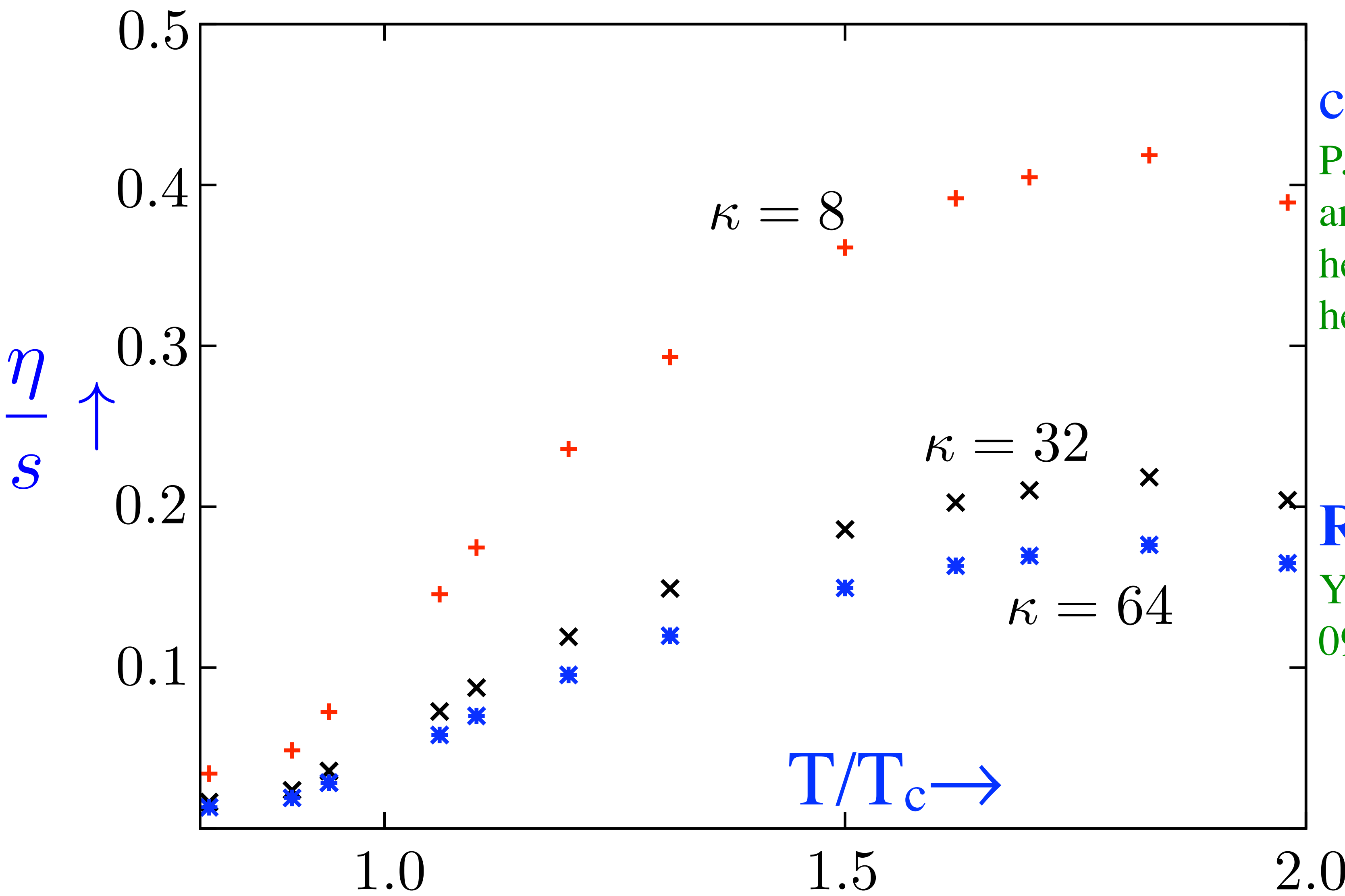
↑ T_c^{chiral} ↑ T_c^{deconf} T→

Lattice: Bazavov and Petreczky + HotQCD, 1110.2160

Shear viscosity *changes* with T

In semi-QGP, η *suppressed* from pert. value
through function **R**(q). Not like kinetic theory
Log sensitivity, through constant ϰ

$$\eta = \frac{c_{\rm pert}\, T^3}{g^4 \, \log(\kappa/g^2 N_c)} \, \mathbf{R}(q)$$



$c_{\rm pert}$:
P. Arnold, G. Moore, and L. Yaffe, hep-ph/0010177, hep-ph/0302165

R(q):
Y. Hidaka & RDP, 0912.0940.

"Bleaching" of color near T_c.

Roughly speaking, as $\langle \text{loop} \rangle \to 0$, *all* colored fields disappear.

Quarks, in fundamental rep. as $\langle \text{loop} \rangle$. Gluons, in adjoint rep., as $\langle \text{loop} \rangle^2$.

Bleaching of color as $T \to T_c$: *robust* consequence of the confinement of color

QGP: quarks *and* gluons. Semi-QGP: dominated by *quarks*, by $\sim \langle \text{loop} \rangle$

Why recombination works at RHIC but not at LHC?

(v_2 /# quarks vs kinetic energy/# quarks)

Suppression of color universal for all fields, *independent* of mass.

Why charm quarks flow the same as light quarks? (single charm vs pions)

An effective theory can provide a bridge from lattice simulations to experiment

Matrix model: two colors

Simple approximation

Two colors: transition 2nd order, vs 1st for $N \geq 3$

Using large N at $N = 2$

Matrix model: SU(2)

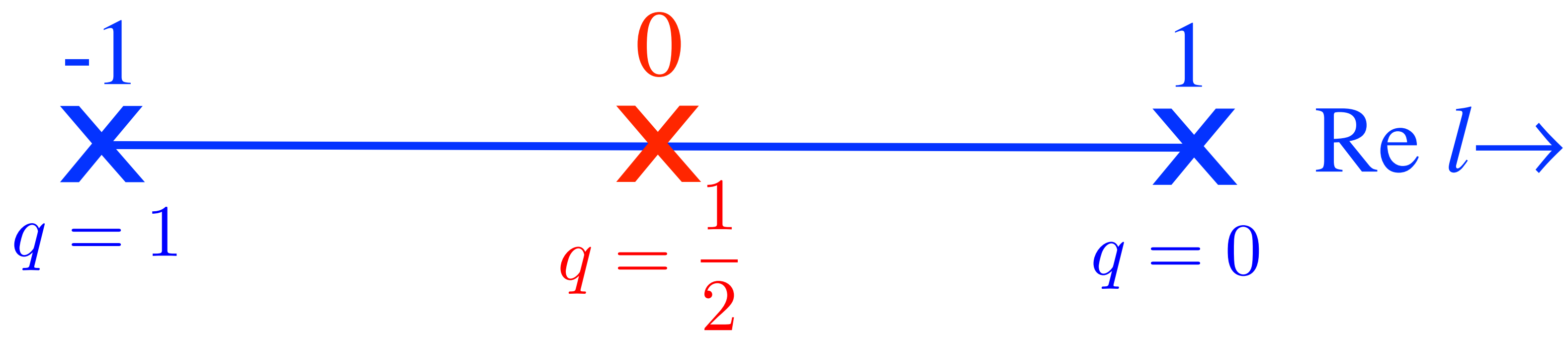
Simple approximation: *constant* $A_0 \sim \sigma_3$, *non*perturbative, $\sim 1/g$:

$$A_0^{cl} = \frac{\pi T}{g}\, q\, \sigma_3 \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \mathbf{L}(q) = \begin{pmatrix} e^{i\pi q} & 0 \\ 0 & e^{-i\pi q} \end{pmatrix}$$

Single dynamical field, q

Loop *l* real. Z(2) degenerate vacua q = 0 and 1:

$$\ell = \cos(\pi q)$$



Point *half*way in between: q = ½ , $l = 0$.

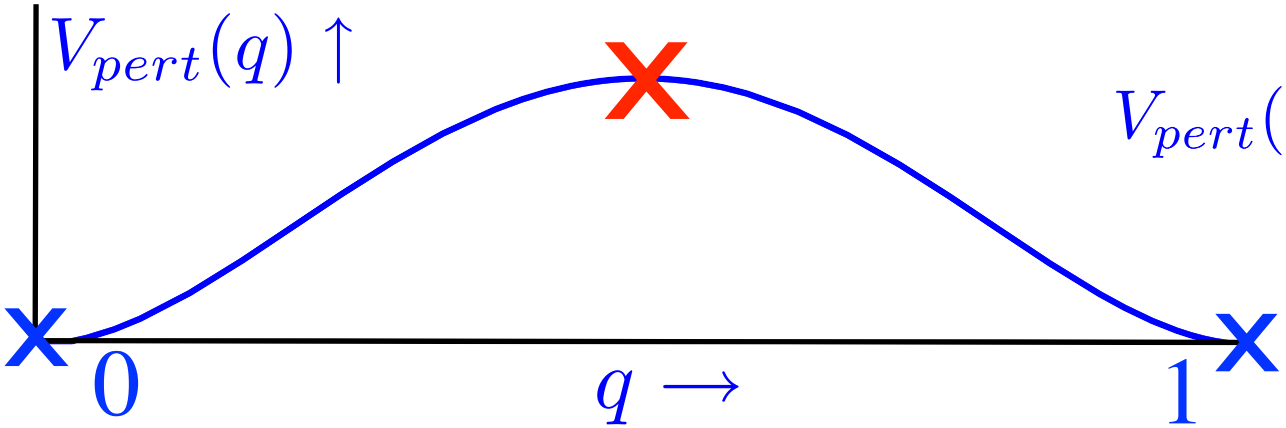
Confined vacuum, $\mathbf{L}_c$,

$$\mathbf{L}_c = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

Classically, A_0^{cl} has zero action: *no* potential for q.

Potential for q, interface tension

Computing to one loop order about A_0^{cl} gives a potential for q: Gross, RDP, Yaffe, '81



$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \, q^2 (1-q)^2$$

Use $V_{pert}(q)$ to compute σ: Bhattacharya, Gocksch, Korthals-Altes, RDP, ph/9205231.

$$V_{tot}(q) = \frac{2\pi^2 T^2}{g^2} \left(\frac{dq}{dz} \right)^2 + V_{pert}(q) \qquad \Rightarrow \sigma = \frac{4\pi^2}{3\sqrt{6}} \frac{T^2}{\sqrt{g^2}}$$

Balancing $S_{cl} \sim 1/g^2$ and $V_{pert} \sim 1$ gives $\sigma \sim 1/\sqrt{g^2}$ (*not* $1/g^2$).

Width interface ~ 1/g, justifies expansion about constant A_0^{cl}. GKA '04: $\sigma \sim \ldots + g^2$

Potentials for the q's

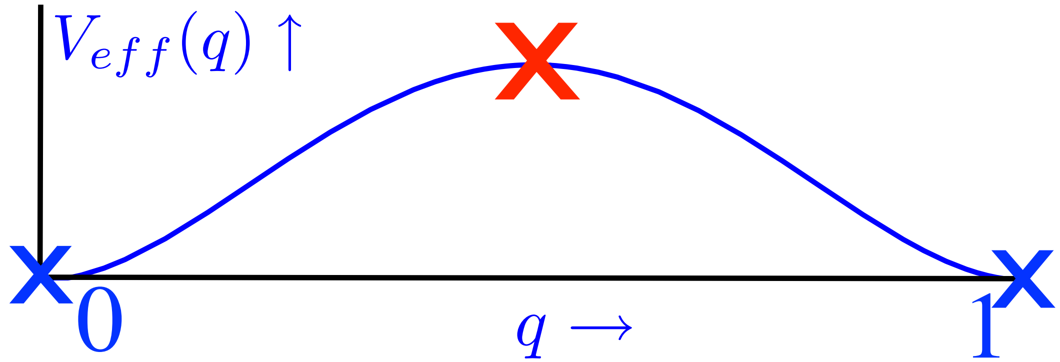
Add *non*-perturbative terms, by *hand*, to generate <q> ≠ 0 :

By hand? $V_{non}(q)$ from: monopoles, vortices...

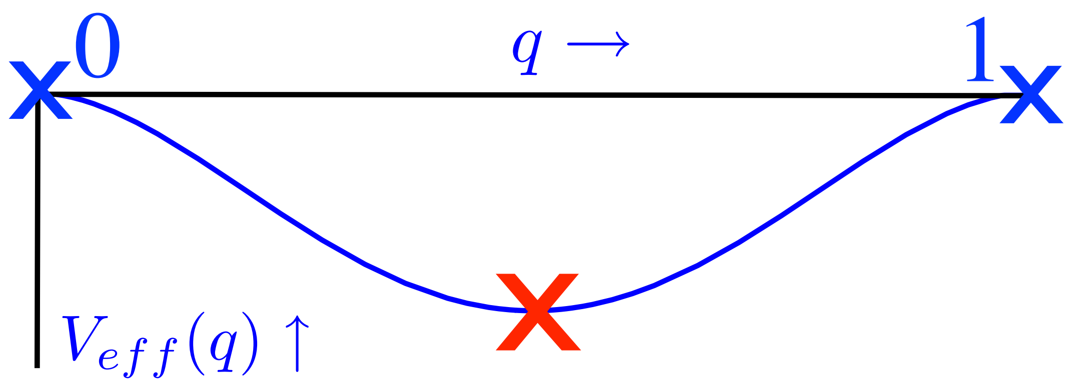
Liao & Shuryak: ph/0611131, 0706.4465, 0804.0255, 0804.4890

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q)$$

$T >> T_c$: $\langle q \rangle = 0,1 \rightarrow$



$T < T_c$: $\langle q \rangle = ½ \rightarrow$



Three possible "phases"

Two phases are familiar:

$\langle q \rangle = 0, 1$: $\langle l \rangle = \pm 1$: "Complete" QGP: usual perturbation theory. $T >> T_c$.

$\langle q \rangle = 1/2$: $\langle l \rangle = 0$: confined phase. $T < T_c$

Also a *third* phase, "partially" deconfined (adjoint Higgs phase)

$0 < \langle q \rangle < 1/2$: $\langle l \rangle < 1$: "semi"-QGP. From some $x\, T_c > T > T_c$ *x?*

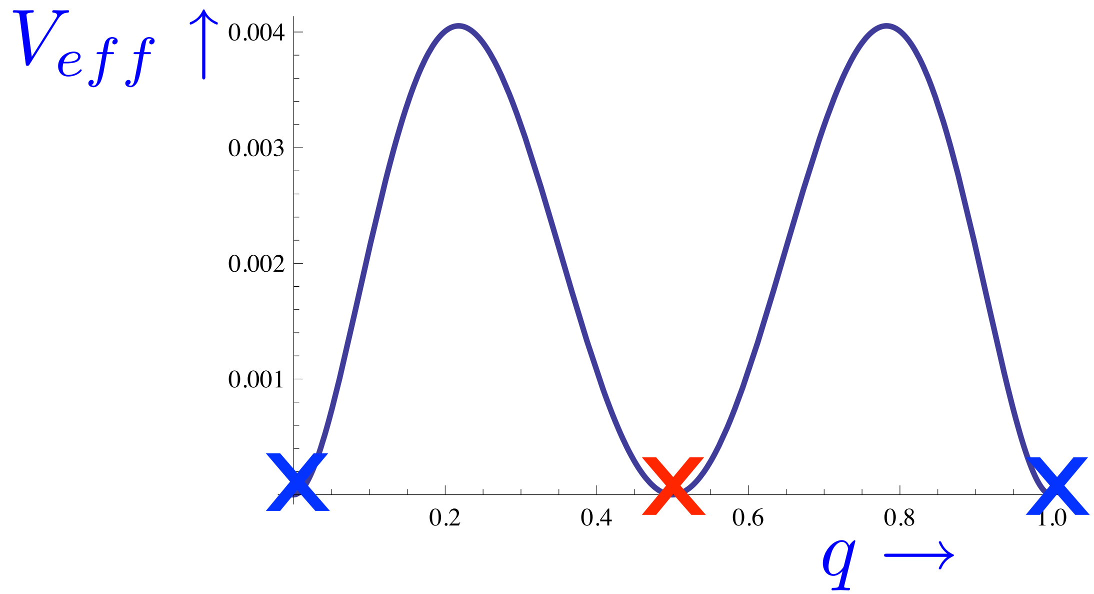
Lattice: *one* transition, to confined phase, at T_c. *No* other transition above T_c.
Still, there is an intermediate phase, the "semi"-QGP

Strongly constrains possible non-perturbative terms, $V_{non}(q)$.

Getting three "phases", one transition

Simple guess: $V_{non} \sim \text{loop}^2$,

$$V_{eff} \sim \frac{a}{\pi^2}(\ell^2 - 1) + q^2(1-q)^2$$
$$\sim q^2(1-a) - 2q^3 + \ldots$$



1st order transition *directly* from complete QGP to confined phase, *not* 2nd

Generic if $V_{non}(q) \sim q^2$ at $q \ll 1$.

Easy to avoid, *if* $V_{non}(q) \sim q$ for small q. Then $\langle q \rangle \neq 0$ at all $T > T_c$.

Imposing the symmetry of $q \leftrightarrow 1 - q$, $V_{non}(q)$ *must* include

$$V_{non}(q) \sim q(1-q)$$

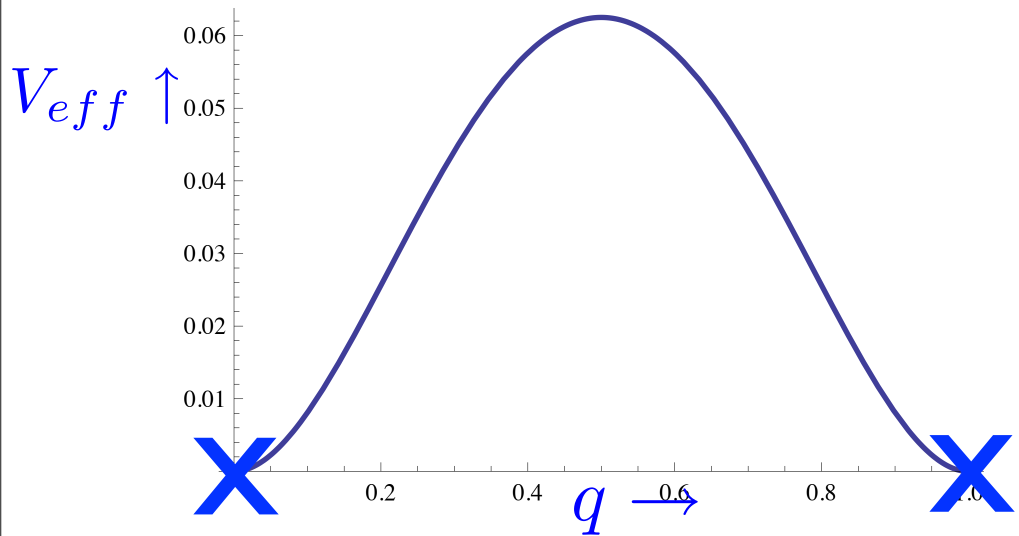
Term ~ q at small q avoids transition from pert. QGP to adjoint Higgs phase

Cartoons of deconfinement

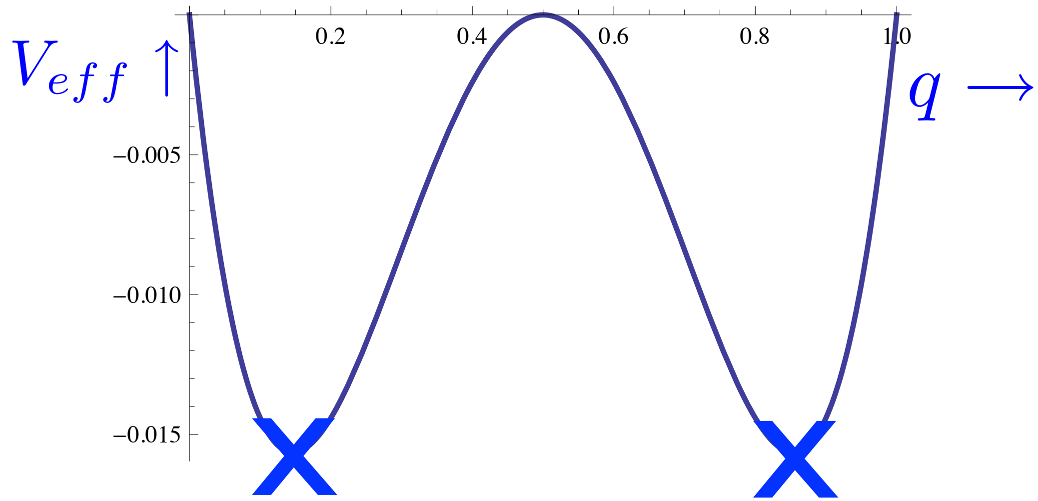
Consider:

$$V_{eff} = q^2(1-q)^2 - \; a\, q(1-q)\,,\; a \sim T_c^2/T^2$$

$\Downarrow$ a = 0: complete QGP



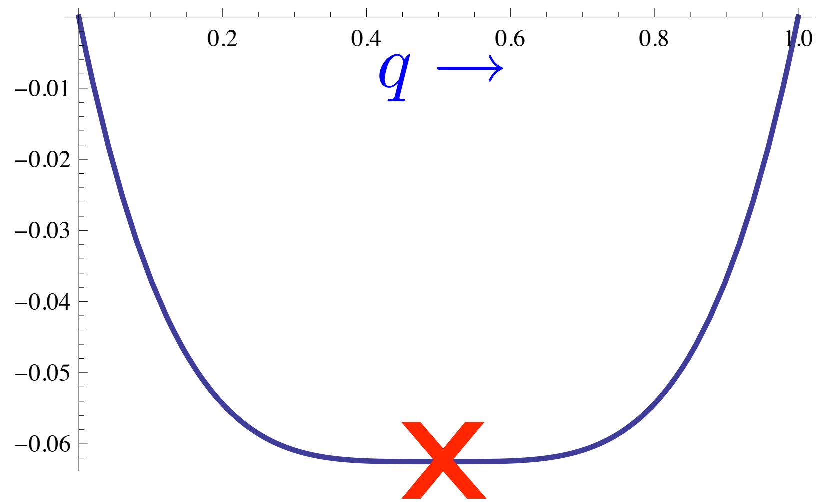
$\Downarrow$ a = ¼: semi QGP



a = ½: T_c=>

Stable vacuum at q = ½

Transition *second* order



0-parameter matrix model, N = 2

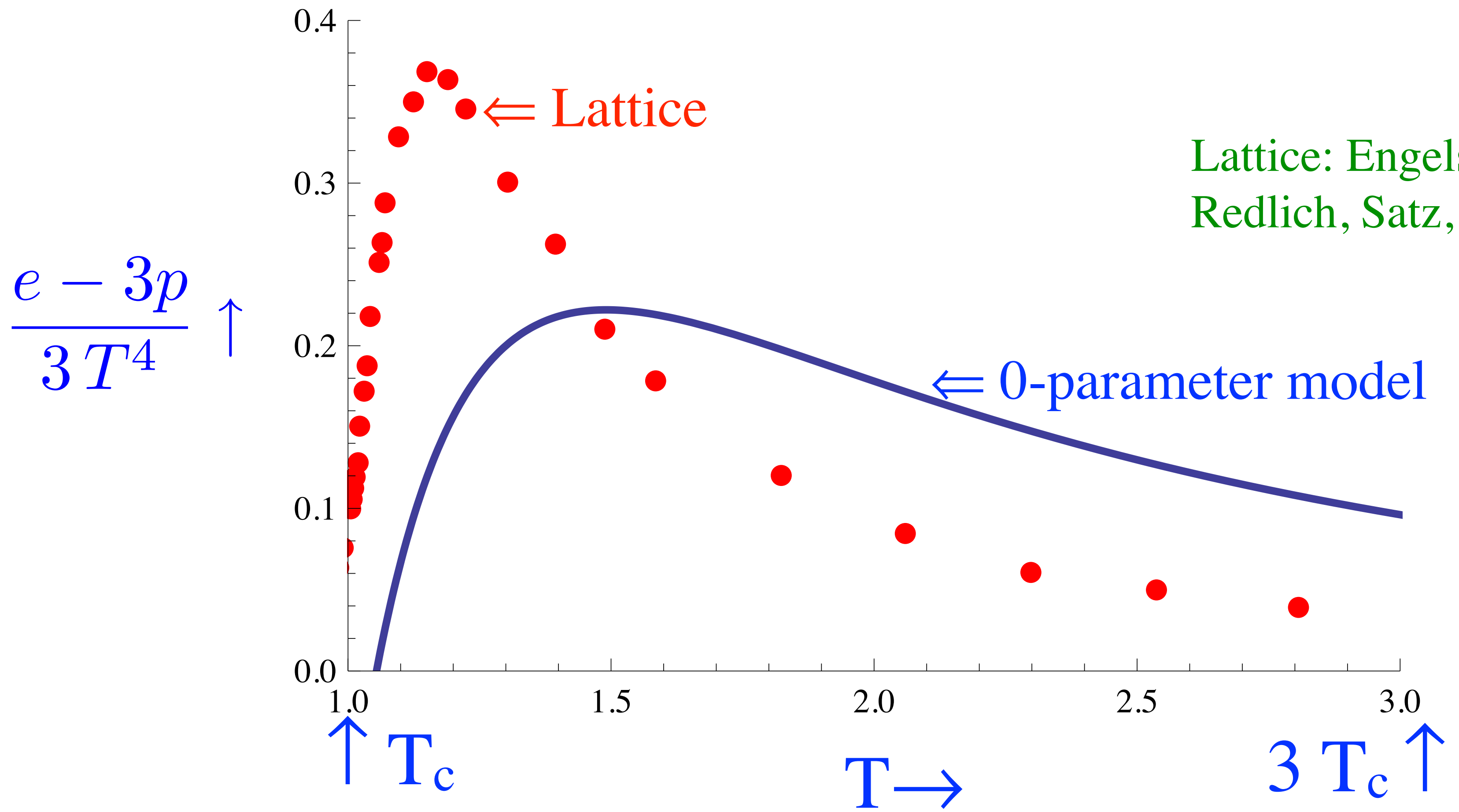
Meisinger, Miller, Ogilvie ph/0108009, MMO:

take $V_{non} \sim T^2$

$$V_{non}(q) = \frac{4\pi^2}{3} T^2 T_c^2 \left(-\frac{c_1}{5} q(1-q) + \frac{c_3}{15} \right)$$

Two conditions: transition occurs at T_c, pressure(T_c) = 0

Fixes c_1 and c_3, *no* free parameters. But *not* close to lattice data *(from '89!)*



1-parameter matrix model, N = 2

Dumitru, Guo, Hidaka, Korthals-Altes, RDP '10: to usual perturbative potential,

$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \left(-\frac{1}{20} + q^2(1-q)^2 \right)$$

Add - *by hand* - a non-pert. potential $V_{non} \sim T^2\, T_c^{\,2}$. *Also* add a term like V_{pert}:

$$V_{non}(q) = \frac{4\pi^2}{3} T^2\, T_c^2 \left(-\frac{c_1}{5}\, q(1-q) - c_2\, q^2(1-q)^2 + \frac{c_3}{15} \right)$$

Now just like any other mean field theory. $\langle q \rangle$ given by minimum of V_{eff}:

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q)$$

$$\left. \frac{d}{dq} V_{eff}(q) \right|_{q = \langle q \rangle} = 0$$

$\langle q \rangle$ depends nontrivially on temperature.

Pressure value of potential at minimum:

$$p(T) = -V_{eff}(\langle q \rangle)$$

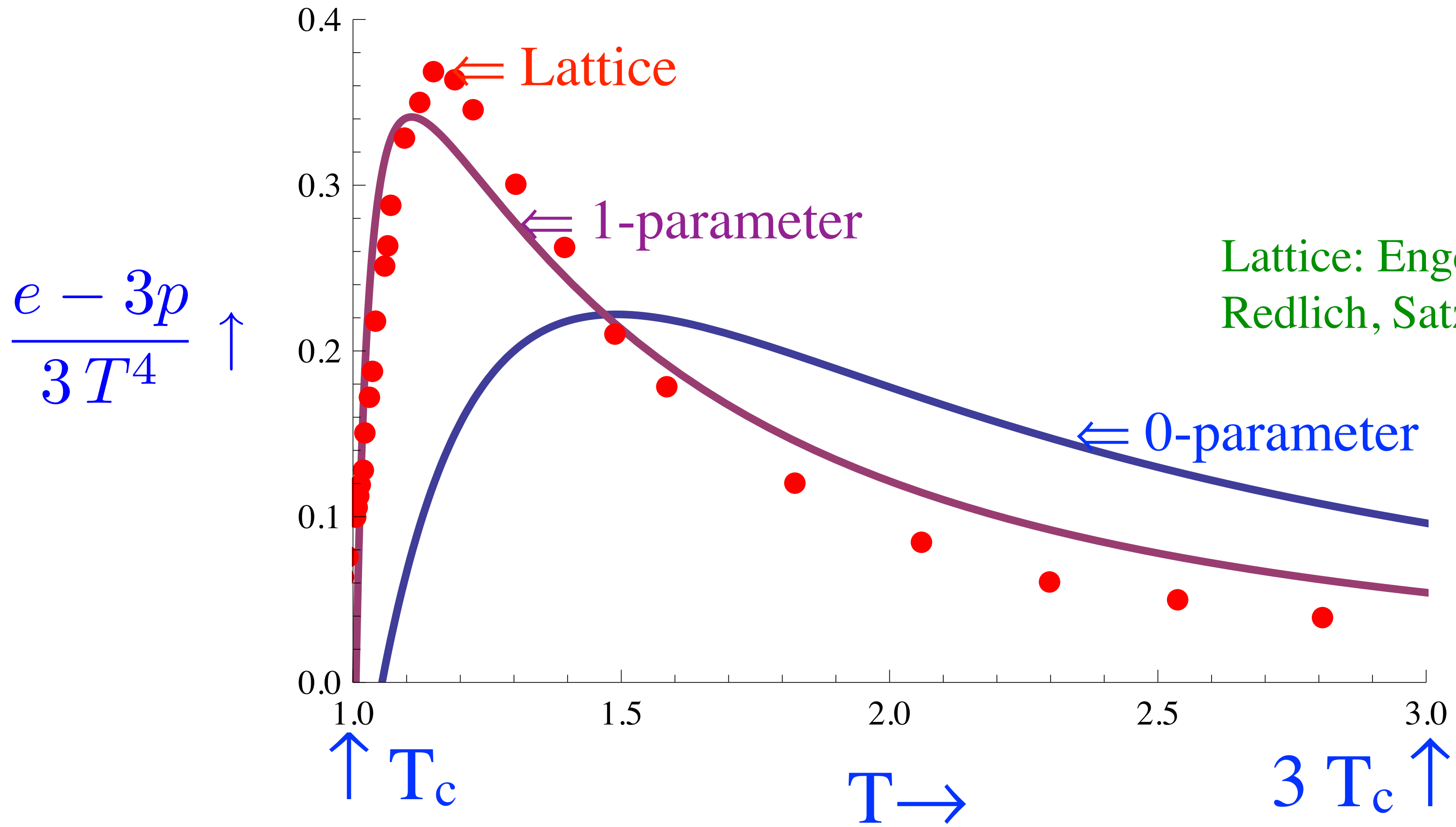
Lattice vs matrix models, N = 2

Choose c_2 to fit e-3p/T^4: optimal choice

$$c_1 = 0.23\,,\; c_2 = .91\,,\; c_3 = 1.11$$

Reasonable fit to e-3p/T^4; also to p/T^4, e/T^4.

N.B.: $c_2 \sim 1$. At T_c, terms $\sim q^2(1-q)^2$ *almost* cancel.



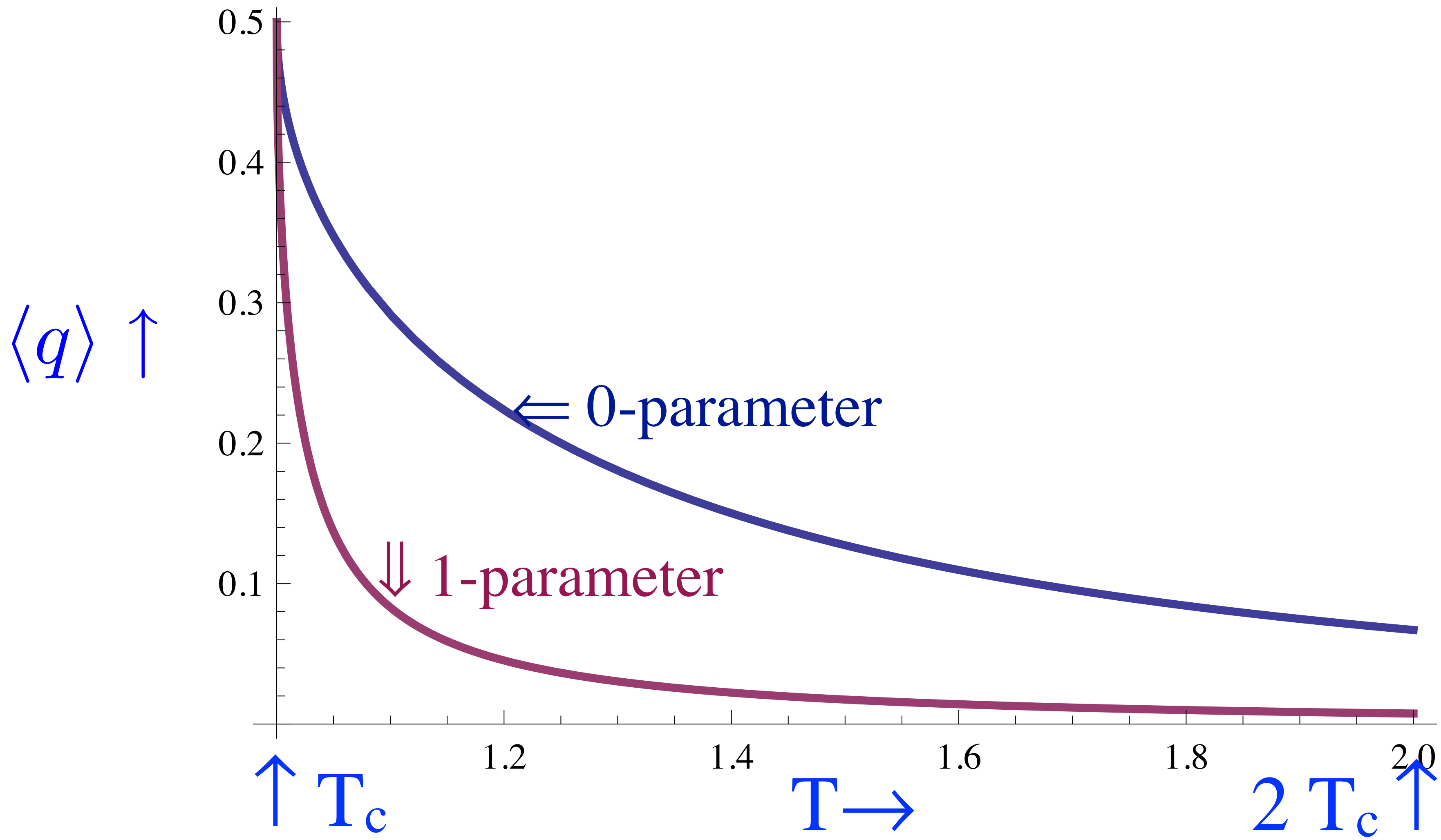
Width of transition region, 0- vs 1-parameter

1-parameter model: get sharper $e-3p/T^4$ because $\langle q \rangle \to 0$ *much* quicker above T_c.

Physically: sharp $e-3p/T^4$ implies region where $\langle q \rangle$ is significant is *narrow*

N.B.: $\langle q \rangle \neq 0$ at all T, but numerically, *negligible* above ~ 1.2 T_c; $p \sim \langle q \rangle^2$.

Above ~1.2 T_c, the T^2 term in the pressure is due *entirely* to the *constant* term, c_3!



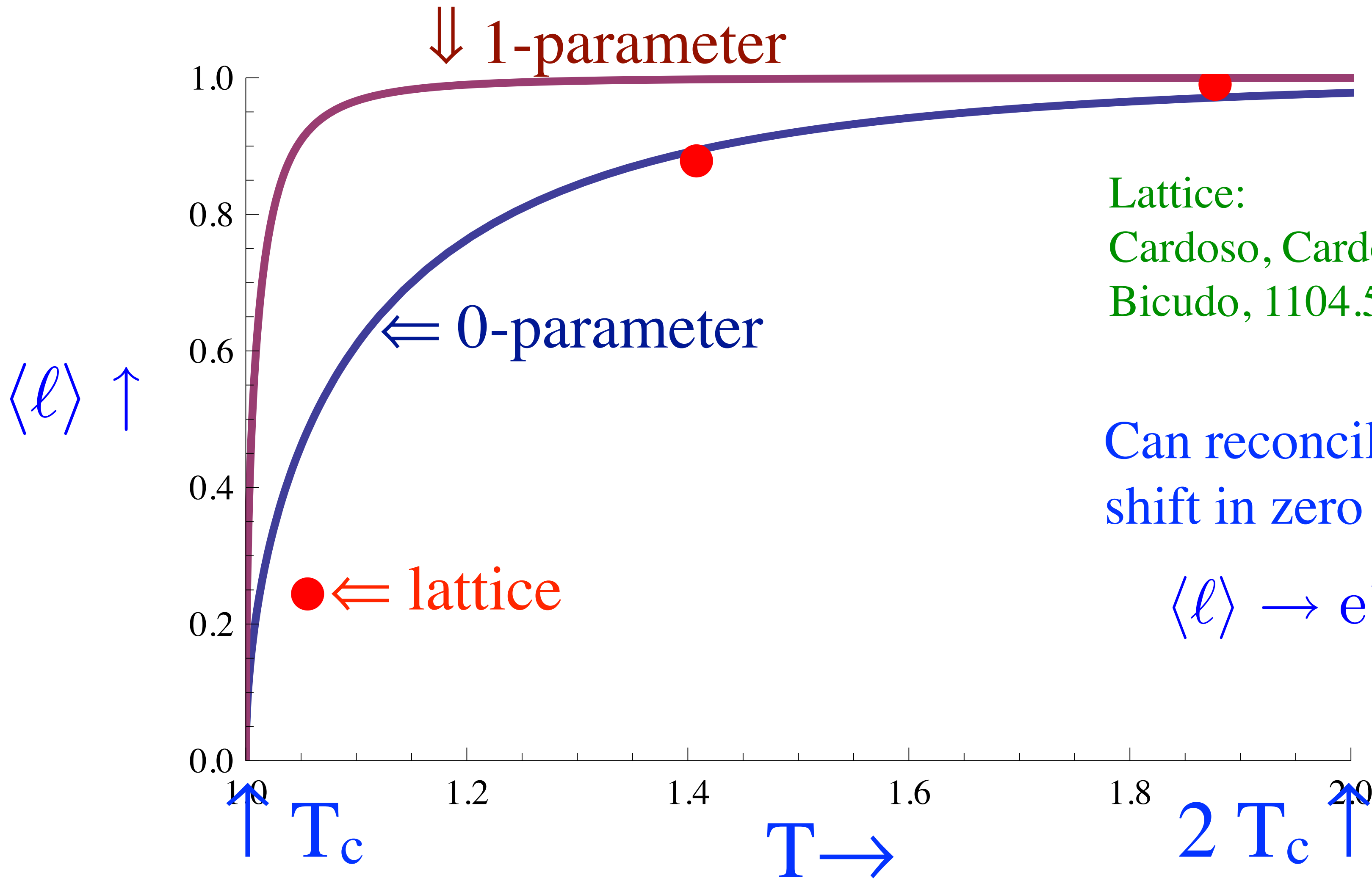
Polyakov loop: 1-parameter matrix model ≠ lattice

Lattice: *renormalized* Polyakov loop. 0-parameter model: close to lattice

1-parameter model: *sharp* disagreement. $\langle l \rangle$ rises to ~ 1 *much* faster - ?

Sharp rise also found using Functional Renormalization Group (FRG):

Braun, Gies, Pawlowski, 0708.2413; Marhauser, Pawlowski, 0812.1144



Lattice:
Cardoso, Cardoso,
Bicudo, 1104.5432

Can reconcile by (arbitrary)
shift in zero point energy

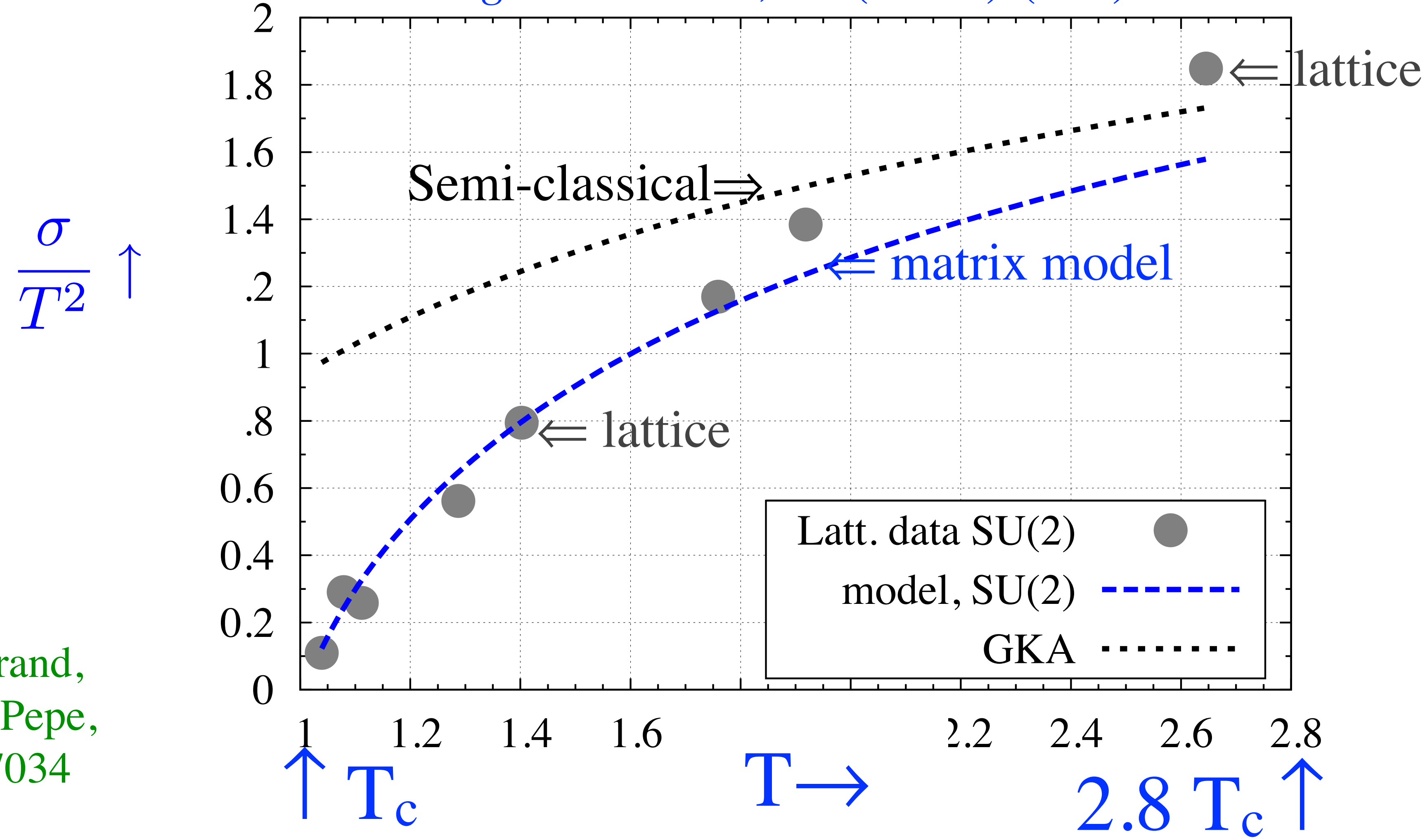
$$\langle \ell \rangle \rightarrow e^{-E_0/T} \langle \ell \rangle$$

Interface tension, N = 2

σ vanishes as $T \to T_c$, $\sigma \sim (t-1)^{2\nu}$.
Ising $2\nu \sim 1.26$; Lattice: ~ 1.32.
Matrix model: ~ 1.5: c_2 important.

$$\sigma(T) = \frac{4\pi^2 T^2}{3\sqrt{6g^2}} \frac{(t^2-1)^{3/2}}{t\,(t^2-c_2)} \, , \, t = \frac{T}{T_c}$$

Semi-class.: GKA '04. *Include* corr.'s $\sim g^2$ in matrix $\sigma(T)$ (ok when $T > 1.2\, T_c$)
N.B.: width of interface *diverges* as $T \to T_c$, $\sim \sqrt{(t^2 - c_2)}/(t^2-1)$.



Lattice:
de Forcrand,
D'Elia, Pepe,
lat/0007034

Lattice: A_0 mass as $T \to T_c$ - *up or down*?

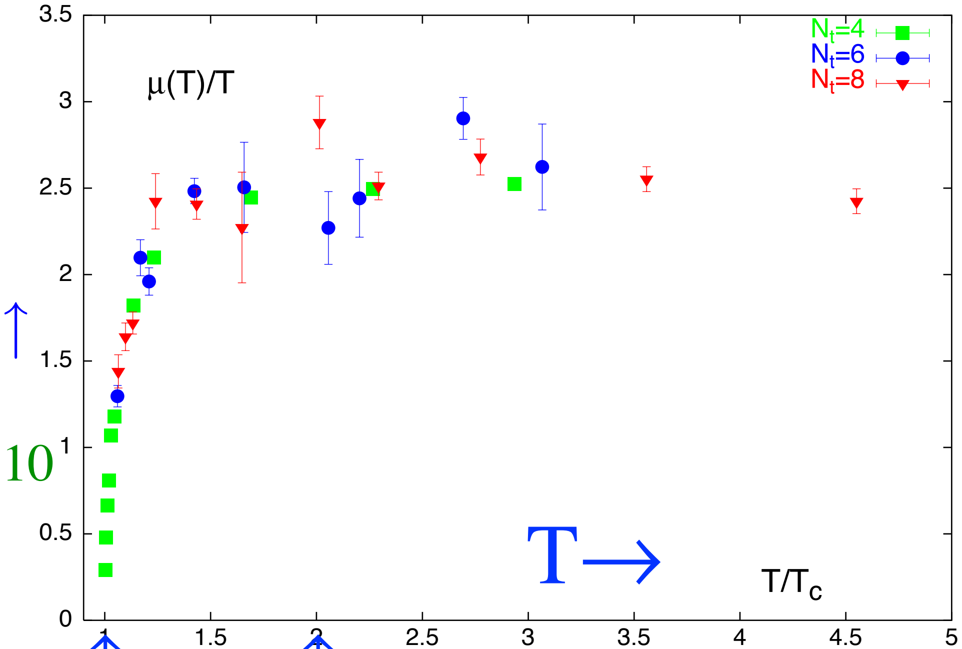
Gauge invariant: 2 pt function of loops:

$$\langle \mathrm{tr}\,\mathbf{L}^\dagger(x)\,\mathrm{tr}\,\mathbf{L}(0)\rangle \sim e^{-\mu x}/x^d$$

μ/T goes *down* as $T \to T_c$

Kaczmarek, Karsch, Laermann, Lutgemeier lat/9908010

$\frac{\mu}{T} \uparrow$



$\uparrow T_c$ $\uparrow 2T_c$

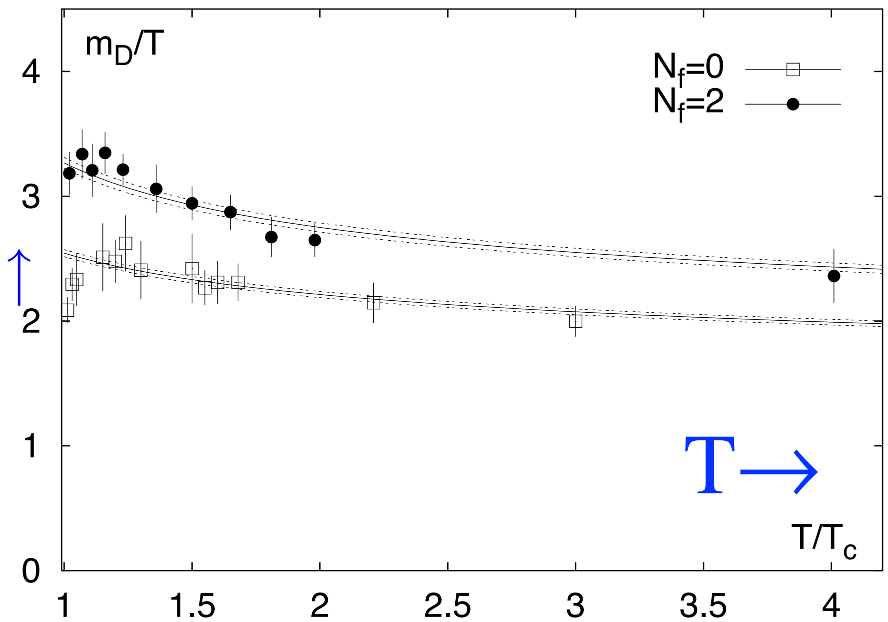
Gauge dependent: singlet potential

$$\langle \mathrm{tr}\,\left(\mathbf{L}^\dagger(x)\,\mathbf{L}(0)\right)\rangle \sim e^{-m_D x}/x$$

m_D/T goes *up* as $T \to T_c$

Cucchieri, Karsch, Petreczky lat/0103009,
Kaczmarek, Zantow lat/0503017
Tuesday: *Tereza Mendes*

$\frac{m_D}{T} \uparrow$



$\uparrow T_c$ $\uparrow 2T_c$

Which way do masses go as $T \to T_c$?
Both are constant above ~ 1.5 T_c.

Adjoint Higgs phase, N = 2

$A_0^{cl} \sim q\, \sigma_3$, so $\langle q \rangle \neq 0$ generates an (adjoint) Higgs phase:

RDP, ph/0608242; Unsal & Yaffe, 0803.0344, Simic & Unsal, 1010.5515

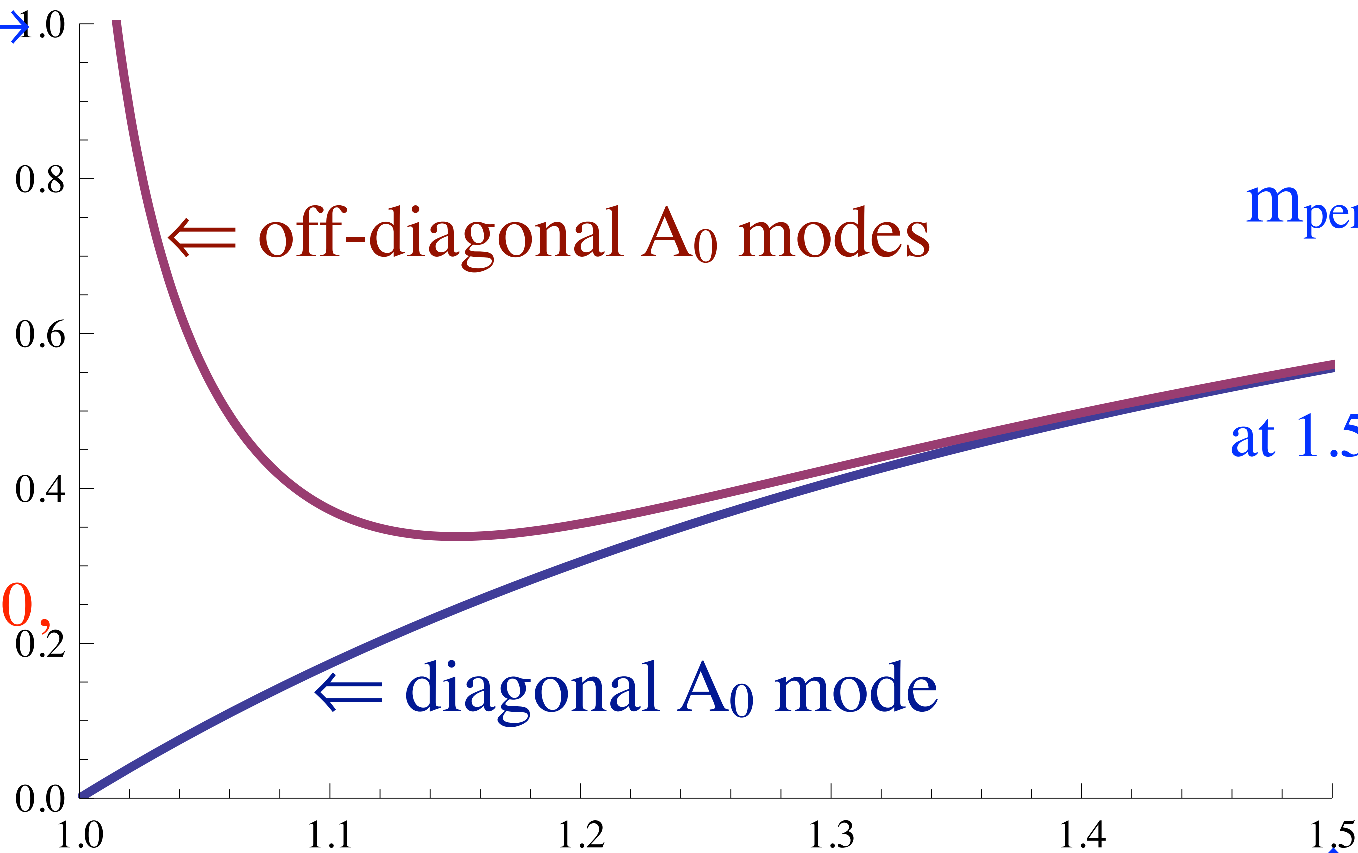
In background field, $A = A_0^{cl} + A^{qu}$: $D_0^{cl} A^{qu} = \partial_0 A^{qu} + i\, g\, [A_0^{cl}, A^{qu}]$

Fluctuations $\sim \sigma_3$ not Higgsed, $\sim \sigma_{1,2}$ Higgsed, get mass $\sim 2\pi T \langle q \rangle$

Hence when $\langle q \rangle \neq 0$, for $T < 1.2\, T_c$, *splitting* of masses:

$1 \rightarrow$

$\dfrac{m_{eff}}{m_{pert}} \uparrow$



$m_{pert} = \sqrt{2/3}\, g\, T$:

$m/m_{pert} \sim .56$ at $1.5\, T_c$, from V_{non}.

At T_c: $m_{diag} = 0$, $m_{off} \sim 2\, m_{pert}$.

$T \rightarrow$

$\uparrow T_c$ $\quad \uparrow 1.2\, T_c$ $\quad 1.5\, T_c \uparrow$

Matrix model: $N \geq 3$

Why the transition is *always* 1st order

One parameter model

Path to Z(3), three colors

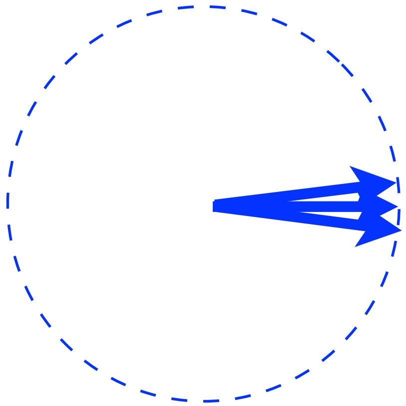
SU(3): *two* diagonal λ's, so *two* q's:

$$A_0 = \frac{2\pi T}{3\,g}\left(q_3\,\lambda_3 + q_8\,\lambda_8\right)$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\;;\;\lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

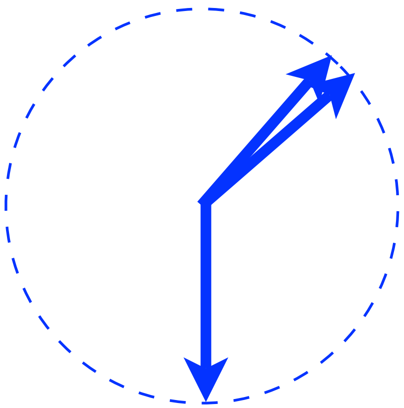
Z(3) paths: move along λ_8, not λ_3: $q_8 \neq 0$, $q_3 = 0$.

$$\mathbf{L} = \mathrm{e}^{2\pi i q_8 \lambda_8/3}$$

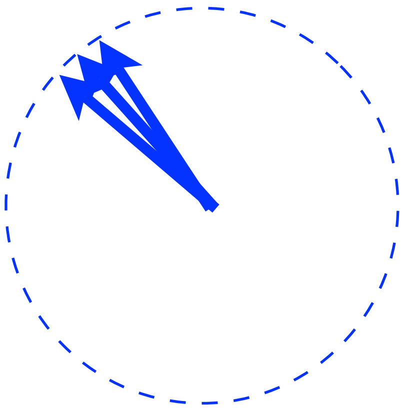


$q_8 = 0$

$\mathbf{L} = \mathbf{1}$



$q_8 = 3/8$



$q_8 = 1$

$\mathbf{L} = \mathrm{e}^{2\pi i/3}\,\mathbf{1}$

Path to confinement, three colors

Now move along λ_3: $\mathbf{L} = \mathrm{e}^{2\pi i q_3 \lambda_3/3}$

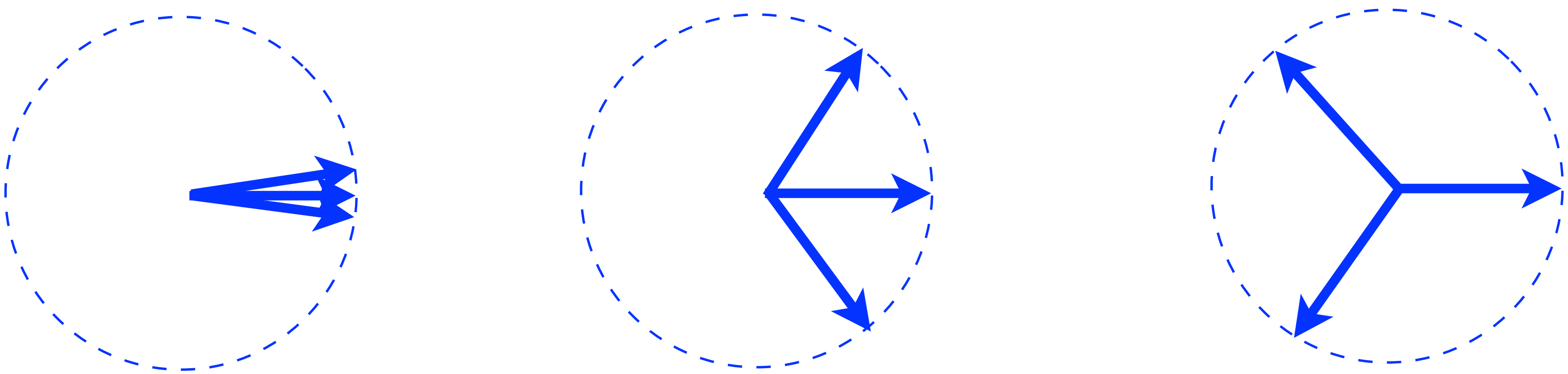
In particular, consider $q_3 = 1$:

Elements of $\mathrm{e}^{2\pi\, i/3}\, \mathbf{L}_c$ *same* as those of $\mathbf{L}_c$.

Hence tr $\mathbf{L}_c$ = tr $\mathbf{L}_c^2 = 0$: $\mathbf{L}_c$ *confining* vacuum

$$\mathbf{L}_c = \begin{pmatrix} \mathrm{e}^{2\pi i/3} & 0 & 0 \\ 0 & \mathrm{e}^{-2\pi i/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Path to confinement: along λ_3, not λ_8, $q_3 \neq 0$, $q_8 = 0$.



$q_3 = 0$ $\quad$ $q_3 = 3/8$ $\quad$ $q_3 = 1$

$\ell = 1$ $\quad$ $\ell \approx .8$ $\quad$ $\ell = 0$

General potential for any SU(N)

Ansatz: constant, diagonal matrix
$i, j = 1 \ldots N$

$$A_0^{ij} = \frac{2\pi T}{g} q_i \, \delta^{ij} \qquad \mathbf{L}_{ij} = \mathrm{e}^{2\pi i \, q_j} \, \delta_{ij}$$

For SU(N), $\Sigma_{j=1 \ldots N} \, q_j = 0$. Hence N-1 independent q_j's, = # diagonal generators.

At 1-loop order, the perturbative potential for the q_j's is

$$V_{pert}(q) = \frac{2\pi^2}{3} T^4 \left(-\frac{4}{15}(N^2 - 1) + \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 \right) \;, \; q_{ij} = |q_i - q_j|$$

As before, *assume* a non-perturbative potential ~ $T^2 \, T_c^2$:

$$V_{non}(q) = \frac{2\pi^2}{3} T^2 T_c^2 \left(-\frac{c_1}{5} \sum_{i,j} q_{ij}(1 - q_{ij}) - c_2 \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 + \frac{4}{15} c_3 \right)$$

Path to confinement, four colors

Move to the confining vacuum along *one* direction, q_j^c:

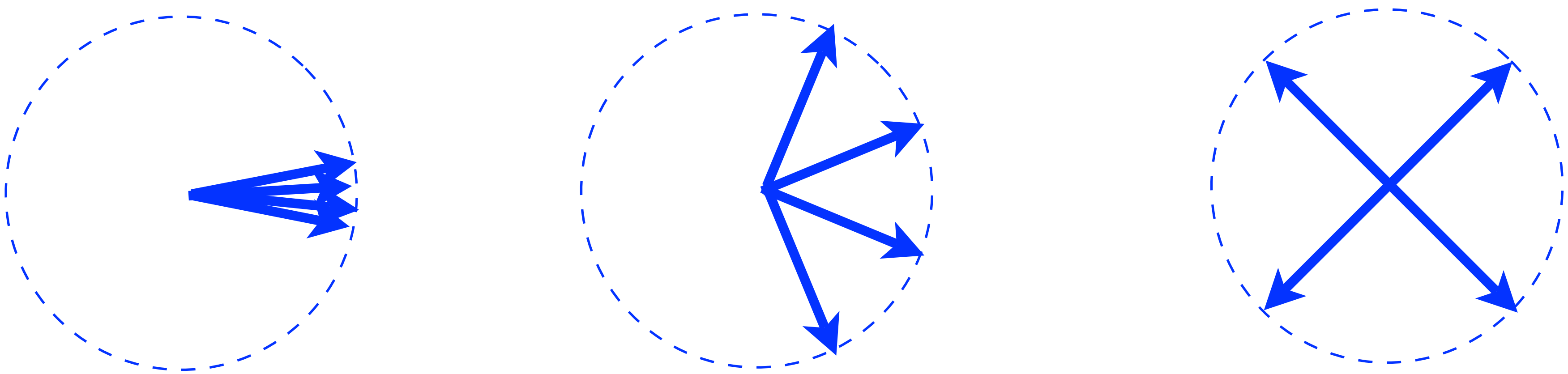
(For general interfaces, need *all* N-1 directions in q_j space)

Perturbative vacuum: q = 0.

Confining vacuum: q = 1.

Four colors:

$$q_j^c = \left(\frac{2j - N - 1}{2N}\right) q \, , \; j = 1 \ldots N$$



$q = 0$ $\quad$ $q = 1/2$ $\quad$ $q = 1$

$\ell = 1$ $\quad$ $\ell \approx .65$ $\quad$ $\ell = 0$

General N: confining vacuum = *uniform* distribution for eigenvalues of **L**

For infinite N, distribution flat.

Cubic term for *all* N ≥ 3, so transition first order

Define $\phi = 1 - q$,
Confining point $\phi = 0$

$$V_{tot} = \frac{\pi^2(N^2-1)}{45}\, T_c^4\, t^2\,(t^2-1)\,\widetilde{V}(\phi,t)\,,\; t = \frac{T}{T_c}$$

$$\widetilde{V}(\phi,t) = -\,m_\phi^2\,\phi^2 - 2\left(\frac{N^2-4}{N^2}\right)\phi^3 + \left(2 - \frac{3}{N^2}\right)\phi^4$$

$$m_\phi^2 = 1 + \frac{6}{N^2} - \frac{c_1}{t^2 - c_2}$$

No term linear in ϕ. Cubic term in ϕ for *all* N ≥ 3.
Along q^c, about $\phi = 0$ there is *no* symmetry of $\phi \to -\phi$ for *any* N ≥ 3.

Hence terms $\sim \phi^3$, and so a first order transition, are *ubiquitous*.
Special to matrix model, with the q_i's elements of Lie *algebra*.

Svetitsky and Yaffe '80: V_{eff}(loop) ⇒ 1st order *only* for N=3; loop in Lie *group*

Also 1st order for N ≥ 3 with FRG: Braun, Eichhorn, Gies, Pawlowski, 1007.2619.

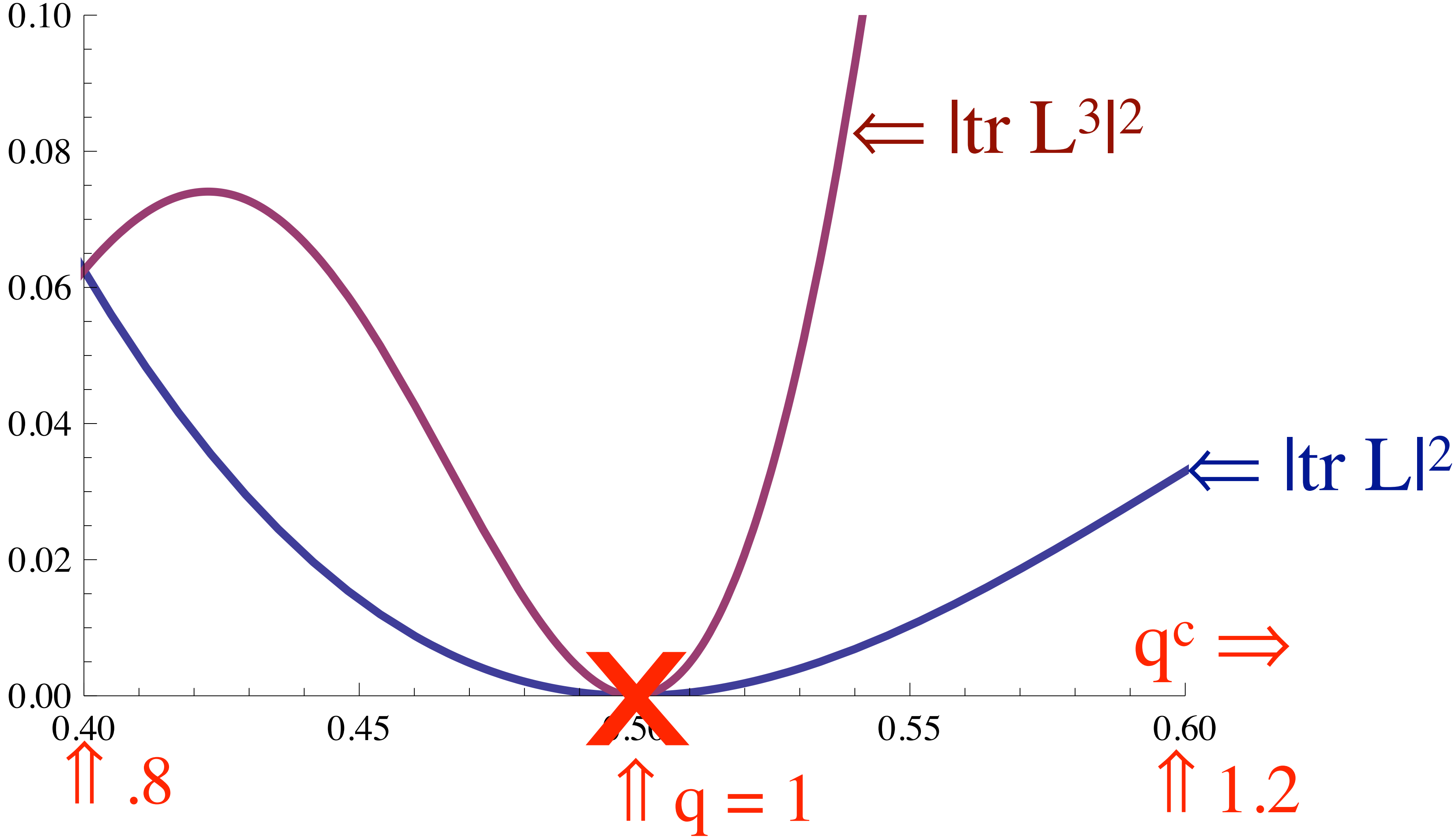
Cubic term for four colors

Construct V_{eff} either from q's, or equivalently, loops: tr $\mathbf{L}$, tr $\mathbf{L}^2$, tr $\mathbf{L}^3$....

N = 4: $|\text{tr}\,\mathbf{L}|^2$ and $|\text{tr}\,\mathbf{L}^3|^2$ *not* symmetric about q = 1, so cubic terms, $\sim (q-1)^3$.

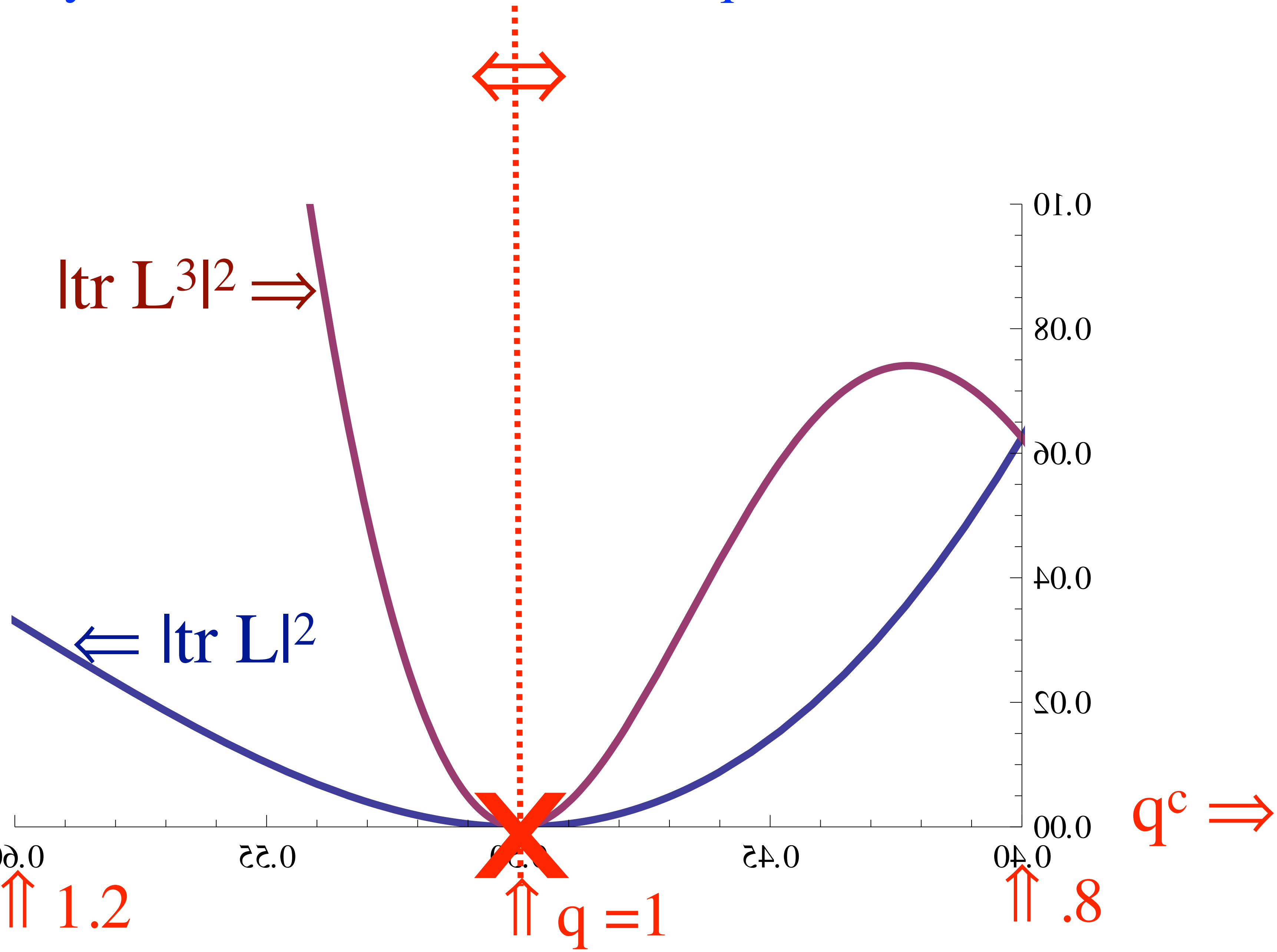
($|\text{tr}\,\mathbf{L}^2|^2$ symmetric, residual Z(2) symmetry)

Cubic terms *special* to moving along q_c in a *matrix* model. *Not* true in loop model



Cubic term for four colors

Asymmetric in reflection about q = 1



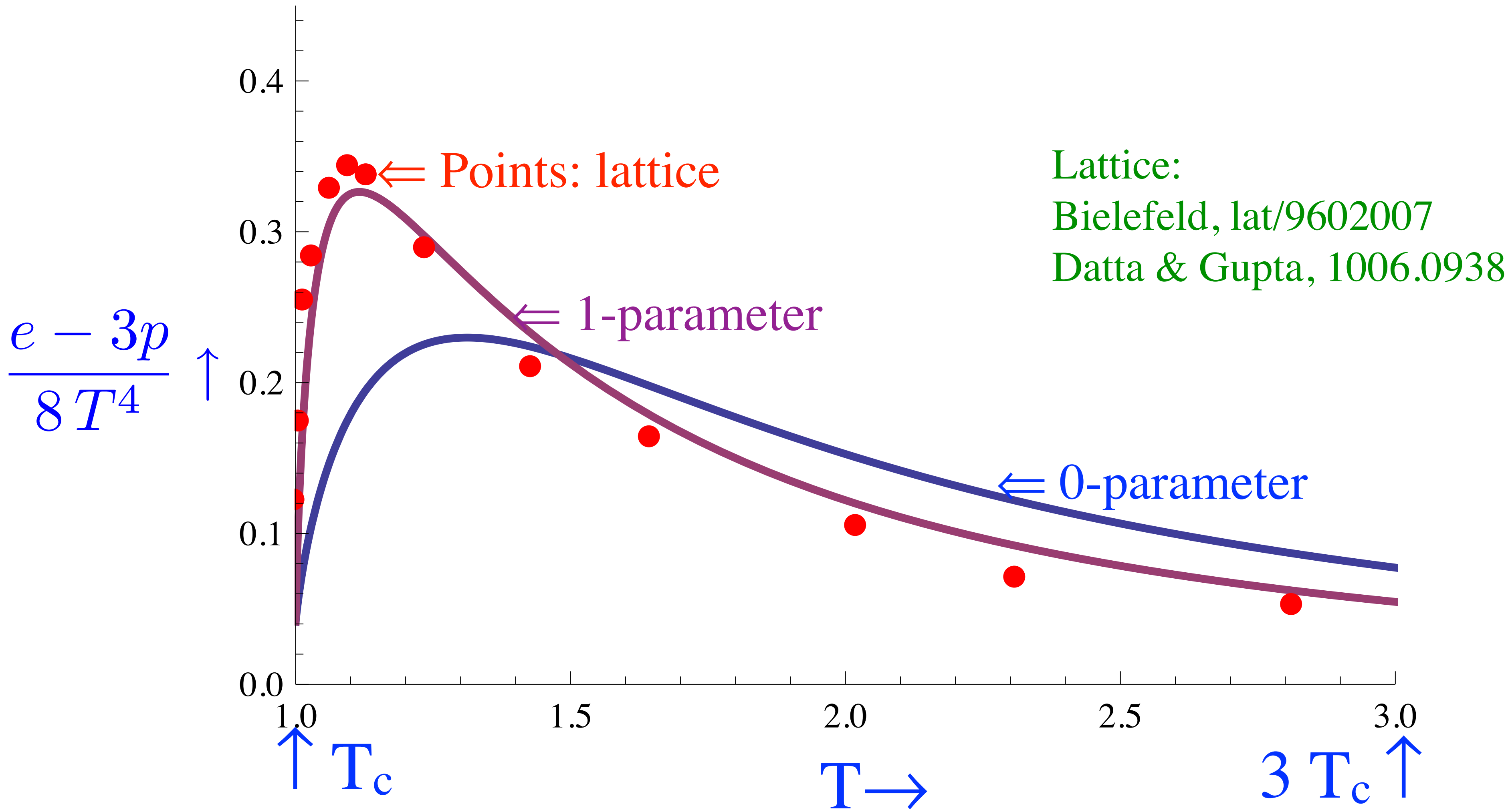
Lattice vs 0- and 1- parameter matrix models, N = 3

Results for N=3 similar to N=2.

0-parameter model way off.

Good fit e-3p/T^4 for 1-parameter model, $c_1 = 0.32\,,\ c_2 = 0.83\,,\ c_3 = 1.13$

Again, $c_2 \sim 1$, so at T_c, terms $\sim q^2(1-q)^2$ *almost* cancel.



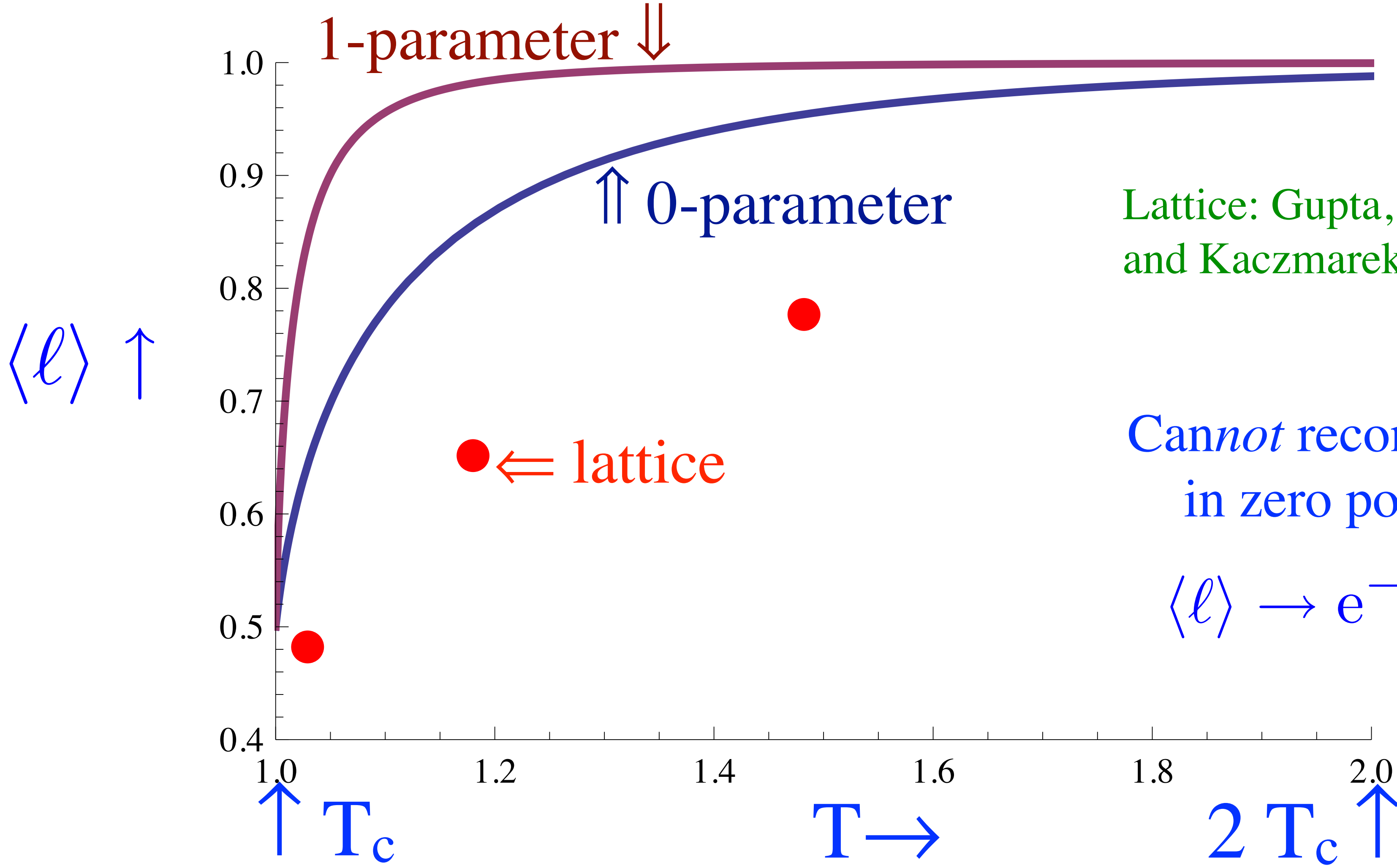
Polyakov loop: matrix models $\neq$ lattice

Renormalized Polyakov loop from lattice does *not* agree with *either* matrix model

$\langle l \rangle - 1 \sim \langle q \rangle^2$: By 1.2 T_c, $\langle q \rangle \sim .05$, negligible.

Again, for $T > 1.2\, T_c$, the T^2 term in pressure due *entirely* to the *constant* term, c_3!

Rapid rise of $\langle l \rangle$ as with FRG: Braun, Gies, Pawlowski, 0708.2413



Lattice: Gupta, Hubner, and Kaczmarek, 0711.2251.

Can*not* reconcile by shift in zero point energy

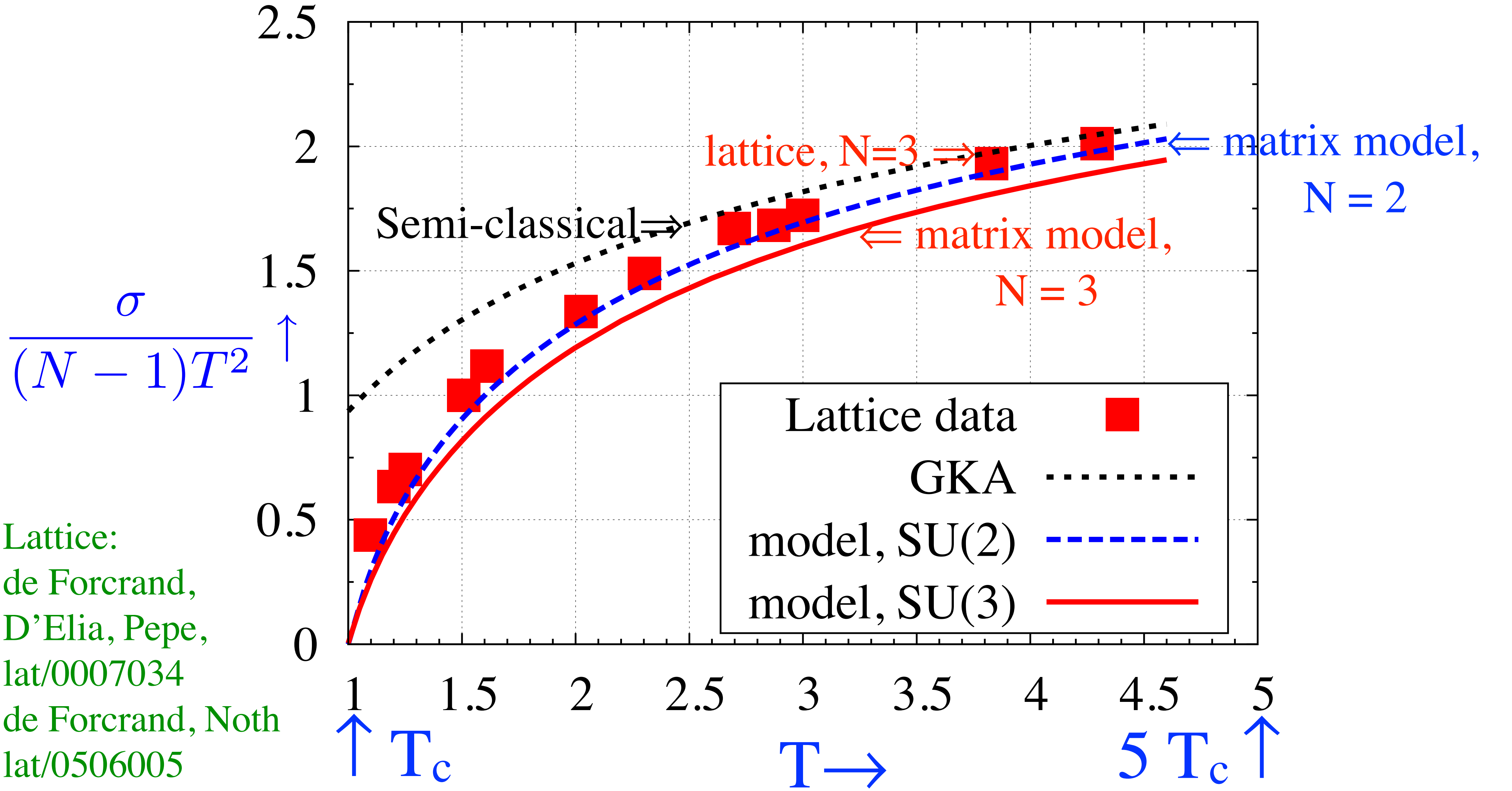
$$\langle \ell \rangle \to e^{-E_0/T} \langle \ell \rangle$$

Interface tension, N = 2 and 3

Order-order interface tension, σ, from matrix model close to lattice.

For T > 1.2 T_c, path along λ_8; for T < 1.2 T_c, along *both* λ_8 and λ_3.

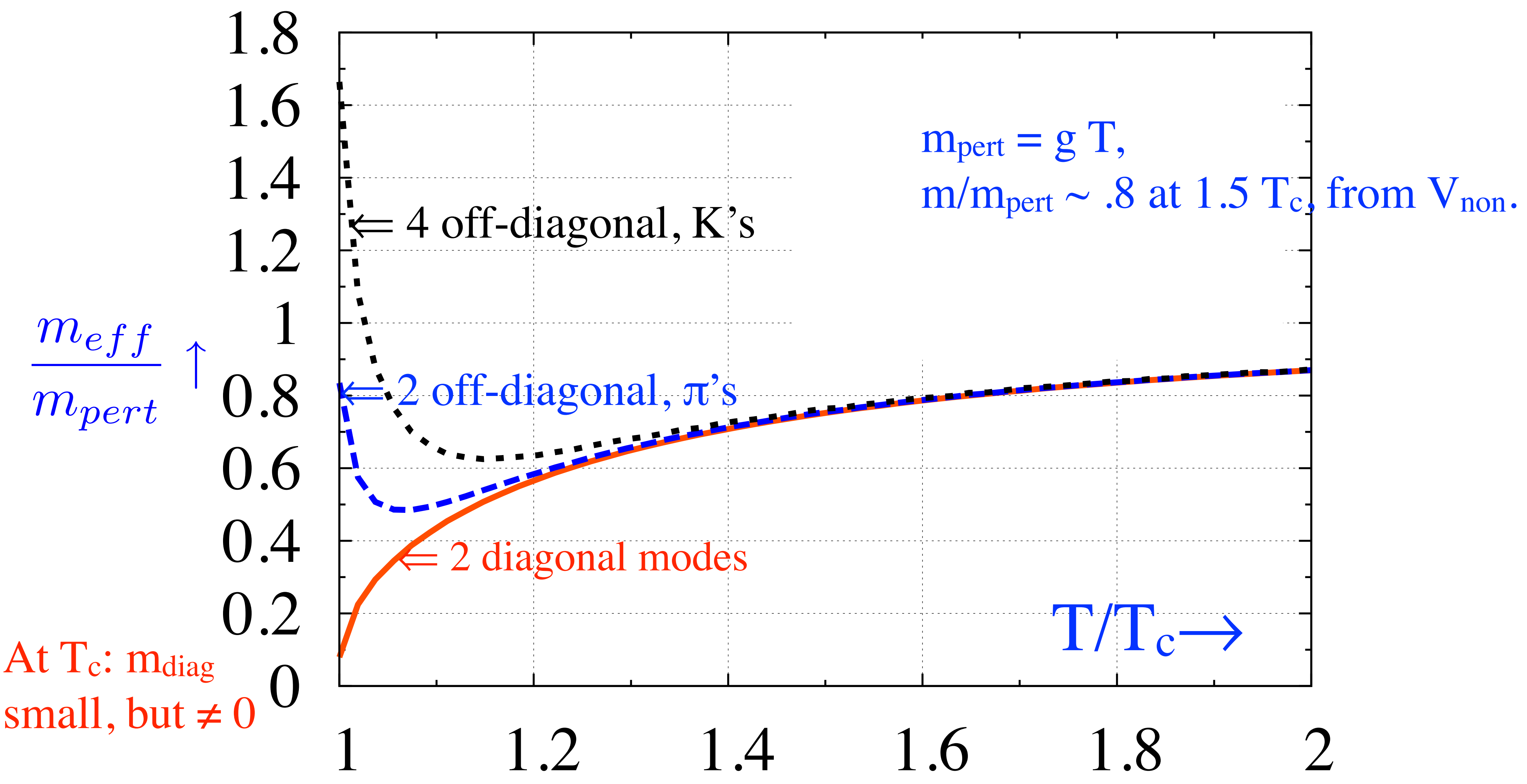
$\sigma(T_c)/T_c^2$ nonzero but *small*, ~ .02. Results for N =2 and N = 3 similar - ?



Lattice:
de Forcrand, D'Elia, Pepe, lat/0007034
de Forcrand, Noth lat/0506005

Adjoint Higgs phase, N = 3

For SU(3), deconfinement along $A_0^{cl} \sim q\, \lambda_3$. Masses $\sim [\lambda_3, \lambda_i]$: two off-diagonal.
Splitting of masses only for $T < 1.2\, T_c$:
Measureable from singlet potential, $\langle \mathrm{tr}\, L^\dagger(x)\, L(0) \rangle$, over *all* x.



Matrix model: $N \geq 3$

To get the latent heat right, two parameter model.

Thermodynamics, interface tensions improve

Latent heat, and a 2-parameter model

Latent heat, $e(T_c)/T_c^4$: 1-parameter model too small:

1-para.: 0.33. BPK: 1.40 ± .1; DG: 1.67 ± .1.

$$c_3(T) = c_3(\infty) + \frac{c_3(1) - c_3(\infty)}{t^2}\,,\; t = \frac{T}{T_c}$$

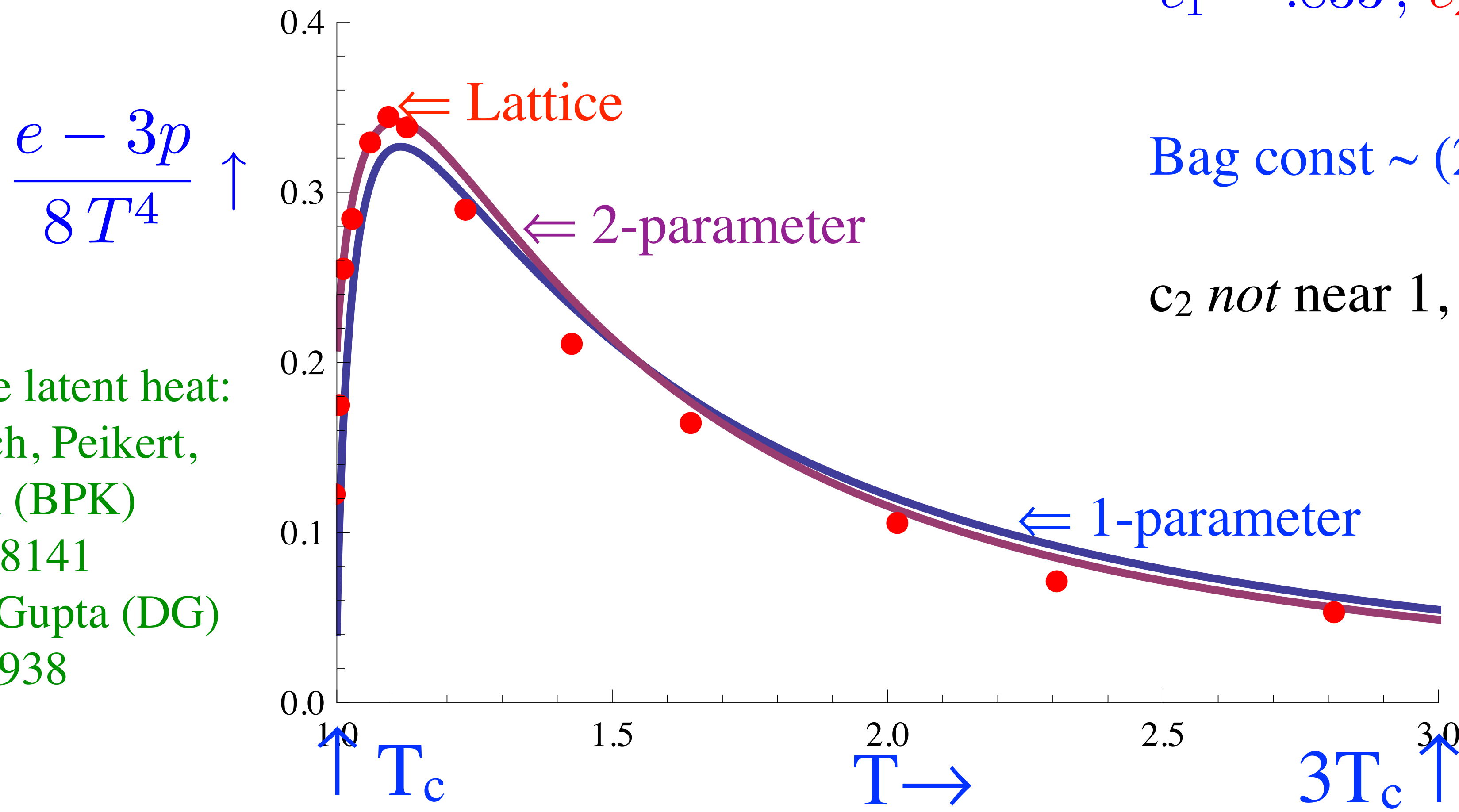
2-parameter model, $c_3(T)$. Like MIT bag constant

WHOT: $c_3(\infty) \sim 1$. *Fit* $c_3(1)$ to DG latent heat

Fits lattice for $T < 1.2\ T_c$, overshoots above.

$$c_3(1) = 1.33\,,\; c_3(\infty) = .95$$

$$c_1 = .833\,,\; c_2 = .552$$



Bag const ~ (262 MeV)^4

c_2 *not* near 1, vs 1-para.

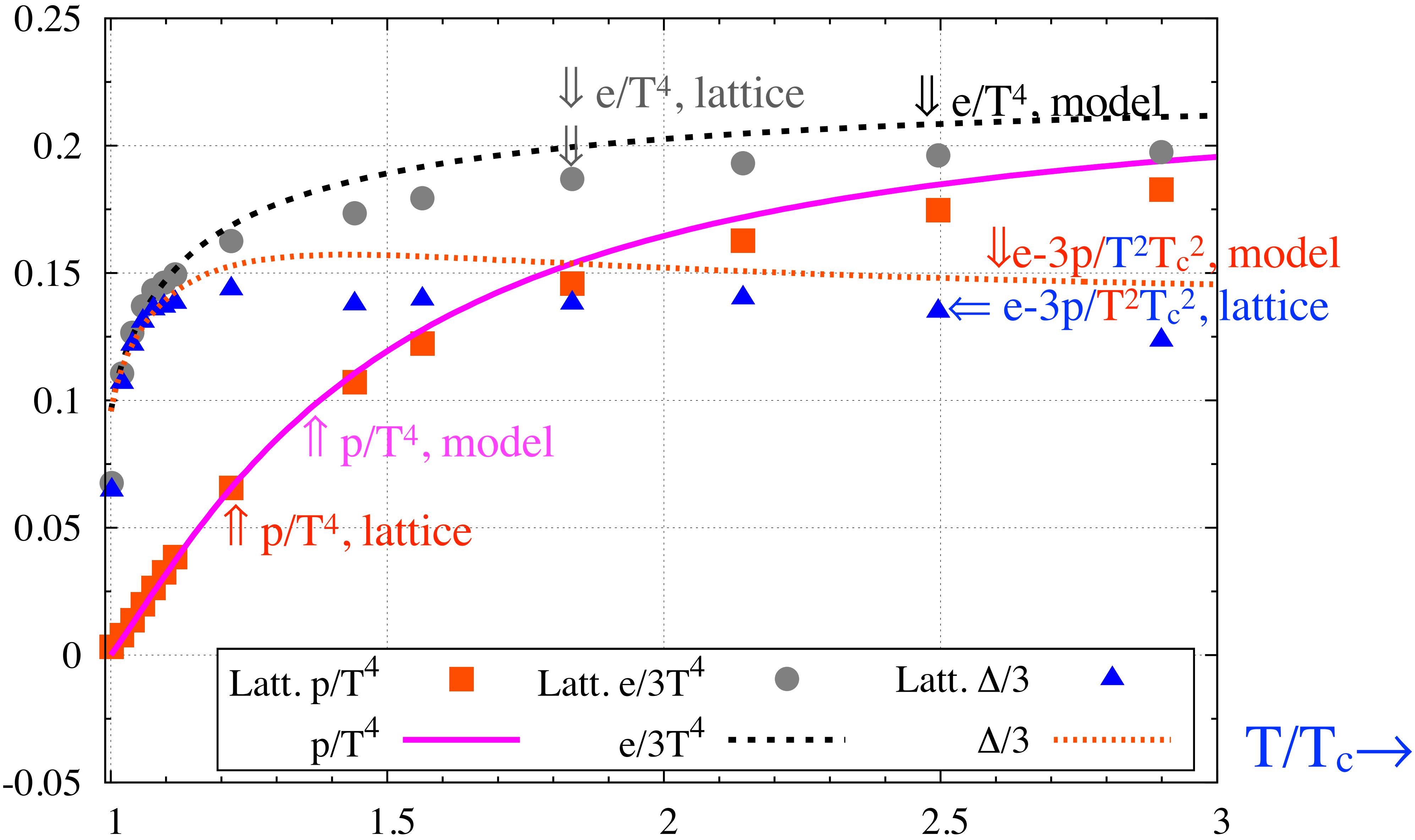
Latttice latent heat:
Beinlich, Peikert, Karsch (BPK)
lat/9608141
Datta, Gupta (DG)
1006.0938

2-parameter model, N = 4

Assume $c_3(\infty) = 0.95$, like N=3. Fit $c_3(1)$ to latent heat, Datta & Gupta, 1006.0938

Order-disorder $\sigma(T_c)/T_c^2 \sim .08$, vs lattice, .12, Lucini, Teper, Wenger, lat/0502003

$$c_3(1) = 1.38\,,\ c_3(\infty) = .95\,,\ c_1 = 1.025\,,\ c_2 = 0.39$$



2-parameter model, N = 6

Order-disorder $\sigma(T_c)/T_c^2 \sim .25$, vs lattice, .39, Lucini, Teper, Wenger, lat/0502003

$$c_3(1) = 1.42\,,\ c_3(\infty) = .95\,,\ c_1 = 1.21\,,\ c_2 = 0.23$$