THE YOUNG MODULUS OF BLACK STRINGS

this is not a bent string

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CHAPTER 1:
THE ART AND CRAFT OF BENDING BLACK BRANES

The Blackfold Approach (I): Myers Perry

\[ ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega_{D-4}^2 + (d\sigma^2 + \sigma^2 d\phi^2) \]

The Blackfold Approach (II): Black Ring

\[ ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega_{D-3}^2 + dz^2 \]
WE HAVE ABSOLUTELY NO IDEA OF WHAT KIND OF HIGHER DIMENSIONAL BLACK HOLES ARE OUT THERE BUT IT SEEMS TO BE A GENERAL FEATURE TO EXHIBIT A LIMIT WHERE THEY BECOME LOCALLY BLACK BRANES.
If the object acts as a source to Einstein’s equations then it must satisfy:

\[ \nabla_\mu T^{\mu\nu} = 0 \]

Projecting this in directions parallel and orthogonal to the worldvolume leads to:

\[ D_a T^{ab} = 0 \]

\[ T^{ab} K_{ab} = 0 \]

\[ T^{ab} = \left( (\epsilon + P)u^a u^b + P \gamma^{ab} \right) \delta^{n+2}(x^\rho - X^\rho(\sigma^a)) \]

\[ \epsilon = -(n + 1)P \]
The equilibrium condition is simply:

\[ \Omega = \frac{1}{\sqrt{n + 1}} \frac{1}{R} \]

The object is described by the mapping functions:

\[ X^t = \tau, \quad X^z = R\phi \]

with intrinsic metric:

\[ \gamma_{ab} d\sigma^a d\sigma^b = -d\tau^2 + R^2 d\phi^2 \]

and Killing vector:

\[ k^a \partial_a = \partial_\tau + \Omega \partial_\phi \]
CHAPTER 2:
BLACK BRANES AS VISCOUS FLUIDS AND ELASTIC SOLIDS
Transport/Response Coefficients (I): Viscosities

The perturbed brane must satisfy:

\[ D_a T^{ab} = 0 \]

To first order in a derivative expansion:

\[ T^{ab} = \left( \epsilon u^a u^b + PP^{ab} - 2\eta \sigma_{ab} - \zeta \phi P^{ab} \right) \delta^{n+2} (x^\rho - X^\rho) \]

\[ \eta = \frac{\Omega (n+1) r_0^{n+1}}{16\pi G}, \quad \zeta = 2\eta \left( \frac{1}{p} + \frac{1}{n+1} \right) \]

Small perturbation of intrinsic parameters

\[ \frac{r_0}{\lambda} \ll 1 \]
The equations of motion are modified to:

\[ D_a T^{ab} = K_c^{\cdot b} \mu \nabla_a d^{ac\mu} \]

\[ T^{ab} K_{ab}^\rho = -K_{ac}^\rho K^{(c} \cdot b_{\lambda} d^{a)b\lambda} - \perp^\rho_\sigma \nabla_b \nabla_a d^{ab\sigma} \]
A real 4D material can be described by a local particle density:

\[ T^{\mu \nu} = \epsilon u^\mu u^\nu + P^{\mu \nu} \]

On the other hand, a p-brane develops dipoles of stress and requires an orthogonal component of the stress-energy tensor: a dipole contribution:

\[ \tilde{T}^{\mu \nu} = \int_{W_{p+1}} d^{p+1}\sigma \left( T^{\mu \nu} \delta^D(x^\mu - X^\mu) + \nabla_\rho (d^{\rho \sigma} u_a^\mu u_b^\nu \delta^D(x^\mu - X^\mu)) \right) \]

Transport/Response Coefficients (IV): Young Modulus

We can measure the dipole from an approximate analytic solution. In general we find the relation between stress and strain (Hookean Idealization):

$$d_{ab} \rho = C(n) Y_{ab} \cd K_{cd} \rho$$

which can be measured in the linearized gravity regime far away from the source:

$$\tilde{h}_{\mu\nu}^{(D)} = \frac{16\pi G \cos \theta}{\Omega(n+1)r^{n+1}} d_{\mu\nu} \rho$$

The Young Modulus for Black Strings is:

$$Y^{zzzz} = Y^{zztt} = -\frac{\Omega(n)(n + 2)}{16\pi G r_0^2} (n^2 + 3n + 4) \xi(n)$$

$$Y^{tzzz} = Y^{tzzt} = 0$$

$$Y^{tttt} = Y^{zzzz} = \frac{\Omega(n)(n + 2)(n + 4)}{16\pi G r_0^2} (3n + 4) \xi(n)$$

Charged bent strings develop a polarization vector in transverse directions to the worldvolume. We can capture this by introducing a dipole correction to the current:

\[ j^{\mu\nu} = \int_{W_{p+1}} d^{p+1}\sigma \left( J^{\mu\nu} \delta^D(x^\mu - X^\mu) + \nabla_\rho (M^{ab\rho} u_a^\mu u_b^\nu \delta^D(x^\mu - X^\mu)) \right) \]

which can be measured using:

\[ \nabla_\chi F^{\lambda\mu\nu} = \hat{j}^{\mu\nu} \]

and hence the electric susceptibility:

\[ M^{ab\hat{\rho}} = \chi e^{ab\hat{\rho}}_{\mu\nu} E^{\mu\nu} \]

Measurement on its way!
CHAPTER 3:
THE END
A Comment and food for thought

- A rather unusual use of jargon when speaking about black hole physics: elasticity, young modulus, stress, strain, susceptibility, etc.

THANK YOU