Holographic three-point function at one loop

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February 21, 2012
Nordic String Theory Meeting 2012
Niels Bohr Institute

based on hep-th 1112.5075
with Troels Harmark and Marta Orselli

Motivations

- Building blocks in conformal field theories are 2-point and 3-point functions of local gauge invariant operators.

$$\langle \mathcal{O}_i(x)\,\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}$$

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\mathcal{O}_k(z)\rangle = \frac{C_{ijk}}{|x-y|^{\Delta_i+\Delta_j-\Delta_k}|y-z|^{\Delta_j+\Delta_k-\Delta_i}|z-x|^{\Delta_i+\Delta_k-\Delta_j}}$$

- In principle all the higher point correlation functions are known by using the OPE:

$$\mathcal{O}_\alpha(x_1)\mathcal{O}_\beta(x_2) \sim \sum_\gamma \frac{C_{\alpha\beta\gamma}}{\mid x_{12} \mid^{\Delta_\alpha+\Delta_\beta-\Delta_\gamma}}\mathcal{O}_\gamma(x_2)$$

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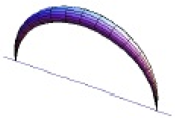
State of the art: String theory side 1

- Protected operators dual to SUGRA modes

[Freedman, Mathur, Matusis and Rastelli, 1998]

[Arutyunov and Frolov, 2000]

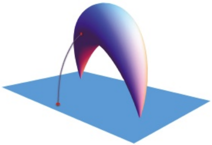
- New techniques for computing 2-point correlation functions (of non BPS operators)



[Buchbinder, 2010]
[Buchbinder and Tseytlin, 2010]
[Janik, Surowka and Wereszczynski, 2010]

State of the art: String theory side 2

- 2 operators are semiclassical and one is dual to a SUGRA mode



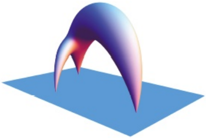
[Zarembo, 2010], [Costa, Monteiro, Santos and Zoakos,2010]
+
[Roiban and Tseytlin, 2010], [Hernandez, 2011], [Ryang, 2011], [Georgiou, 2011],[Russo and Tseytlin, 2011], [Park and Lee, 2011],[Buchbinder and Tseytlin, 2011],[Buchbinder and Tseytlin, 2011],[Bak, Chen and Wu,2011],[AB, Kristjansen, Young and Zoubos, 2011],[Arnaudov, Rashkov and Vetso, 2011],[Escobedo, Gromov, Sever and Vieira, 2011], [Hernandez, 2011], [Ahn and Bozhilov , 2011]

- Geodesic approximation for the 3 operators

[Klose and McLoughlin, 2011]

State of the art: String theory side 3

- Contributions to the 3-point function of 3 heavy operators



[Janik and Wereszczynski, 2011]
[Kazama and Komatsu, 2011]
[Buchbinder and Tseytlin, 2011]

Figures from Zarembo's paper

State of the art: Gauge theory side 1

- BMN operators

[Kristjansen,Plefka, Semenoff and Staudacher, 2002]
[Constable, Freedman, Headrick and Minwalla, 2002]
[Chu, Khoze and Travaglini, 2002]
[Beisert,Kristjansen,Plefka, Semenoff and Staudacher, 2002]

- Spin chain approach

[Roiban and Volovich, 2004]
[Okuyama and Tseng, 2004]
[Alday, David, Gava and Narain, 2005]
[Escobedo, Gromov, Sever and Vieira, 2011]
[Gromov, Sever and Vieira, 2011]

- Various operators in BMN limit (2 BPS and 1 non BPS)

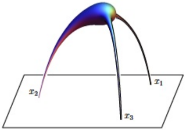
[Georgiou, Gili and Russo, 2009]

- 2 Schur polynomials and 1 chiral primary

[A.B., Kristjansen, Young and Zoubos, 2011]

State of the art: Gauge theory side 2

- 2 operators are light and 1 is heavy



[Gromov, Sever and Vieira, 2011]

- Three point function at one loop for scalar operators up to lenght five

[Georgiou, Gili, Grossardt and Plefka, 2012]

Tree level

Tree level computation [Escobedo, Gromov, Sever and Vieira, 2011]

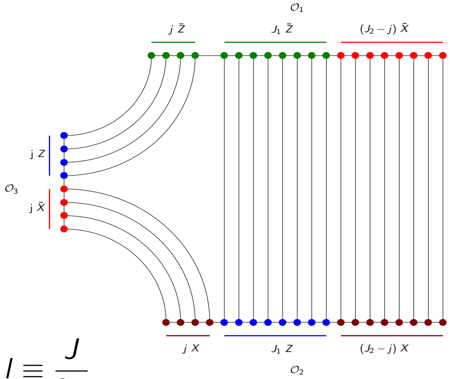
Agreement for the tree level part of the three point structure constant computed both at weak and strong coupling of 2 heavy operators and a light chiral primary operator (in the Frolov-Tseytlin limit).

$$C_{123}^{(0)} = \frac{1}{N} \frac{j!J}{\sqrt{(2j-1)!}} \int_0^{2\pi} \frac{d\sigma}{2\pi} (\bar{u}_1 u_2)^j$$

What happens at one loop?

We compute the one loop correction to the structure constants on the gauge theory side and compare this to the corresponding correction on the string theory side.

- All three operators are single trace operators made out of the three complex scalars Z, X and Y.
- Each operator is in an $SU(2)$ sector of $\mathcal{N} = 4$ SYM.
- Non extremal $\Delta_1 \neq \Delta_2 + \Delta_3$.



$$W^i = (Z, X)\,,\quad \bar{W}_i = (\bar{Z}, \bar{X})\,,\quad l \equiv \frac{J}{2\pi}$$

$$\mathcal{O}_1(x_1) = \mathcal{N}_1 \bar{\mathbf{u}}_{i_1}\left(\frac{k+1}{l}\right)\bar{\mathbf{u}}_{i_2}\left(\frac{k+2}{l}\right)\cdots\bar{\mathbf{u}}_{i_L}\left(\frac{k}{l}\right) : \mathrm{tr}(\bar{W}^{i_1}\bar{W}^{i_2}\cdots\bar{W}^{i_J}) : (x_1)$$

$$\mathcal{O}_2(x_2) = \mathcal{N}_2 \mathbf{v}^{j_1}\left(\frac{k+1}{l}\right)\mathbf{v}^{j_2}\left(\frac{k+2}{l}\right)\cdots\mathbf{v}^{j_L}\left(\frac{k}{l}\right) : \mathrm{tr}(W_{j_1}W_{j_2}\cdots W_{j_J}) : (x_2)$$

$$\mathcal{O}_3(x_3) = \mathcal{N}_3 : \mathrm{tr}(\mathrm{sym}(\bar{X}^j Z^j)) : (x_3)$$

One loop

The structure constant C_{123} has a λ' expansion

$$C_{123} = C_{123}^{(0)} + \lambda' C_{123}^{(1)} + \mathcal{O}(\lambda'^2)$$

where $\lambda' = \lambda/J^2$.
In order to compute $C_{123}^{(1)}$ there are 2 corrections to be considered:

1. Two loop contribution to the effective sigma model description $\rightarrow \mathcal{O}(\lambda)$ correction to the external wave function.
2. One-loop diagrams with two legs in one of the three operators and the other two legs in two different operators.

Wave function correction

- We use coherent state representation $\rightarrow$ very easy to take into account this contribution.
- Solve the Landau Lifshitz up to two loops:

$$\mathbf{u} = \mathbf{u}^{(0)} + \lambda' \mathbf{u}^{(1)} + \lambda'^2 \mathbf{u}^{(2)} + \mathcal{O}(\lambda'^3)$$

- Assume that the function entering in the one-loop result for C_{123} is the full **u**, substitute the **u** in:

$$C_{123} = \frac{1}{N} \frac{j! J}{\sqrt{(2j-1)!}} \int_0^{2\pi} \frac{d\sigma}{2\pi} (\bar{u}_1 u_2)^j$$

and extract the contribution of order λ'.

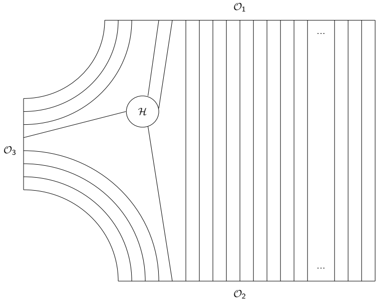
One loop diagram

Insertion of the one loop Hamiltonian with two legs in one of the operators and the other two legs in two different operators.
We use the prescription of

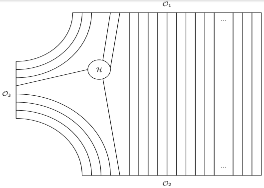
[Okuyama and Tseng, 2004], [Roiban and Volovich, 2004], [Alday, David, Gava and Narain, 2005]

$$\boxed{C^{(1)}_{123} = \frac{1}{32\pi^2} B \sum_{k=1}^{J} \left(\frac{(\bar{u}^1 v_1)(\frac{k}{l})}{(\bar{\mathbf{u}}\cdot\mathbf{v})(\frac{k}{l})} \right)^j \left(f^1_{23}(k) + f^2_{31}(k) + f^3_{12}(k) \right)}$$

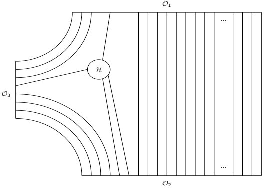
with $\boxed{B \equiv \prod_{m=1}^{J} \bar{\mathbf{u}}(\frac{m}{l}) \cdot \mathbf{v}(\frac{m}{l})}$



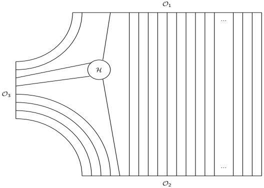
$$\boxed{f^1_{23}(k)} = -\frac{\bar{u}^{i_1}(\frac{k+j+1}{l})\bar{u}^{i_2}(\frac{k+j}{l})v_{j_1}(\frac{k+j+1}{l})\delta^3_{j_2}}{(\bar{u}\cdot v)(\frac{k+j+1}{l})\bar{u}^3(\frac{k+j}{l})}\mathcal{H}^{j_1 j_2}_{i_1 i_2} - \frac{\bar{u}^{i_1}(\frac{k+1}{l})\bar{u}^{i_2}(\frac{k}{l})\delta^3_{j_1}v_{j_2}(\frac{k}{l})}{\bar{u}^3(\frac{k+1}{l})(\bar{u}\cdot v)(\frac{k}{l})}\mathcal{H}^{j_1 j_2}_{i_1 i_2}$$



$$\boxed{f^2_{31}(k)} = -\frac{\delta^{i_1}_1\bar{u}^{i_2}(\frac{k+j+1}{l})v_{j_1}(\frac{k+j}{l})v_{j_2}(\frac{k+j+1}{l})}{v_1(\frac{k+j}{L})(\bar{u}\cdot v)(\frac{k+j+1}{L})}\mathcal{H}^{j_1 j_2}_{i_1 i_2} - \frac{\bar{u}^{i_1}(\frac{k}{L})\delta^{i_2}_1 v_{j_1}(\frac{k}{l})v_{j_2}(\frac{k+1}{l})}{(\bar{u}\cdot v)(\frac{k}{l})v_1(\frac{k+1}{l})}\mathcal{H}^{j_1 j_2}_{i_1 i_2}$$



$$\boxed{f^3_{12}(k)} = -\frac{\bar{u}^{i_1}(\frac{k+j}{l})\delta^{i_2}_1\delta^3_{j_1}v_{j_2}(\frac{k+j}{l})}{\bar{u}^3(\frac{k+j}{l})v_1(\frac{k+j}{l})}\mathcal{H}^{j_1 j_2}_{i_1 i_2} - \frac{\delta^{i_1}_1\bar{u}^{i_2}(\frac{k+1}{l})v_{j_1}(\frac{k+1}{l})\delta^3_{j_2}}{\bar{u}^3(\frac{k+1}{l})v_1(\frac{k+1}{l})}\mathcal{H}^{j_1 j_2}_{i_1 i_2} = 0$$



$$\boxed{\mathcal{H}^{j_1 j_2}_{i_1 i_2} = 2(I-P)^{j_1,j_2}_{i_1,i_2}\ , \quad I^{j_1 j_2}_{i_1 i_2} = \delta^{j_1}_{i_1}\delta^{j_2}_{i_2}\ , \quad P^{j_1 j_2}_{i_1 i_2} = \delta^{j_2}_{i_1}\delta^{j_1}_{i_2}}$$

Using that:

- $\bar{\mathbf{u}}$ and $\mathbf{v}$ vary slowly
- $j \ll J$
- Taylor expansion to evaluate $\mathbf{u}$ and $\mathbf{v}$ at two different values of σ
- $\mathbf{u} \simeq \mathbf{v}$

we get

$$f^1_{23}(k) \sim -2\left[\frac{1}{l^2}\bar{\mathbf{u}}' \cdot \mathbf{u}' - \frac{j}{l^2}\left(\frac{\bar{u}^{1\prime}(\frac{k}{l})}{\bar{u}^1(\frac{k}{l})}\right)'\right]$$

$$f^2_{31}(k) \sim -2\left[\frac{1}{l^2}\bar{\mathbf{u}}' \cdot \mathbf{u}' - \frac{j}{l^2}\left(\frac{u_2'(\frac{k}{l})}{u_2(\frac{k}{l})}\right)'\right]$$

- Summing over k

$$C^{(1)}_{123} = -\frac{\lambda'}{2\,N}\frac{j!\,J}{\sqrt{(2j-1)!}}\int_0^{2\pi}\frac{d\sigma}{2\pi}(\bar{u}_1 u_2)^j\left(\partial_\sigma \bar{\mathbf{u}} \cdot \partial_\sigma \mathbf{u} - \frac{j}{4}\partial^2_\sigma(\log(\bar{u}_1 u_2))\right)$$

Gauge theory result

Taking into account the tree level result and the wave function correction, the final expression for the structure constant is

$$\boxed{C_{123} = \frac{1}{N}\frac{j!J}{\sqrt{(2j-1)!}}\int_0^{2\pi}\frac{d\sigma}{2\pi}(\bar{u}_1 u_2)^j\left[1-\frac{\lambda'}{2}\left(\partial_\sigma\bar{\mathbf{u}}\cdot\partial_\sigma\mathbf{u}-\frac{j}{4}\partial_\sigma^2(\log(\bar{u}_1 u_2))\right)\right]+\mathcal{O}(\lambda'^2)}$$

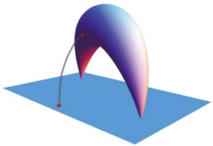
String theory side

We use the prescription of [Zarembo, 2010] to compute the structure constant C_{123} for 2 semiclassical operators and a light chiral primary (dual to a SUGRA mode)

$$C_{123} = c_j \frac{\sqrt{\lambda}}{N} \int_{-\infty}^{+\infty} d\tau_e \int_0^{2\pi} \frac{d\sigma}{2\pi} \frac{(\bar{U}_1 U_2)^j}{\cosh^{2j}(\frac{\tau_e}{\kappa})} \left[\frac{2}{\kappa^2 \cosh^2(\frac{\tau_e}{\kappa})} - \frac{1}{\kappa^2} - \partial_a \bar{\mathbf{U}} \cdot \partial^a \mathbf{U} \right]$$

$c_j = \frac{(2j+1)!}{2^{2j+2} j! \sqrt{(2j-1)!}}$

$\mathbf{U}(\tau, \sigma)$ are complex sphere embedding coordinates constrained by $\bar{\mathbf{U}} \cdot \mathbf{U} = 1$ and we write $\mathbf{U}(\sigma, \tau) = e^{i\tau/\kappa} \mathbf{u}(\sigma, \tau)$ where κ is a gauge constant.



Frolov-Tseytlin limit

The Frolov-Tseytlin limit [Frolov and Tseytlin, 2003],[Kruczenski, 2003] in our notation is

$$\boxed{\kappa \to 0\ , \quad \frac{1}{\kappa}\partial_\tau \mathbf{U} \quad \text{fixed}\ , \quad \partial_\sigma \mathbf{U} \quad \text{fixed}}$$

Starting from the bosonic sigma model it is possible to solve perturbatively in κ Virasoro constraints and determine κ in terms of λ'

$$\boxed{\kappa^2 = \lambda' = \frac{\lambda}{J^2}}$$

The expansion in κ coincides precisely with the weak coupling expansion in λ.

String theory result

In the Frolov-Tseytlin limit we can compute $\partial_a \bar{\mathbf{U}} \cdot \partial^a \mathbf{U}$ ending up with

$$C_{123} = c_j \frac{\sqrt{\lambda}}{N} \int_{-\infty}^{+\infty} d\tau_e \int_0^{2\pi} \frac{d\sigma}{2\pi} \frac{(\bar{u}_1 u_2)^j}{\cosh^{2j}(\frac{\tau_e}{\kappa})} \left[\frac{1}{\kappa^2 \cosh^2(\frac{\tau_e}{\kappa})} - \partial_\sigma \bar{\mathbf{u}} \cdot \partial_\sigma \mathbf{u} + \mathcal{O}(\kappa^2) \right]$$

After performing τ_e integrals we get

$$\boxed{C_{123} = \frac{J}{N} \frac{j!}{\sqrt{(2j-1)!}} \int_0^{2\pi} \frac{d\sigma}{2\pi} (\bar{u}_1 u_2)^j \left[1 - \lambda' \frac{2j+1}{2j} \partial_\sigma \bar{\mathbf{u}} \cdot \partial_\sigma \mathbf{u} + \mathcal{O}(\lambda'^2) \right]}$$

Note: We can set $\tau = 0$ in u since the factor $\frac{1}{cosh^{2j+2}(\frac{\tau_e}{\kappa})}$ localizes the integral around $\tau_e = \kappa$.

Comparison

GAUGE THEORY RESULT

$$\boxed{C_{123} = \frac{1}{N}\frac{j!J}{\sqrt{(2j-1)!}}\int_0^{2\pi}\frac{d\sigma}{2\pi}(\bar{u}_1 u_2)^j\left[1 - \frac{\lambda}{2J^2}\left(\partial_\sigma \bar{\mathbf{u}}\cdot\partial_\sigma \mathbf{u} - \frac{j}{4}\partial_\sigma^2(\log(\bar{u}_1 u_2))\right)\right] + \mathcal{O}(\lambda^2)}$$

STRING THEORY RESULT

$$\boxed{C_{123} = \frac{1}{N}\frac{j!J}{\sqrt{(2j-1)!}}\int_0^{2\pi}\frac{d\sigma}{2\pi}(\bar{u}_1 u_2)^j\left(1 - \frac{\lambda}{2J^2}\frac{(2j+1)}{j}\partial_\sigma \bar{\mathbf{u}}\cdot\partial_\sigma \mathbf{u}\right) + \mathcal{O}(\lambda^2/J^4)}$$

DISAGREEMENT AT ONE LOOP!

Comments

- Do we expect agreement?
- Prescription used on the gauge theory side $\rightarrow$ make numerical checks as in [Georgiou, Gili, Grossardt and Plefka, 2012]
- Two semiclassical states approximated by the same coherent state BUT $\mathcal{O}_1$ can not be exactly the conjugate of $\mathcal{O}_2$ otherwise $C_{123} = 0$ by R-charge conservation.[Escobedo, Gromov, Sever and Vieira, 2011]

Comments

- While at order λ $\mathcal{O}_1$ and $\mathcal{O}_2$ can be described in the long wave approximation using coherent states, at order λ^2 one has to use a linear combination of a coherent state and a spin-flipped coherent state [Kruczenski, Ryzhov and Tseytlin, 2004]

$$\boxed{\mathcal{O}_i(x_i) = \mathcal{O}_i^{(0)}(x_i) + \mathcal{O}_i^{(1)}(x_i).}$$