The spectrum of initial fluctuations and thermalization in the little Bang

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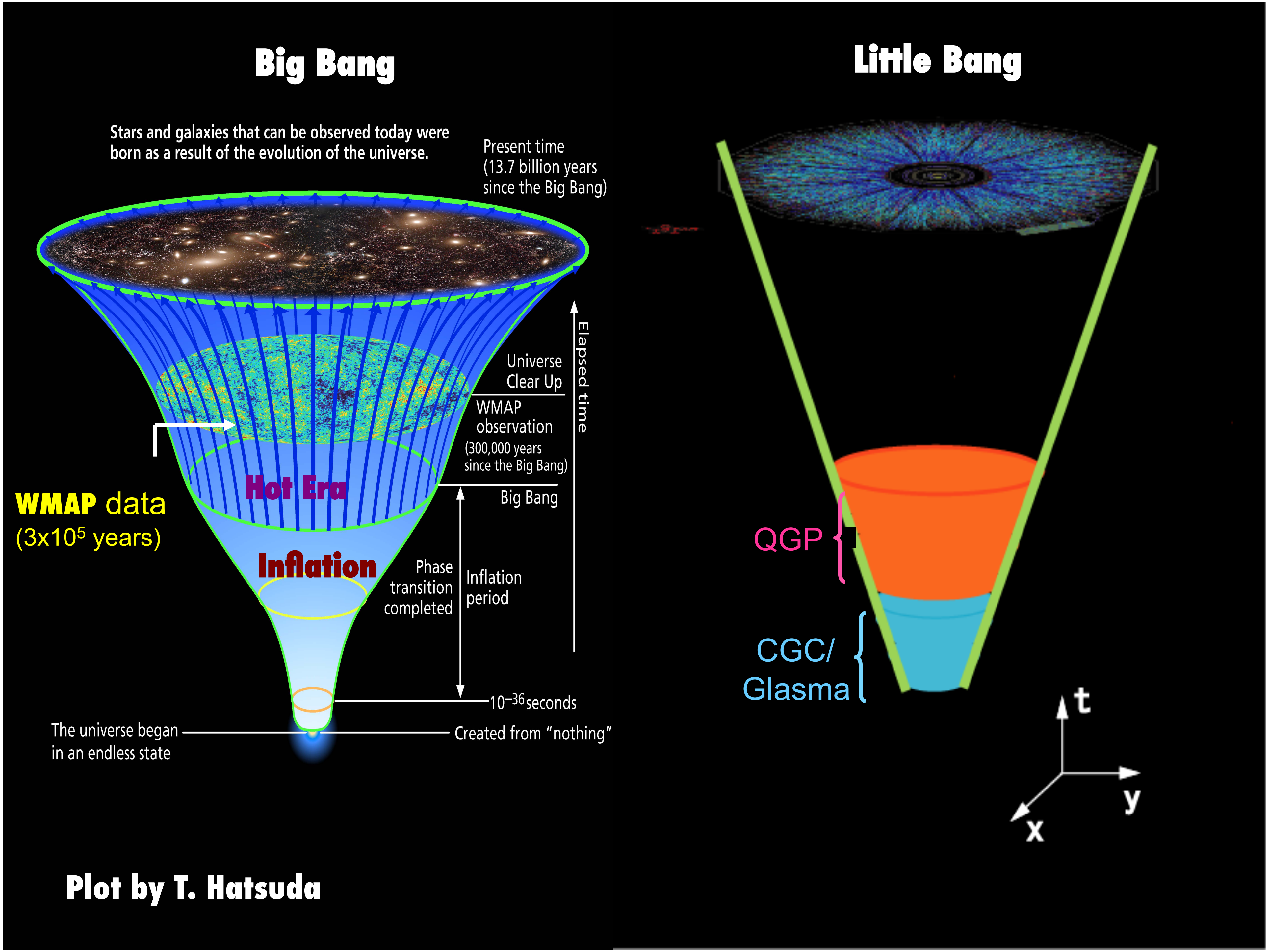
Discovery Center Workshop, Nov. 7-9, 2011

Talk Outline

- Briefly: Strong fields at weak coupling-the classical paradigm

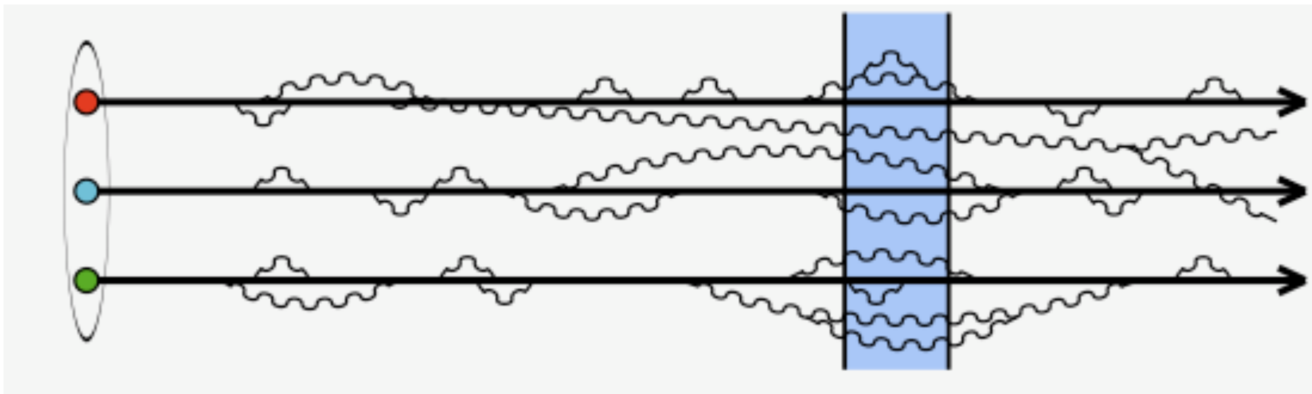
- Quantum effects: factorization, evolution, decoherence, isotropization…

- The problem of thermalization: a solution?

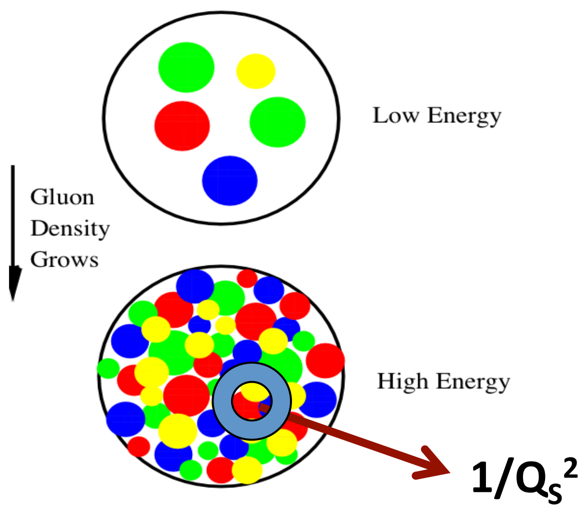


Big Bang
Little Bang
Stars and galaxies that can be observed today were born as a result of the evolution of the universe.
Present time (13.7 billion years since the Big Bang)
Elapsed time
Universe Clear Up
WMAP observation (300,000 years since the Big Bang)
Big Bang
Hot Era
WMAP data (3x10^5 years)
Inflation
Phase transition completed
Inflation period
10^-36 seconds
The universe began in an endless state
Created from "nothing"
QGP
CGC/ Glasma
t
y
x
Plot by T. Hatsuda

Gluon Saturation in a nucleus: classical coherence from quantum fluctuations



Wee parton fluctuations time dilated on strong interaction time scales



The gluon density saturates at a maximal value of ~ $1/\alpha_S$ ➔ gluon saturation

Large occupation # => classical color fields

|P>_pert ➔ |P>_classical

Many-body high energy QCD:
The Color Glass Condensate

Gelis,Iancu,Jalilian-Marian,RV:
Ann. Rev. Nucl. Part. Sci. (2010), arXiv: 1002.0333

- **QCD light front EFT framework of static light front color sources ρ^a and dynamical gauge fields A^a_μ**

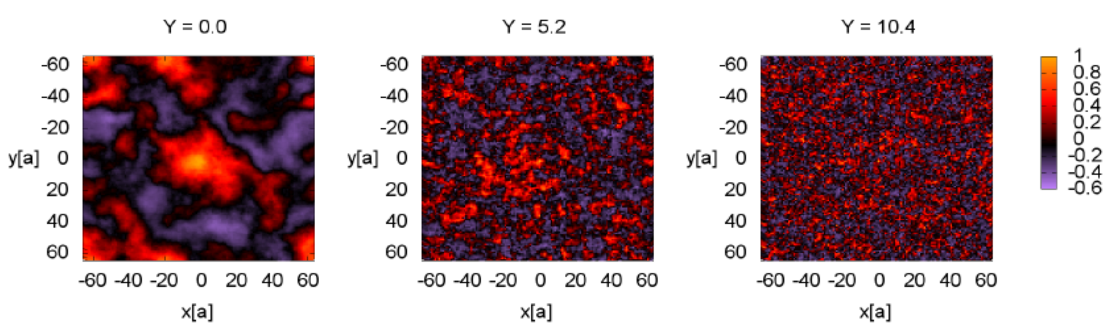
$$\langle \mathcal{O} \rangle_Y = \int [d\rho] W_Y[\rho]\, \mathcal{O}$$

- **Require observables be independent of separation of fast (large x) & slow (small x) modes: functional RG for "density matrices" W[ρ]**

$$\frac{\partial W_Y[\rho]}{\partial Y} = \mathcal{H}[\rho]\, W_Y[\rho]$$

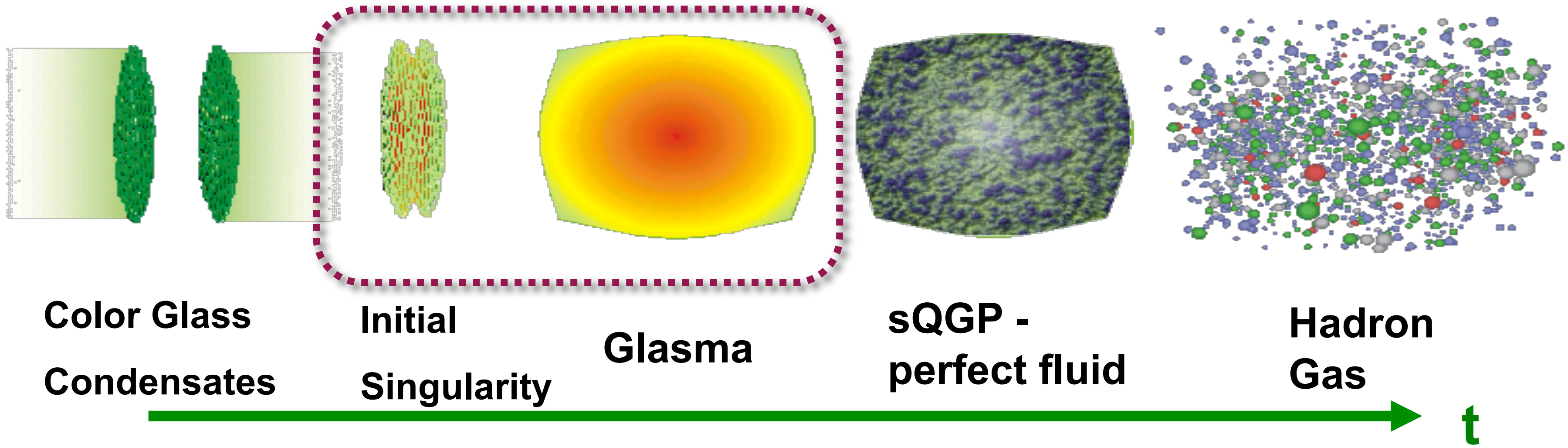
- **JIMWLK Hamiltonian-describes "Fokker-Planck" –like evolution of multi-parton (Wilson line) correlators**

-can be solved by Langevin techniques on 2+1-D lattices



Rummukainen,Weigert (2003)
Dumitru,Jalilian-Marian,Lappi, Schenke,RV,
arXiv:1108.4764

Quantum decoherence from classical coherence



Glasma (\Glahs-maa\): *Noun:* non-equilibrium matter between CGC & QGP

Computational framework

Schwinger-Keldysh formalism: for strong time dependent sources ($\rho \sim 1/g$) , computation of inclusive quantities can be formulated as an *initial value problem*

Power counting:

Gelis,RV NPA (2006)

- LO: $O(1/g^2)$ but all multiple scatterings $(g\rho)^n$
- NLO: $O(1)$ but all multiple scatterings $(g\rho)^n$

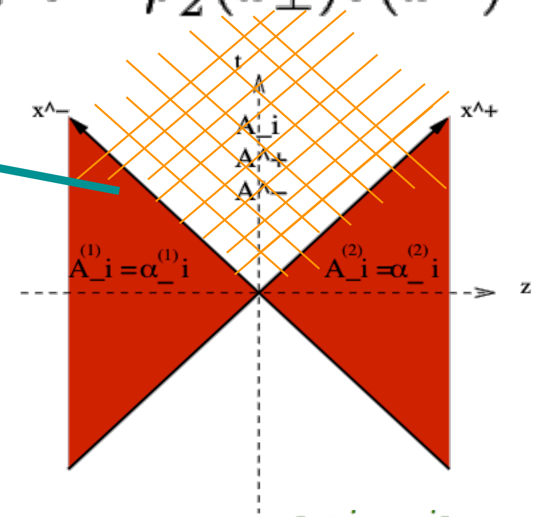
Spoiler alert: divergent contributions in rapidity and proper time modify power counting at NLO: these have to be resummed

The Glasma at LO:Yang-Mills eqns. for two nuclei

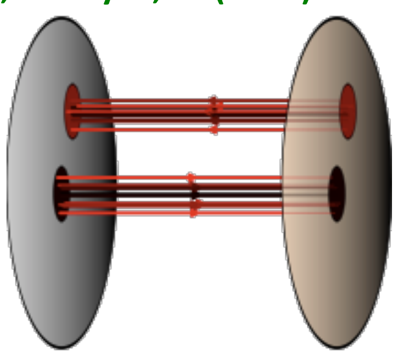
$O(1/g^2)$ and all orders in $(g\rho)^n$

$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_1^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_2^a(x_\perp)\delta(x^+)$$

Glasma initial conditions from matching classical CGC wave-fns on light cone



Kovner,McLerran,Weigert; Krasnitz, RV; Lappi
Lappi,Srednyak,RV (2010)



$$\nabla \cdot E = \rho_{\mathrm{electric}} \qquad \rho_{\mathrm{electric}} = ig[A^i, E^i]$$

$$\nabla \cdot B = \rho_{\mathrm{magnetic}} \qquad \rho_{\mathrm{magnetic}} = ig[A^i, B^i]$$

Boost invariant flux tubes of size with || color E & B fields
-generate Chern-Simons charge

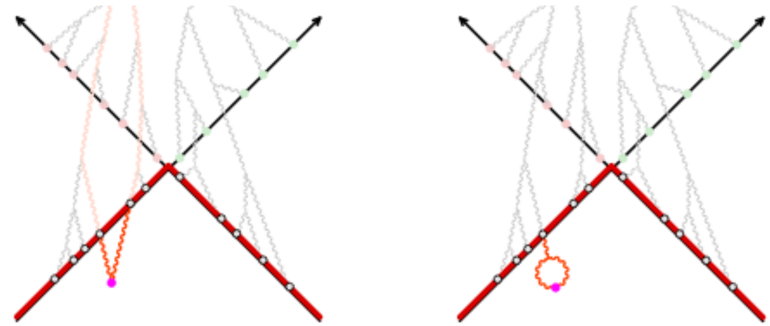
However, this results in very anisotropic distributions for $\tau \sim 1/Q_S$

Factorization of quantum fluctuations

Gelis,Lappi,RV (2008,2009)

O(1) and all orders in $(g\rho)^n$

Divergent contributions at NLO in rapidity in the respective nuclei can be factorized

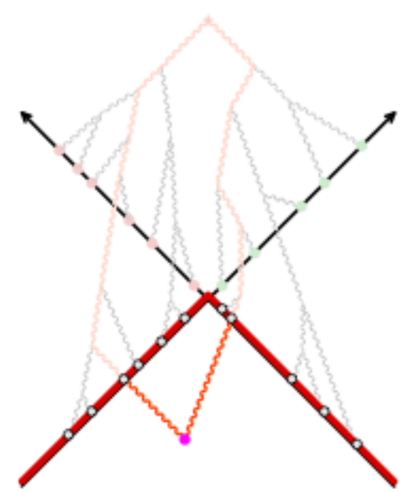


Initial value problem in Schwinger-Keldysh

$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}} \mathcal{G}(\vec{u},\vec{v})\, \mathcal{T}_u\mathcal{T}_v + \int_{\vec{u}} \beta(\vec{u})\, \mathcal{T}_u\right]\mathcal{O}_{\rm LO}$$

$\mathcal{G}(\vec{u},\vec{v})$ and $\beta(\vec{u})$ can be computed on the initial Cauchy surface

$\mathcal{T}_u = \dfrac{\delta}{\delta A(\vec{u})}$ linear operator on initial surface

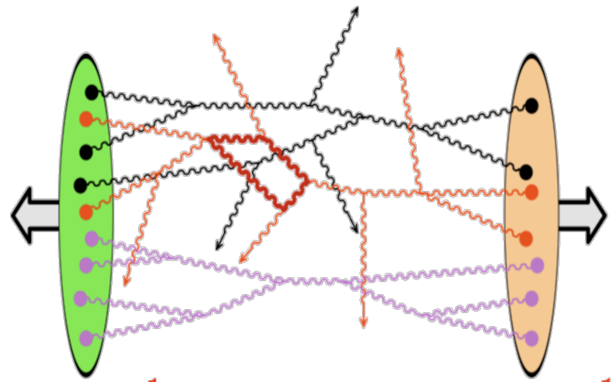


Contributions across both nuclei are finite-no log divergences => JIMWLK factorization

$$\mathcal{O}_{\rm NLO} = \left[\ln\left(\frac{\Lambda^+}{p^+}\right)\mathcal{H}_1 + \ln\left(\frac{\Lambda^-}{p^-}\right)\mathcal{H}_2\right]\mathcal{O}_{\rm LO}$$

Factorization in the Glasma

$$T^{\mu\nu}_{\rm LO} = \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta} - F^{\mu\lambda} F^{\nu}_{\lambda} \qquad {\rm O}\left(\frac{Q_S^4}{g^2}\right)$$



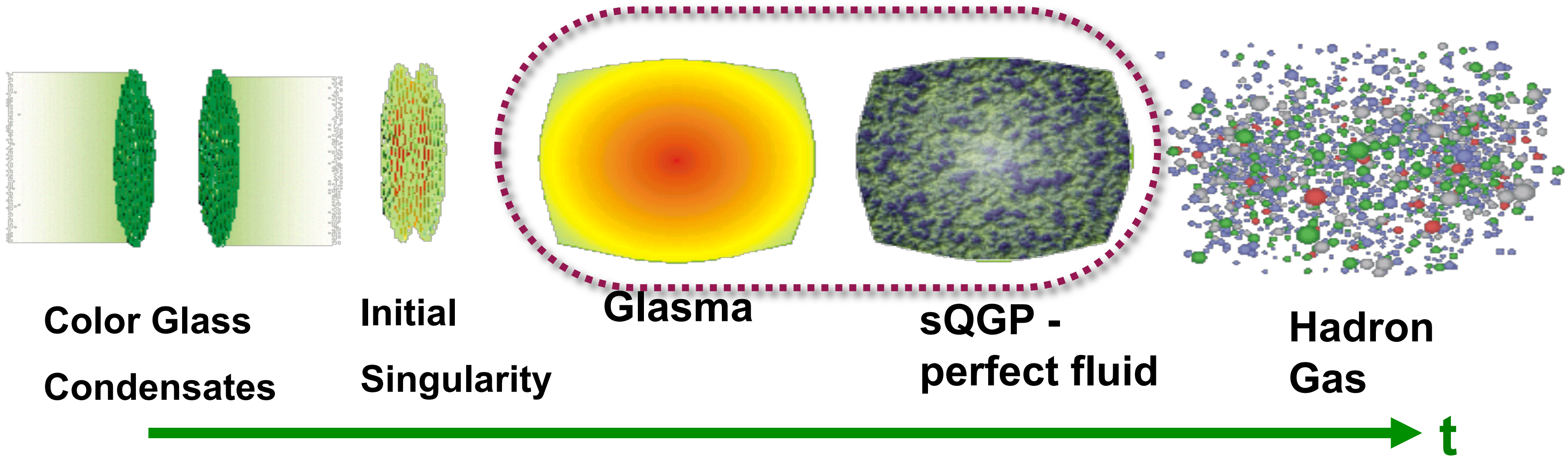
ε=20-40 GeV/fm³ for τ=0.3 fm @ RHIC

$$\langle T^{\mu\nu}(\tau, \underline{\eta}, x_\perp)\rangle_{\rm LLog} = \int [D\rho_1 d\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\, T^{\mu\nu}_{\rm LO}(\tau, x_\perp)$$

$$Y_1 = Y_{\rm beam} - \eta\,;\; Y_2 = Y_{\rm beam} + \eta$$

Glasma factorization => universal "density matrices W" ⊗ "matrix element"

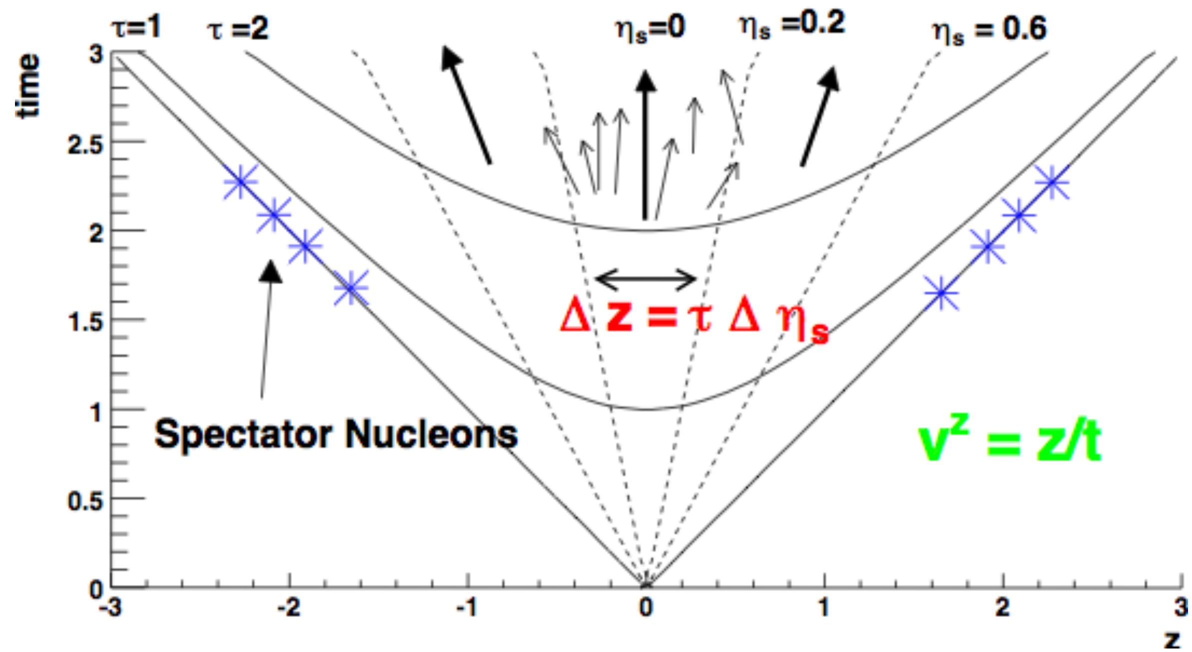
Quantum fluctuations in the Glasma



Two kinds of important quantum fluctuations:

a) Before the collision: p_η=0 modes – factorized into the wavefunction

a) After the collision $p_\eta \neq 0$; hold the key to early time dynamics



τ=1
τ =2
ηs=0
ηs =0.2
ηs = 0.6
time
Δ z = τ Δ ηs
Spectator Nucleons
vz = z/t
z

The problem of thermalization

- **Large v_2 –elliptic flow – appears to require early thermalization/perfect fluidity ($\eta/s \sim 1/4\pi$).**

- **But *naive* kinetic theory gives** $\tau_{\mathrm{therm.}} \sim \frac{1}{\alpha_S^2} \frac{1}{Q_S} >> \frac{1}{Q_S}$

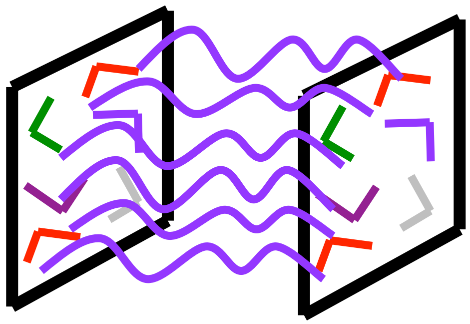
- **Our weak coupling (non-perturbative!) framework leads to hydrodynamic behavior outside the framework of kinetic theory.**

- **Treatment similar to enhancement of quantum fluctuations due to parametric resonances during inflation**

Kofman, Linde, Starobinsky
Micha, Tkachev

From Glasma to Plasma

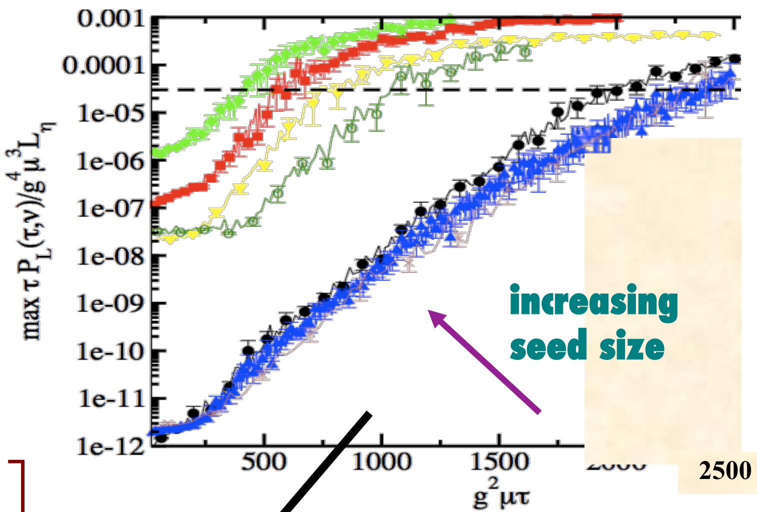
Romatschke,RV
Fukushima,Gelis,McLerran



Recall…

Quant. fluct.
grow exponentially
after collision

As large as classical
field at 1/Qs !



Gelis,Lappi,RV

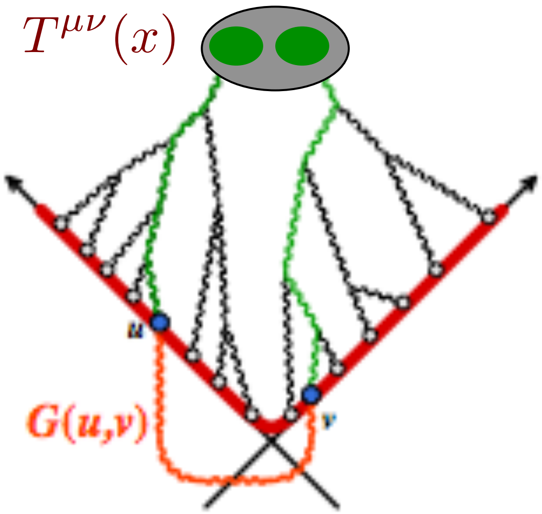
$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}} \mathcal{G}(\vec{u},\vec{v})\ \mathcal{T}_u\mathcal{T}_v + \int_{\vec{u}} \beta(\vec{u})\ \mathcal{T}_u\right]\mathcal{O}_{\rm LO}$$

For $p_\eta \neq 0$ modes: $\mathbf{T}_u\mathcal{A}(x) \sim \dfrac{\delta\mathcal{A}(x)}{\delta\mathcal{A}(0,y)} \sim \exp\left(\sqrt{Q_s\tau}\right)$

- **Requires resummation of ``secular" divergences to all orders in pert. theory**

$$\left[g\exp\left(\sqrt{Q_S\tau}\right)\right]^n$$

Spectrum of initial quantum fluctuations

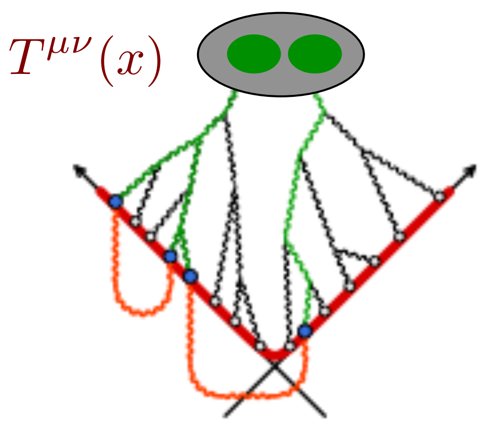


Dusling,Gelis,RV (2011)

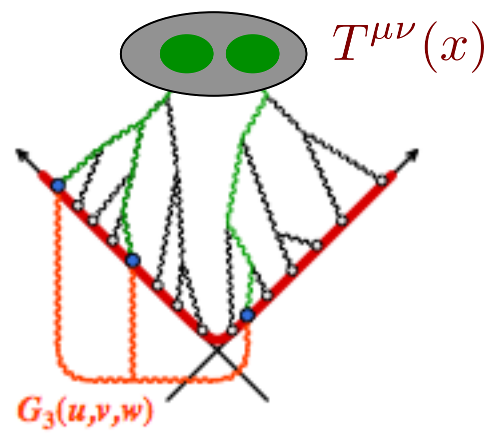
$$\mathcal{G}^{\mu\nu} = \int \frac{d^3k}{(2\pi)^3 2E_k} a^{\mu}_{-k}(\vec{u})\; a^{\nu}_{+k}(\vec{v})$$

$$\left[\frac{\delta^2 S_{\rm YM}}{\delta A^\mu A^\nu}\right]_{\rm A=A_{cl}} a^{\nu}_{\pm k} = 0$$

$$\lim_{x^0\to-\infty} a^{\mu}_{\pm k,\lambda a}(x) = \epsilon^\mu(k)\; T^a\; e^{\pm ik\cdot x}$$



Higher orders:



$$(g\exp(\sqrt{Q_S\tau}))^4 \sim O(1)$$

$$g(g\exp(\sqrt{Q_S\tau}))^3 \sim O(g)$$

Spectrum of initial fluctuations

Dusling,Gelis,RV (2011)

$$T^{\mu\nu}_{\rm resum} = \exp\left[\frac{1}{2}\int_{\tau=0^+} d^3u\; d^3v\; G(u,v)\cdot \mathcal{T}_u\mathcal{T}_v\right] T^{\mu\nu}_{\rm LO}$$

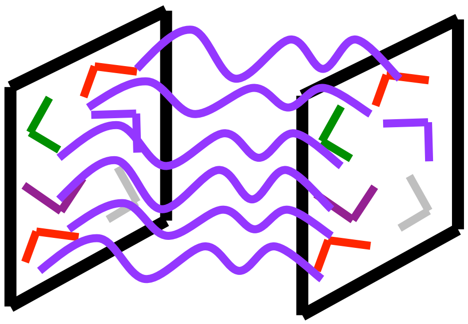
$$= \int [da_0(u)]\; F_{\rm init}[a_0]\; T_{\rm LO}[A_{\rm cl}+a_0]$$

$$\propto \exp\left[-\frac{1}{2}\int_{\tau=0^+} d^3u\; d^3v\; a_0(u)(G^{\mu\nu})^{-1}a_0(v)\right]$$

- $G^{\mu\nu}$ *has now been computed at* $\tau=0^+$ *in* $A^\tau=0$ *gauge.*

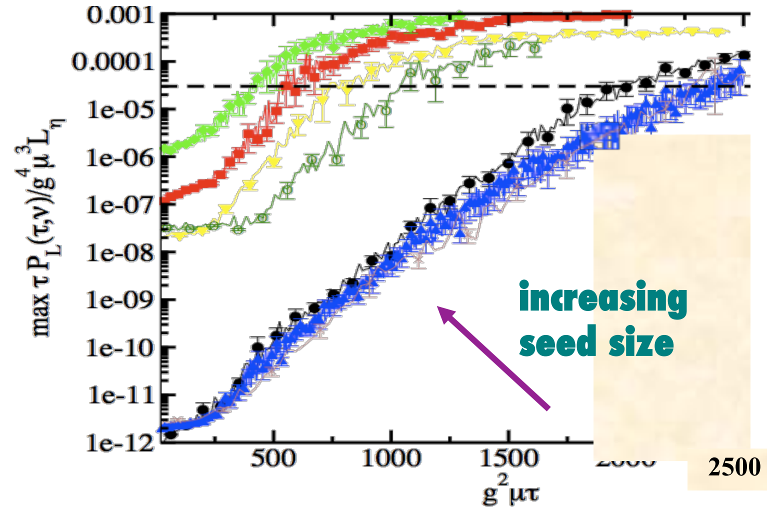
- ***Real time simulations in QCD are feasible…and will be done***

Quantum chaos in the quantum fluid ?



Quant. fluct.
grow exponentially
after collision

Romatschke,RV
Fukushima,Gelis,McLerran



Leading quantum corrections to all orders give:

$$\langle\langle T^{\mu\nu}\rangle\rangle_{\mathrm{LLx+Linst.}} = \int [D\rho_1][D\rho_2]\, W_{\mathrm{Y_{beam}-Y}}[\rho_1]\, W_{\mathrm{Y_{beam}+Y}}[\rho_2]$$
$$\times \int [da(u)]\; F_{\mathrm{init}}[a]\; T^{\mu\nu}_{\mathrm{LO}}[A_{\mathrm{cl}}(\rho_1,\rho_2)+a]$$

Path integral over multiple initializations of classical trajectories in one event can lead to quasi-ergodic "eigenstate thermalization"

Berry; Srednicki; Rigol et al.; …

Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

"Toy" example: scalar Φ^4 theory

Gaussian random variable

$$\langle c_{\nu k} c_{\mu l} \rangle = 0$$

$$\langle c_{\nu k} c^*_{\mu l} \rangle = 2\pi\delta(\nu - \mu)\delta_{kl}$$

$$\phi(\tau, \eta, x_\perp) = \phi_{\text{cl.}}(\tau, x_\perp) + \frac{1}{2}\int \frac{d\nu}{2\pi}\, d\mu_k\, c_{\nu k}\, e^{i\nu\eta}\chi_k(x_\perp) H_{i\nu}(\lambda_k \tau) + c.c$$

Satisfies the equation

$$[-\partial_\perp^2 + V''(\phi_0)]\chi_k = \lambda_k^2\, \chi_k$$

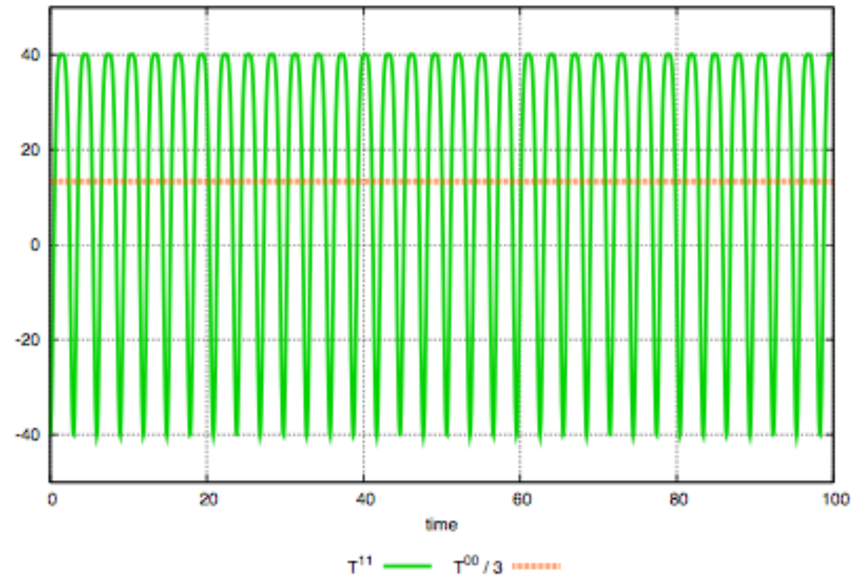
- **These quantum modes satisfy the criteria conjectured by Berry (and developed by others) as essential for thermalization of a quantum fluid**

Hydrodynamics from quantum fluctuations

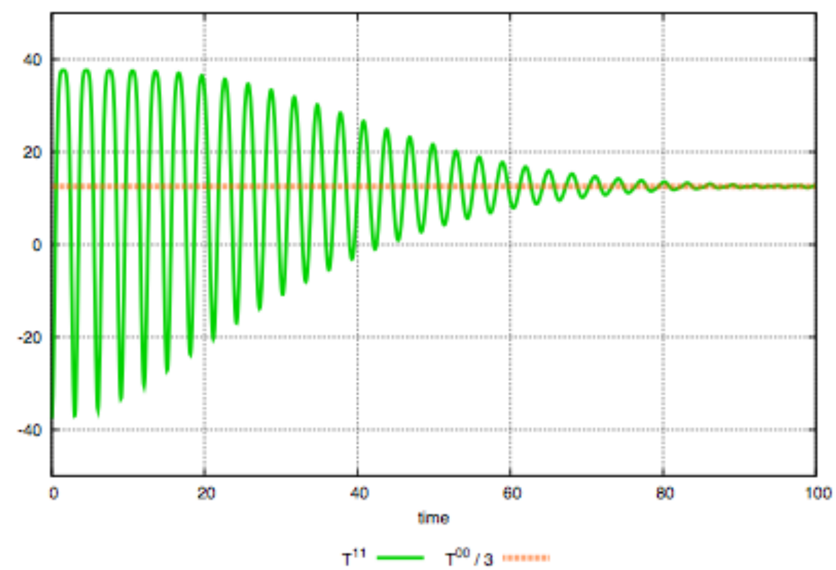
Dusling,Epelbaum,Gelis,RV (2011)

scalar Φ^4 theory:

Energy density and pressure without averaging over fluctuations



Energy density and pressure after averaging over fluctuations

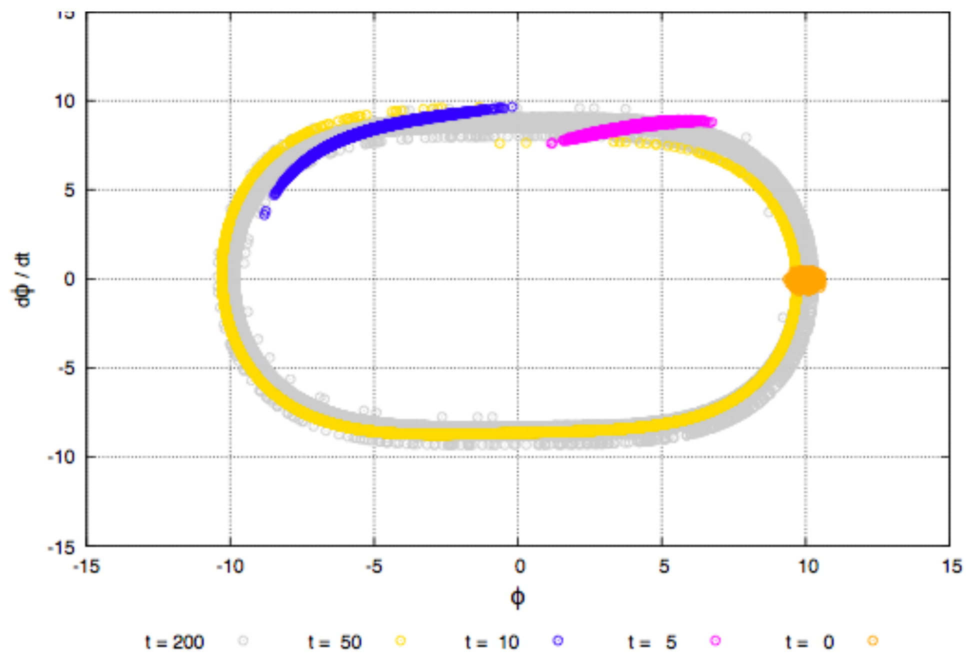


Converges to single valued relation "EOS"

Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

Anatomy of phase decoherence:



$\Delta\Theta = \Delta\omega\, t$

$T_{period} = 2\pi / \Delta\omega$

$\Rightarrow T_{period} \cong 18.2 / g\, \Delta\Phi_{max}$

Different field amplitudes from different initializations of the classical field

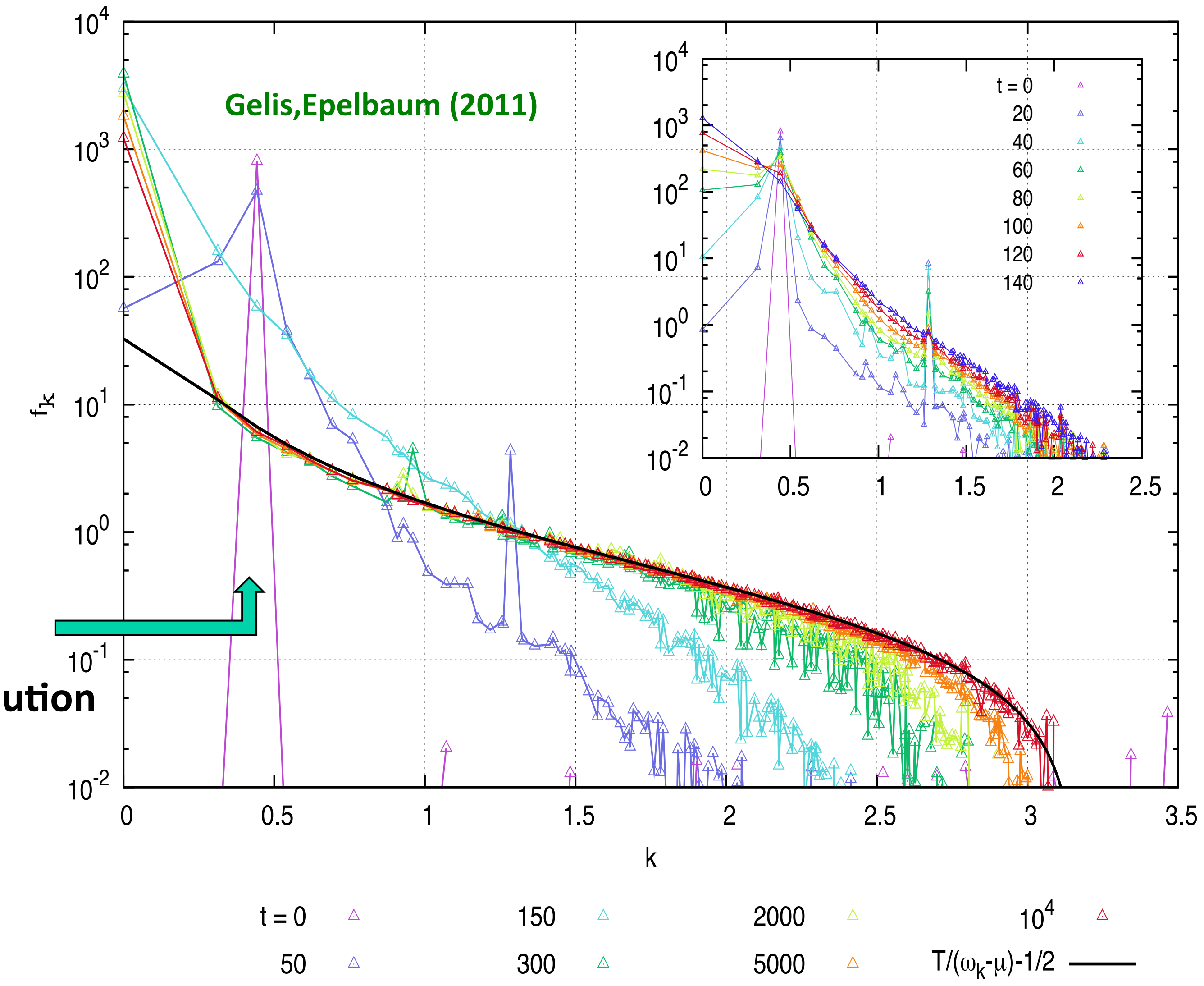
$$\langle T^\mu_\mu \rangle = \int d\phi\, d\dot\phi\, \rho_t(\phi,\dot\phi)\, T^\mu_\mu(\phi,\dot\phi) \equiv \int dE\, d\theta\, \tilde\rho_t(E,\theta)\, T^\mu_\mu(E,\theta)$$

$$t \overset{\approx}{\to} \infty \quad \int dE\, \tilde\rho_t(E) \underbrace{\int d\theta\, T^\mu_\mu(E,\theta)} = 0$$

$$\int d\theta\, T^\mu_\mu(E,\theta) = \frac{2\pi}{T}\int_t^{t+T} d\tau\, T^\mu_\mu(\phi(\tau),\dot\phi(\tau)) = 0$$

Because $T_\mu{}^\mu$ for scalar theory is a total derivative and ϕ is periodic

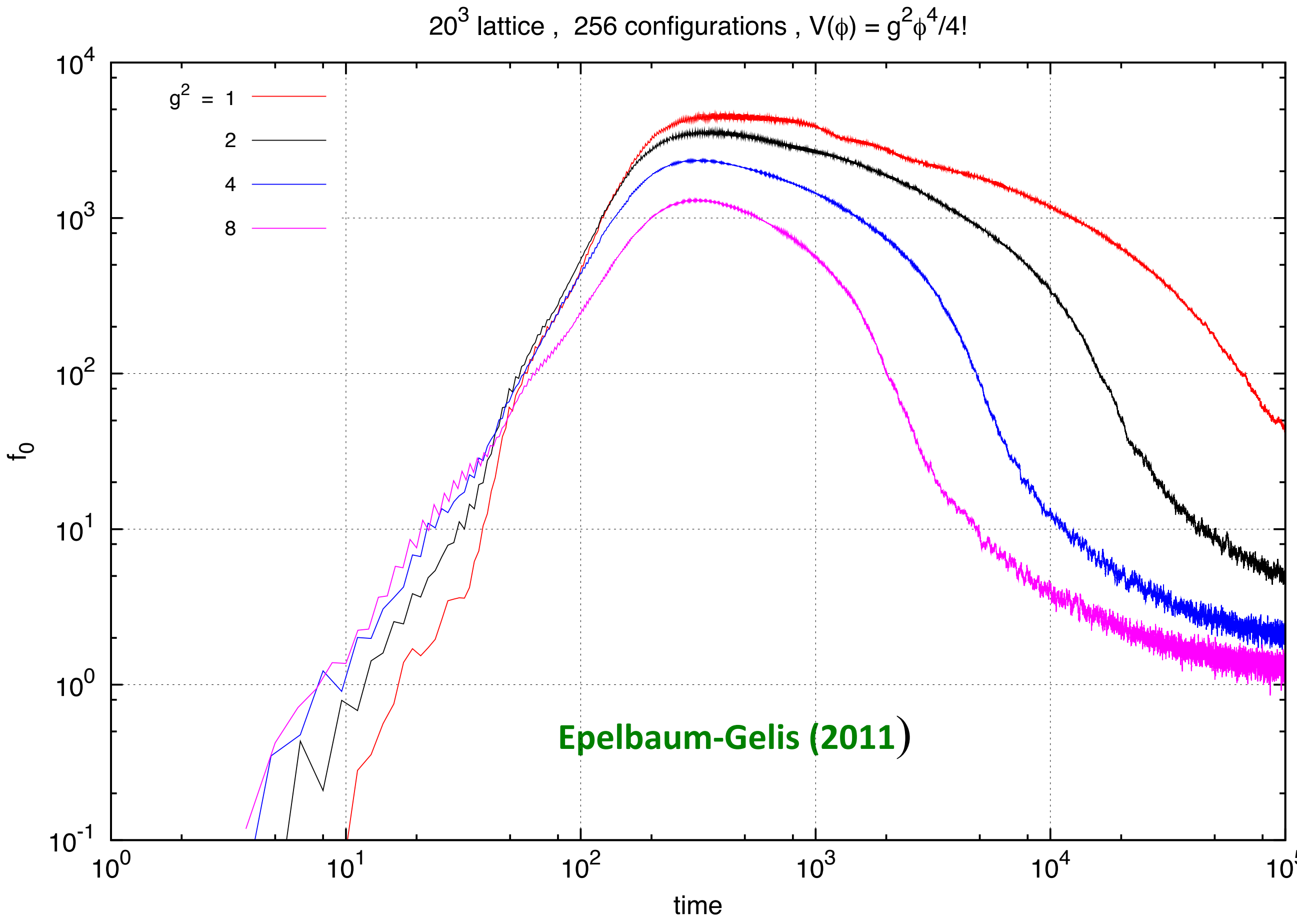
Quasi-particle occupation number



Initial mode distribution

System becomes over occupied relative to a thermal distribution…

Bose-Einstein condensate ?



Also familiar in discussions of preheating in inflation Khlebnikov,Tkachev (1996)

Bose-Einstein Condensation and Thermalization

Blaizot,Gelis,Liao,McLerran,RV: arXiv:1107.5295v2

Assumption: Evolution of “classical” fields in the Glasma can be matched to a quasi-particle transport description

See also, Mueller, Son (2002)
Jeon (2005)

All estimates are “parametric”: $\alpha_S \ll 1$

System is over-occupied: $n \approx Q_S^3/\alpha_s$; $\varepsilon = Q_S^4/\alpha_S$

$\Rightarrow n \bullet \varepsilon^{-3/4} \approx 1/\alpha_S^{1/4} \gg 1$

In a thermal system, $n \bullet \varepsilon^{-3/4} = 1$

If a system is over-occupied near equilibrium and elastic scattering dominates, it can generate a Bose-Einstein condensate

Bose-Einstein Condensation and Thermalization

$$n_{\rm eq} = \int_{\mathbf{p}} f_{\rm eq}(\mathbf{p}) \;;\; \varepsilon_{\rm eq} = \int_{\mathbf{p}} \omega_{\mathbf{p}}\, f_{\rm eq}(\mathbf{p})$$

$$f_{\rm eq}(\mathbf{p}) = \frac{1}{e^{\beta(\omega_p - \mu)} - 1}$$

In a many-body system, gluons develop a mass $\omega_{p=0} = m \approx \alpha_S^{1/2}\, T$

μ and β (=1/T) are adjusted to reproduce n_{eq} and ε_{eq} . But $\mu \leq m$ for positive def. f_{eq}
If over-occupation persists, system develops a condensate

$$f_{\rm eq}(\mathbf{p}) = n_c \delta^3(\mathbf{p}) + \frac{1}{e^{\beta(\omega_p - m)} - 1}$$

$$n_c = \frac{Q_s^3}{\alpha_S}\left(1 - \alpha_S^{1/4}\right)$$

As $\alpha_S \rightarrow 0$, most particles go into the condensate

$$\varepsilon_c = m\, n_c \approx \alpha_S^{1/4}\, T^4 << T^4$$

It however carries a small fraction of the energy density…

Transport in the Glasma

$$\frac{df}{dt} \equiv \partial_\tau f - \frac{p_z}{\tau}\partial_{p_z} f = C[f]$$

Assuming high occupancy (not ignoring (1+f) factors),
"Landau" equation for small angle 2 ➔ 2 scattering:

$$\frac{df}{dt}|_{\rm coll} \sim \frac{\Lambda_S^2 \Lambda}{p^2}\partial_p \left\{ p^2 \left[\frac{df}{dp} + \frac{\alpha_S}{\Lambda_S} f(p)(1+f(p)) \right] \right\}$$

Λ_S is scale below which distribution is ~1/α_S; Λ is a hard scale – typical momentum.

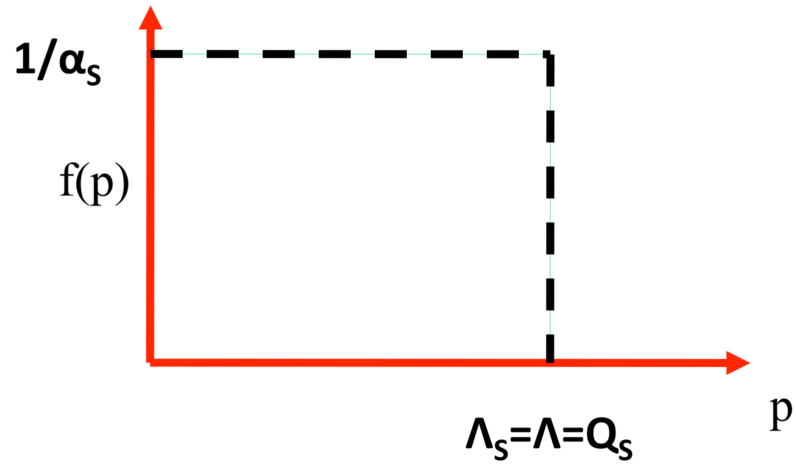
This is satisfied by a distribution where

$$f \sim \frac{1}{\alpha_S}\,; p < \Lambda_S \qquad \sim \frac{1}{\alpha_S}\frac{\Lambda_S}{p}\,; \Lambda_S < p < \Lambda \qquad \sim 0\,; \Lambda < p$$

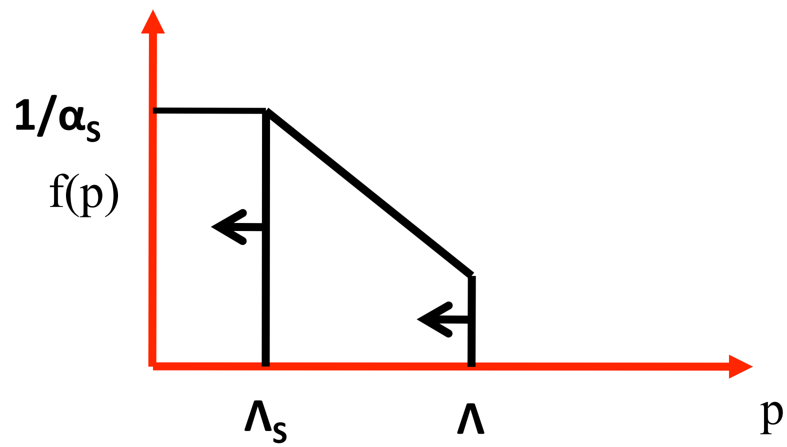
Λ_S and Λ are dynamical scales determined by the transport equation

Transport in the Glasma

At $\tau \sim 1/Q_S$



At $\tau > 1/Q_S$



When $\Lambda_S = \alpha_S \Lambda$, the system thermalizes;
one gets the ordering of scales: $\Lambda = T$, $m = \Lambda \Lambda_S = \alpha^{1/2} T$, $\Lambda_S = \alpha_S T$

Thermalization: from Glasma to Plasma

From moments of the transport eqn., $\tau_{coll} = \Lambda / \Lambda_S^2$

<u>Consider first a fixed box</u>: Energy conservation gives $\Lambda^3 \Lambda_S$ = constant

Scattering time is itself a function of time: $(Q_S \tau_{coll})^a = Q_S t$

From the moments, the only consistent choice is $\tau_{coll} \sim t$

From these two conditions, $\Lambda_S \sim Q_s \left(\frac{t_0}{t}\right)^{3/7}$ $\Lambda \sim Q_s \left(\frac{t}{t_0}\right)^{1/7}$

Thermalization time: $t_{\text{therm.}} \sim \frac{1}{Q_S}\left(\frac{1}{\alpha_S}\right)^{7/4}$

Also, Kurkela, Moore (2011))

Number density decreases slowly with time as $(t_0/t)^{1/7}$

Energy density of the condensate remains small – decreases as $(t_0/t)^{1/7}$

Entropy density $s = \Lambda^3$ increases and saturates at t_{therm} as T^3

$N_{quark} \sim \Lambda^3 = N_{gluon} (\Lambda^2 \Lambda_S/\alpha_S)$ at t_{therm} when $\Lambda_S = \alpha_S \Lambda$

Thermalization: from Glasma to Plasma

Expanding box:

$$\partial_\tau \varepsilon + \frac{(\varepsilon + P_L)}{\tau} = 0 \qquad \varepsilon_g(t) \sim \varepsilon(t_0)\left(\frac{t_0}{t}\right)^{1+\delta}$$

Estimate for collision time – only depends on RHS of Collision integral – stays the same

$$\Lambda_S \sim Q_S\left(\frac{t_0}{t}\right)^{(4+\delta)/7} \qquad \Lambda \sim Q_S\left(\frac{t_0}{t}\right)^{(1+2\delta)/7}$$

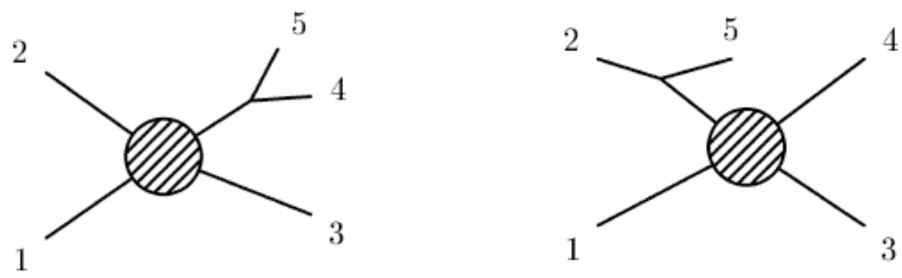
Thermalization time t_{therm} $= \dfrac{1}{Q_S}\left(\dfrac{\tau_0}{\tau}\right)^{7/(3-\delta)}$

For δ= -1, recover fixed box results…

A condensate can still form in the expanding case for δ > 1/5

$$n_c = \frac{Q_s^3}{\alpha_S}\left(\frac{t_0}{t}\right)\left[1 - \left(\frac{\tau_0}{\tau}\right)^{(-1+5\delta)/7}\right]$$

Role of inelastic processes ?



Wong (2004)
Mueller,Shoshi,Wong (2006)

Power counting for n ➔ m processes contributions to the collision integral

Vertices contribute α_S^{n+m-2}

Factor of $(\Lambda_S/\alpha_S)^{n+m-2}$ from distribution functions

Screened infrared singularity: $(1/\Lambda\ \Lambda_S)^{n+m-4}$

Remaining phase space integrals Λ^{n+m-5}

Net result is $\tau_{inelas} \sim \Lambda / \Lambda_S^2 = \tau_{elas}$

At most parametrically of the same order as elastic scattering.
So a transient Bose-Einstein condensate can form.

Numerical simulations will be decisive

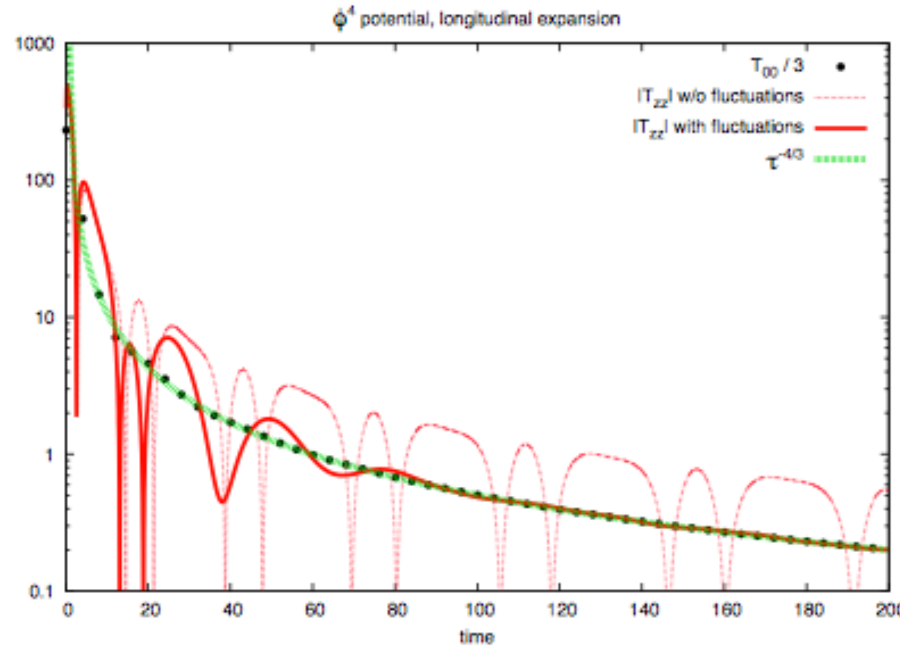
Dusling,Epelbaum,Gelis,RV, in progress
Blaizot, Liao, McLerran

Summary

- Thermalization in QCD: a subtle and beautiful problem in quantum field theory
- Much progress…many open questions
- Key to understanding many phenomena at RHIC and LHC

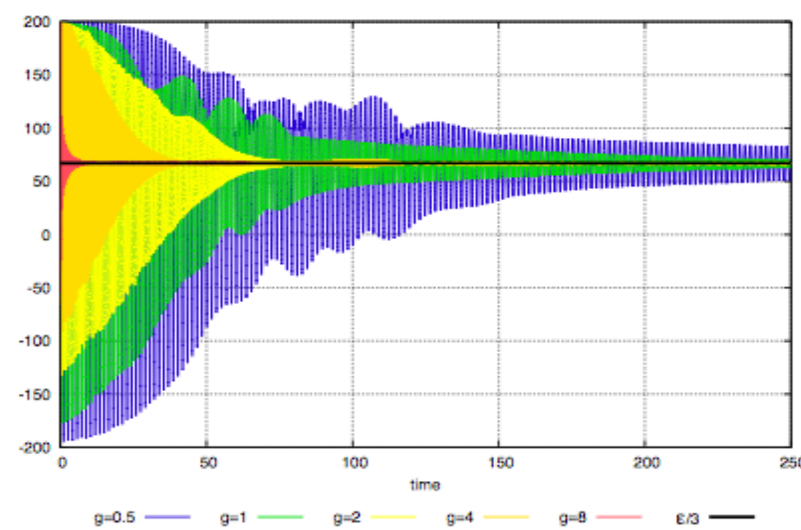
Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)



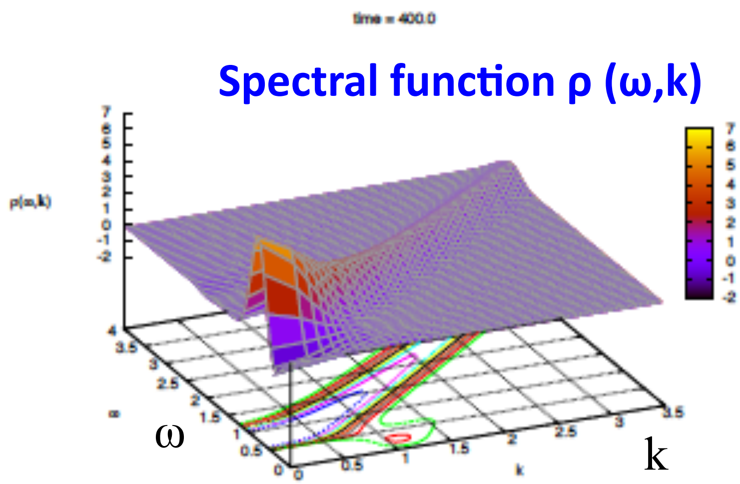
For 1-d expansion recover
$\varepsilon \sim \tau^{-4/3}$ behavior of ideal hydrodynamics

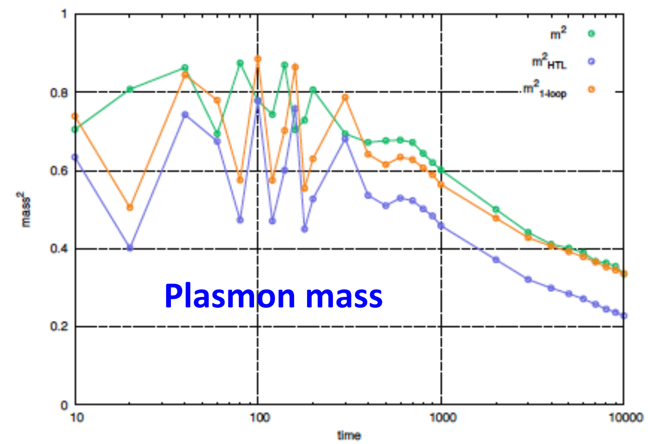
Convergence with increasing g

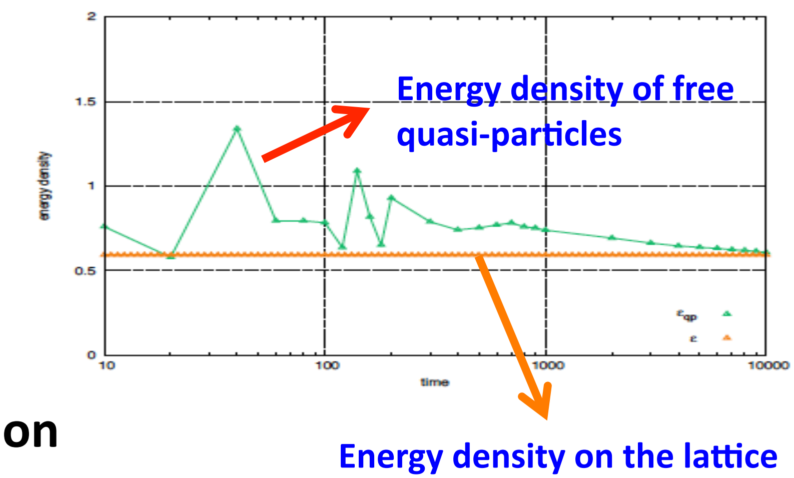


Quasi-particle description?

Epelbaum,Gelis (2011)







- At early times, no quasi-particle description

- May have quasi-particle description at late times. Effective kinetic "Boltzmann" description in terms of interacting quasi-particles at late times ?