

Lattice QCD at non zero chemical potential - status and challenges

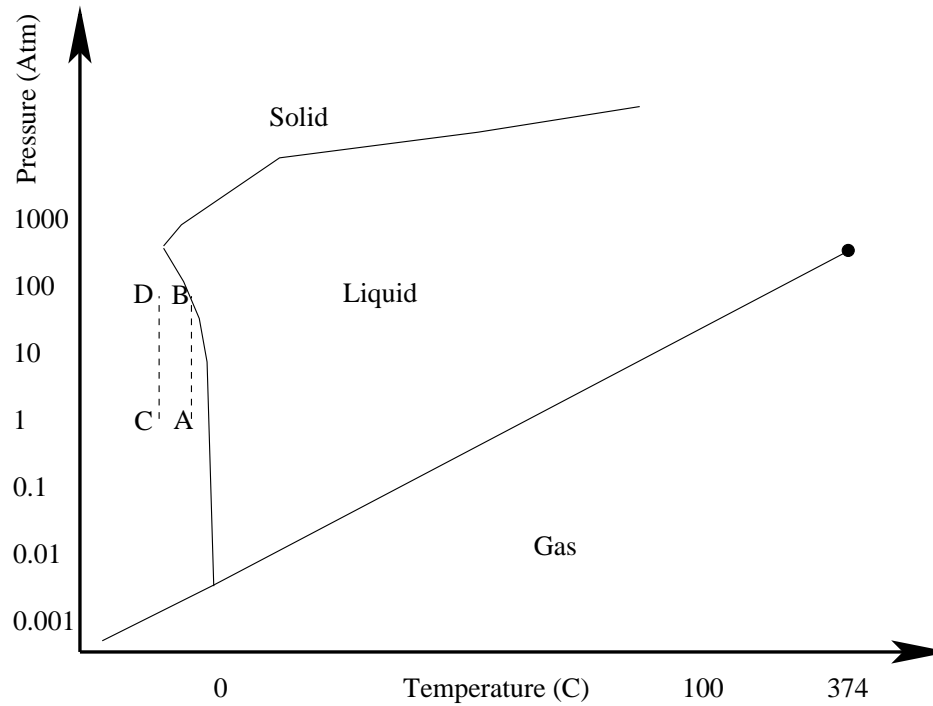
Kim Splittorff

Niels Bohr Institute / DFF - Sapere Aude

Heavy ions: Experiments Confront Theory, the Discovery Center, November 8, 2011

Alternative motivation !

Snowballs



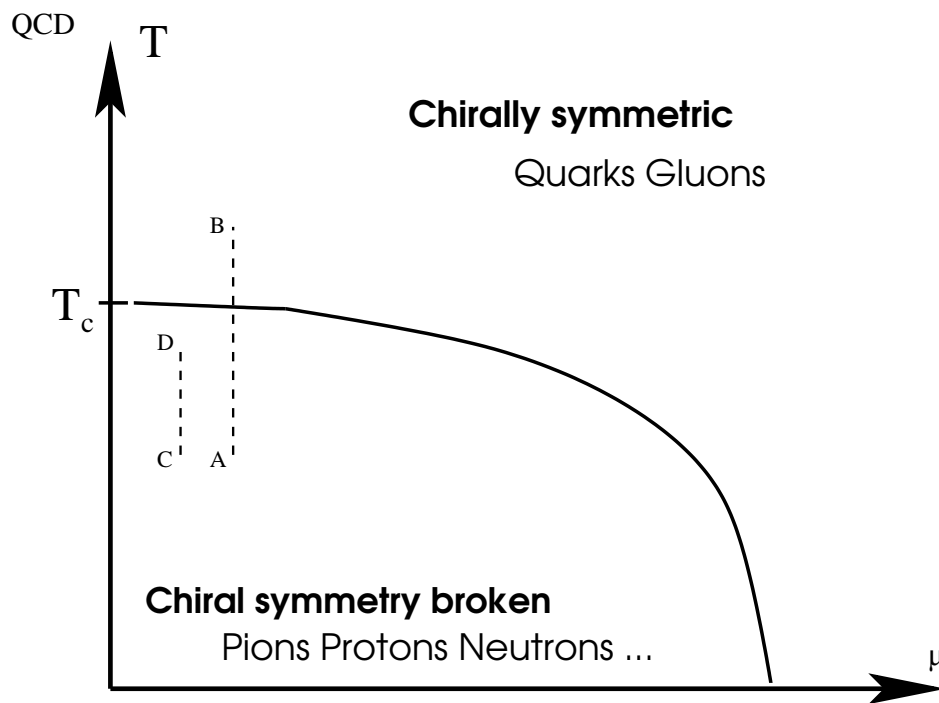
$A \rightarrow B \rightarrow A = \text{Snowball}$

$C \rightarrow D \rightarrow C = \text{Fluff}$

Scott A. Sandford NASA Ames Research Center

<http://www.astrosociety.org/pubs/mercury/9801/snowball.html>

Fireballs



$A \rightarrow B \rightarrow A = \text{Jets gets eaten}$

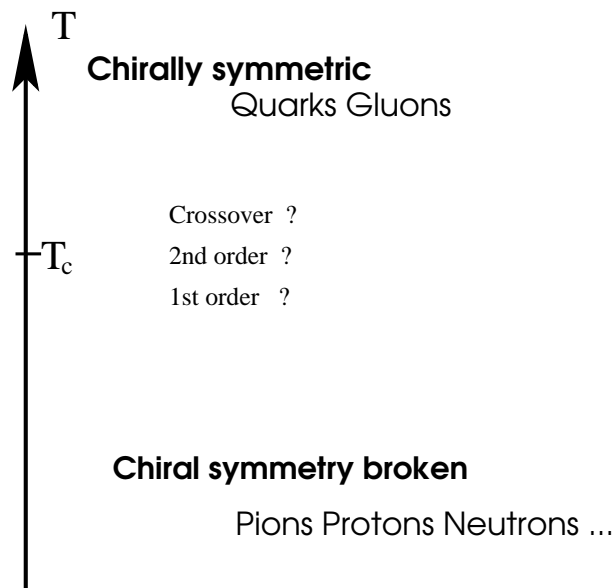
$C \rightarrow D \rightarrow C = \text{Fluff}$

Can lattice QCD help understand the phase diagram ?

- **Success story $\mu = 0$**
- **How to include μ and the sign problem**
- **$\mu \neq 0$ status, challenges and progress**

The baryon/anti-baryon symmetric world

$(\mu = 0)$



Landau theory: $N_f \geq 2$ 1st order ($m_f = 0$)

Answer depends on values of m_f and $U_A(1)$ anomaly

Lattice: @ physical quark masses **crossover**

Pisarski, Wilczek, PRD29 (1984) 338

Aoki, Endrodi, Fodor, Katz, Szabo, Nature 443 (2006) 675

de Forcrand, Philipsen JHEP 0701 (2007) 077

$$N > 0$$

Here: Fact which we adopt into QCD

Grand canonical approach: *Fix μ determine N*

$$N = \frac{1}{V} \partial_{\mu} \log Z(\mu)$$

How to include μ in Z

μ is conjugate variable to N

$$\mu N = \mu \langle q^\dagger q \rangle = \mu \langle \bar{q} \gamma_0 q \rangle$$

$$\mathcal{L}_{\text{QCD}} = \bar{q}(D_\eta \gamma_\eta + \mu \gamma_0 + m)q + \text{Gluons}$$

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μ enters as the 0th component of the gauge field

$$\mathcal{L}_{\text{Lattice QCD}} = \dots + e^{a\mu} \bar{q}_x \gamma_0 U_{x, x+\hat{0}} q_{x+\hat{0}} + e^{-a\mu} \bar{q}_{x+\hat{0}} \gamma_0 U_{x+\hat{0}, x}^\dagger q_x + \dots$$

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Works fine for free quarks

Hasenfratz, Karsch, PLB 125 (1983) 308

The sign problem

$$Z_{N_f=2} = \int dA \det^2(D + \mu\gamma_0 + m) e^{-S_{\text{YM}}}$$

Anti Hermitian   Hermitian

$$\det^2(D + \mu\gamma_0 + m) = |\det(D + \mu\gamma_0 + m)|^2 e^{2i\theta}$$

The measure is not real and positive

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No Monte Carlo sampling of A_η at $\mu \neq 0$

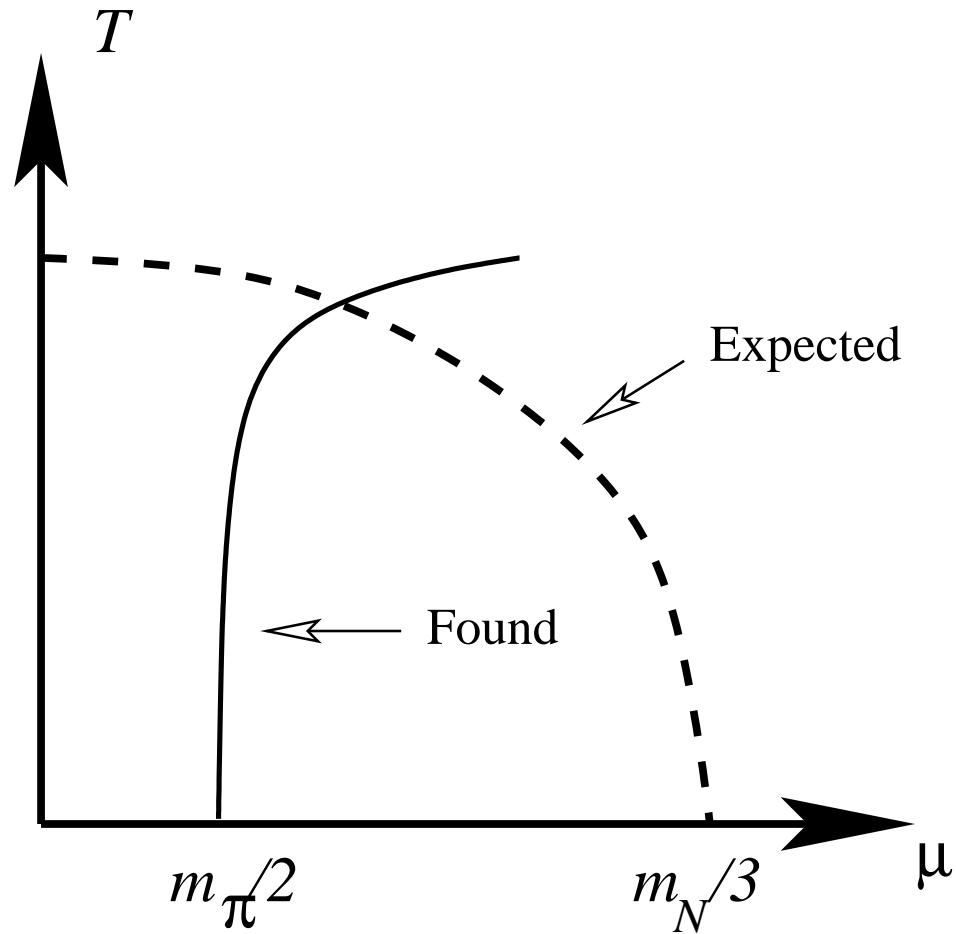
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What if we simply ignore $e^{2i\theta}$?

If we ignore $e^{2i\theta}$: *phase quenched* QCD



Kogut, Sinclair Phys.Rev.D77:114503,2008

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$$|\det(D + \mu\gamma_0 + m)|^2 = \det(D + \mu\gamma_0 + m) \det(D - \mu\gamma_0 + m)$$

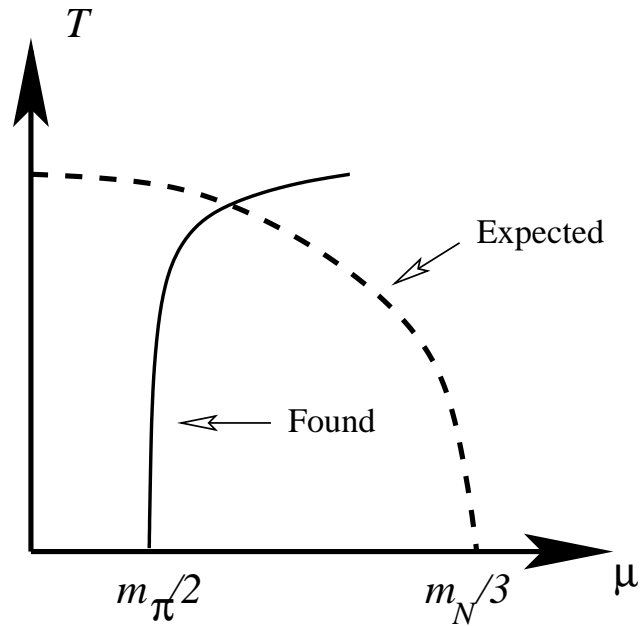
ISOSPIN CHEMICAL POTENTIAL

The QCD Phase Diagram at nonzero isospin chemical potential

The pions have nonzero isospin \Rightarrow BEC of pions

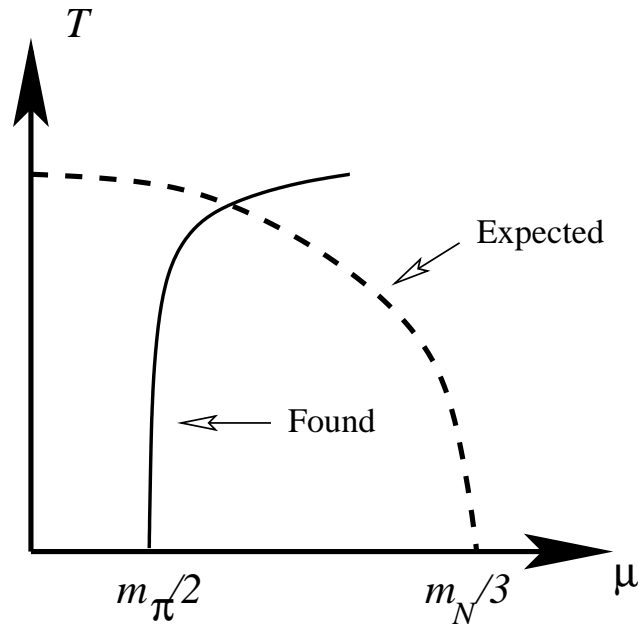
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The QCD Phase Diagram at nonzero isospin chemical potential

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Conclude: The phase $e^{2i\theta}$ is *highly* relevant for the phase diagram

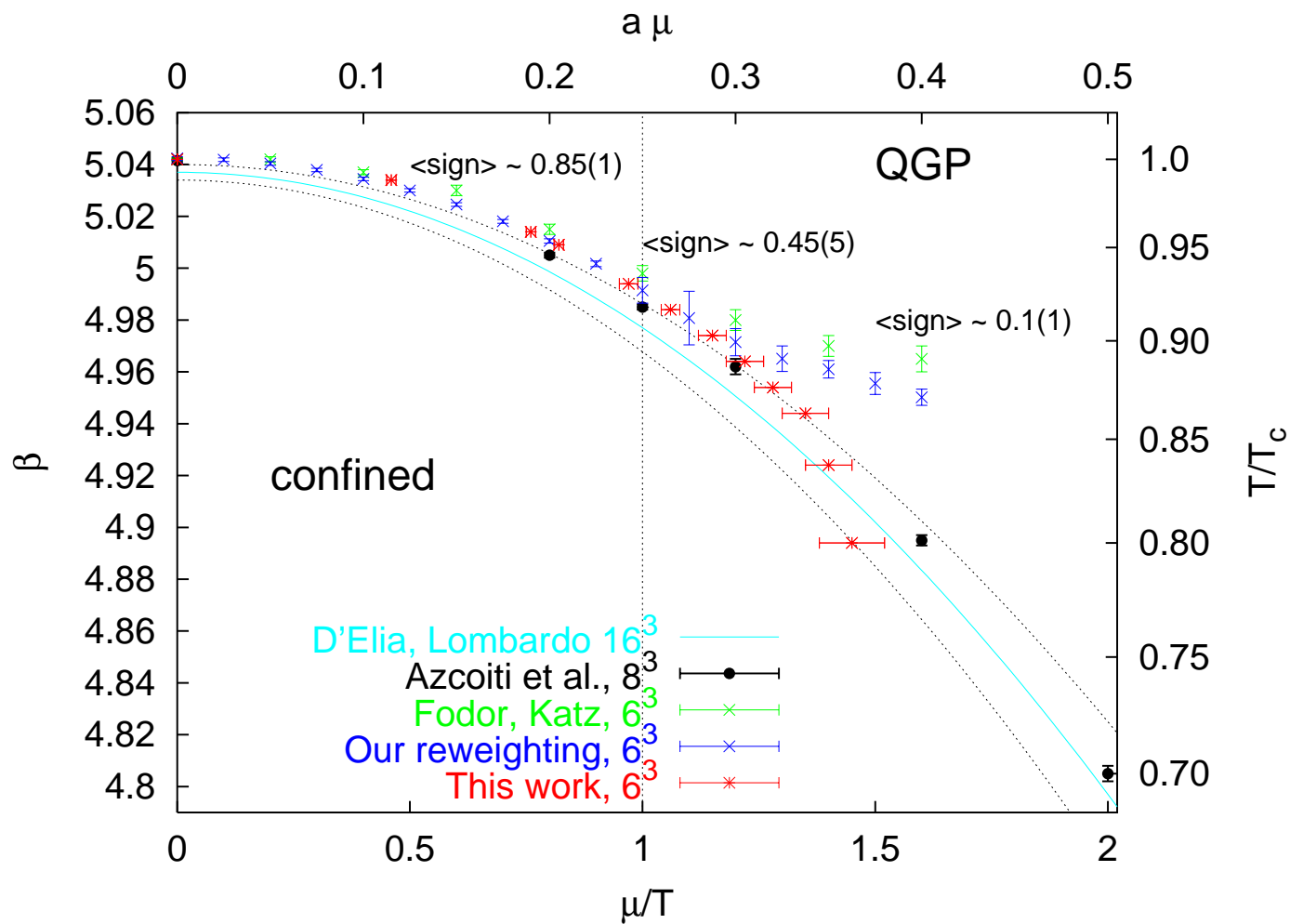
Son, Stephanov Phys.Rev.Lett. 86 (2001) 592

Kogut, Sinclair Phys.Rev.D77:114503,2008

So include the phase factor in Lattice QCD

Method	Idea	Challenge
Reweighting	Absorb the sign in the observable	Exponential cancellations
Taylor expansion	Expand at $\mu = 0$	Higher order terms
Imaginary μ	Determine the analytic function	Control the extrapolation
Density of states	Use the distribution of the phase	Determine the distribution
Canonical ensemble	Work at fixed baryon number	Fix the baryon number

The curvature and the average sign



de Forcrand Philipsen JHEP PoS LAT2005 (2005) 016

Progress: Karsch B-J Schaefer Wagner Wambach arXiv:1110.6038

Particular challenging for $\mu > m_\pi/2$

1) $\langle e^{2i\theta} \rangle \sim e^{-V(1-m_\pi^2/4\mu^2)^2 \mu^2 F_\pi^2}$

2) New Banks Casher relation (essential for $SB_\chi S$)

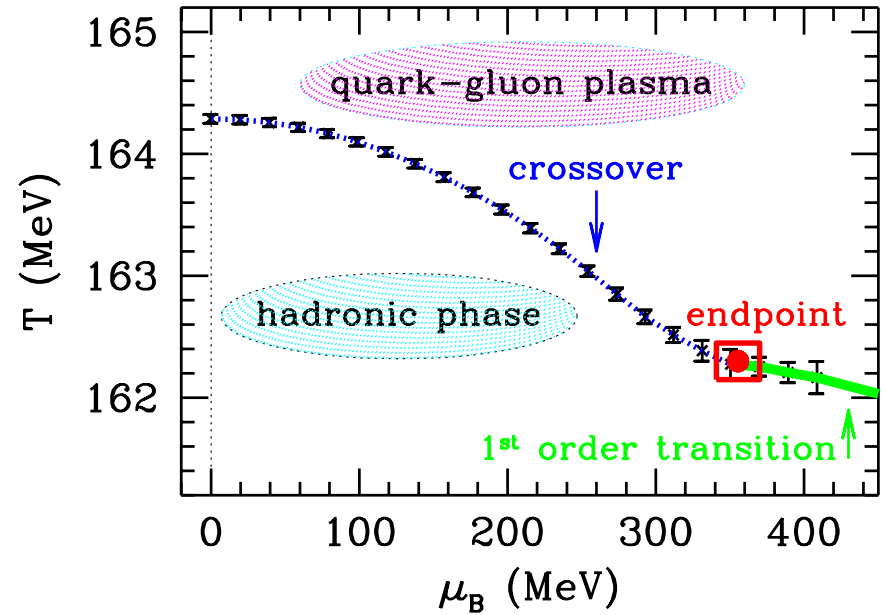
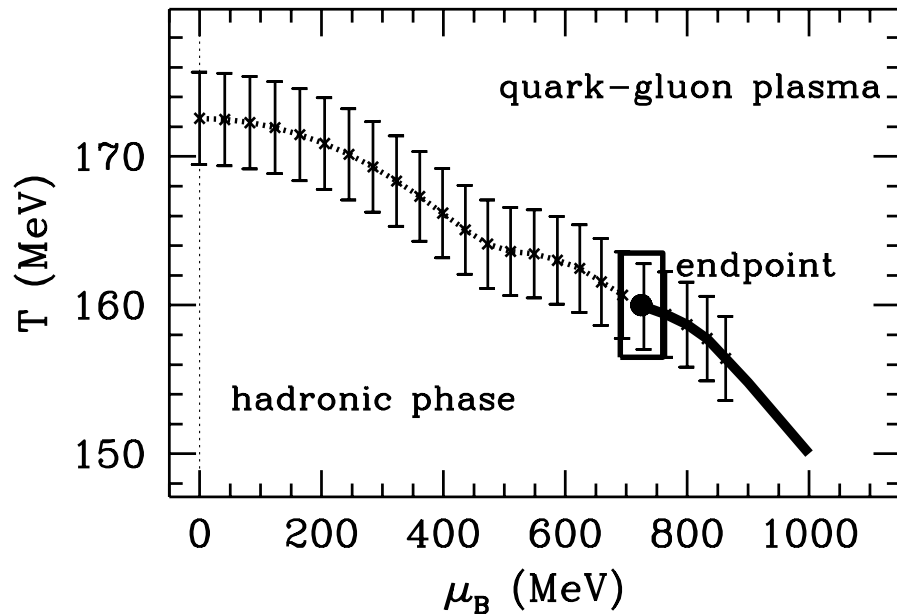
3) $\bar{\psi}\psi$ and n_B fluctuate wildly (Lorentzian dist)

reviews

Splittorff Verbaarschot arXiv:0809.4503

Lombardo, Splittorff, Verbaarschot arXiv:0912.4410

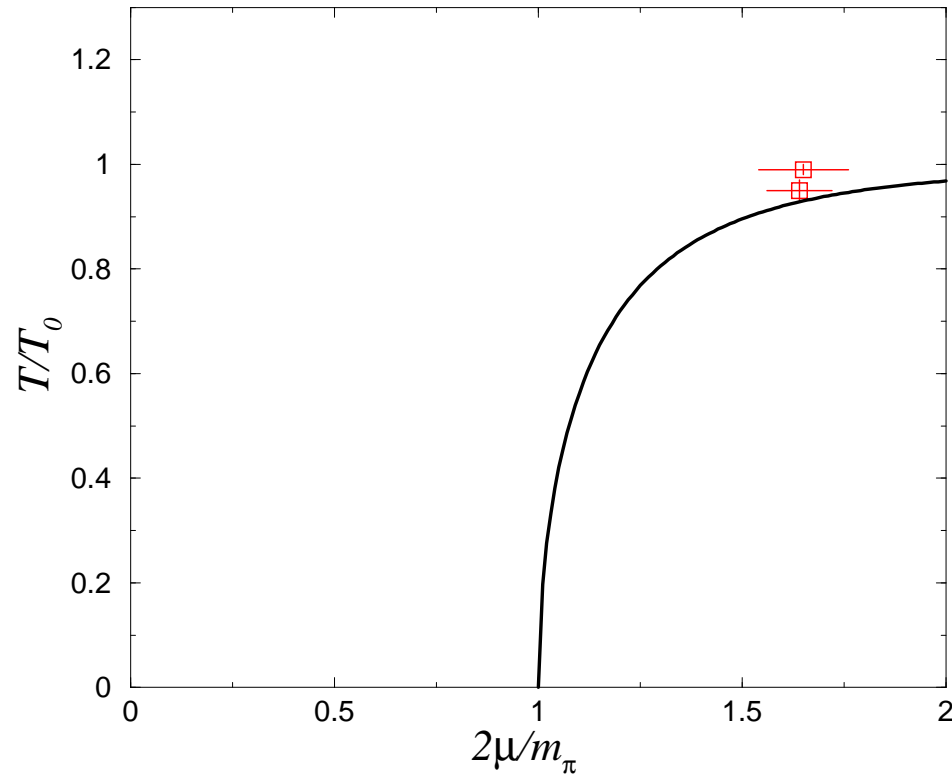
Lattice measurement of endpoint (Reweighting)



Fodor Katz JHEP 0203:014,2002; JHEP 0404:050,2004

Lattice measurement of endpoint (Rewieghting)

- Scale the axis by T_C and $m_\pi/2$



Fodor Katz JHEP 0203:014,2002; JHEP 0404:050,2004
Splittorff, hep-lat/0505001, PoS LAT2006:023,2006

Philipsen 0710.1217

Progress: Subset method

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Method	Idea	Challenge
Subset method	Sum over subset of configs	Apply it work for QCD

Bloch, PRL.107:132002,2011

Progress: Complex Langevin

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Method	Idea	Challenge
Complex Langevin	Stochastic flow in complex plane	Optimization

Problems for CL are 'orthogonal' to $\langle e^{2i\theta} \rangle \ll 1$

Parisi, Phys. Lett. 131 B (1983) 393

Aarts, Splittorff, JHEP 1008:017,2010

Aarts, Seiler, Stamatescu Phys.Rev.D81:054508,2010

Progress: Complex Langevin

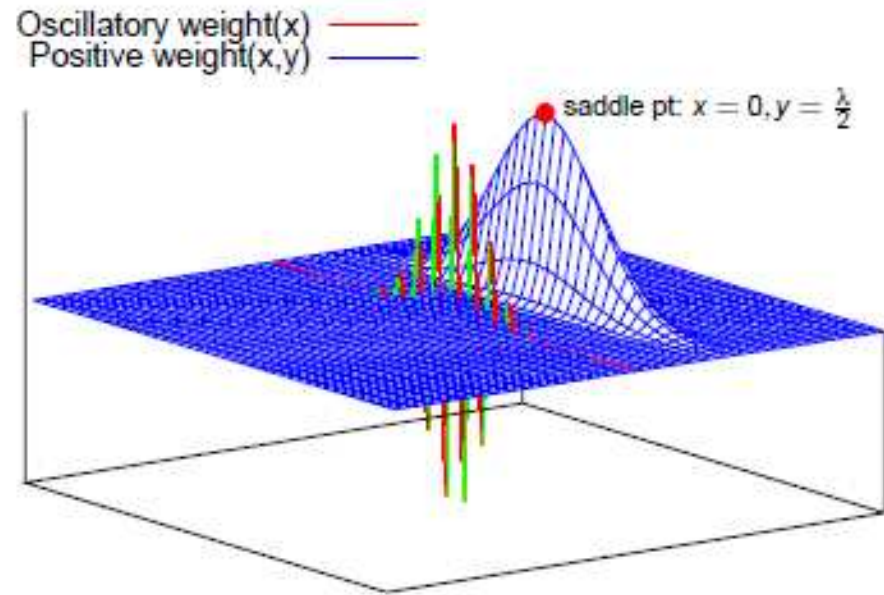


Illustration by Philippe de Forcrand

Progress: Complex Langevin

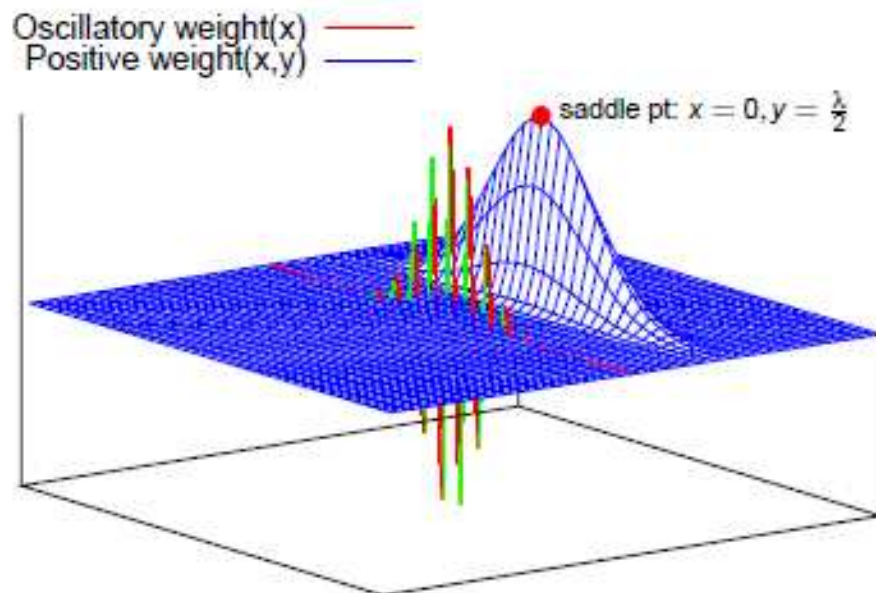


Illustration by Philippe de Forcrand

1) shift by $\mathcal{O}(V)$ in imaginary direction

2) Amplitude: real axis $\mathcal{O}(\exp(V))$; complex plane $\mathcal{O}(1)$

Aarts personal correspondence (2009)
de Forcrand PoS (LAT2009), 10

Lombardo Splittorff Verbaarschot arXiv:0910.5482

Conclusions

Interplay between lattice QCD and analytic QCD has

- 1) allowed us to understand the sign problem
- 2) understand lattice results
- 3) solid progress towards a solution

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